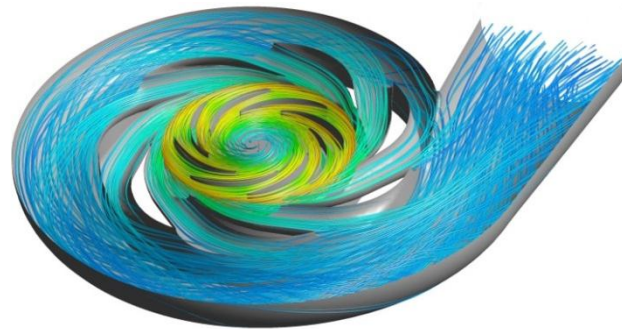


CHAPTER 4

Motion of Fluid Particles and Streams



Dr. Khalil Mahmoud ALASTAL

Objectives of this Chapter:

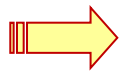
- Introduce concepts necessary to **analyze fluids in motion**.
- Identify differences between **Steady/unsteady, uniform/non-uniform, compressible/incompressible** flow.
- Demonstrate **streamlines** and **stream tubes**.
- Introduce **laminar and turbulent flow**.
- Introduce the **Continuity principle** through conservation of mass and **control volumes**.

Introduction:

- Fluid motion can be predicted in the same way as the motion of solids by use of the **fundamental laws of physics** and the **physical properties of the fluid**.
- Some fluid flow is very **complex**:
 - Spray behind a car.
 - waves on beaches.
 - hurricanes and tornadoes.
- All can be analyzed with varying degrees of success (in some cases hardly at all!).
- There are many common situations which analysis gives very accurate predictions.

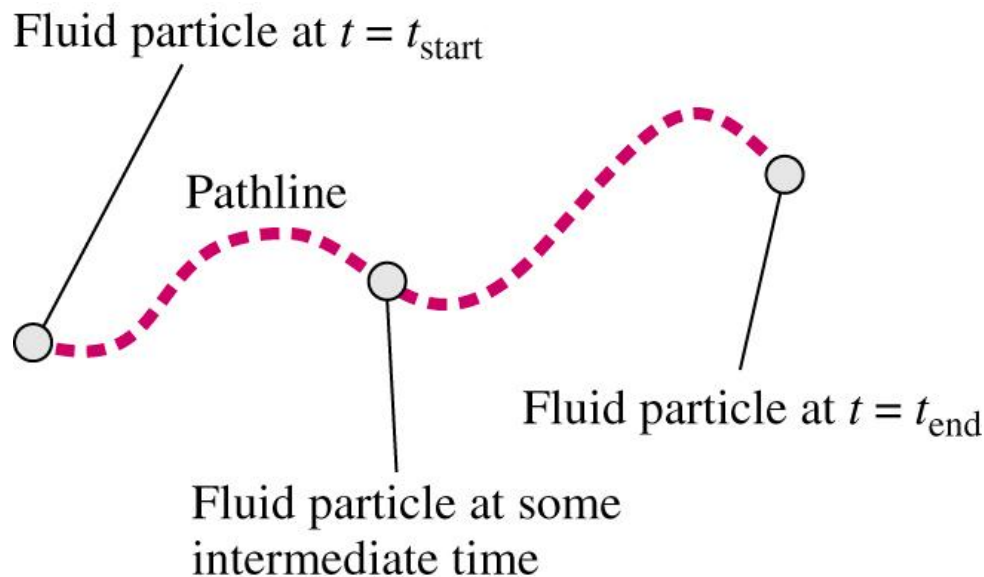
4.1 Fluid Flow

Pathline: line traced by a given particle as it flows from one point to another.



If an individual particle of fluid is colored, or otherwise rendered visible, it will describe the **pathline**.

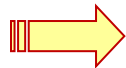
Which is the trace showing the position at successive intervals of time of particle which started from a given point.



A **Pathline** is the actual path traveled by an individual fluid particle **over some time period**.

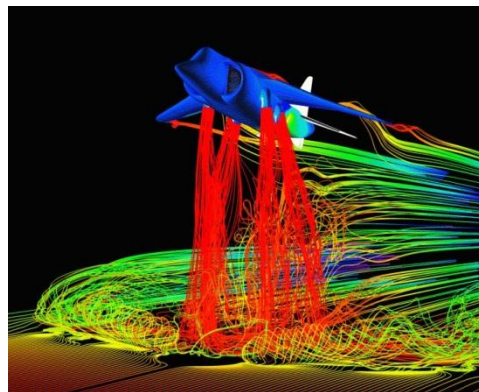
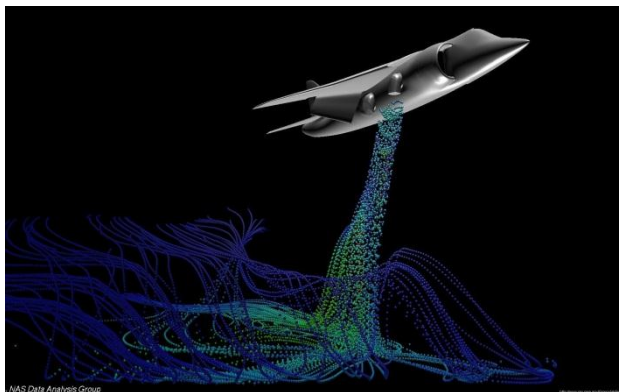
Streaklines: (filament line)

A laboratory tool used to obtain **instantaneous photographs** of marked particles that all passed through a given flow field at some earlier time.



Instead of coloring an individual particle the flow pattern is made visible by injecting a stream of dye into a liquid, or smoke into a gas, the result will be ***streakline or filament line***.

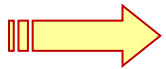
Which gives an instantaneous picture of the positions of all the particles which have passed through the particular point at which the dye is being injected.



A **Streakline** is the locus of fluid particles that have passed sequentially through a prescribed point in the flow.

Streamline:

It is an **imaginary curve** in the fluid across which, at a given instant, there is no flow. thus the velocity of every particle of fluid along the stream line is tangential to it at that moment



This can be done by drawing lines joining points of equal velocity - **velocity contours**.

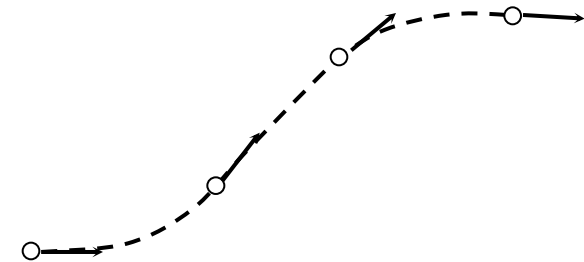
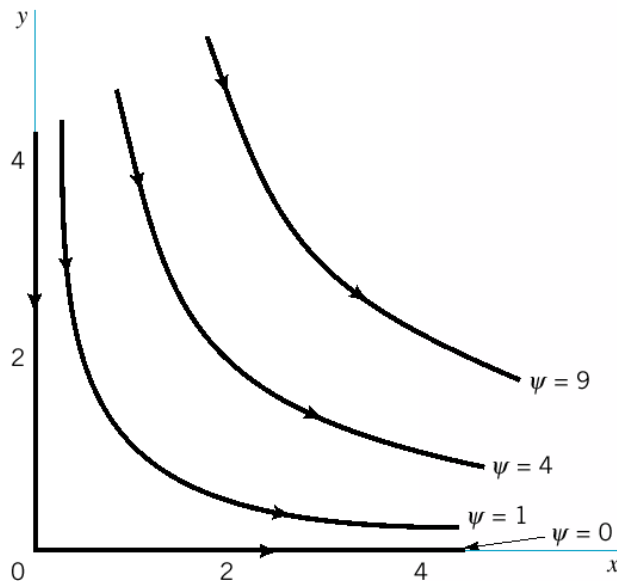
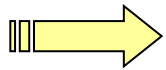
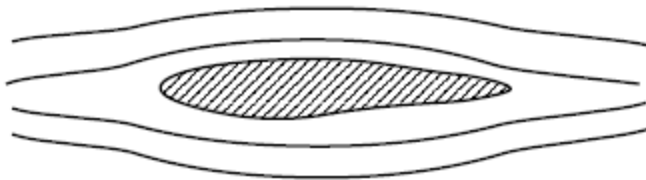


Figure 3.5: A streamline

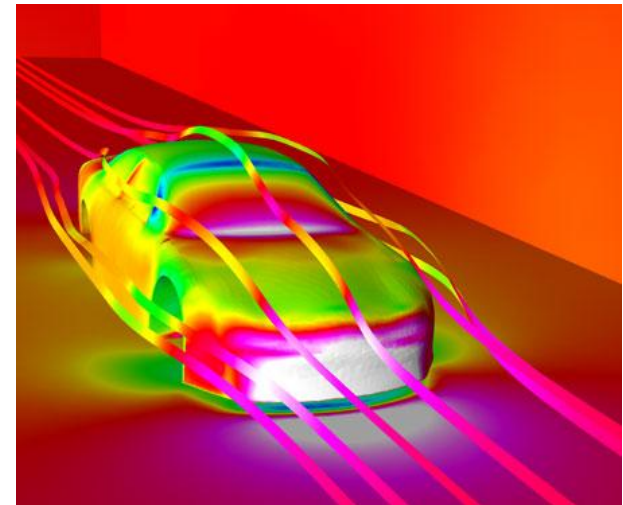
When fluid is flowing past a solid boundary, e.g. the surface of an aerofoil or the wall of a pipe, fluid obviously does not flow into or out of the surface. So very close to a boundary wall the flow direction must be parallel to the boundary.



Close to a solid boundary, streamlines are parallel to that boundary



2-D streamlines around a cross-section of an aircraft wing shaped body



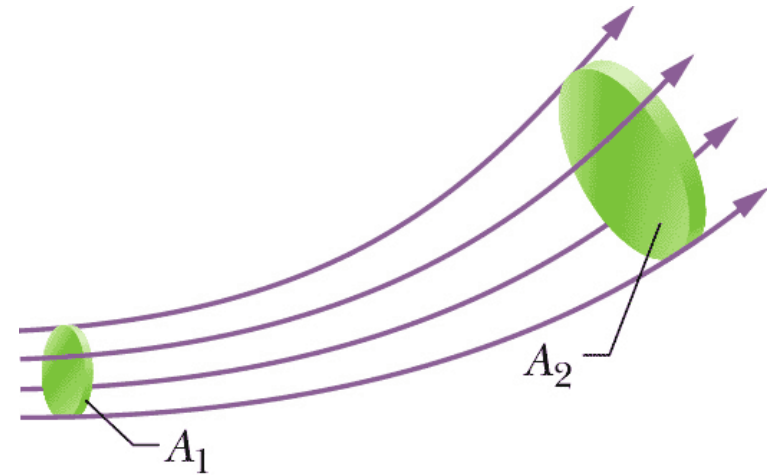
Surface pressure contours and streamlines

Notes about streamlines:

- Because the fluid is moving in the same direction as the streamlines, **fluid can not cross a streamline.**
- **Streamlines can not cross each other.** If they were to cross this would indicate two different velocities at the same point. This is not physically possible.
- The above point implies that any particles of fluid starting on one streamline will stay on that same streamline throughout the fluid.

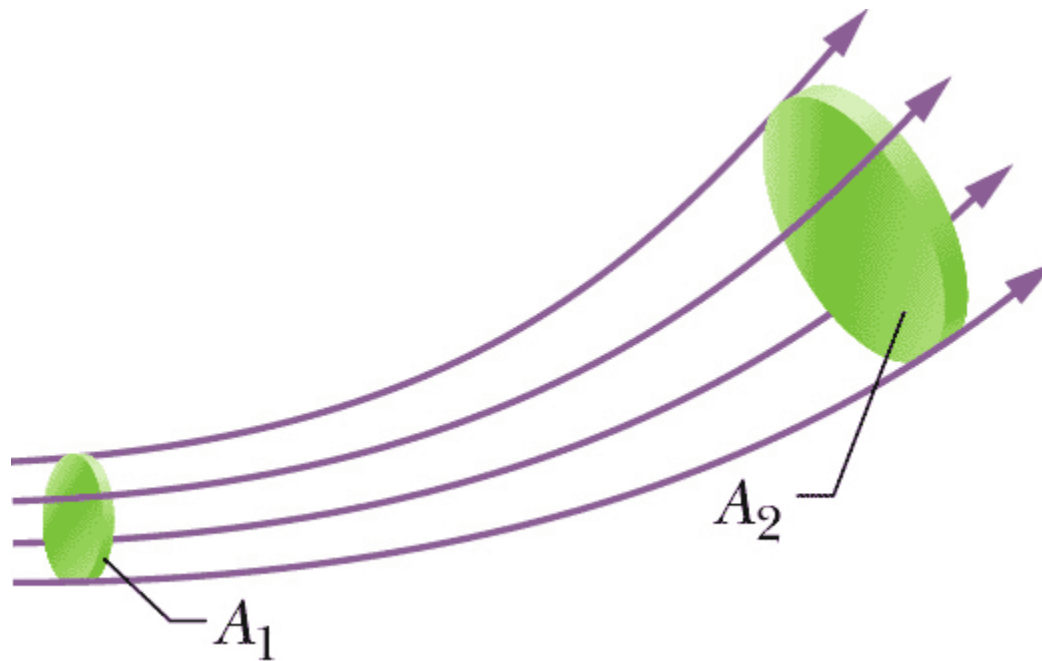
Streamtube:

- A circle of points in a flowing fluid each has a streamline passing through it.
- These streamlines make a tube-like shape known as a streamtube
- A useful technique in fluid flow analysis is to consider only a part of the total fluid in isolation from the rest.



Notes:

- The "*walls*" of a streamtube are made of streamlines.
- As we have seen above, fluid cannot flow across a streamline, so fluid cannot cross a streamtube wall.
- The streamtube can often be viewed as a solid walled pipe.
- A streamtube is not a pipe - it differs in unsteady flow as the walls will move with time. And it differs because the "*wall*" is moving with the fluid.



Streamtube is an imaginary tube whose boundary consists of streamlines.

4.2 Uniform Flow and Steady Flow

Fluid flow may be classified as:

- **uniform:**
Flow conditions (velocity, pressure, cross section or depth) are the same at every point in the fluid.
- **non-uniform:**
Flow conditions are not the same at every point.
- **steady:**
Flow conditions may differ from point to point but DO NOT change with time.
- **unsteady:**
If at any point in the fluid, the conditions change with time, the flow is described as unsteady.

Fluid flowing under normal circumstances - a river for example :

Conditions vary from point to point we have non-uniform flow.

If the conditions at one point vary as time passes then we have unsteady flow.

Combining the above we can classify any flow into one of four type:

Steady uniform flow:

- Conditions: do not change with position in the stream or with time.
- Example: the flow of water in a pipe of constant diameter at constant velocity.

Steady non-uniform flow:

- Conditions: change from point to point in the stream but do not change with time.
- Example: flow in a tapering pipe with constant velocity at the inlet-velocity will change as you move along the length of the pipe toward the exit.

Unsteady uniform flow:

- At a given instant in time the conditions at every point are the same, but will change with time.
- Example: a pipe of constant diameter connected to a pump pumping at a constant rate which is then switched off.

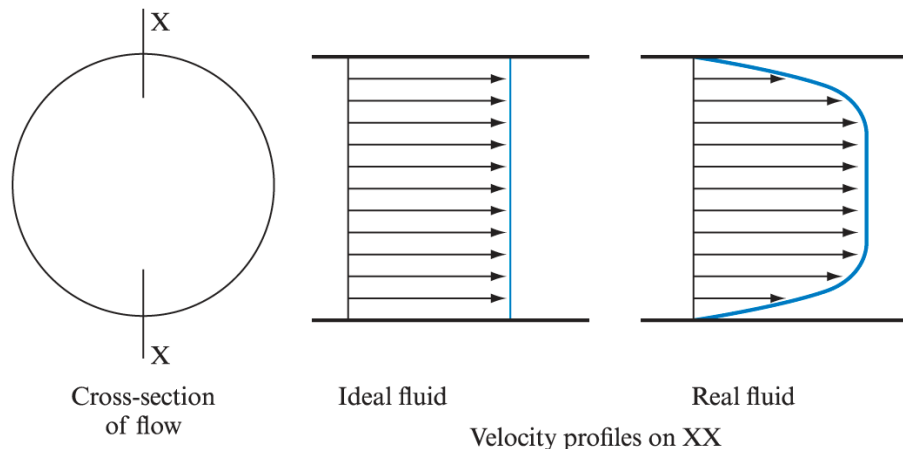
Unsteady non-uniform flow:

- Every condition of the flow may change from point to point and with time at every point.
- Example: waves in a channel.

4.4 Real and Ideal Fluids:

Ideal fluids:

- An ideal fluid is one which **has no viscosity**.
- Since there is no viscosity, there is no shear stress between adjacent fluid layers and between the fluid layers and the boundary.
- In reality there is no fluid which is ideal, however in certain cases the fluid is assumed to be ideal. And thus greatly simplify the mathematical solution.



Real fluids:

- A real fluid is one which **possesses viscosity**. (tend to “stick” to solid surfaces)
- As soon as motion takes place, shearing stresses come into existence.

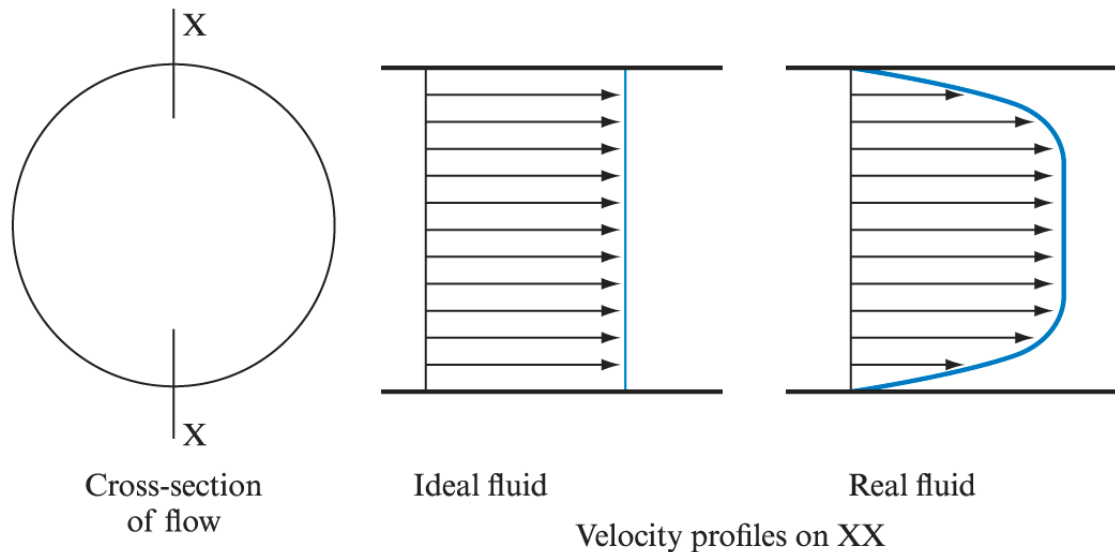
4.5 Compressible or Incompressible:

- **All fluids are compressible** - even water - their density will change as pressure changes.
- Under steady conditions, and provided that the changes in pressure are small, it is usually possible to **simplify analysis of the flow by assuming it is incompressible** and has constant density.
- **Liquids** are quite difficult to compress - so under most steady conditions they are treated as incompressible.
- **Gasses**, on the contrary, are very easily compressed, it is essential in most cases to treat these as compressible, taking changes in pressure into account.

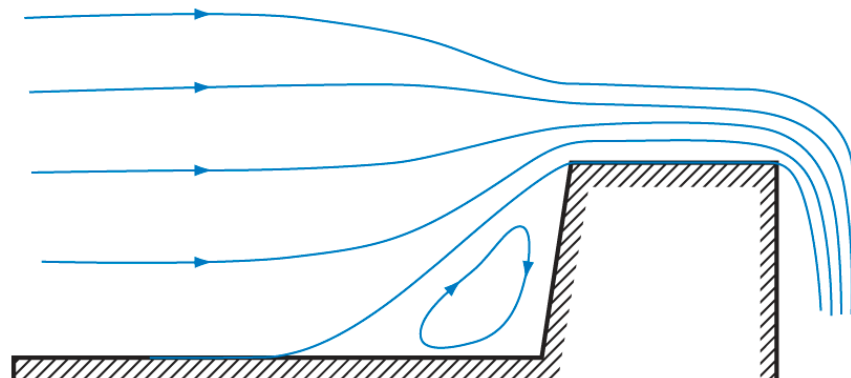
4.6 One, Two and Three-Dimensional Flow:

- In general **all fluids flow three-dimensionally**, with pressures and velocities and other flow properties varying in all directions.
- In many cases the greatest changes only occur in two directions or even only in one.
- In these cases changes in the other direction can be effectively ignored making analysis much more simple.

- **Flow is *one dimensional*** if the flow parameters (such as velocity, pressure, depth etc.) at a given instant in time **only vary in the direction of flow and not across the cross-section**. The flow may be unsteady, in this case the parameter vary in time but still not across the cross-section.
- An example of one-dimensional flow is the flow in a pipe. Note that since flow must be zero at the pipe wall - yet non-zero in the centre - there is a difference of parameters across the cross-section.



- **Flow is *two-dimensional*** if it can be assumed that the flow parameters **vary in the direction of flow and in one direction at right angles to this direction.**
- Streamlines in two-dimensional flow are curved lines on a plane and are the same on all parallel planes.
- An example is flow over a weir for which typical streamlines can be seen in the figure below. Over the majority of the length of the weir the flow is the same - only at the two ends does it change slightly. Here correction factors may be applied.

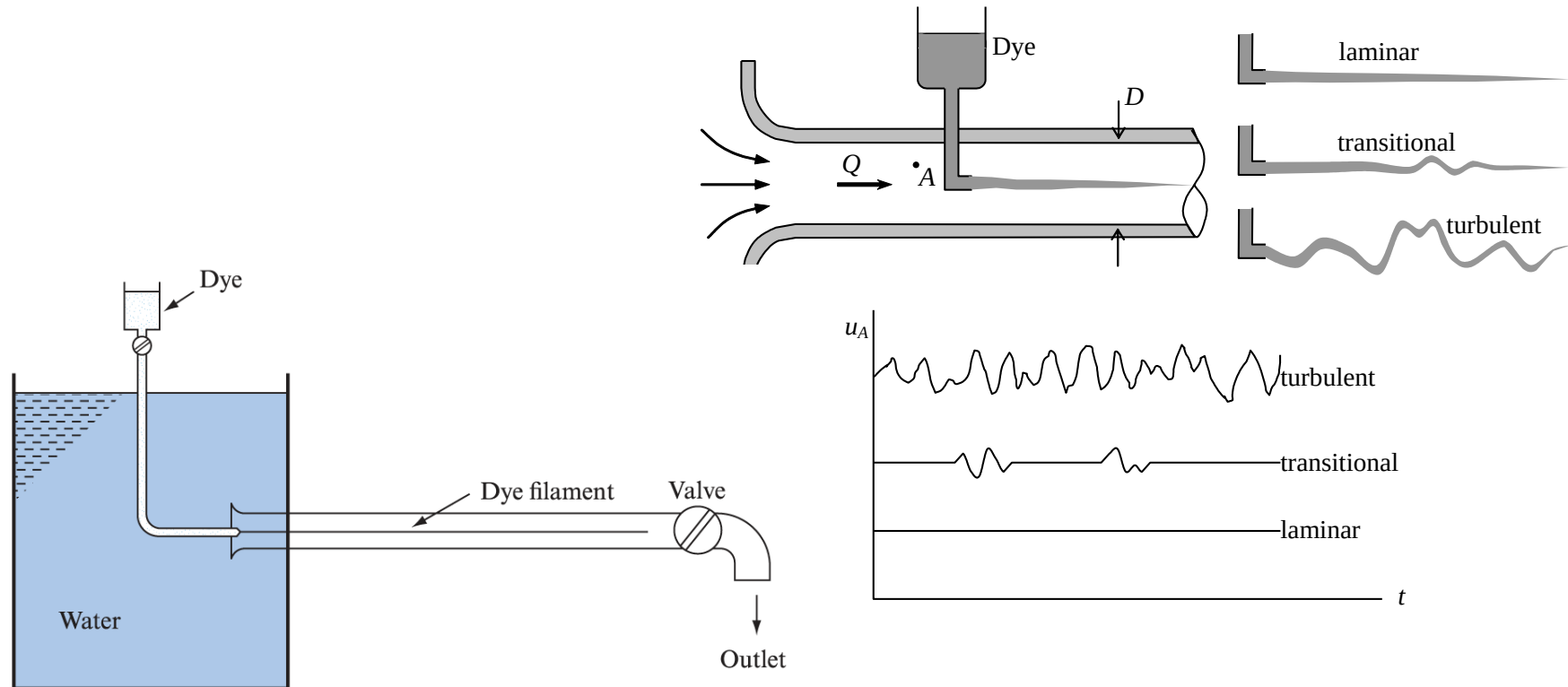


Two-dimensional flow over a weir.

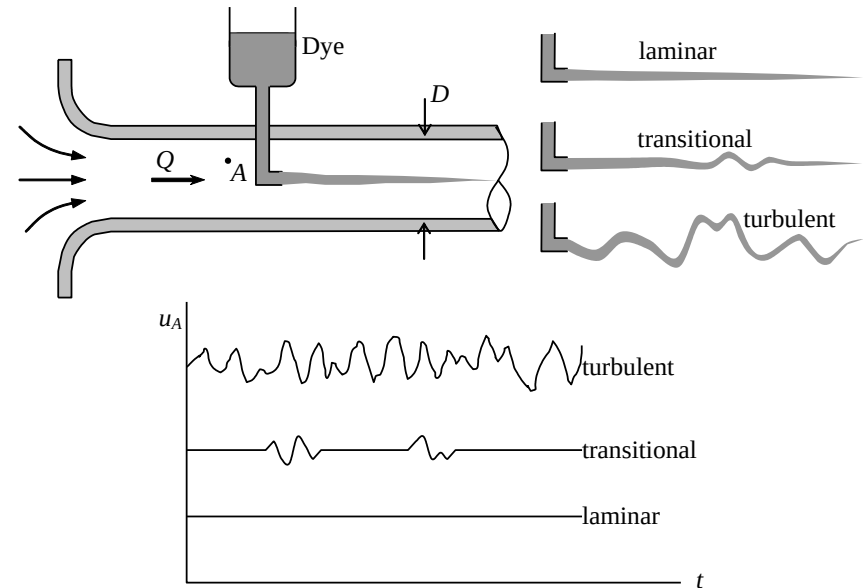
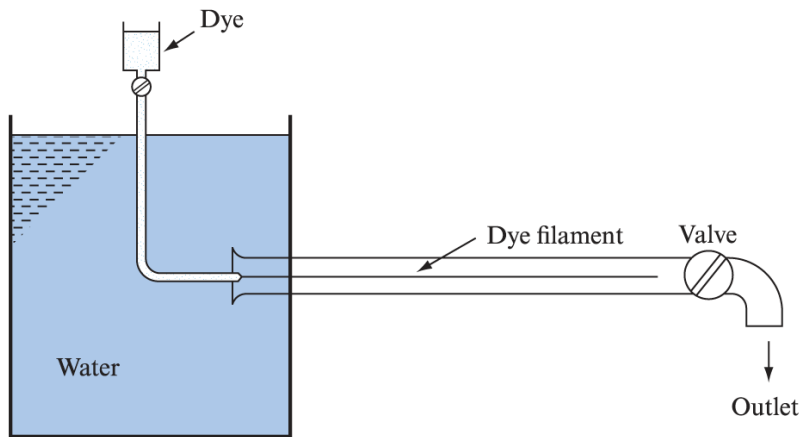
4.10 Laminar and Turbulent Flow:

- Flow in pipes can be divided into two different regimes: *laminar* and *turbulence*.
- The experiment to differentiate between both regimes was introduced in 1883 by **Osborne Reynolds** (1842–1912), an English physicist who is famous in fluid experiments in early days.

The Reynolds' experiment:



Experiment for Differentiating Flow Regime



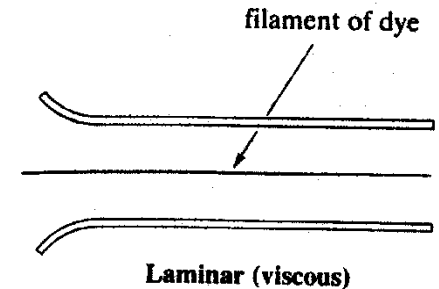
In this experiment:

- a filament of **dye was injected** to the flow of water.
- The discharge was carefully controlled, and passed through a **glass tube** so that observations could be made.
- Reynolds discovered that the **dye filament would flow smoothly** along the tube as long as the discharge is low.
- By gradually increased the discharge, a point is reached where the **filament became wavy**.
- A small further increase in discharge will cause vigorous eddy motion, and the **dye mixed completely with water**.

THREE distinct patterns of flow were recognized:

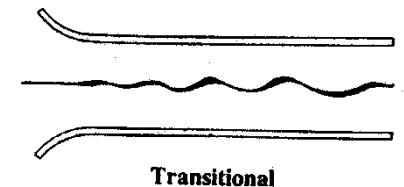
Viscous or Laminar:

in which the fluid particles appear to move in definite smooth parallel path with no mixing.



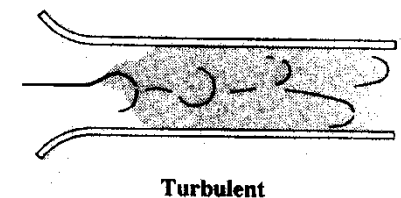
Transitional:

in which some unsteadiness becomes apparent (the wavy filament).



Turbulent:

in which the flow incorporates an eddying or mixing action. The motion of a fluid particle within a turbulent flow is complex and irregular, involving fluctuations in velocity and directions.



- **Reynolds experiment also revealed that** the initiation of turbulence was a function of **fluid velocity, viscosity, and a typical dimension**.
- This led to the formation of the dimensionless Reynolds Number (Re).

$$Re = \frac{\text{inertia forces}}{\text{viscous forces}} = \frac{\rho V D}{\mu} = \frac{V D}{\nu}$$

where

V = mean velocity

μ = dynamic viscosity

ρ = density

D = pipe diameter

ν = kinematic viscosity

Re , is a non-dimensional number (**no units**)

Laminar flow: $Re < 2000$

Transitional flow: $2000 < Re < 4000$

Turbulent flow: $Re > 4000$

Physical meaning of Reynolds Number:

$$\text{Re} = \frac{\text{inertia forces}}{\text{viscous forces}} = \frac{\rho V D}{\mu}$$

It can be interpreted that:

- When the inertial forces dominate over the viscous forces (when the fluid is flowing faster and Re is larger) then the flow is turbulent.
- When the viscous forces are dominant (slow flow, low Re) they are sufficient enough to keep all the fluid particles in line, then the flow is laminar.

Summary

Laminar flow	Transitional flow	Turbulent flow
• $Re < 2000$; • 'low' velocity; • Dye does not mix with water; • Fluid particles move in straight lines; • Simple mathematical analysis possible; • Rare in practice in water systems.	• $2000 > Re < 4000$; • 'medium' velocity; • Dye stream wavers in water - mixes slightly.	• $Re > 4000$; • 'high' velocity; • Dye mixes rapidly and completely; • Particle paths completely irregular; • Average motion is in the direction of the flow; • Changes/fluctuations are very difficult to detect; • Mathematical analysis very difficult - so experimental measures are used; and • Most common type of flow.

Example:

If the pipe and the fluid have the following properties:

- water density $\rho = 1000 \text{ kg/m}^3$
- pipe diameter $d = 0.5\text{m}$
- (dynamic) viscosity, $\mu = 0.55 \times 10^{-3} \text{ Ns/m}^2$
- We want to know the maximum velocity when the Re is 2000.

$$Re = \frac{\rho V D}{\mu} \qquad 2000 = \frac{1000 V 0.5}{0.55 \times 10^{-3}}$$

$$V = 0.0022 \text{ m/s}$$

- **In practice:** it very rarely occurs in a piped water system - the velocities of flow are much greater.
- Laminar flow occurs in situations with fluids of greater viscosity - e.g. in bearing with oil as the lubricant.

Example:

- Oil of viscosity 0.05 kg/m.s and density 860 kg/m³ flows in a 0.1 m diameter pipe with a velocity of 0.6 m/s. Determine the type of flows.

Solution:

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{860 \times 0.6 \times 0.1}{0.05} = 1032$$

$$\text{Re} = 1032 < 2000 \quad \therefore \quad \text{laminar flow}$$

4.11 Discharge and Mean Velocity:

Discharge:

- The total quantity of fluid flowing in unit time past any cross section of a stream is called the *discharge* or *flow* at that section.

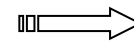
It can be measured either:

- in terms of mass (mass rate of flow, $\dot{m}$)
- or in terms of volume (volume rate of flow, Q)

1. Mass flow rate:

- It is a method to measure the rate at which water is flowing along a pipe.
- It is the mass of fluid flowing per unit time.

$$\dot{m} = \frac{dm}{dt} = \frac{\text{mass of fluid}}{\text{time taken to collect the fluid}}$$



$$\text{Time} = \frac{\text{mass of fluid}}{\text{mass flow rate}}$$

Example:

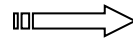
An empty bucket weighs 2.0kg. After 7 seconds of collecting water the bucket weighs 8.0kg, then:

$$\dot{m} = \frac{dm}{dt} = \frac{\text{mass of fluid}}{\text{time taken to collect the fluid}} = \frac{8-2}{7} = 0.857 \text{ kg/s}$$

2. Volume flow rate - Discharge :

- It is another method to measure the rate at which water is flowing along a pipe. It is **more commonly** use
- The discharge is the volume of fluid flowing per unit time.

$$Q = \frac{\text{volume of fluid}}{\text{time}}$$



$$\text{Time} = \frac{\text{volume of fluid}}{\text{discharge}}$$

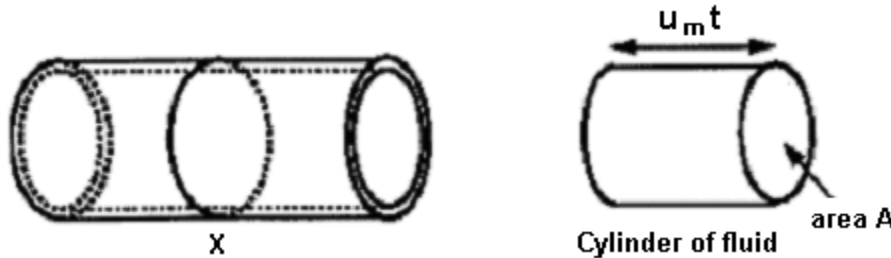
- Also Note that: $\dot{m} = \rho Q$

Example:

If the density of the fluid in the above example is 850 kg/m^3 , then:

$$Q = \frac{\dot{m}}{\rho} = \frac{0.857}{850} = 1.008 \times 10^{-3} \text{ m}^3/\text{s} = 1.008 \text{ l/s}$$

3. Discharge and Mean Velocity



Discharge in pipe

- If the area of cross section of the pipe at point **X** is A , and the mean velocity here is u_m , during a time t , a cylinder of fluid will pass point **X** with a volume $A \times u_m \times t$. The volume per unit time (the discharge) will thus be :

$$Q = \frac{\text{volume}}{\text{time}} = \frac{A \times u_m \times t}{t} \implies Q = A \times u_m$$

$$\text{or: } u_m = \frac{Q}{A}$$

$$\text{Let: } u_m = V \implies V = \frac{Q}{A}$$

This idea, that mean velocity multiplied by the area gives the discharge, applies to all situations - not just pipe flow.

Example:

If the cross-section area, A , is $1.2 \times 10^{-3} \text{ m}^2$ and the discharge, Q is 24 l/s , then the mean velocity, u_m , of the fluid is:

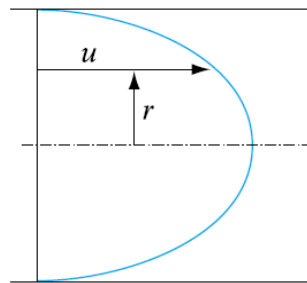
$$u_m = V = \frac{Q}{A} = \frac{24 \times 10^{-3}}{1.2 \times 10^{-3}} = 2 \text{ m/s}$$

- Note how carefully we have called this the *mean* velocity. This is because the velocity in the pipe is not constant across the cross section.
- Crossing the centre line of the pipe, the velocity is zero at the walls, increasing to a maximum at the centre then decreasing symmetrically to the other wall.
- If u is the velocity at any radius r , the flow dQ through an annular element of radius r and thickness dr will be:

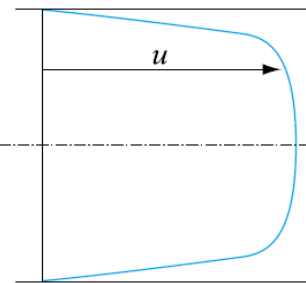
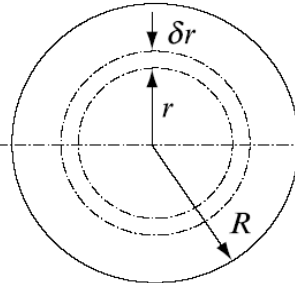
$$dQ = \text{Area of element} \times \text{Velocity}$$

$$= 2\pi r dr \times u$$

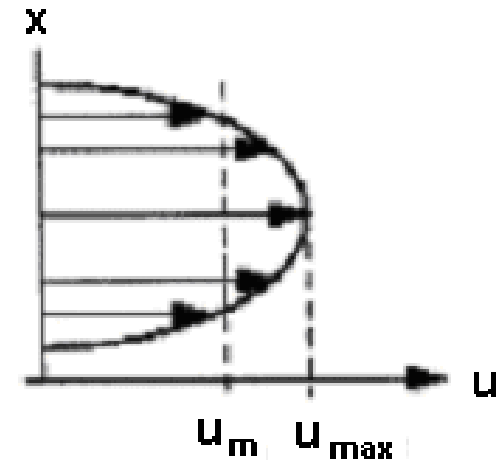
$$Q = 2\pi \int_0^R u r dr$$



(a) Laminar flow



(b) Turbulent flow

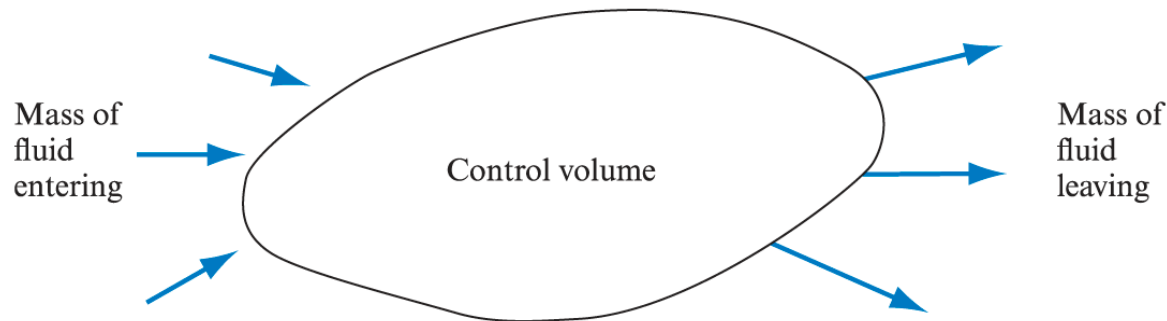


A typical velocity profile across a pipe for laminar flow

- If the relation between u and r can be established, this integral can be evaluated (or can be evaluated numerically)

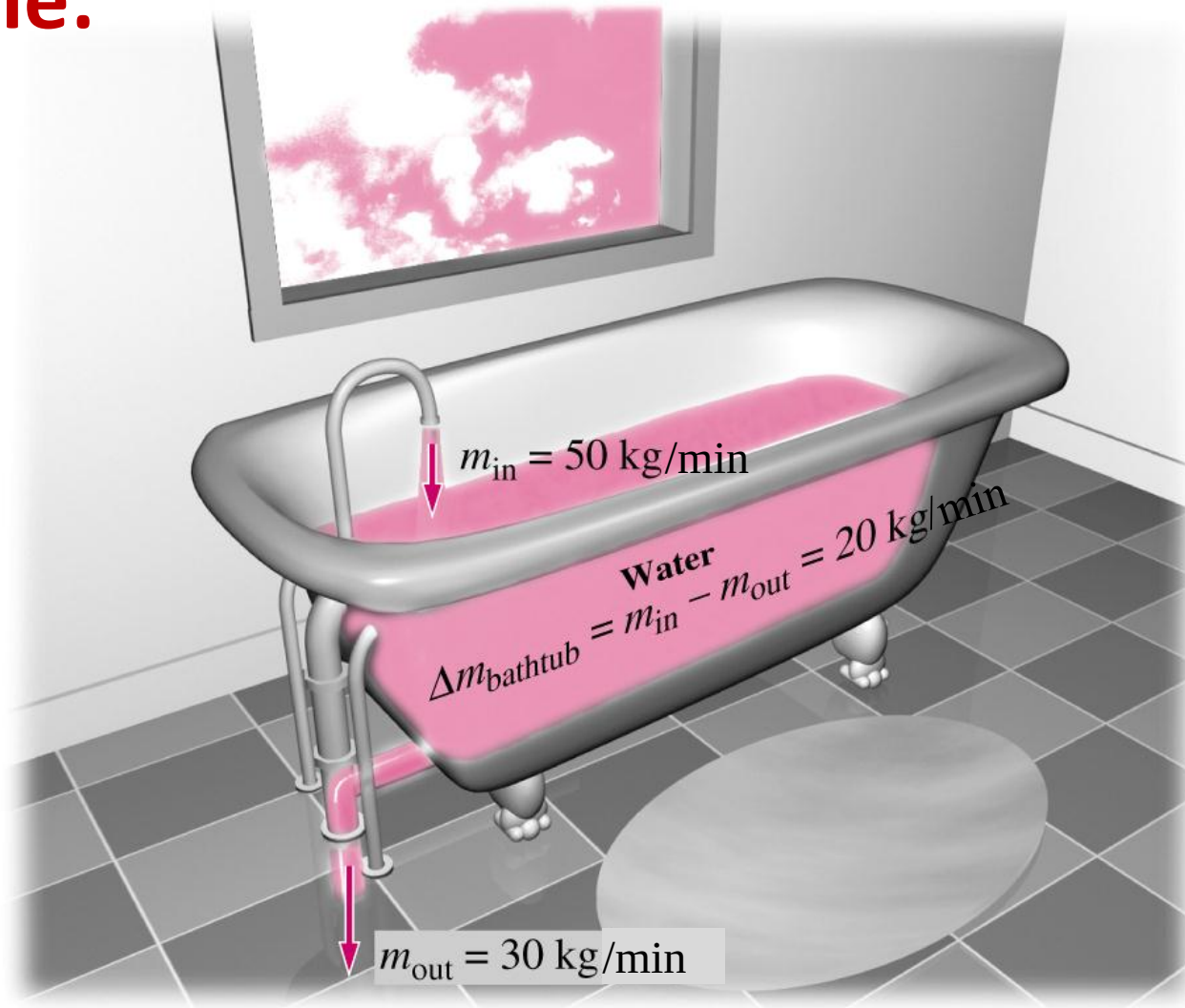
4.12 Continuity in Flow:

- This principle of *conservation of mass* says
Matter cannot be created or destroyed
- We use it in the analysis of flowing fluids.
- The principle is applied to fixed region in the flow, known as ***control volumes*** (or surfaces), like that in the figure Shown:
- And for any control volume:



$$\text{Mass entering per unit time} = \text{Mass leaving per unit time} + \text{Increase of mass in the control volume per unit time}$$

Example:



- **For steady flow** there is no increase in the mass within the control volume, so:

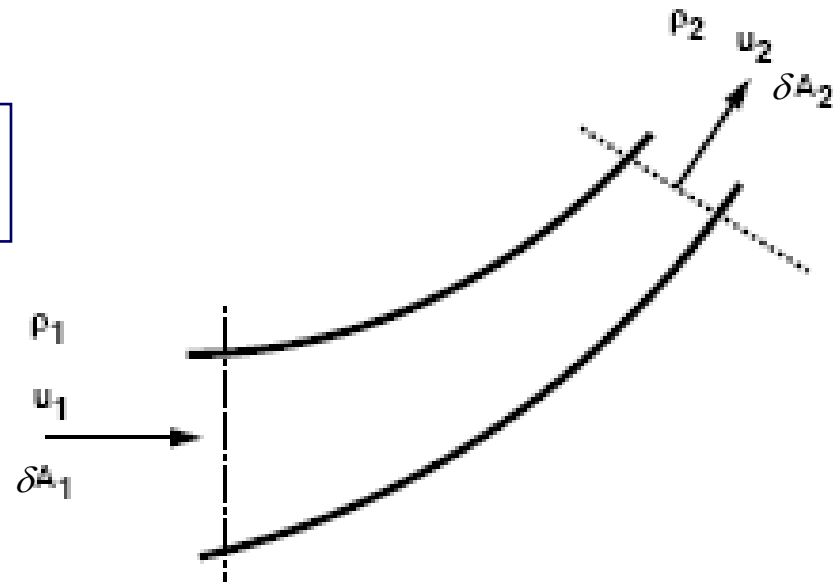
Mass entering per unit time = Mass leaving per unit time

- This can be applied to a streamtube such as that shown below.
- No fluid flows across the boundary made by the streamlines so mass only enters and leaves through the two ends of this streamtube section.

**Mass entering per unit time at end 1 =
Mass leaving per unit time at end 2**

$$\rho_1 \delta A_1 u_1 = \rho_2 \delta A_2 u_2$$

*This is the continuity
equation for the of
compressible fluid*



- The flow of fluid through a real pipe (or any other vessel) will vary due to the presence of a wall - in this case we can use the ***mean velocity*** and write:

$$\rho_1 A_1 \bar{u}_1 = \rho_2 A_2 \bar{u}_2 = \text{constant} = \dot{m}$$

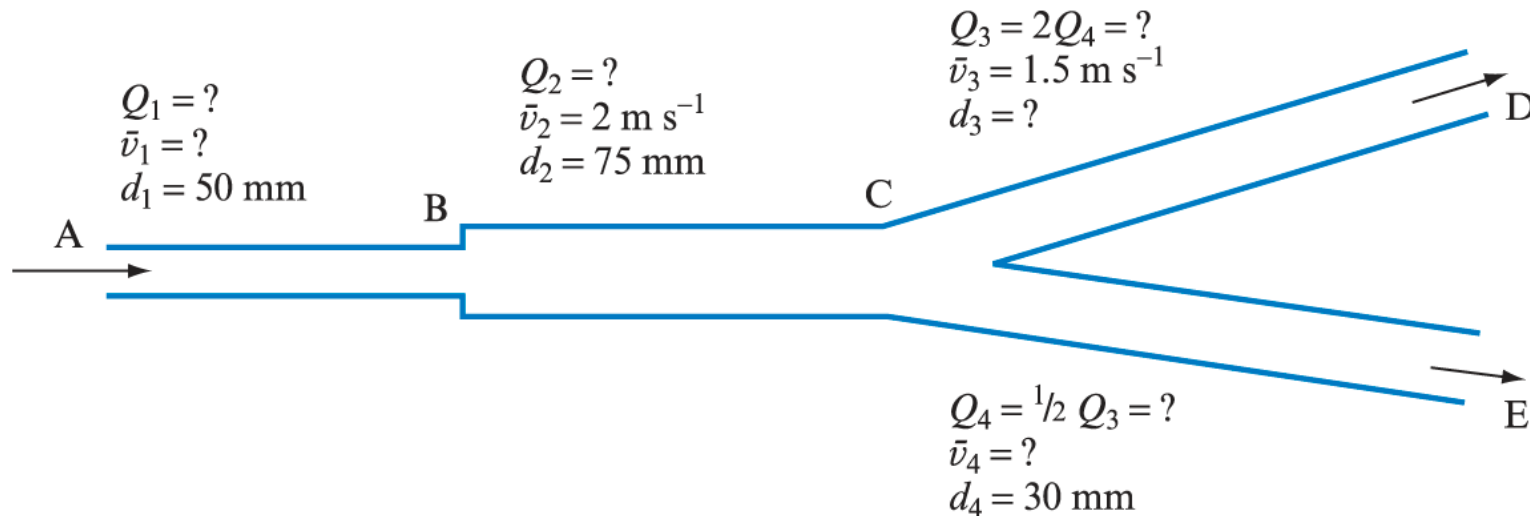
- When the fluid can be considered incompressible, i.e. the density does not change, $\rho_1 = \rho_2 = \rho$ so:

$$A_1 \bar{u}_1 = A_2 \bar{u}_2 = \text{constant} = Q$$

- This is the form of the continuity equation most often used.
- This equation is a very powerful tool in fluid mechanics and will be used **repeatedly** throughout the rest of this course.

Example:(Ex 4.2, page 106 Textbook)

Find the missing values



Example:

