

CHAPTER 3

Static Forces on Surfaces

Buoyancy

Dr. Khalil Mahmoud ALASTAL

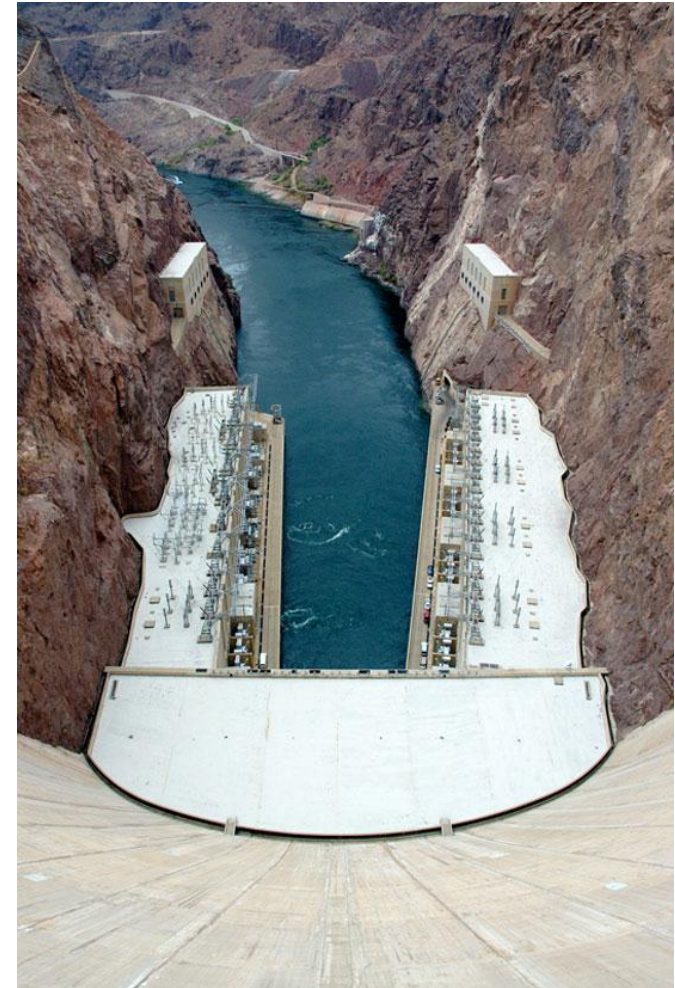


Hoover Dam

Hoover Dam



- Construction of Hoover Dam began in 1931 and was completed in 1935.
- Hoover Dam, located on the Colorado River, has a length of 1,244 feet and a height of 726 feet.



Objectives of this Chapter:

1. Compute the **hydrostatic pressures** and **forces** on submerged surfaces in a static fluid.
2. Use the principle of static equilibrium to **solve for the forces involves in buoyancy** problems.
3. Define the **condition for stability** of submerged and floating bodies.

Introduction:

- When a surface is submerged in a fluid, **forces develop on the surface due to the fluid.**
- The determination of these **forces is important in the design** of storage tanks, ships, dams and other hydraulic structures.

From previous lectures

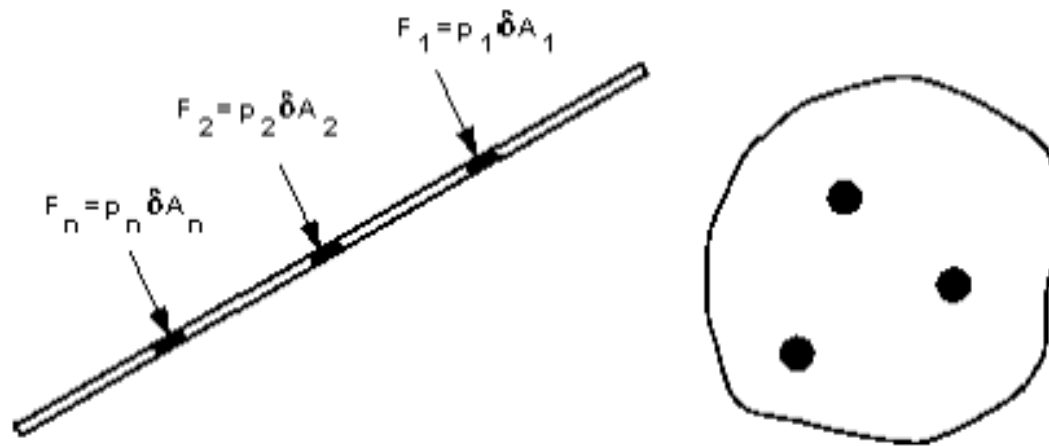
- For Fluid at rest, we knew that the **force must be perpendicular to the surface.**
- We knew that the **pressure will vary linearly with depth.**

3.1 Action of Fluid on a Surface

- Pressure is defined as force per unit area. If a pressure p acts on a small area δA then the force exerted on that area will be:

$$F = p\delta A$$

- Since the **fluid is at rest** the force will **act at right-angles** to the surface.

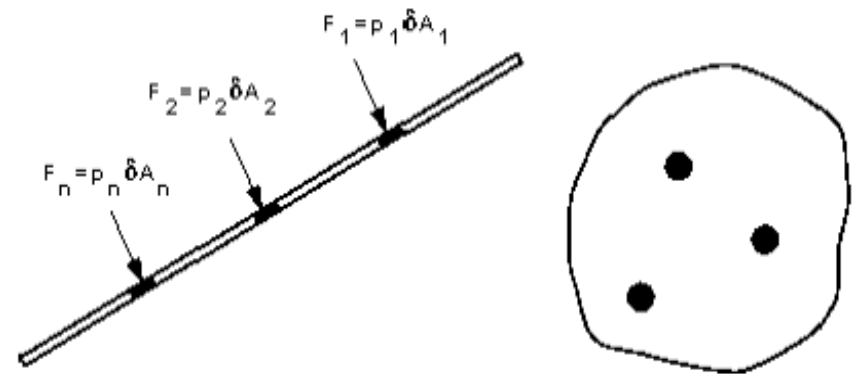


General submerged plane

- The total area is made up of many elemental areas.
- The force on each elemental area is always normal to the surface but, in general, each force is of different magnitude as the pressure usually varies.
- We can find the total or **resultant** force, R , on the plane by summing up all of the forces on the small elements i.e.

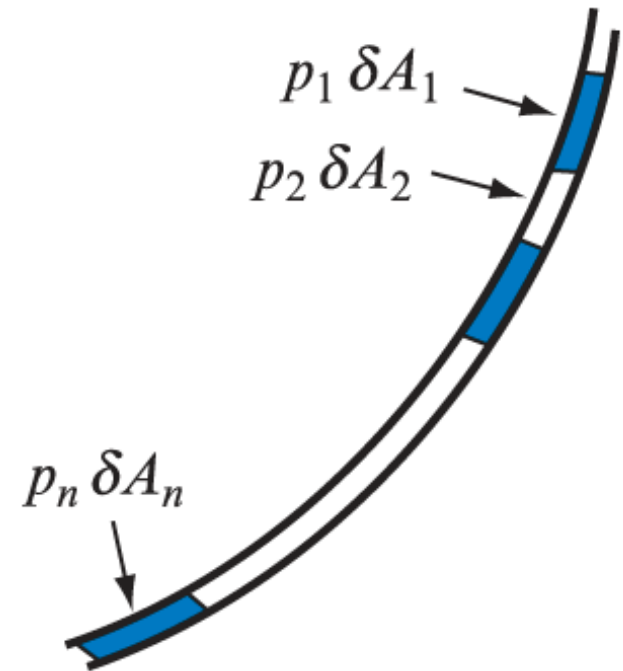
$$R = p_1 \delta A_1 + p_2 \delta A_2 + \dots + p_n \delta A_n = \sum p \delta A$$

If the surface is a **plane** the force can be represented by one single **resultant force**, acting at right-angles to the plane through the **centre of pressure**.



Curved submerged plane

- If the surface is curved, each elemental force will be a **different magnitude** and in different **direction** but still normal to the surface of that element.
- The resultant force can be found by **resolving all forces into orthogonal co-ordinate** directions to obtain its magnitude and direction.
- This will always be **less than** the sum of the individual forces, $\sum p \delta A$



3.2 Resultant Force and Center of Pressure on a Plane Surface under Uniform Pressure

For horizontal plane submerged in a liquid, **the pressure, p , will be equal at all points of the surface.**

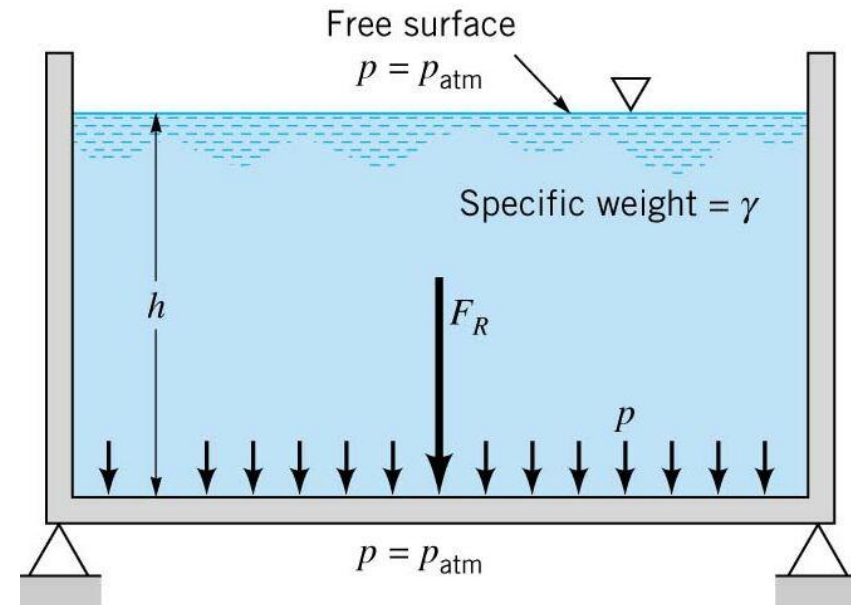
Simplest Case: Tank bottom with a uniform pressure distribution

$$p = \rho gh = \gamma h$$

Now, the resultant Force:

$$R = pA$$

A = area of the plane surface (Tank Bottom)

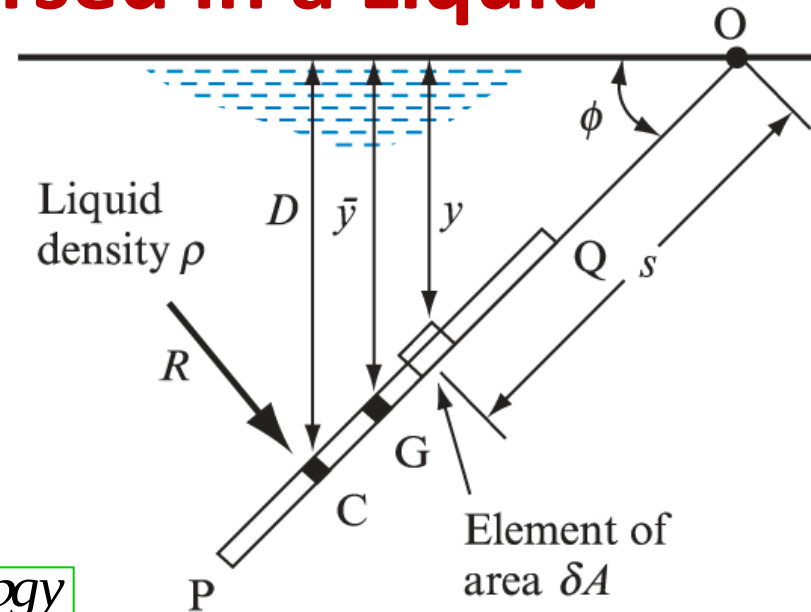


This force will act vertically downward and the center of pressure will be the **centroid of the surface**

3.3 Resultant Force and Center of Pressure on a Plane Surface Immersed in a Liquid

- Shown a plane surface PQ of an area A submerged in a liquid of density, ρ , and inclined at an angle ϕ to the free surface.
- Considering one side only, there will be a force due to fluid pressure, acting on each element of area δA ,
- The pressure at a distance y below the free surface can be written as: $p = \rho gy$
- Force on elemental area δA : $\delta R = p\delta A = \rho gy\delta A$
- The resultant force acting on the plane can be found by summing all the forces on the small element:

$$R = \Sigma p\delta A = \Sigma \rho gy \delta A$$



$$R = \Sigma p \delta A = \Sigma \rho g y \delta A$$

Assuming that ρ and g are constant

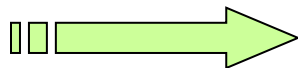
$$R = \rho g \Sigma y \delta A$$

- The quantity $\Sigma y \delta A$ is the first moment of area under the surface PQ about the free surface of the liquid

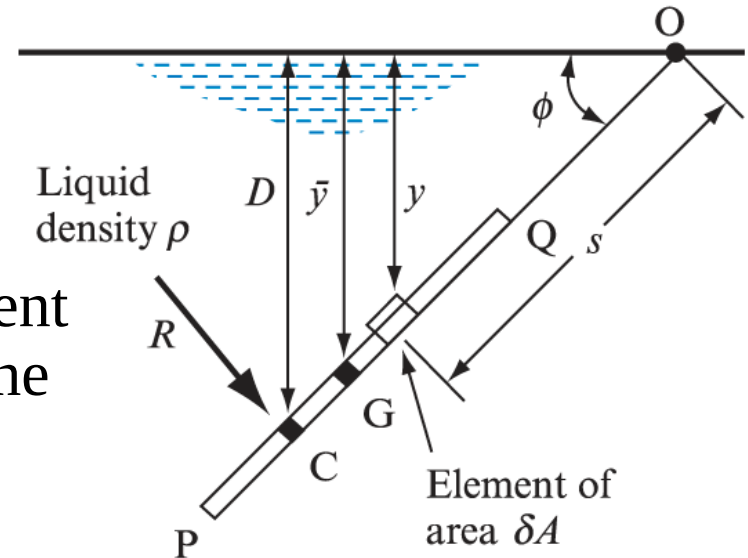
$$\Sigma y \delta A = A \bar{y}$$

where

- A = the area of the whole immersed surface and
- $\bar{y}$ = vertical distance from the free surface to the centroid of the area, G , of the immersed surface.



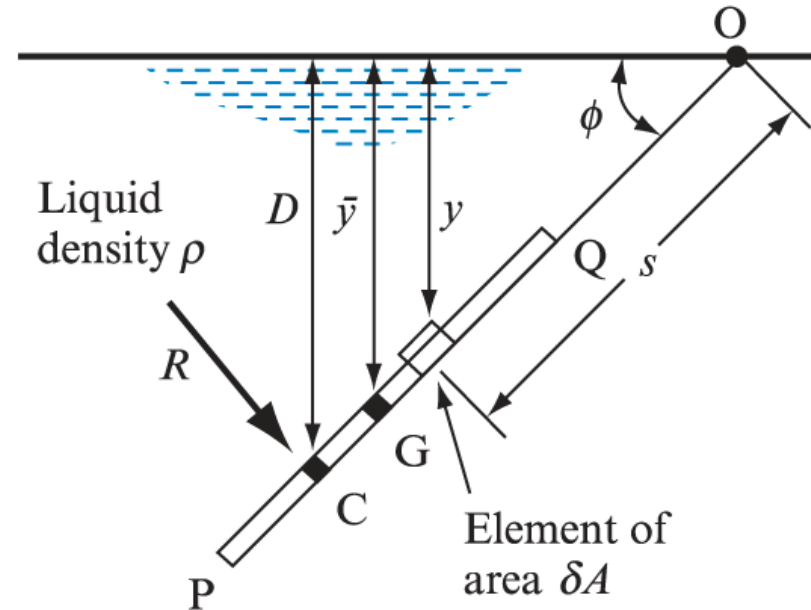
$$\text{Resultant Force} = R = \rho g A \bar{y}$$



Remember: Centroid of the area is defined as the point at which the area would be balanced if suspended from that point.

$$R = \rho g A \bar{y}$$

- It may be noted that the resultant force, R , is **independent** of the angle of inclination ϕ as long as the depth of the centroid $\bar{y}$ is unchanged.
- The point of application of the resultant force on the submerged area is called the center of pressure.
- This resultant force will act perpendicular to the immersed surface at the center of pressure, C at some depth D below the free surface.



- The vertical depth of the center of pressure, D , below the free surface can be found using the following:

$$D = \sin^2 \phi \left(\frac{I_o}{A\bar{y}} \right) \quad \text{or} \quad D = \sin^2 \phi \left(\frac{I_G}{A\bar{y}} \right) + \bar{y}$$

where

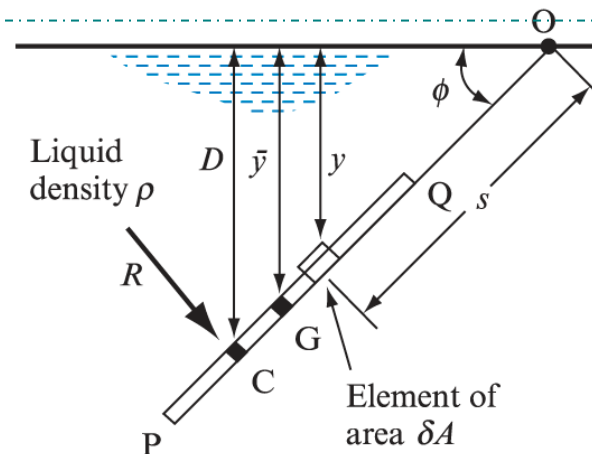
- I_G = second moment of plane area about an axis through its centroid G parallel to the free surface
- A = the area of the whole immersed surface
- $\bar{y}$ = vertical distance from the free surface to the centroid of the area A

The above equation implies that the center of pressure is always below the centroid.

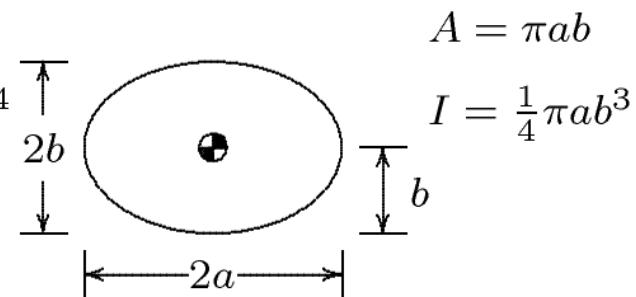
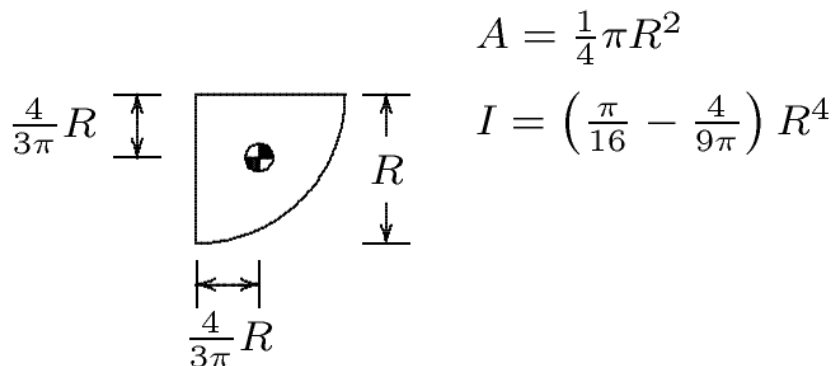
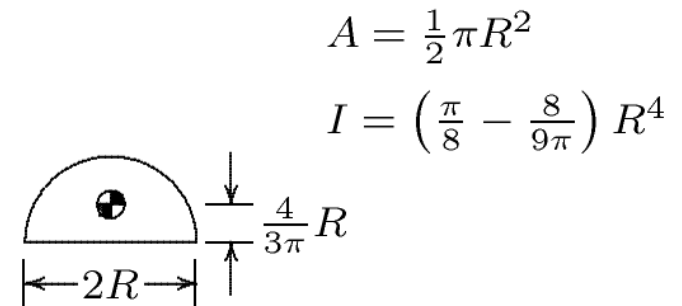
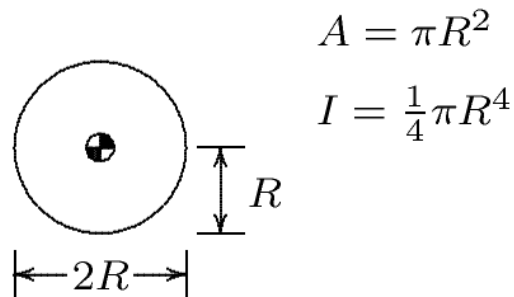
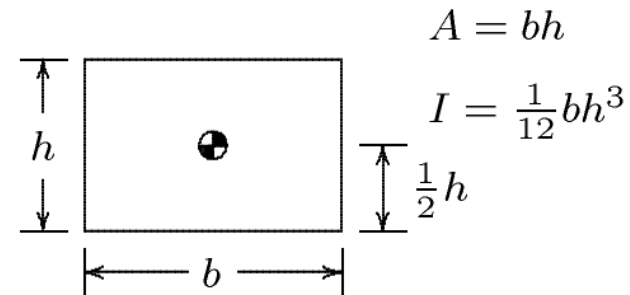
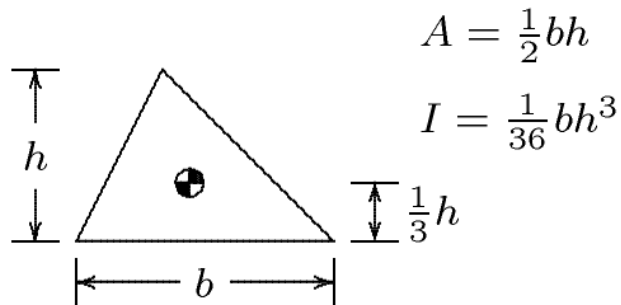
Remember:

Parallel axis theorem

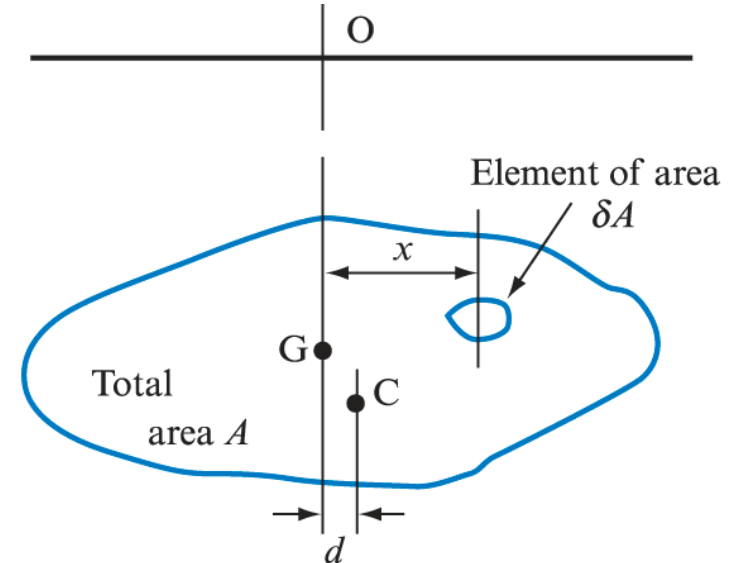
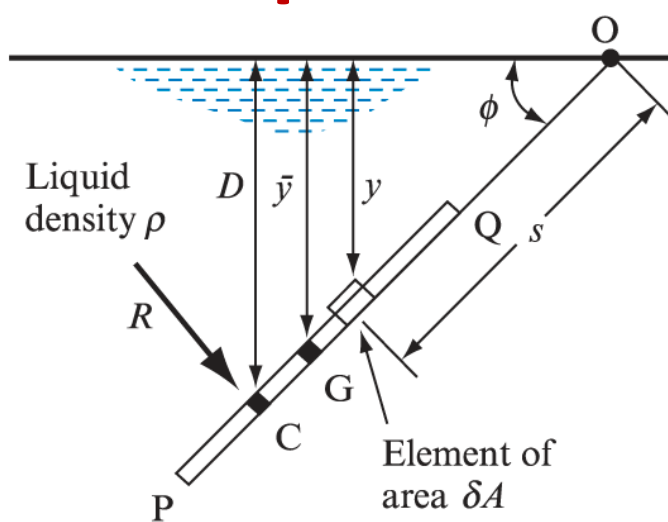
$$I_o = I_G + Ad^2$$



The second moment of area about a line through the centroid of some common shapes



Lateral position of Centre of Pressure



- If the shape is symmetrical the centre of pressure lies on the line of symmetry.
- But if it is not symmetrical its position must be found by taking moments about the line OG.

$R \times d = \text{Sum of moments of the forces on all elements about OG}$

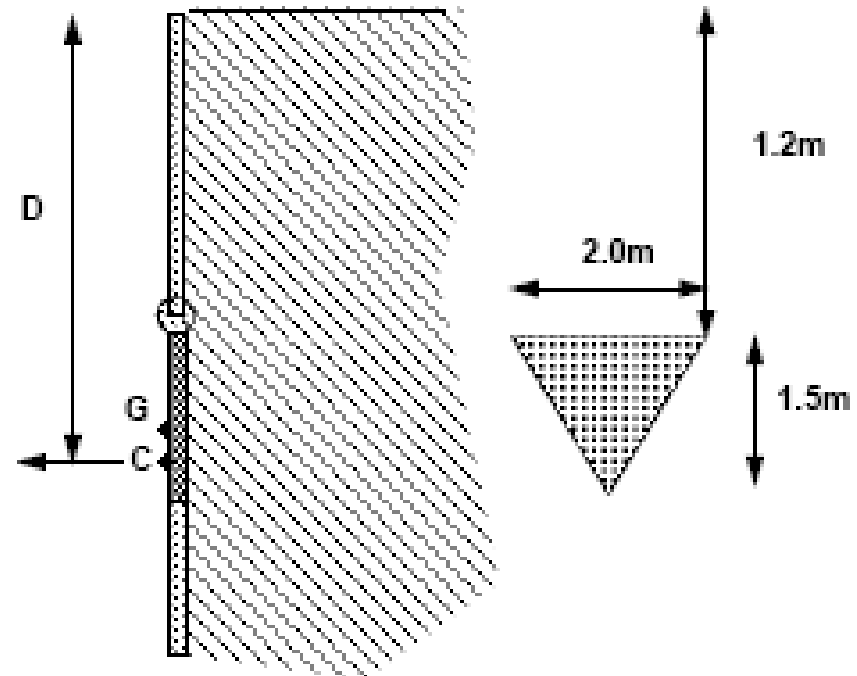
$$= \sum (\rho g y \delta A) x$$

But $R = \rho g \bar{y} A$

$$\Rightarrow d = \frac{\sum y x \delta A}{A \bar{y}}$$

Example:

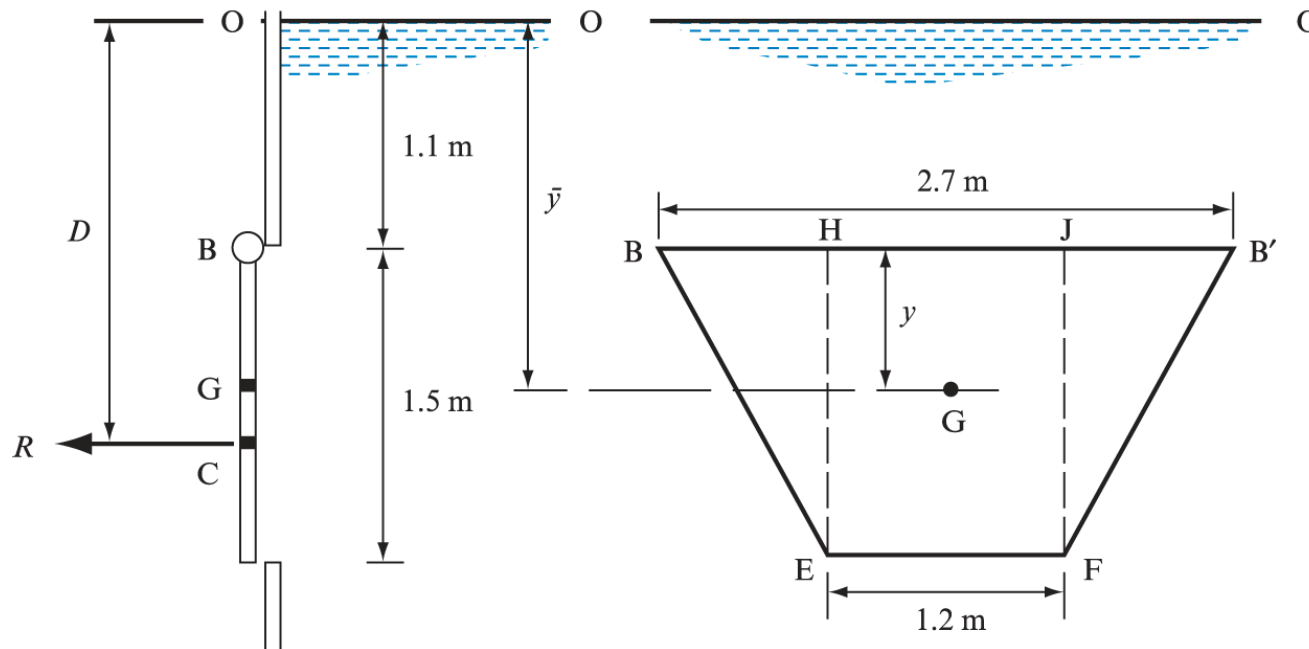
Find the moment required to keep this triangular gate closed on a tank which holds water.



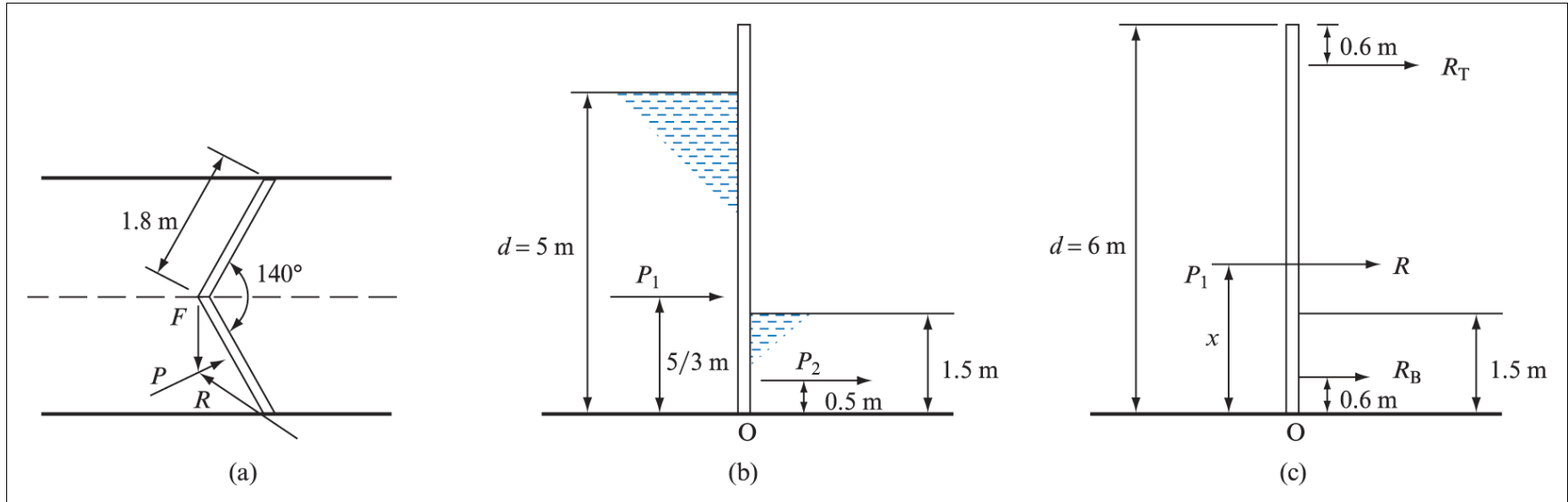
Example: (Ex 3.1, page 65 Textbook)

A trapezoidal opening in the vertical wall of a tank is closed by a flat plate which is hinged at its upper edge. The plate is symmetrical about its centreline and is 1.5 m deep. Its upper edge is 2.7 m long and its lower edge is 1.2 m long. The free surface of the water in the tank stands 1.1 m above the upper edge of the plate.

Calculate the moment about the hinge line required to keep the plate closed.



Example: (Ex 3.2, page 67 Textbook)

HW


The angle between a pair of lock gates is 140° and each gate is 6 m high and 1.8 m wide, supported on hinges 0.6 m from the top and bottom of the gate. If the depths of water on the upstream and downstream sides are 5 m and 1.5 m , respectively, **estimate the reactions at the top and bottom hinges.**

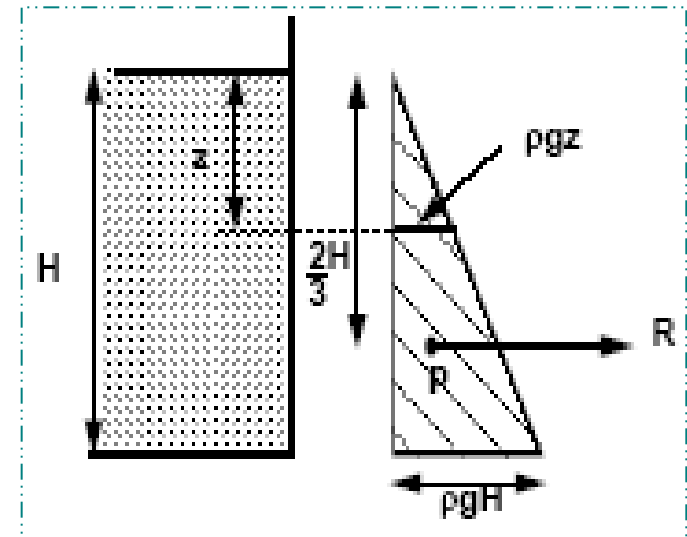
3.4 Pressure diagrams

- For vertical walls of constant width it is possible to find the resultant force and centre of pressure graphically using a ***pressure diagram***.
- We know the relationship between pressure and depth:

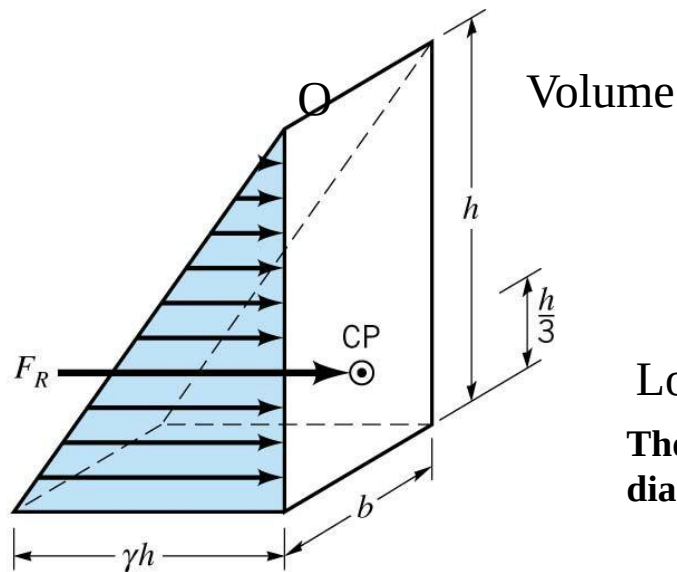
$$p = \rho g z$$

Consider the tank shown:

- We can draw the shown diagram (graphical representation of the (gauge) pressure change with depth on one of the vertical walls.)
- It increases linearly from zero at the surface by $p = \rho g y$, to a maximum at the base of $P = \rho g H$.
- This is known as a **pressure diagram**.



Pressure Diagram: A graphical interpretation of the forces due to a fluid acting on a plane area. The “volume” of fluid acting on the wall is the pressure diagram and equals the resultant force acting on the wall.



Resultant Force:

$$F_R = \text{volume}$$

$$F_R = \frac{1}{2}(\rho gh)(h)(b)$$

$$F_R = \frac{1}{2}(\rho gh^2)b$$

Location of the Resultant Force, C:

The location is at the centroid of the volume of the pressure diagram.

Center of Pressure:

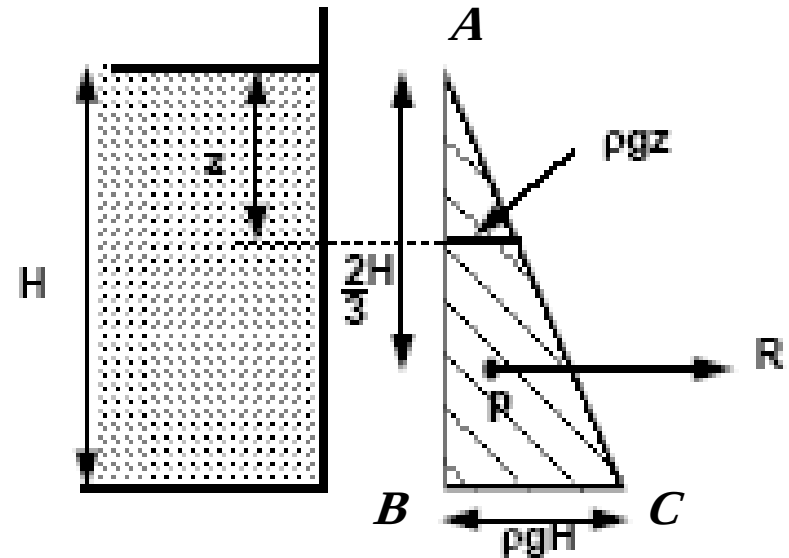
$$\left(\frac{b}{2}, \frac{2h}{3} \right)$$

- The area of this triangle represents the **resultant force per unit width** on the vertical wall, (N/m). So:

$$Area = \frac{1}{2} AB \times BC = \frac{1}{2} H \rho g H = \frac{1}{2} \rho g H^2$$

Resultant force per unit width

$$R = \frac{1}{2} \rho g H^2 \text{ (N/m)}$$



- This force acts through the centroid of the pressure diagram. For a triangle, the centroid is located at $2/3$ its height, thus the resultant force acts at a depth of $2/3 H$ from the surface.
- The total resultant force can be obtained by multiplying the above equation with the width of the surface, B .

$$R = \frac{1}{2} \rho g H^2 B \text{ (N)}$$

This can be checked against the previous method:

$$R = \rho g A \bar{y}$$

$$\Rightarrow R = \rho g (H \times 1) \frac{H}{2} = \frac{1}{2} \rho g H^2 \quad \checkmark$$

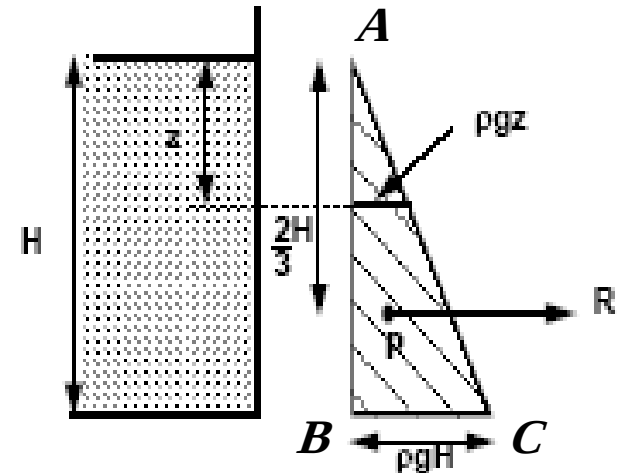
$$D = \sin^2 \phi \left(\frac{I_o}{A \bar{y}} \right)$$

or

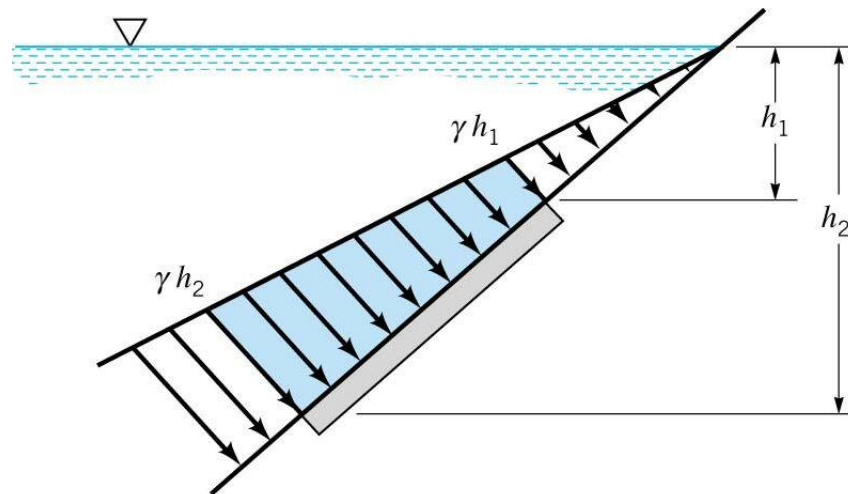
$$D = \sin^2 \phi \left(\frac{I_g}{A \bar{y}} \right) + \bar{y}$$

$$\Rightarrow \phi = 90^\circ, \sin \phi = 1, \bar{y} = H/2, I_g = H^3/12$$

$$D = \left(\frac{H^3/12}{(H \times 1) \times (H/2)} \right) + \frac{H}{2} = \frac{H}{6} + \frac{H}{2} = \frac{4H}{6} = \frac{2H}{3} \quad \checkmark$$



- If the plane surface is inclined and submerged below the surface, the pressure diagram is drawn perpendicular to the immersed surface and will be a straight line extending from $p=0$ at the free surface to $p=\rho gh_2$ at depth h_2 .
- As the immersed surface does not extend to the free surface, the resultant force R is represented by the shaded area, instead of the whole triangle, and acts through the centroid P of this shaded area.



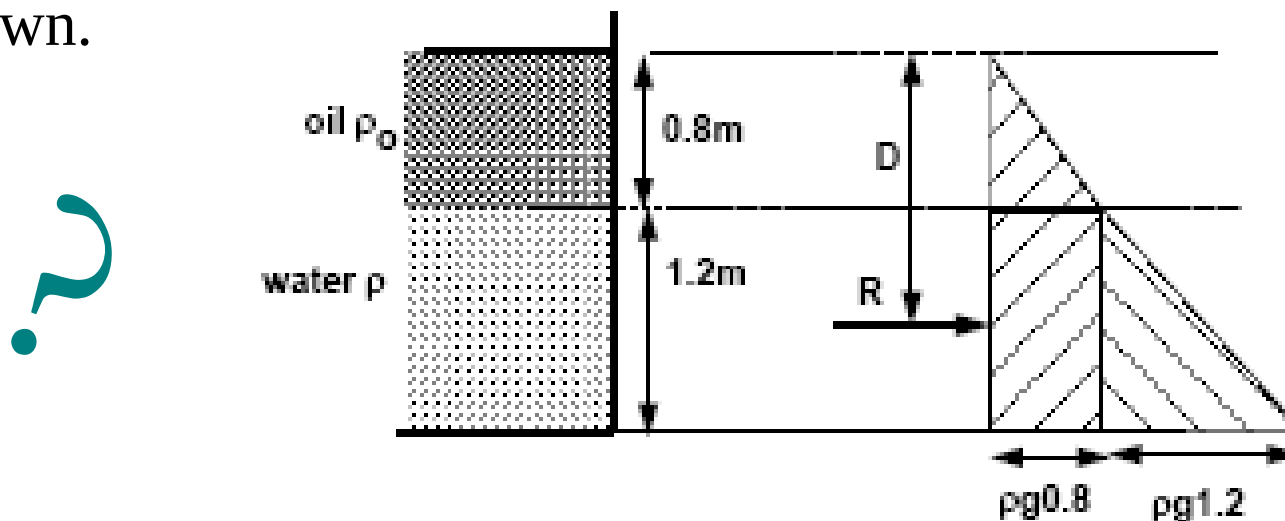
Note

- More complex pressure diagrams can be drawn for non-rectangular or non-vertical planes but it is usually far easier to use the moments method.

- The same pressure diagram technique can be used when combinations of liquids are held in tanks (e.g. oil floating on water) with position of action found by taking moments of the individual resultant forces for each fluid.

For example:

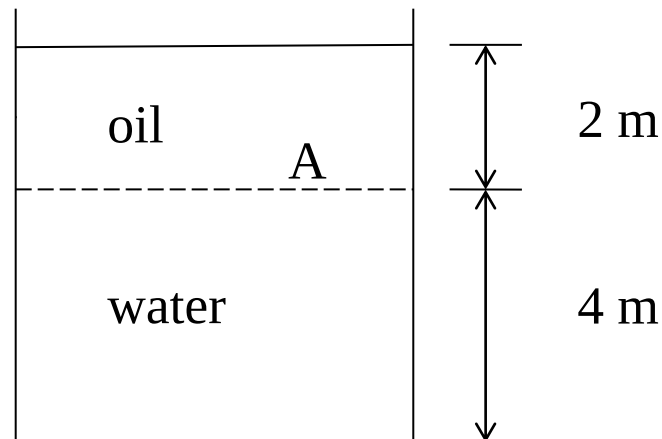
- Find the position and magnitude of the resultant force on this vertical wall of a tank which has oil floating on water as shown.



Example:

A 6-m deep tank contains 4 m of water and 2-m of oil as shown in the diagram below.

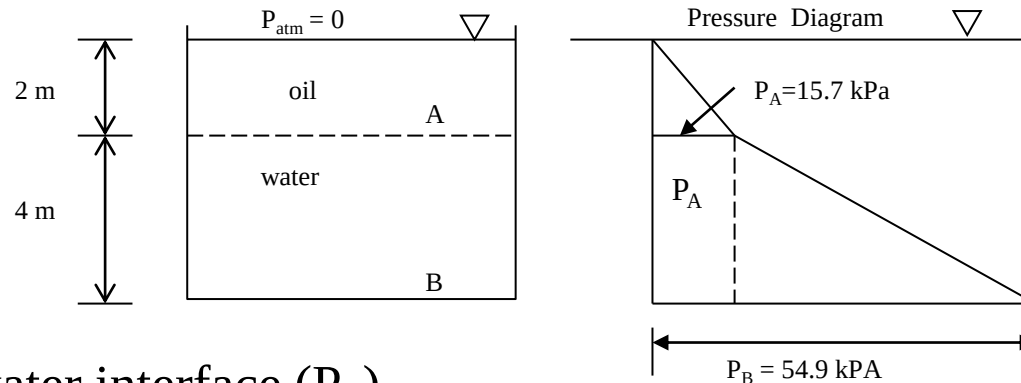
- Determine the pressure at point A and at the bottom of the tank.
- Draw the pressure diagram.



$$\rho_{\text{water}} = 1000 \text{ kg/m}^3$$

$$\text{SG of oil} = 0.98$$

Solution:



Pressure at oil water interface (P_A)

$$\begin{aligned}
 P_A &= P_{atm} + P_{oil} \text{ (due to 2 m of oil)} \\
 &= 0 + \rho_{oil}gh_{oil} = 0 + 0.98 \times 1000 \times 9.81 \times 2 \\
 &= 15696 \text{ Pa}
 \end{aligned}$$

$$P_A = 15.7 \text{ kPa (gauge)}$$

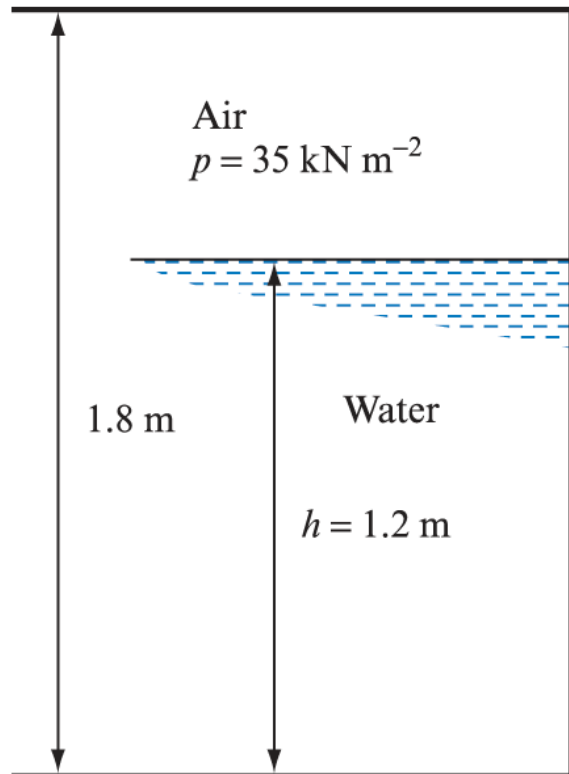
Pressure at the bottom of the tank;

$$\begin{aligned}
 P_B &= P_A + \rho_{water}gh_{water} \\
 P_B &= 15.7 \times 1000 + 1000 \times 9.81 \times 4 \\
 &= 54940 \text{ Pa}
 \end{aligned}$$

$$P_B = 54.9 \text{ kPa (gauge)}$$

Example: (Ex 3.3, page 70 Textbook)

A closed tank, rectangular in plan with vertical sides, is 1.8m deep and contains water to a depth of 1.2m. Air is pumped into the space above the water until the air pressure is 35kPa .



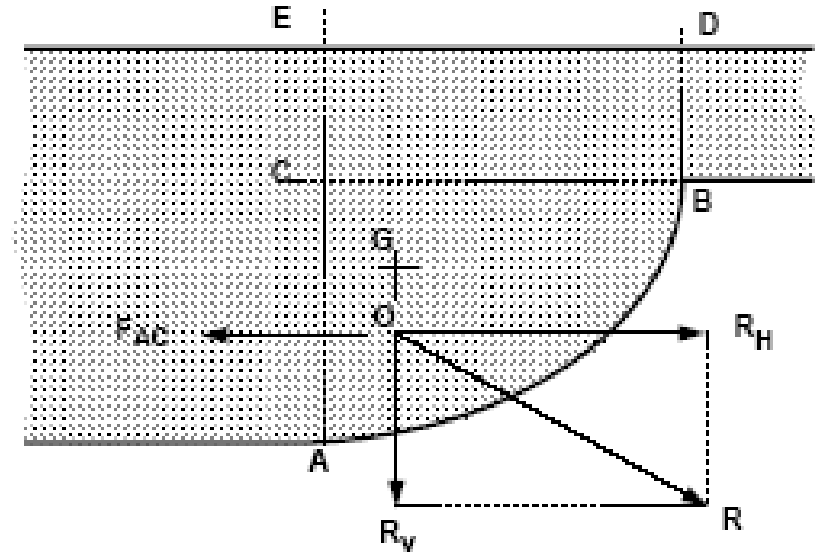
Determine the resultant force on this wall and the height of the center of pressure above the base

The length of the wall of tank is 3m

3.5 Force on a Curved Surface due to Hydrostatic Pressure

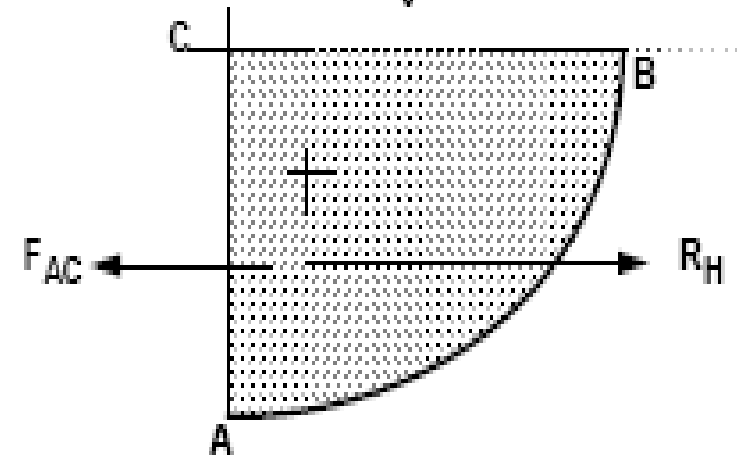
- ☒ Many surfaces in dams, pumps, pipes or tanks are curved
- ☒ General theory of plane surfaces does not apply to curved surfaces
- ☒ As stated above, if the surface is curved the **forces on each element of the surface will not be parallel** and must be combined using some vectorial method.
- ☒ It is most straightforward to calculate the **horizontal** and **vertical** components.
- ☒ Then combine these forces to obtain the resultant force and its direction.

- In the diagram below liquid is resting on top of a curved base.
- The element of fluid ABC is equilibrium (as the fluid is at rest).



Consider the Horizontal forces

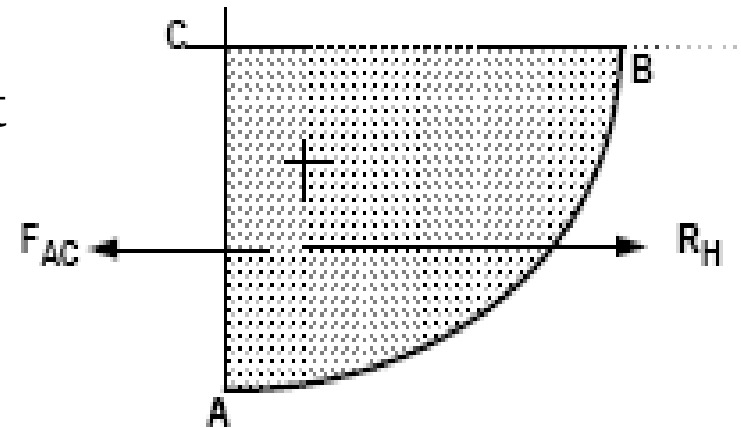
- The sum of the horizontal forces is zero.
- F_{AC} must be equal and opposite to R_H



The resultant horizontal force of a fluid above a curved surface is:
 $R_H =$ Resultant force on the projection of the curved surface onto a vertical plane.

We know that:

- The force on a vertical plane must act horizontally (as it acts normal to the plane).
- R_H must act through the same point.



So we can say

R_H acts horizontally through the centre of pressure of the projection of the curved surface onto a vertical plane.

Note:

We can use **the pressure diagram method** to calculate the position and magnitude of the resultant horizontal force on a curved surface.

Consider the Vertical forces

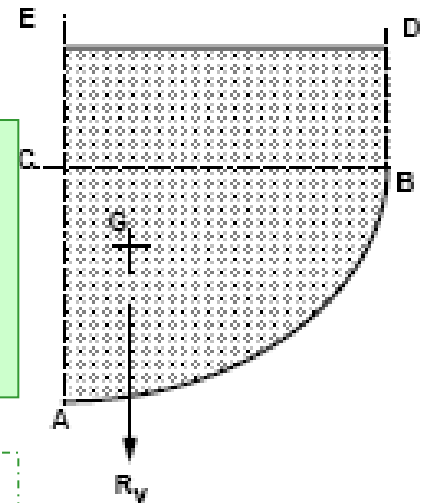
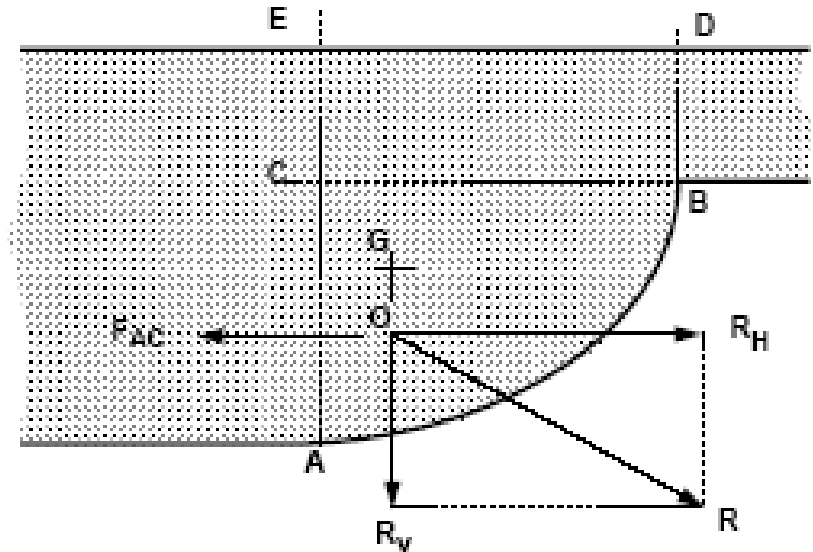
- The sum of the vertical forces is zero.
- There are no shear force on the vertical edges, so the vertical component can only be due to the weight of the fluid.



The resultant vertical force of a fluid above a curved surface is:

R_V = Weight of fluid directly above the curved surface.

It will act vertically downward through the centre of gravity of the mass of fluid.



Resultant force

- The overall resultant force is found by combining the vertical and horizontal components vectorially:

$$R = \sqrt{R_H^2 + R_V^2}$$

- And acts through O at an angle of ν .
- The angle the resultant force makes to the horizontal is:

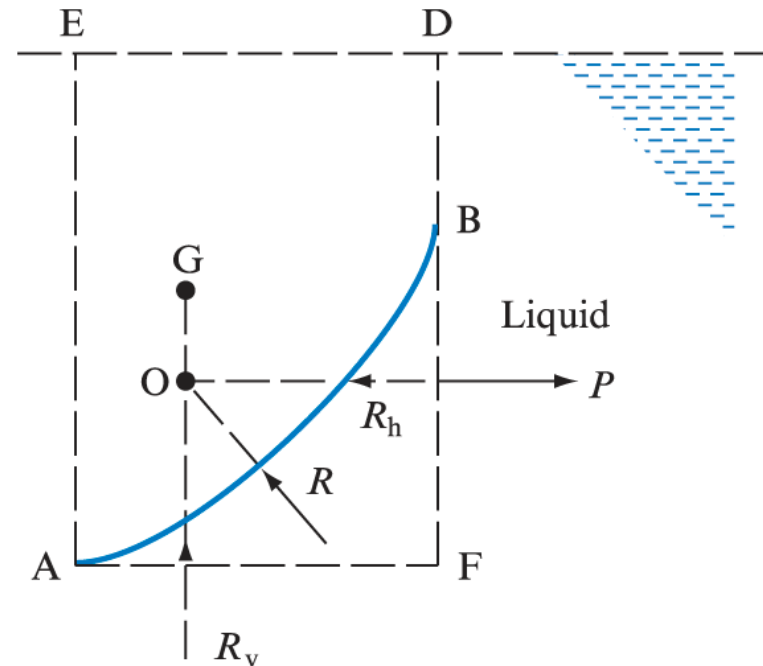
$$\theta = \tan^{-1} \left(\frac{R_V}{R_H} \right)$$

- The position of O is the point of intersection of the horizontal line of action of R_H and the vertical line of action of R_V .

What are the forces if the fluid is *below* the curved surface?

Consider the Horizontal forces

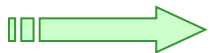
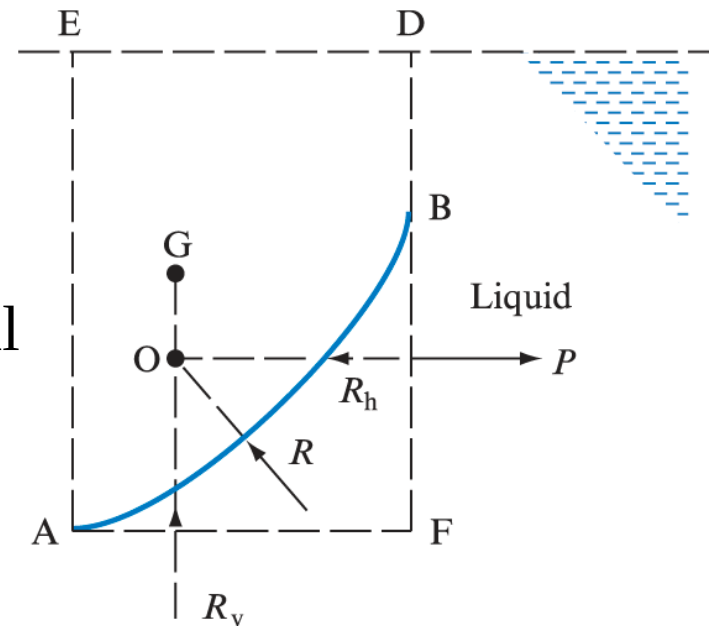
- There are two horizontal forces on the element which is in equilibrium:
- The horizontal reaction force R_H
- The pressure force on the vertical plane FB .



The resultant horizontal force of a fluid below a curved surface is:
 $R_H =$ Resultant force on the projection of the curved surface onto a vertical plane.

Consider the Vertical forces

- If the curved surface were removed and the area it were replaced by the fluid, the whole system would be in equilibrium.
- Thus the force required by the curved surface to maintain equilibrium is equal to that force which the fluid above the surface would exert - i.e. the weight of the fluid.



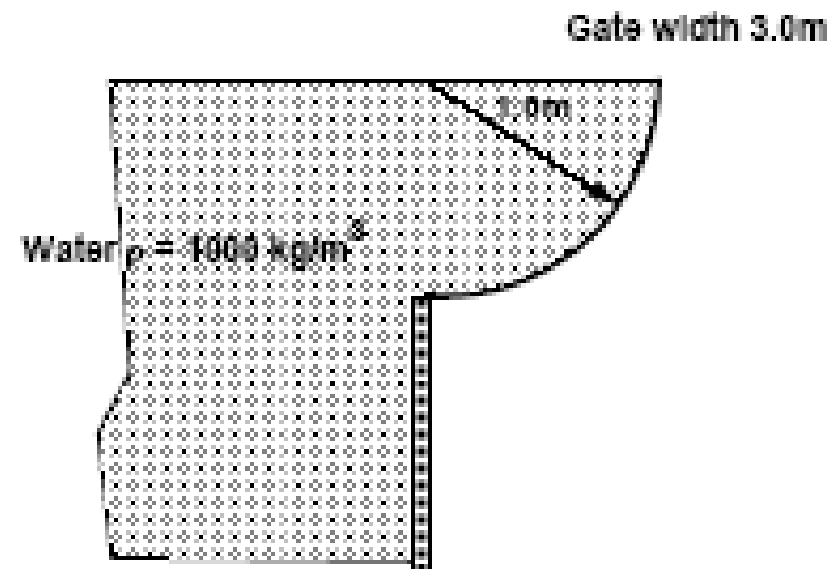
The resultant vertical force of a fluid below a curved surface is:

R_v = Weight of the *imaginary* volume of fluid vertically above the curved surface.

What vertical force would keep this in equilibrium?

Example:

Find the magnitude and direction of the resultant force of water on a quadrant gate as shown below.

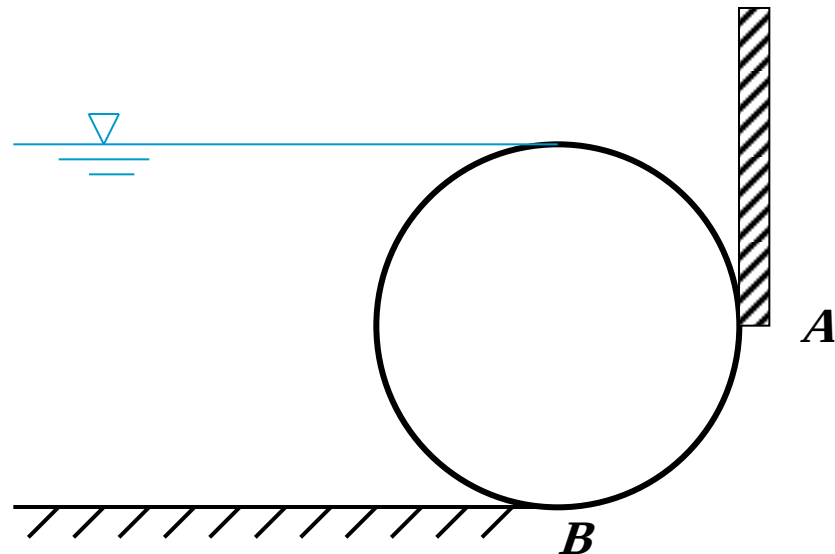


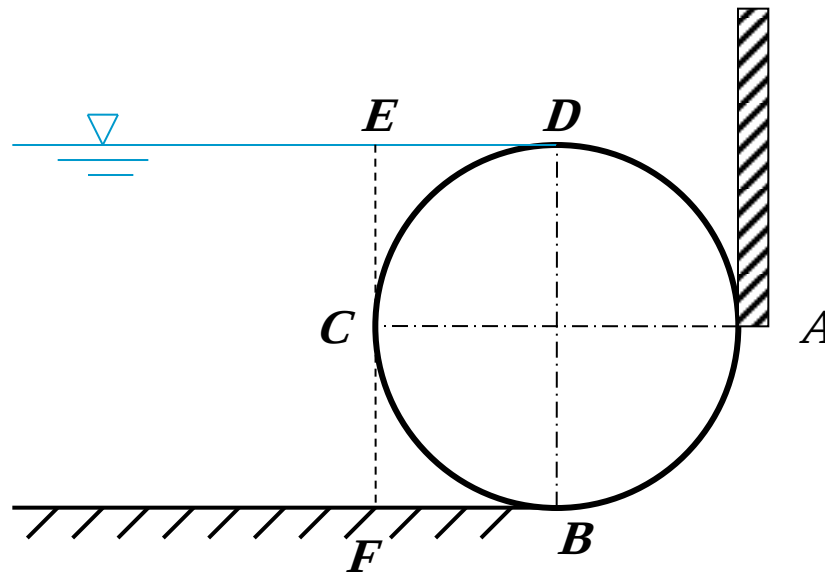
Example:

A 2m long cylinder lies as shown in the figure (diameter = 2m also) , holding back oil of relative density 0.8. If the cylinder has a mass of 3250 kg.

Find:

- the reaction at A
- the reaction at B



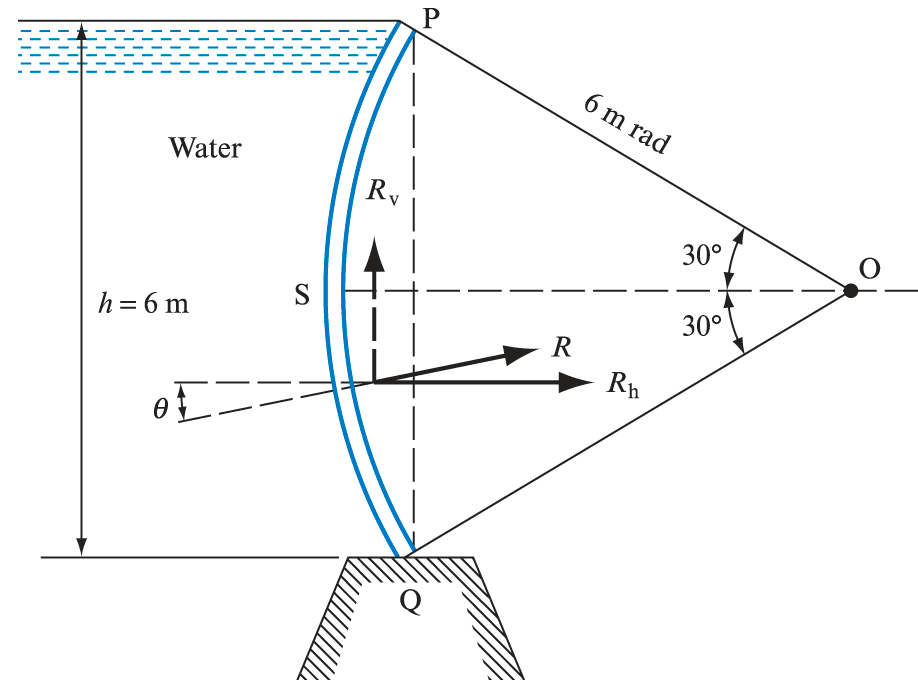


Example: (Ex 3.4, page 72 Textbook)

HW

A sluice gate is in the form of a circular arc of radius 6m as shown.

Calculate the magnitude and direction of the resultant force on the gate, and the location with respect to O of a point on its line of action.



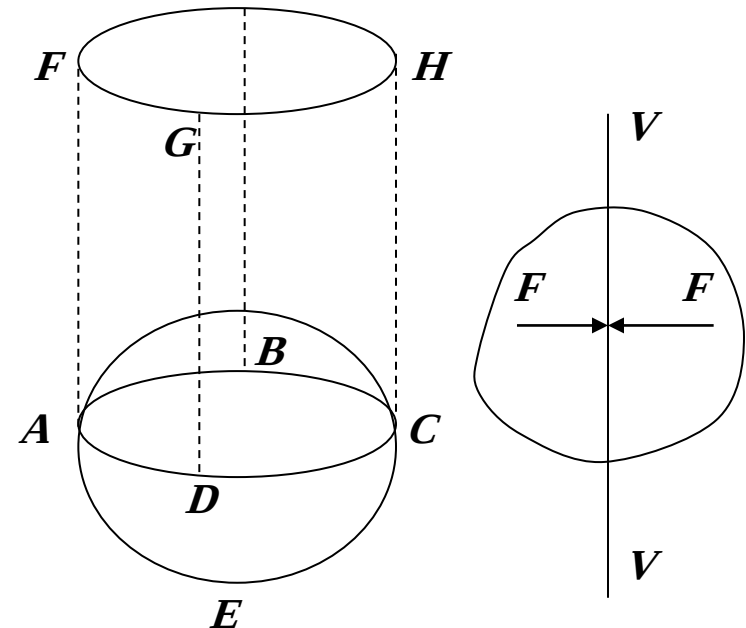
3.6 Buoyancy

- When a body is submerged or floating in a static fluid, the resultant force exerted on it by the fluid is called the buoyancy force.
- Buoyant force can be defined as the resultant fluid force which acts on a fully submerged or floating body.

- Consider the vertical plane VV
- The projected area of each of the two sides on this plane will be equal.
- The horizontal forces F will be equal and opposite.



No resultant horizontal force on the body due to the pressure of the surrounding fluid



In the vertical direction:

- A force is exerted by the fluid on the immersed body called the buoyancy or upthrust force.
- Upthrust on the body = Upward force on lower surface ADEC
 - Downward force on the upper surface ABCD
 = Weight of volume of fluid AECDGFH
 - Weight of volume of fluid ABCDGFH
 = Weight of volume of fluid ABCDE

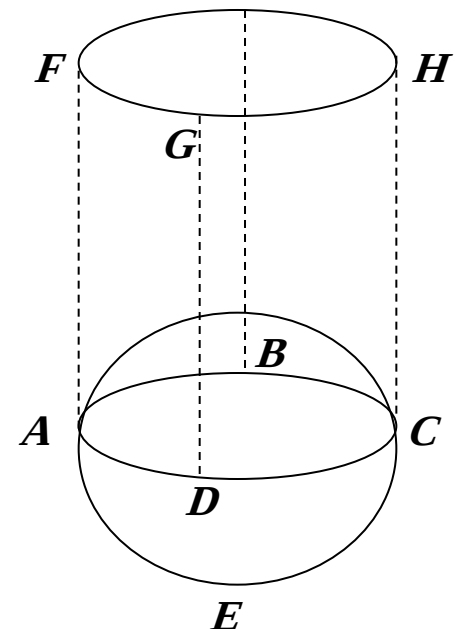


Upthrust on the body = Weight of fluid displaced by the body

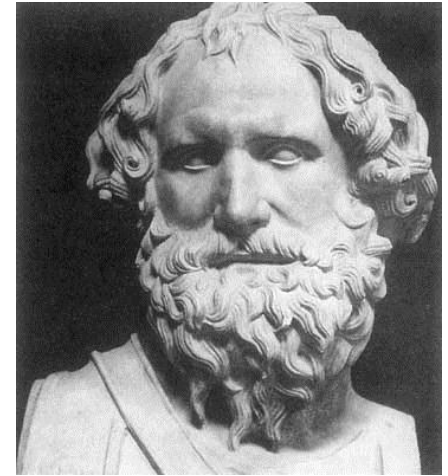
- This force will act vertically upward through the centroid of the volume of fluid displaced, known as **the centre of buoyancy**



Archimedes' principle



Buoyancy: Archimedes' Principle



Archimedes (287-212 BC)

Archimedes' Principle states that the buoyant force has a magnitude equal to the weight of the fluid displaced by the body and is directed vertically upward.

$$R = F_b = \rho_{Fluid} g V_{Displaced\ by\ Body}$$

If a body is immersed in two different fluid as shown

- Upthrust on upper part:

$$R_1 = \rho_1 g V_1$$

Acting through G_1 , the centroid of V_1

- Upthrust on lower part:

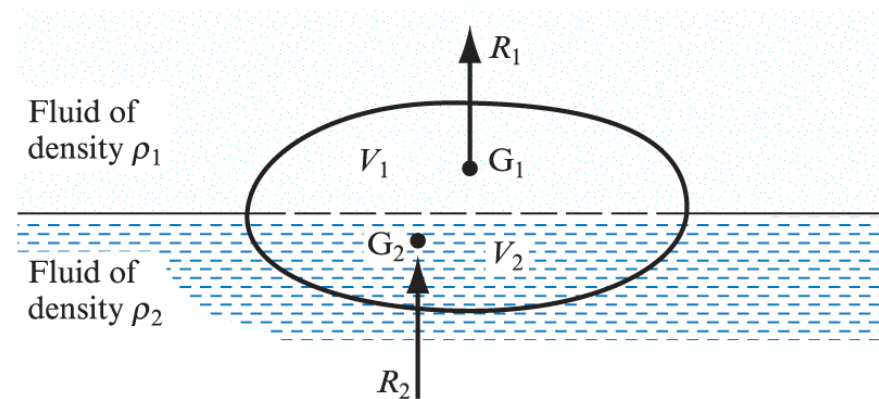
$$R_2 = \rho_2 g V_2$$

Acting through G_2 , the centroid of V_2



- Total Upthrust on the body:

$$R = R_1 + R_2 = \rho_1 g V_1 + \rho_2 g V_2$$



Example: (Ex 3.5, page 75 Textbook)

A rectangular pontoon has a width B of 6m, a length L of 12m, and a draught D of 1.5m in fresh water ($\rho = 1000 \text{ kg/m}^3$).

Calculate:

- a) The weight of the pontoon.
- b) Its draught in sea water ($\rho = 1025 \text{ kg/m}^3$)
- c) The load that can be supported by the pontoon in fresh water if the maximum draught permissible is 2 m

3.7 Equilibrium of floating bodies

When the body (Ship) floats in vertical equilibrium in a liquid, the forces presented are:

1. The upthrust force R acting through the center of buoyancy B .
2. The weight of the body W acting through its center of gravity G .

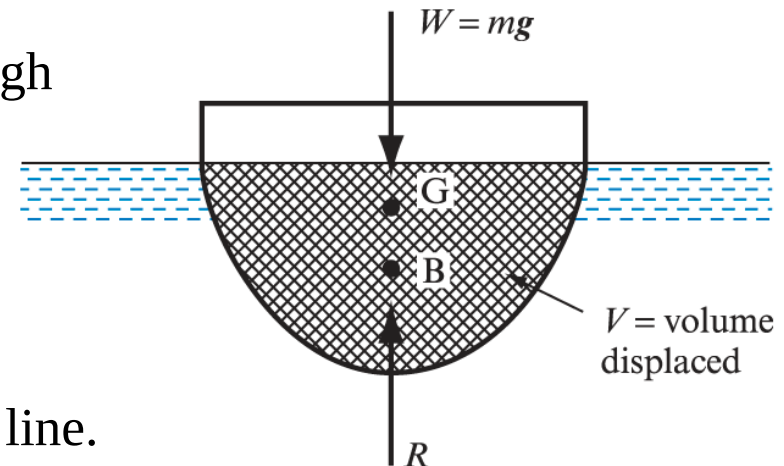
For equilibrium:

1. R and W must be equal.
2. R and W must act in the same straight line.

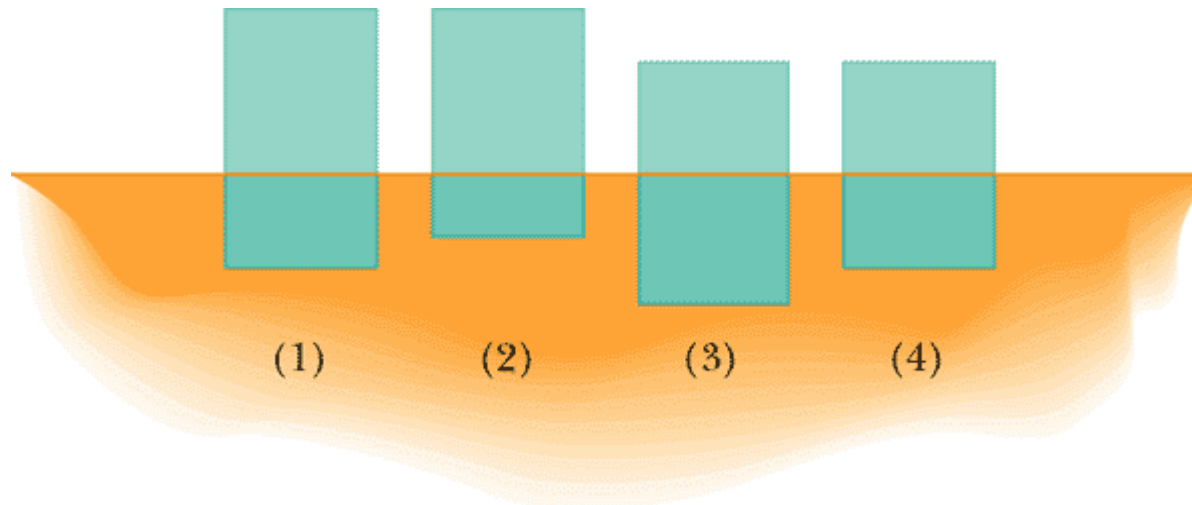
$$W = R$$

$$mg = \rho Vg \implies$$

$$V = \frac{m}{\rho}$$



Example:



Four solid objects are floating in corn syrup.
Rank the objects according to their density, greatest first.

The equilibrium of a body may be:

- Stable
 - Unstable
 - Neutral
- Depending upon whether, when given a small displacement, it tends to return to the equilibrium position, move further from it or remain in the displaced position.

For floating body (such as ship) stability is of major importance.

3.8 Stability of a Submerged Body

Remember

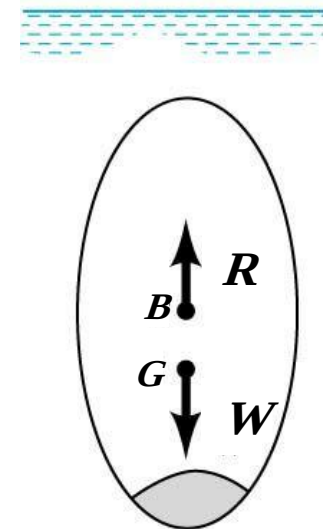
Stable Equilibrium: if when displaced returns to equilibrium position.

Unstable Equilibrium: if when displaced it returns to a new equilibrium position.

- The upthrust force R acting through the center of buoyancy B .
- The weight of the body W acting through its center of gravity of the body G .

Whatever the orientation of the body, these two points will remain in the same positions relative to the body.

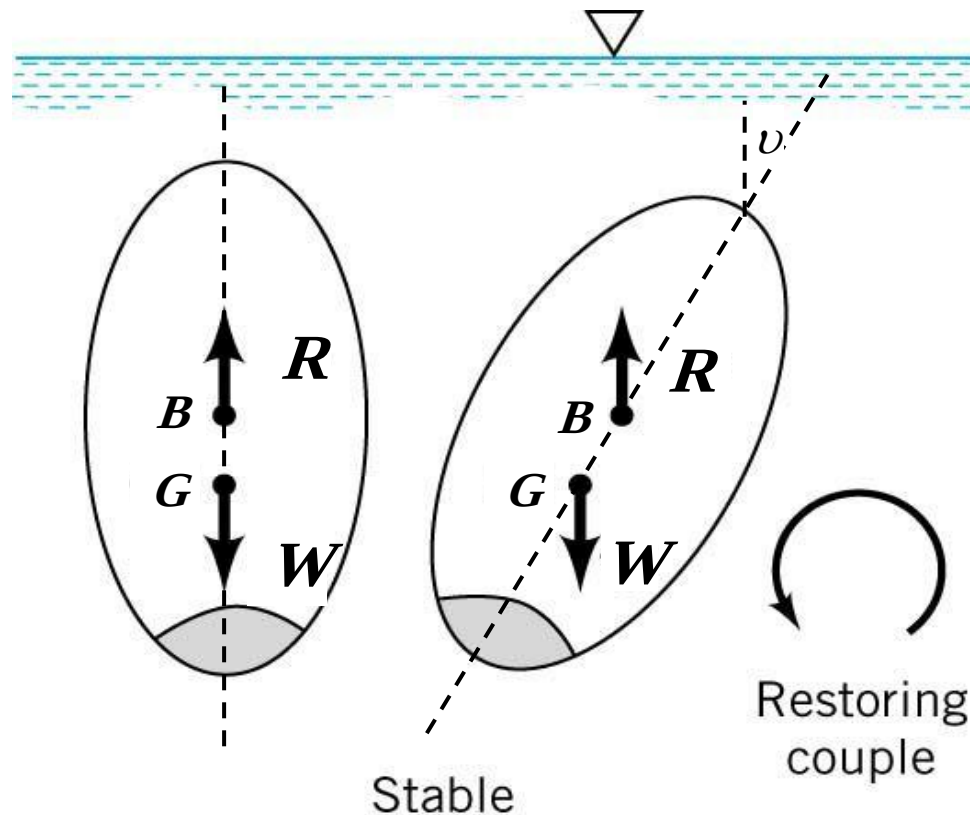
Note: as the body is totally immersed, the shape of the displaced fluid is not altered when the body is tilted and so the centre of buoyancy remains unchanged relative to the body.



It can be seen that: A small angular displacement ν from the equilibrium position will generate a moment $W.BG. \nu$.

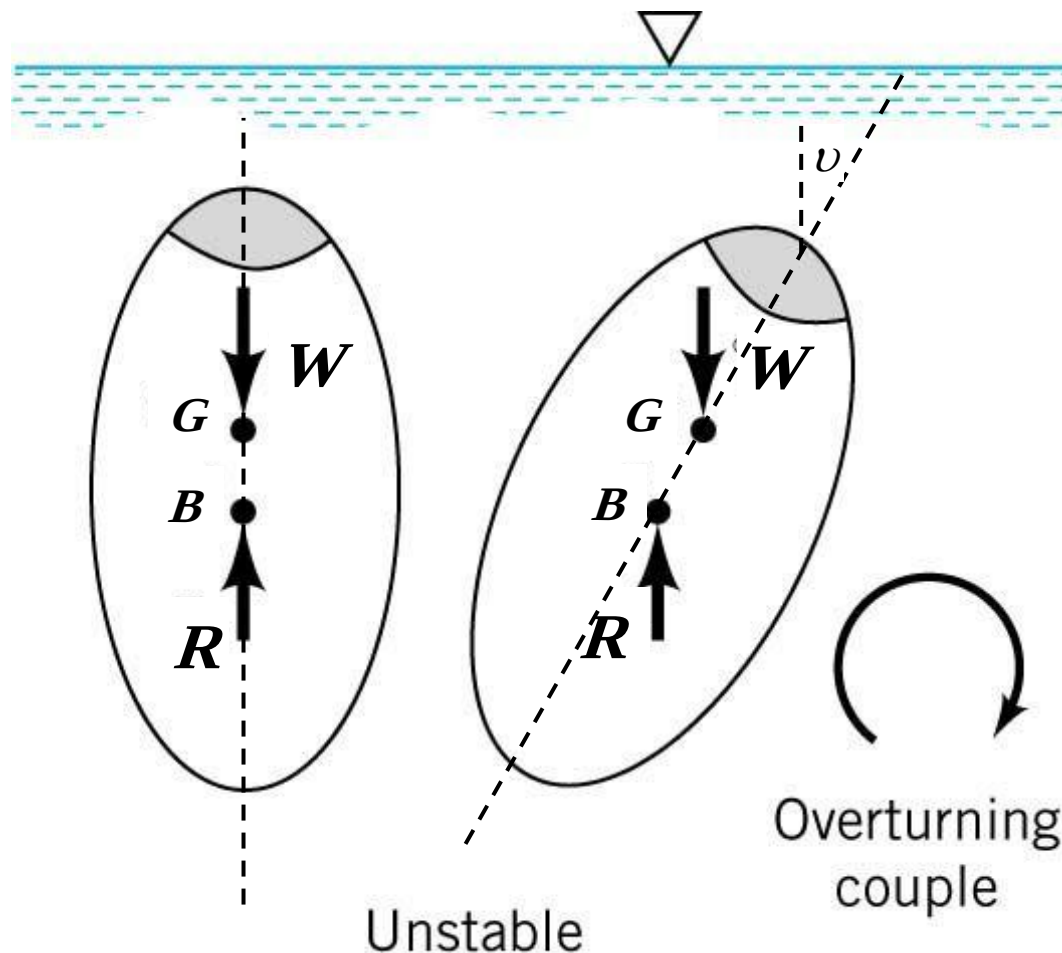
Stable

If G is below B , this will generate **righting moment** and the body will tend to return to its equilibrium position

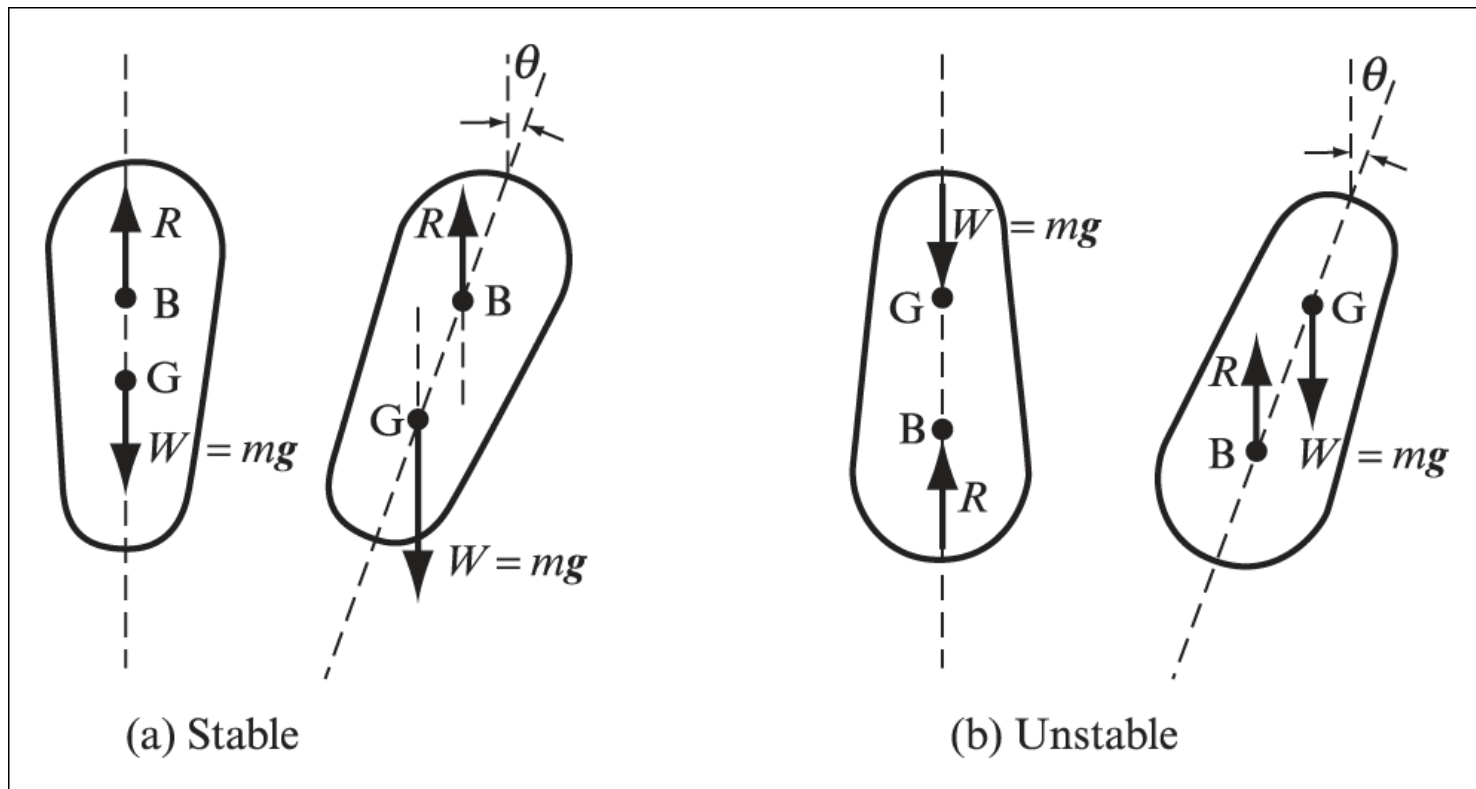


Unstable

If G is above B , this will be an **overturning moment** and the body is unstable



Summary



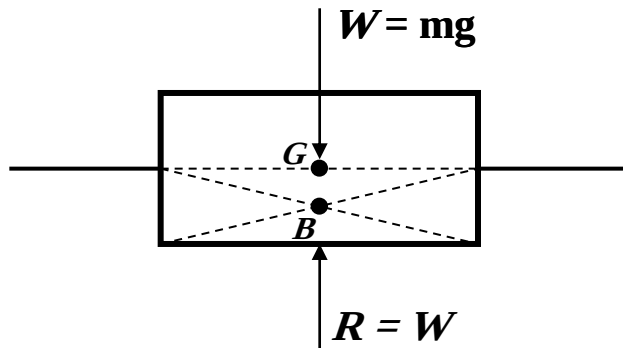
3.9 Stability of Floating Bodies

- It is **slightly more complicated** as the location of the center of buoyancy can change:
- The volume of liquid remains unchanged since $R=W$, but the **shape of this volume changes** and therefore its centre of buoyancy.
- When the body is displaced through an angle ν the center of buoyancy move from B to B_1 and a turning moment is produced.
- Let **M (metacentre)** is the point at which the line of action of the upthrust R cuts the original vertical line through G , then:

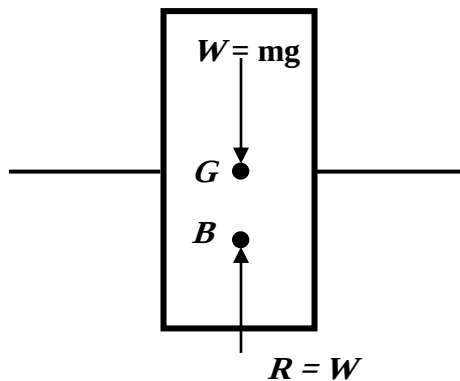
$$\text{Moment generated} = W \times GM \times \theta$$

- GM known as the **metacentric height**

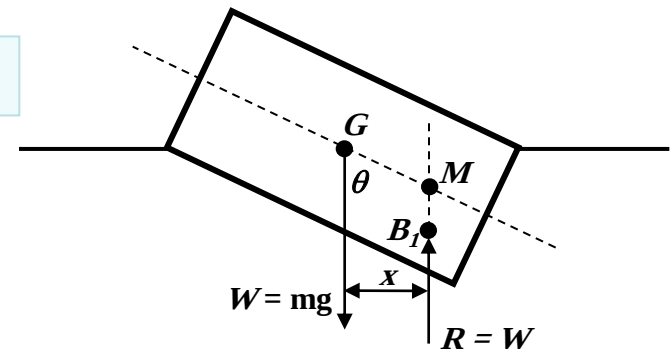
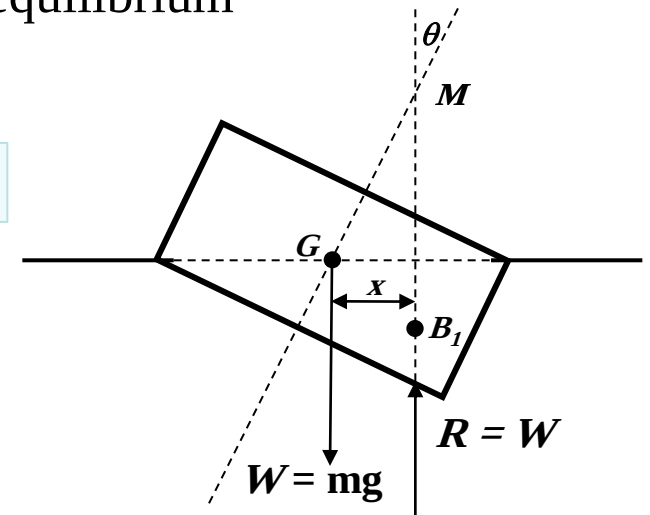
- If M lies above G , a righting moment is produced, equilibrium is stable and GM is regarded as positive.
- If M lies below G , an overturning moment is produced, equilibrium is unstable and GM is regarded as negative.
- If M coincides with G , the body is in neutral equilibrium



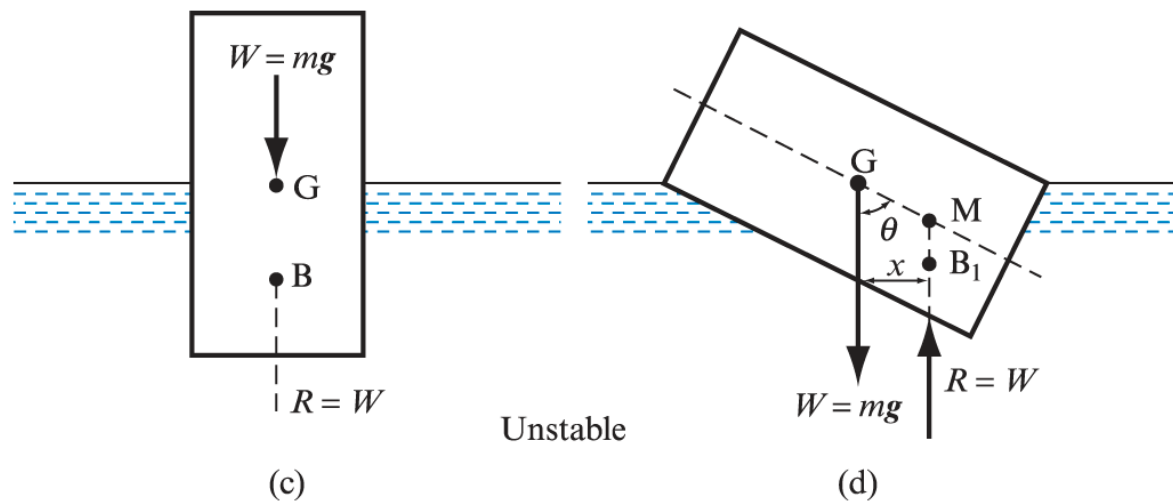
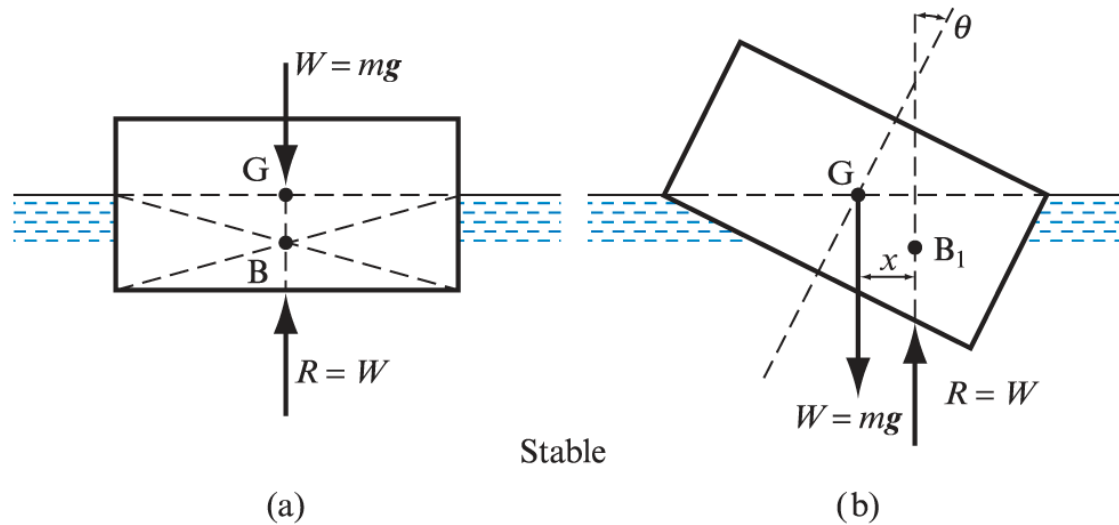
Stable



Unstable



Summary



3.10 Determination of Metacentric Height

- The metacentric height of a vessel can be measured if the angle of tilt θ caused by moving a load P a known distance x across the deck is measured.

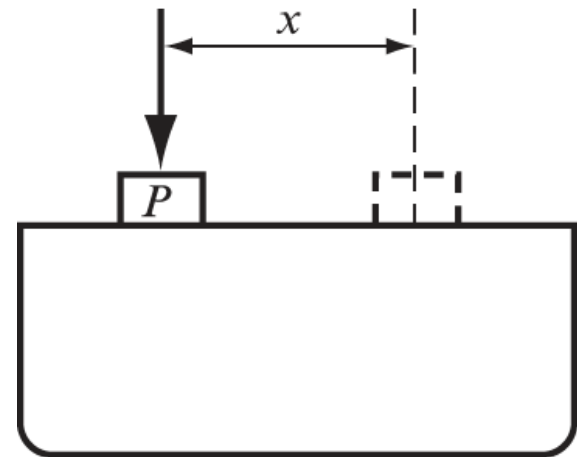
The overturning moment due to movement of load P = Righting moment

$$\Rightarrow W \times GM \times \theta = Px$$

$$\Rightarrow GM = \frac{Px}{W\theta}$$

Where:

W : is the weight of the vessel including P



3.11 Determination of the Position of Metacentre Relative to Centre of Buoyancy

When a vessel is tilted through a small angle θ then:

- The centre of buoyancy B will move to B' .
- The total weight displaced remains unchanged.

The distance BB' can be calculated as follows:

$$BB' = \frac{\theta I}{V}$$

The metacentric radius BM :

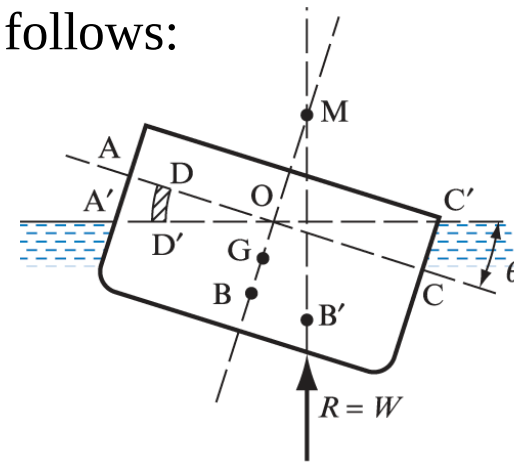
$$BM = \frac{BB'}{\theta} = \frac{I}{V}$$

Where:

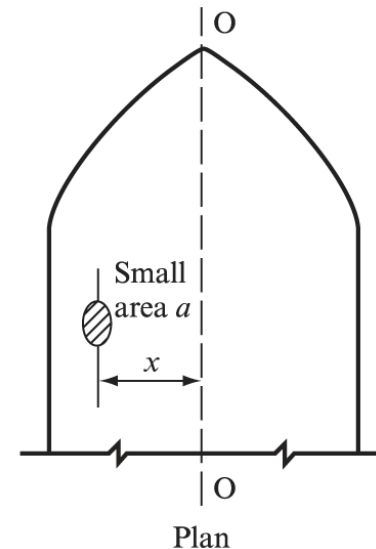
θ : tilted angle of the vessel

I : second moment of area of the waterline plane about OO

V : Volume of liquid displaced



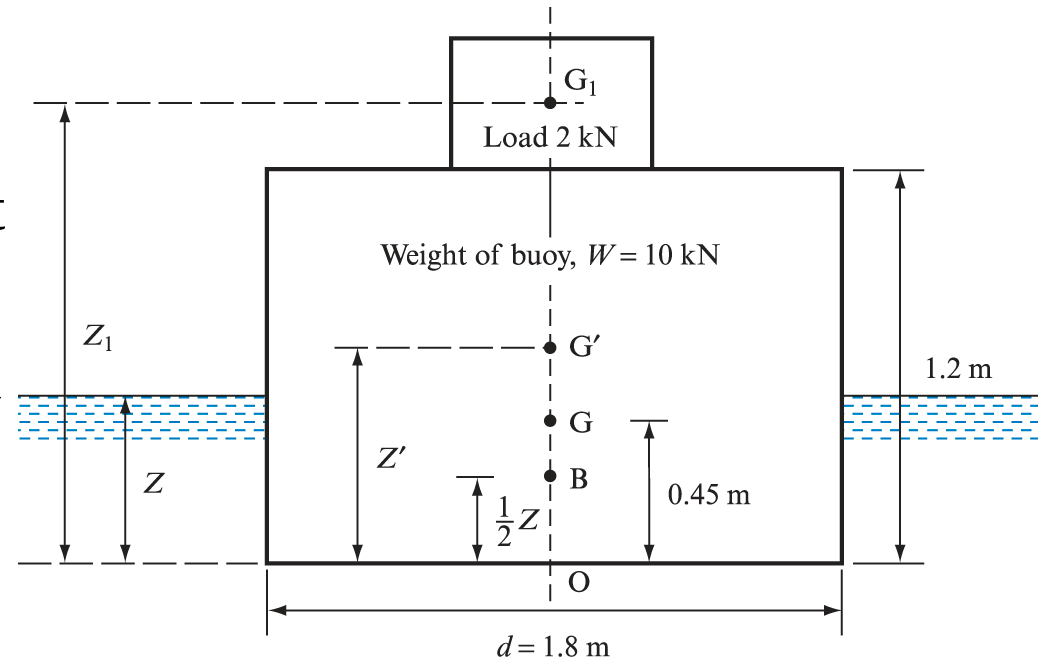
Cross-section



Plan

Example: (Ex 3.6, page 80 Textbook)

A cylindrical buoy 1.8-m in diameter, 1.2-m high and weighting 10-kN floats in salt water of density 1025-kg/m³. Its center of gravity is 0.45-m from the bottom. If a load of 2-kN is placed on the top.



Find the ***maximum height*** of the center of gravity of this load above the bottom if the buoy is to remain in stable equilibrium

Example:

- A circular door having a diameter of 4 m is positioned at the inclined wall as shown in Fig. which forms part of a large water tank. The door is mounted on a shaft which acts to close the door by rotating it and the door is restrained by a stopper. If the depth of the water is 10 m at the level of the shaft, Calculate:
 - Magnitude of the hydrostatic force acting on the door and its centre of pressure,
 - The moment required by the shaft to open the door.
- Use $\rho_{\text{water}} = 998 \text{ kg/m}^3$ and $g = 9.81 \text{ m/s}^2$.

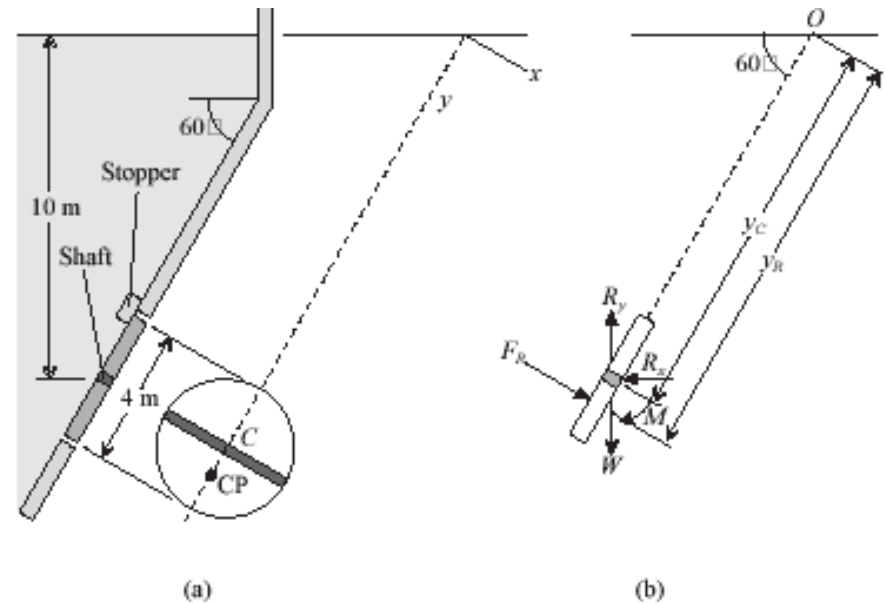


Figure E2.3

- (a) The magnitude of the hydrostatic force F_R is

$$\begin{aligned}
 F_R &= \rho g y A \\
 &= (998)(9.81)(10) \left[\frac{1}{4} \pi (4)^2 \right] \\
 &= 1.230 \times 10^6 \text{ N} \\
 &= \underline{1.23 \text{ MN}}
 \end{aligned}$$

- For the coordinate system shown in Figure E2.3(b), since circle is a symmetrical shape, $I_{xy} = 0$, then $x_R = 0$. For y coordinate,

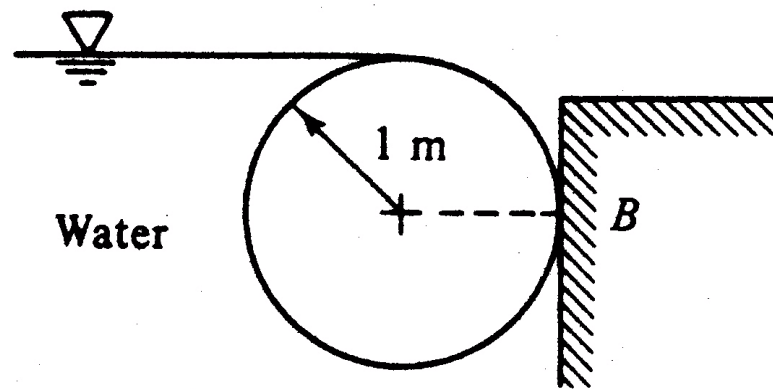
$$\begin{aligned}
 y_R &= \frac{I_{xx}}{y_C A} + y_C = \frac{\frac{1}{4} \pi R^4}{\pi y_C R^2} + y_C \\
 &= \frac{\frac{1}{4} \pi (2)^4}{\pi (10/\sin 60^\circ)(2)^2} + \frac{10}{\sin 60^\circ} \\
 &= \underline{11.6 \text{ m}}
 \end{aligned}$$

- Use moment equilibrium $\Sigma M = 0$ about the shaft axis. With reference to Figure E2.3(b), the moment M required to open the door is:

$$\begin{aligned}
 M &= F_R (y_R - y_C) \\
 &= (1.230 \times 10^5) (0.0866) \\
 &= 1.065 \times 10^5 \text{ N} \cdot \text{m} \\
 &= \underline{107 \text{ kM} \cdot \text{m}}
 \end{aligned}$$

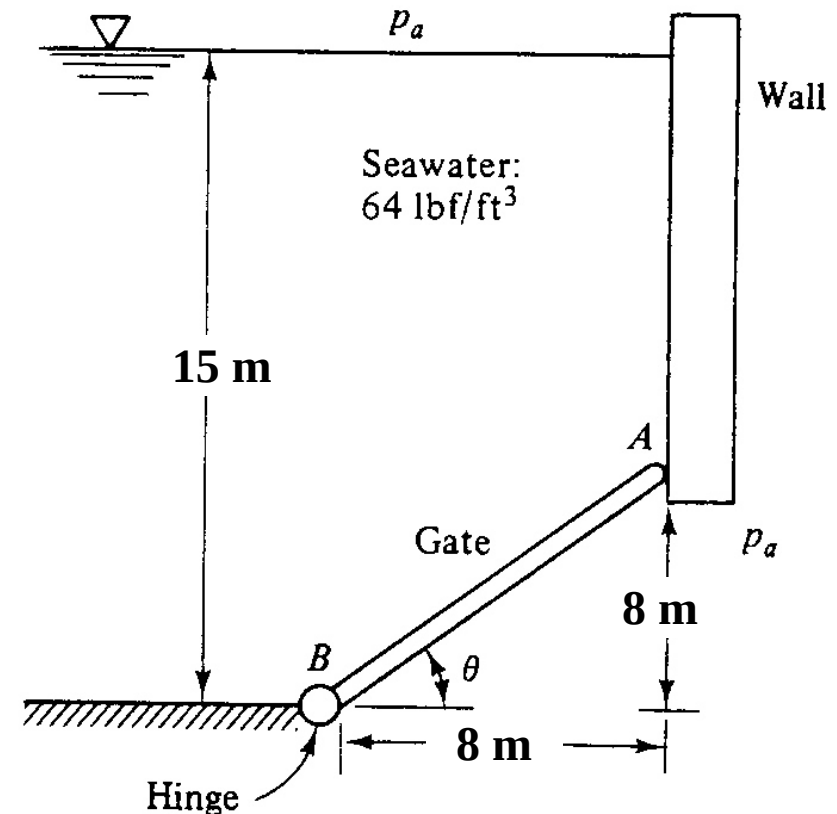
Example:

- The 2-m diameter cylinder shown is 5m long into the paper and rests in static equilibrium against smooth wall at point B .
- Compute the weight of the cylinder assume zero wall friction at point B



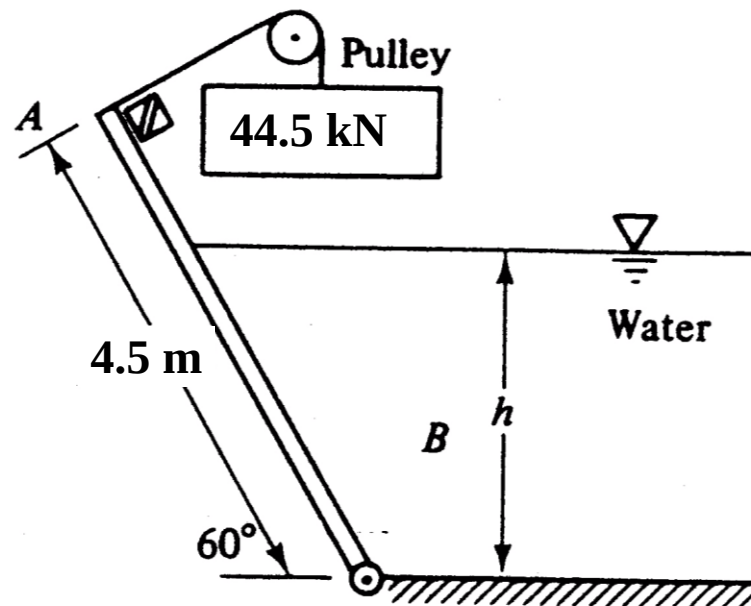
Example:

- A gate 5m wide is hinged at point B and rests against a smooth wall at point A ,
Compute:
- The force on the gate due to water
- The horizontal force P exerted by the wall at point A



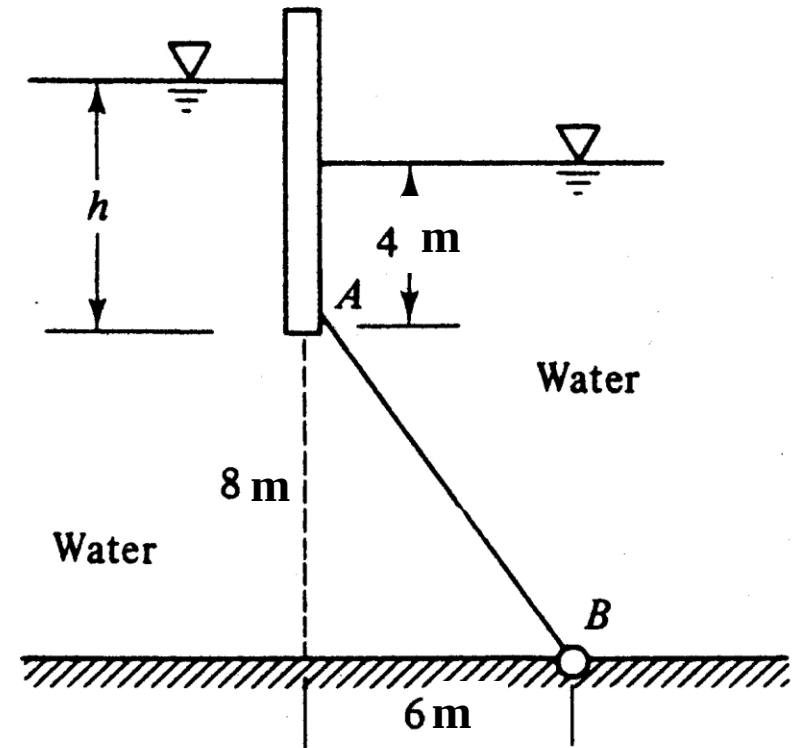
Example:

- The gate AB is 4.5 m long and 3 m wide into the paper and hinged at B with a stop at A . Neglecting the weight of the gate, compute the water level h for which the gate will start to fall



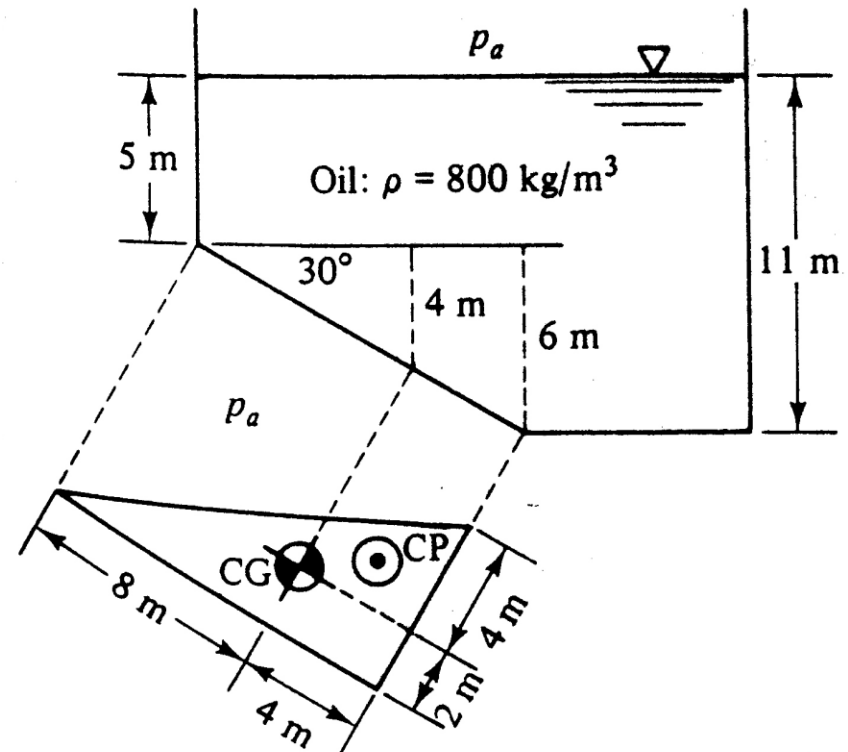
Example:

- Gate AB is 5 m into the paper and weights 30 kN when submerged.
- It is hinged at B and rests against a smooth wall at A .
- determine the water level h at the left which will just cause the gate to open

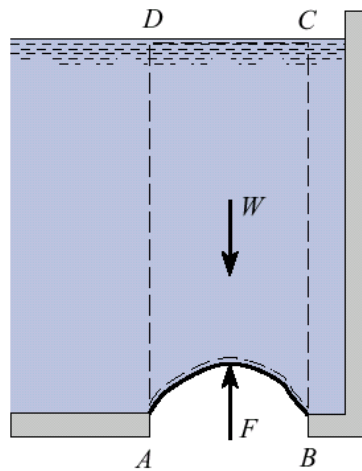


Example:

- A tank has right triangular gate near the bottom, Compute:
- The hydrostatic force on the gate
- the center of pressure on the gate

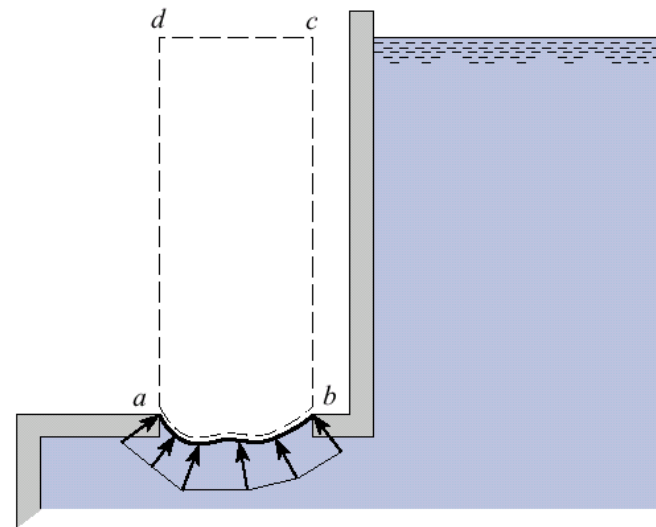


Example:



(a)

$$F = \gamma \mathcal{V}_{ABCD} = W$$

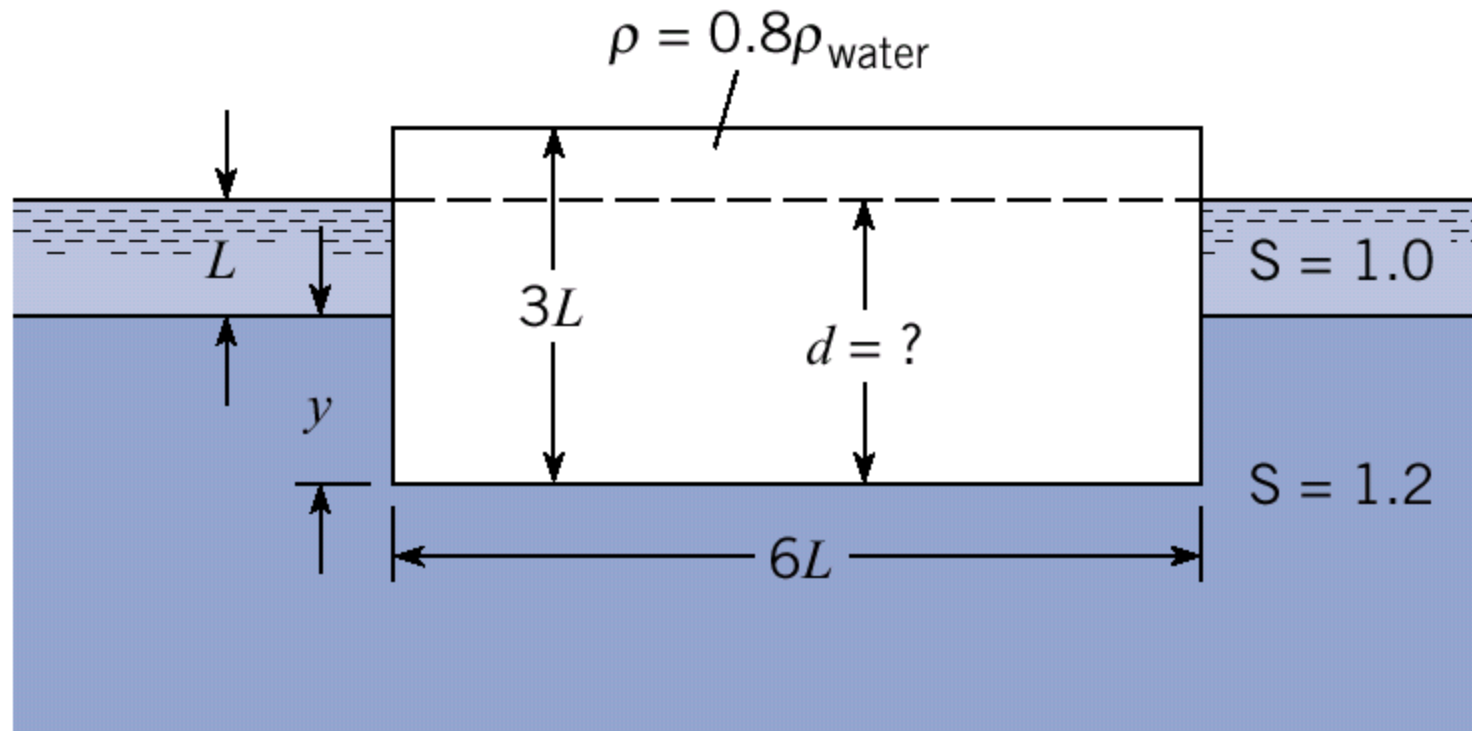


(b)

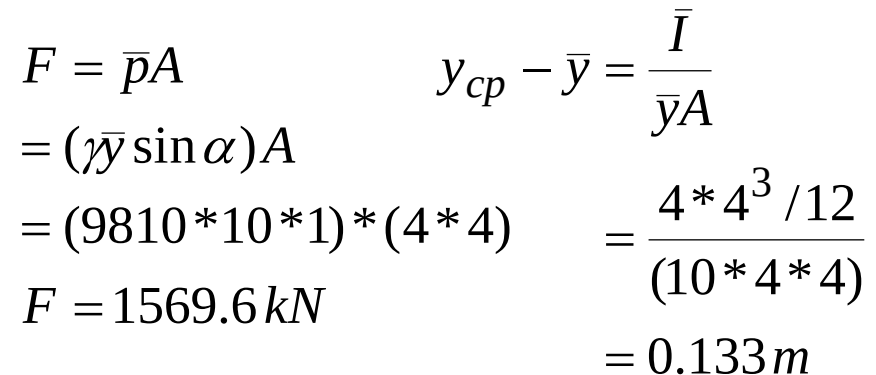
$$F = \gamma \mathcal{V}_{abcd} = W$$

Example:

To what depth d will this rectangular block (with density 0.8 times that of water) float in the two-liquid reservoir?



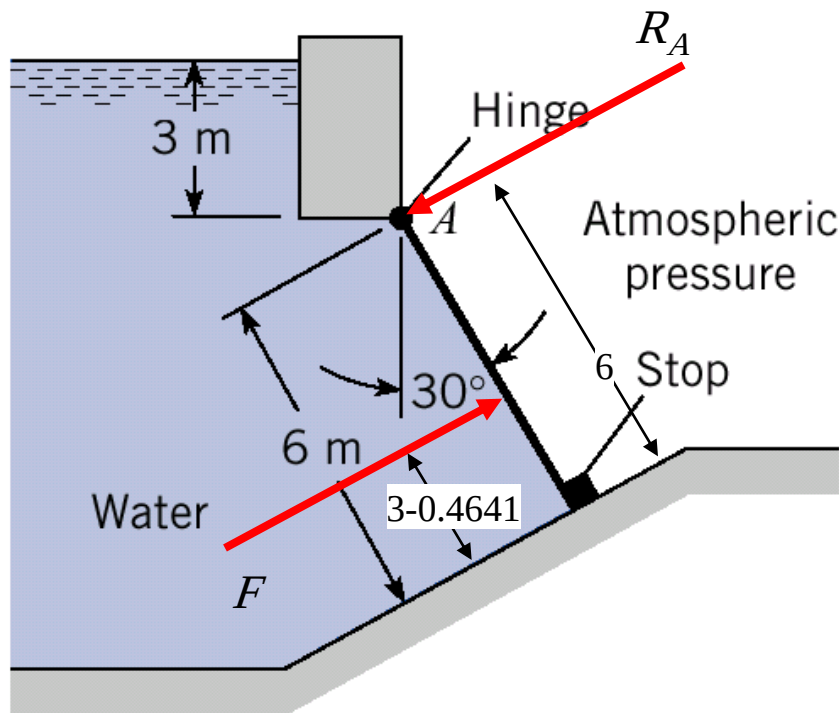
Find: Force of block on gate



$$F_{b,g} = 104.378 \text{ kN}$$

Example:

The gate shown is rectangular and has dimensions 6 m by 4 m. What is the reaction at point A? Neglect the weight of the gate.



$$F = \bar{p}A = (\gamma \bar{y} \sin \alpha)A$$

$$= 9810 * (3 + 3 \cos 30) * (4 * 6)$$

$$= 1,318,000 \text{ N}$$

$$y_{cp} - \bar{y} = \frac{\bar{I}}{\bar{y}A} = \frac{4 * 6^3 / 12}{(6.464 * 24)}$$

$$= 0.4641 \text{ m}$$

$$\sum M = 0$$

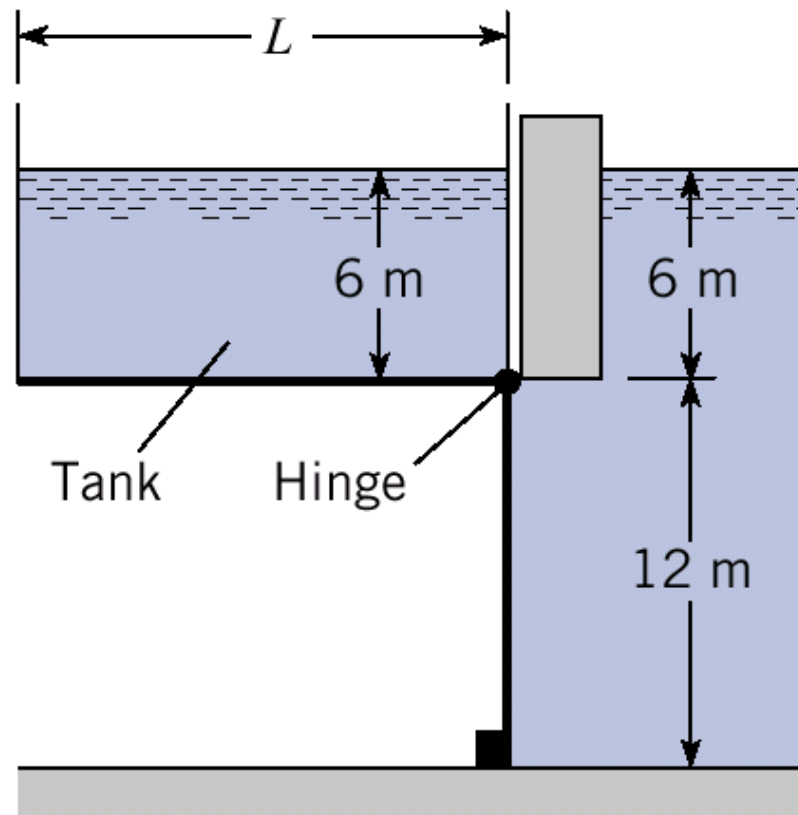
$$= 6R_A - (3 - 0.4641)F$$

$$R_A = \frac{3 - 0.4641}{6} F$$

$$= (0.42265) 1318 \text{ kN}$$

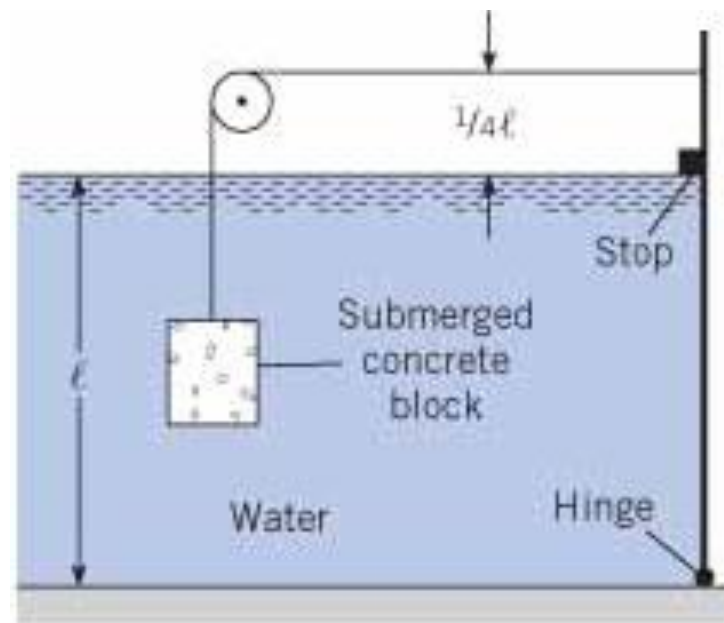
$$R_A = 557.05 \text{ kN}$$

Example:



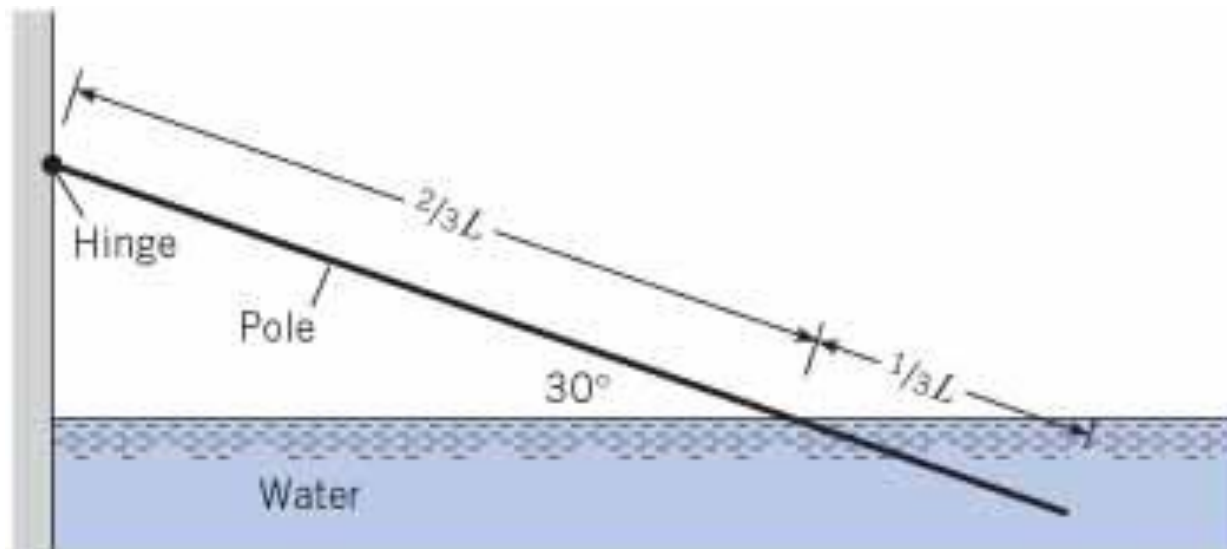
Example:

Determine the minimum volume of concrete ($\gamma = 23.6 \text{ kN/m}^3$) needed to keep the gate (1 m wide) in a closed position, with $\ell = 2 \text{ m}$. Note the hinge at the bottom of the gate.



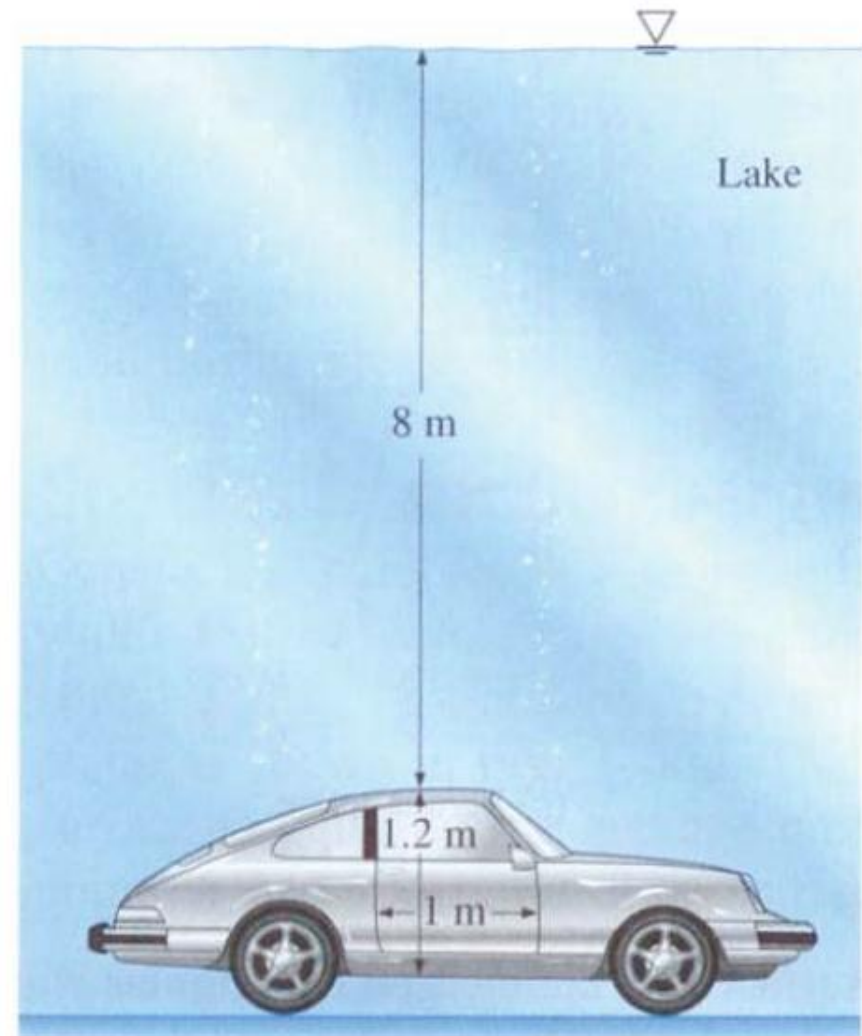
Example:

The partially submerged wood pole is attached to the wall by a hinge as shown. The pole is in equilibrium under the action of the weight and buoyant forces. Determine the density of the wood.



Example:

- A heavy car plunges into a lake during an accident and lands at the bottom of the lake on its wheels
- The door is 1.2 m high and 1 m wide, and the top edge of the door is 8 m below the free surface of the water
- Determine the **hydrostatic force** on the door and the location of the **pressure center**, and **discuss** if the driver can open the door

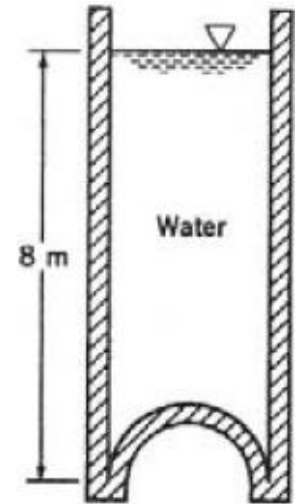


Example:

- Few assumptions should be made in order to facilitate the solution:
 - The bottom surface of the lake is horizontal
 - The passenger cabin is well-sealed so that no water leaks inside
 - The door can be approximated as a vertical rectangular plate
 - The pressure in the passenger cabin remains at atmospheric value since there is no water leaking in and no compression of the air inside

Example:

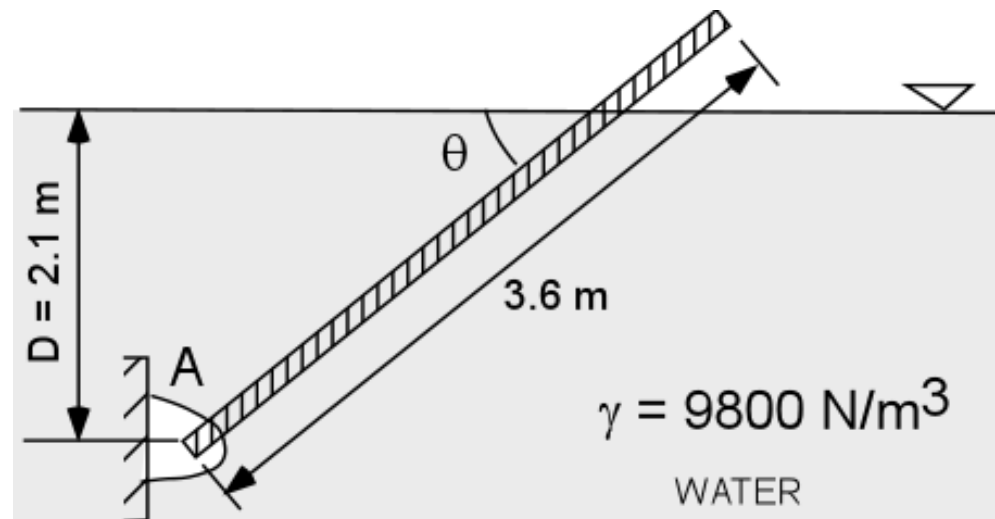
A 3-m-diameter open cylindrical tank contains water and has a hemispherical bottom as shown. Determine the magnitude, line of action, and direction of the force of the water on the curved bottom.



Quiz:

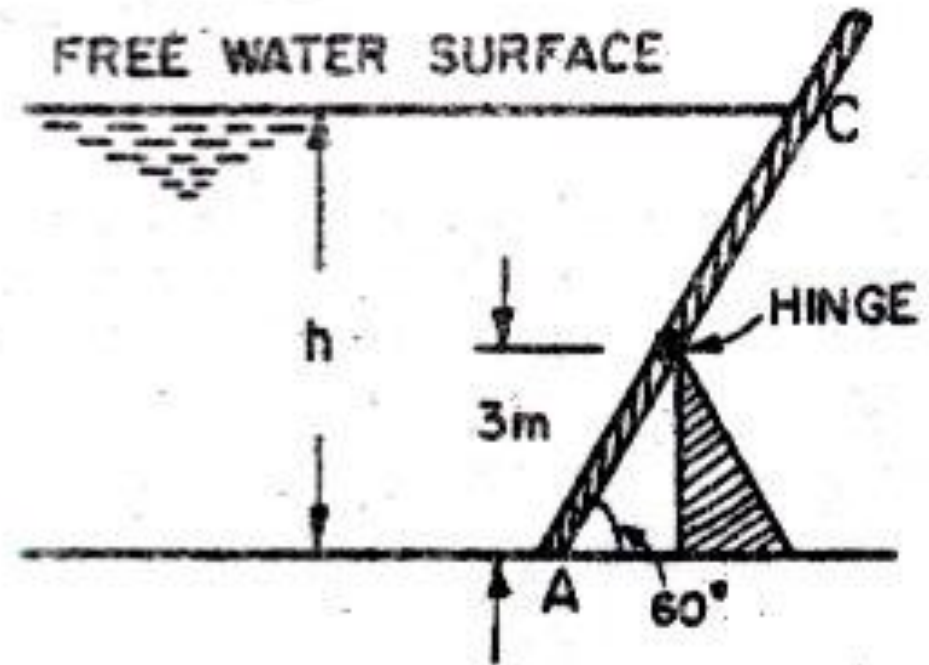
$0.15 \times 0.15 \times 3.6$ m uniform wooden bar of weights 700N, hinged at point A in water as shown in the figure. Determine:

- (a) the angle θ .
- (b) the depth (D) required to allow the bar to float vertically? (Bonus)



Quiz:

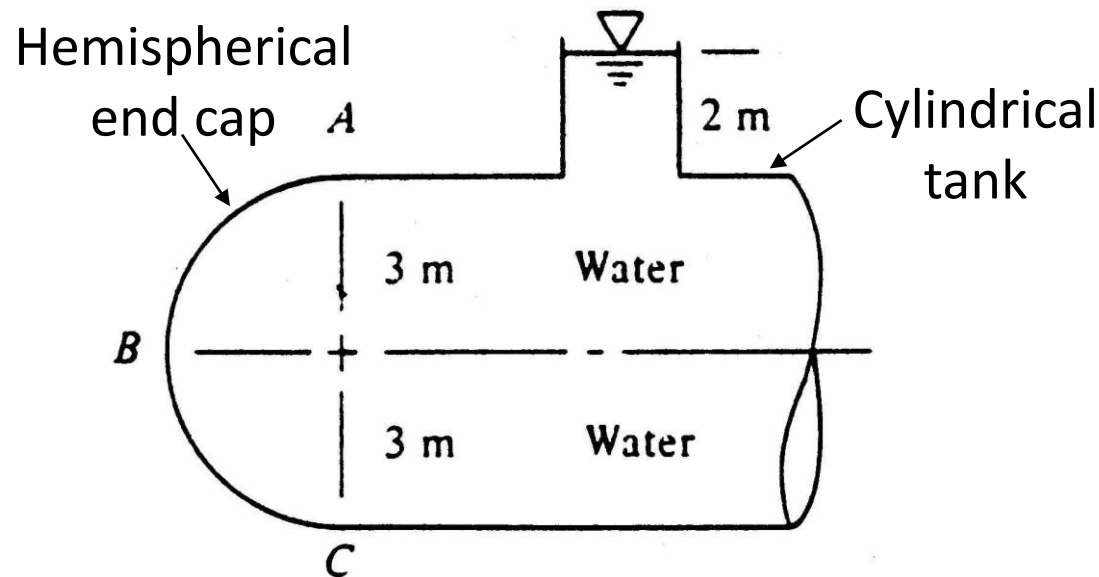
A gate supporting water is shown in the figure below. Find the height h of the water so that the gate tips about the hinge. Take the width of the gate as unity.



Quiz:

The cylindrical tank shown has a hemispherical end cap ABC contains water as shown in the figure.

Compute the resultant force of the fluid on the end cap ABC .

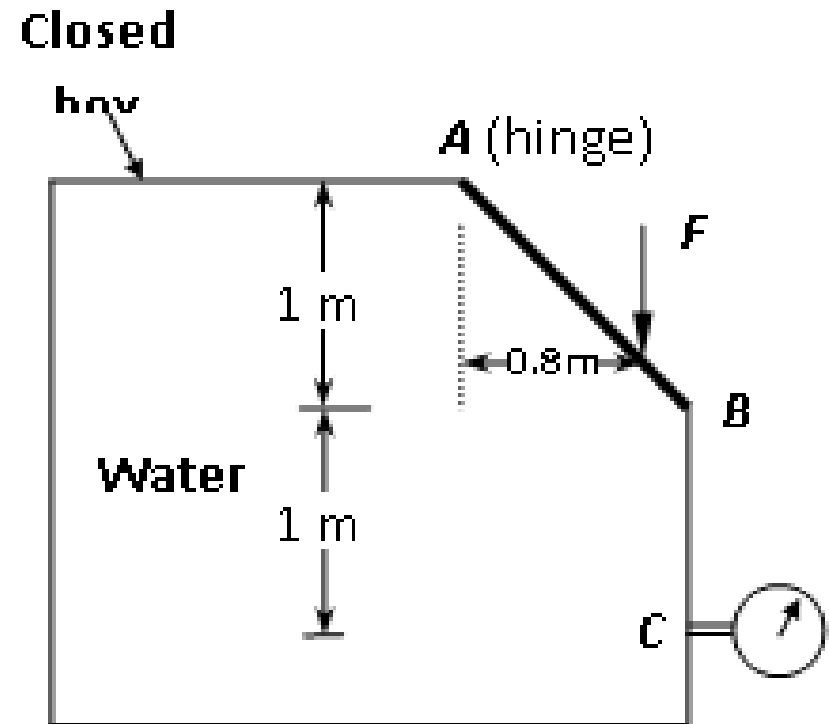


If required: volume of sphere = $V = \frac{4}{3}\pi R^3$ where R = radius of sphere

Quiz:

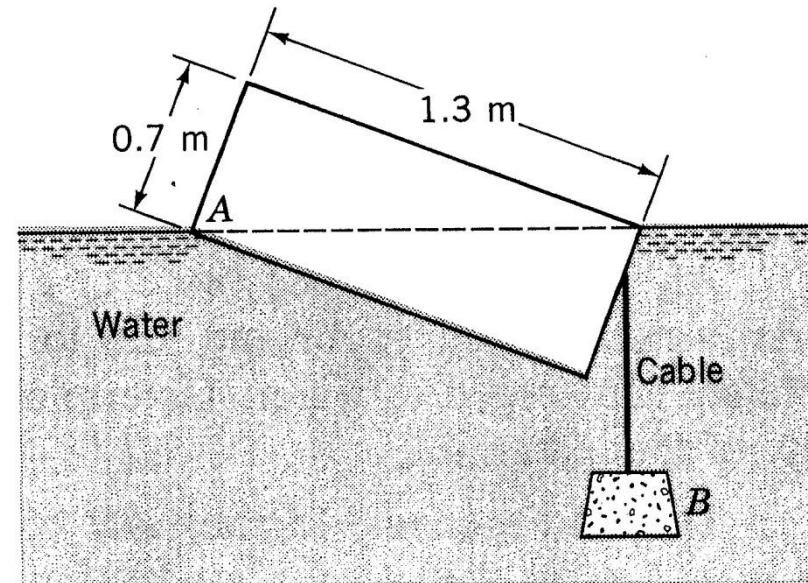
Calculate the minimum force F required to keep the cover AB of the box shown **closed**.

The box is 1 m wide (into the paper) and the gauge pressure at point C is 40 kPa. The cover is inclined at an angle of 45°



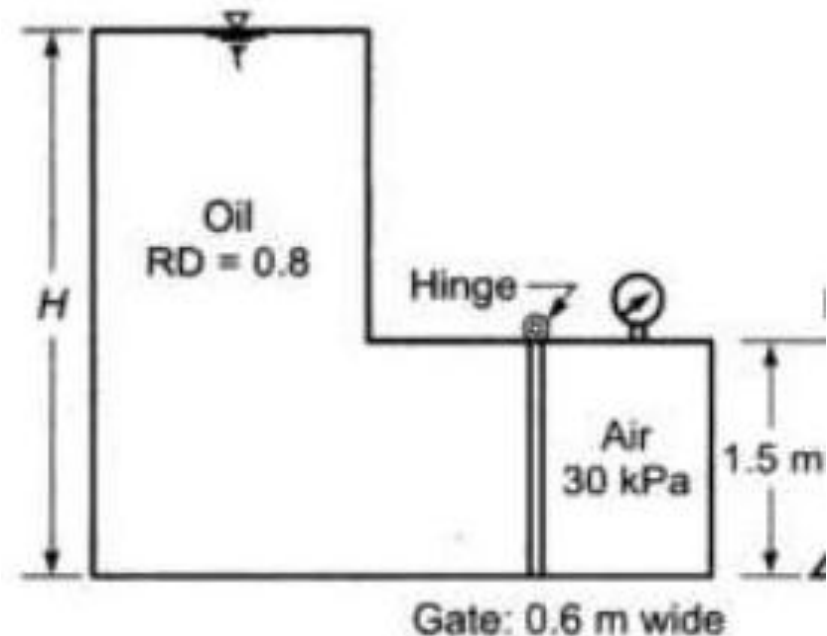
Quiz:

The homogeneous wooden block **A** shown in the figure is 0.7 by 0.7 by 1.3 m and weighs 2400 N. The concrete block **B** has a specific weight = 23.6 kN/m^3 is suspended from **A** by means of a cable causing **A** to float in the position indicated. **Determine volume of the block B.**

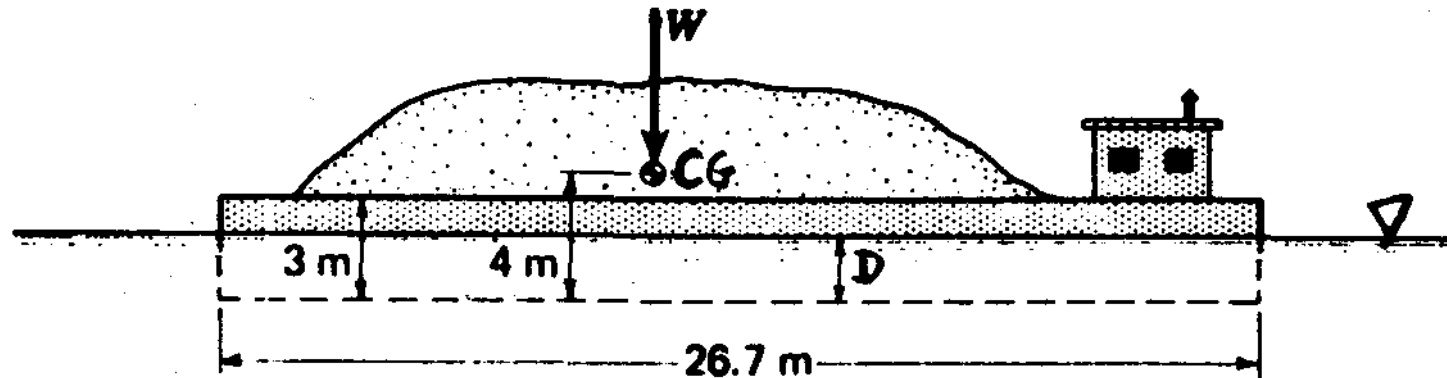


Quiz:

For the system shown, **Calculate** the height H of oil at which the rectangular hinged gate will just begin to rotate counterclockwise. The gate is 1.5m height by 0.6 m wide (into the paper).



Quiz:



The barge shown in the figure above has the form of a parallelepiped having dimensions 10m by 26.7m by 3m. After loading, the total weight of the barge is 4450 kN with a center of gravity 4m measured from the bottom.

For a rotation about its longest centerline, **determine**:

- the location of the metacenter (M) measured from the bottom of the barge.
- whether the barge is stable or not.