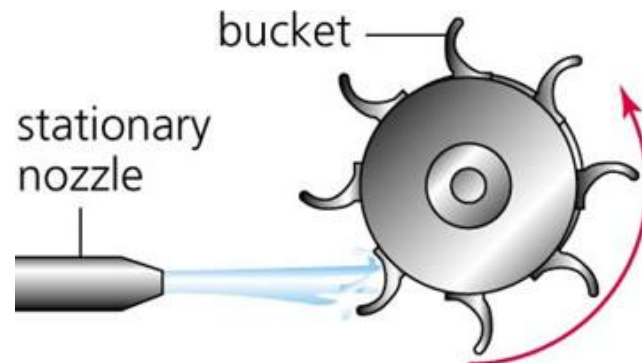


CHAPTER 5

Momentum Equation and its Applications



Dr. Khalil Mahmoud ALASTAL

Objectives of this Chapter:

- Introduce the momentum equation for a fluid.
- Demonstrate how the momentum equation and principle of conservation of momentum is used to predict forces induced by flowing fluids.

5.1 Momentum and Fluid Flow

- We have all seen moving fluids exerting forces:
 - The lift force on an aircraft is exerted by the air moving over the wing.
 - A jet of water from a hose exerts a force on whatever it hits.
- In fluid mechanics the analysis of motion is performed in the same way as in solid mechanics - by use of **Newton's laws of motion**.
- Account is also taken for the special properties of fluids when in motion.
- The momentum equation is a statement of Newton's Second Law and relates **the sum of the forces acting on an element of fluid to its acceleration or rate of change of momentum**.

- From solid mechanics you will recognize $F = ma$
- In fluid mechanics it is not clear what mass of moving fluid, we should use a different form of the equation.

Newton's 2nd Law can be written:

The Rate of change of momentum of a body is equal to the resultant force acting on the body, and takes place in the direction of the force.

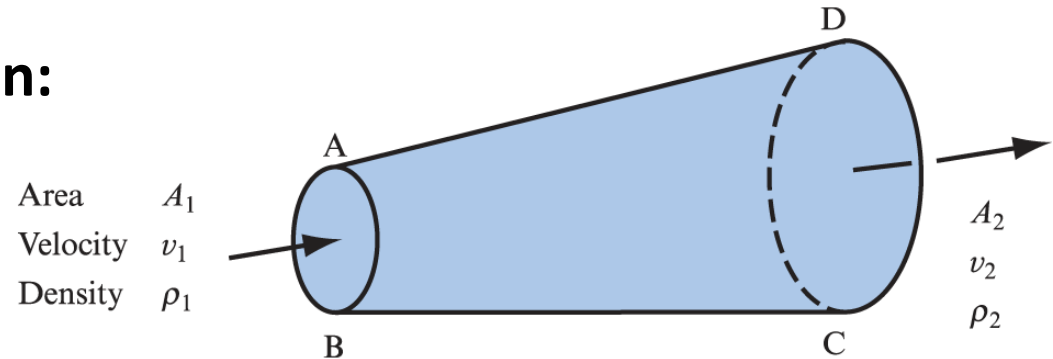
- In mechanics, the momentum of particle or object is defined as:

$$\text{Momentum} = mv$$

- To determine the rate of change of momentum for a fluid we will consider a streamtube (assuming *steady non-uniform* flow).

From continuity equation:

- $\rho_1 A_1 v_1 = \rho_2 A_2 v_2 = \dot{m}$



- The rate at which momentum exits face $CD =$

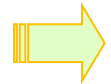
$$\rho_2 A_2 v_2 v_2$$

- The rate at which momentum enters face $AB =$

$$\rho_1 A_1 v_1 v_1$$

- The rate of change of momentum across the control volume

$$\rho_2 A_2 v_2 v_2 - \rho_1 A_1 v_1 v_1 = \dot{m} v_2 - \dot{m} v_1 = \dot{m}(v_2 - v_1)$$



The rate of change of momentum across the control volume:

$$\dot{m}(v_2 - v_1) = \text{Mass flow rate} \times \text{Change of velocity}$$

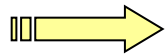
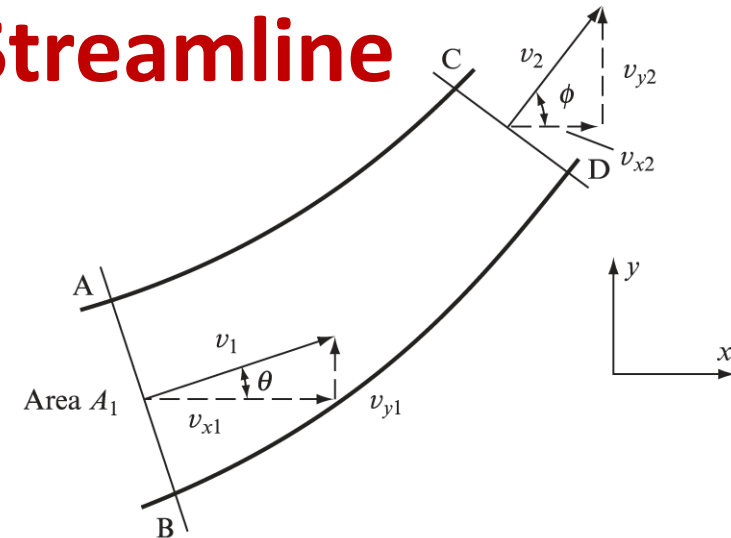
- And according to Newton's second law, this change of momentum per unit time will be caused by a force F , Thus:

$$F = \dot{m}(v_2 - v_1)$$
$$F = \rho Q(v_2 - v_1)$$

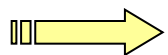
- This is the resultant force **acting on the fluid** in the ***direction of motion***.
- By Newton's third law, the fluid will exert an equal and opposite reaction on its surroundings

5.2 Momentum Equation for Two and Three dimensional Flow along a Streamline

- Both **momentum** and **force** are **vector quantities** → they can be resolving into components in the x and y directions.



$$\begin{aligned}
 F_x &= \text{Rate of change of momentum in } x \text{ direction} \\
 &= \text{Massflow rate} \times \text{Change of velocity in } x \text{ direction} \\
 &= \dot{m}(v_2 \cos \phi - v_1 \cos \theta) = \dot{m}(v_{x2} - v_{x1})
 \end{aligned}$$



$$\begin{aligned}
 F_y &= \text{Rate of change of momentum in } y \text{ direction} \\
 &= \text{Massflow rate} \times \text{Change of velocity in } y \text{ direction} \\
 &= \dot{m}(v_2 \sin \phi - v_1 \sin \theta) = \dot{m}(v_{y2} - v_{y1})
 \end{aligned}$$



$$F_x = \dot{m}(v_{x2} - v_{x1})$$

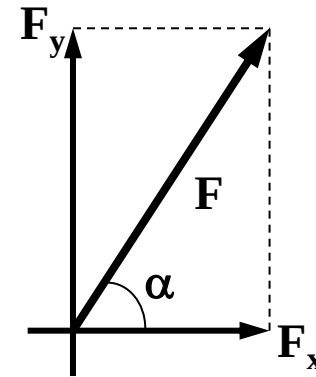
$$F_y = \dot{m}(v_{y2} - v_{y1})$$

- These components can be combined to give the resultant force:

$$F = \sqrt{F_x^2 + F_y^2}$$

- And the **angle** of this force:

$$\alpha = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$



- For a **three-dimensional** (x, y, z) system we then have an extra force to calculate and resolve in the z direction.

In summary :

The total force **exerted on** the fluid in a control volume in a given direction = Rate of change of momentum in the given direction of fluid passing through the control volume

$$F = \dot{m}(v_{out} - v_{in})$$
$$F = \rho Q(v_{out} - v_{in})$$

Note:

- The value of F is positive in the direction in which v is assumed to be positive

This force is made up of three components:

- $F_1 = F_R$ = Force exerted in the given direction on the fluid by any ***solid body*** touching the control volume
- $F_2 = F_B$ = Force exerted in the given direction on the fluid by ***body force*** (e.g. gravity)
- $F_3 = F_P$ = Force exerted in the given direction on the fluid by ***fluid pressure*** outside the control volume

So we say that the total force, F_T , is given by the sum of these forces:

$$F_T = F_R + F_B + F_P = \dot{m}(v_{out} - v_{in})$$

- The force **exerted by** the fluid **on** the solid body touching the control volume is equal and opposite to F_R . So the reaction force, R , is given by:

$$R = -F_R$$

Application of the Momentum Equation

We will consider the following examples:

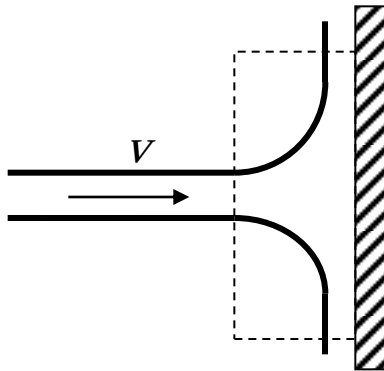
- Impact of a jet on a plane surface
- Force due to flow round a curved vane
- Force due to the flow of fluid round a pipe bend.
- Reaction of a jet.

Step in Analysis:

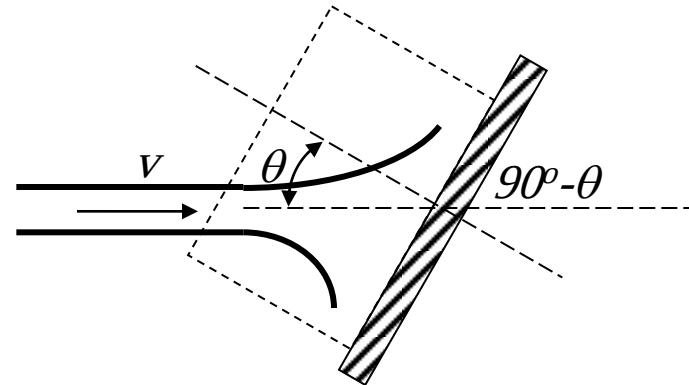
1. Draw a control volume
2. Decide on co-ordinate axis system
3. Calculate the total force
4. Calculate the pressure force
5. Calculate the body force
6. Calculate the resultant force

5.5 Force Exerted by a jet striking a Flat Plate

- Consider a jet striking a flat plate that may be **perpendicular** or **inclined** to the direction of the jet.
- This plate may be **moving** in the initial direction of the jet.



Plate, normal to jet



Plate, inclined to jet, angle $(90^\circ - \theta)$

- It is helpful to consider components of the velocity and force vectors **perpendicular and parallel** to the surface of the plate.

- The general term of the jet velocity component normal to the plate can be written as:

$$v_{normal} = (v - u) \cos \theta$$

- The mass flow entering the control volume

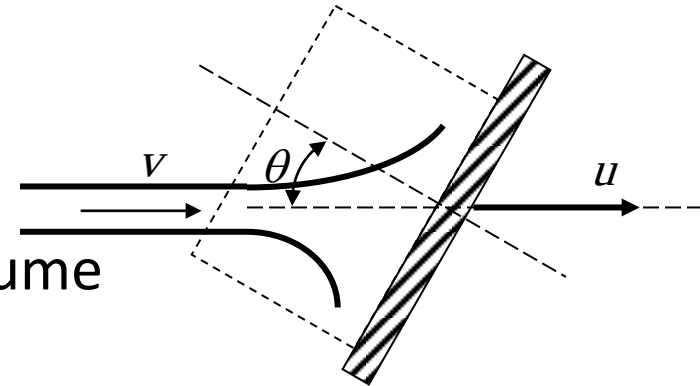
$$\dot{m} = \rho A(v - u)$$

- If the plate is stationary:

$$\dot{m} = \rho A v$$

- Thus the rate of change of momentum normal to the plate:

$$\text{Rate of change of momentum} = \rho A(v - u)(v - u) \cos \theta$$



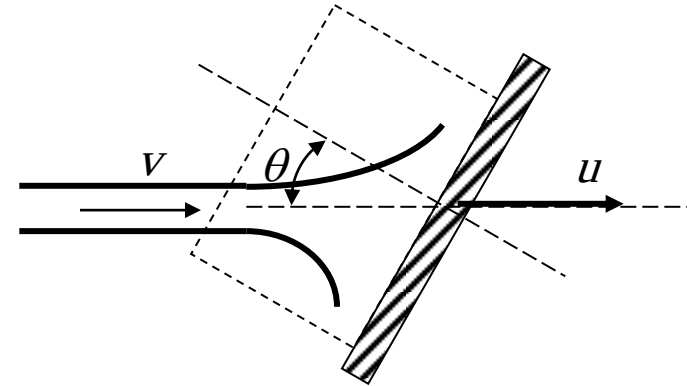
$$= \rho A v^2 \cos \theta$$

if the plate is stationary and inclined

$$= \rho A v^2$$

if the plate is both stationary and perpendicular

- Force exerted normal to the plate = The rate of change of momentum normal to the plate:



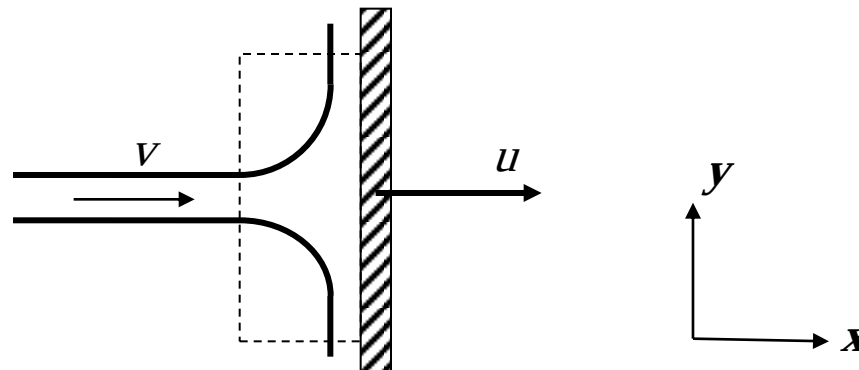
$$\text{Force exerted normal to the plate} = \rho A(v - u)(v - u) \cos \theta$$

- There will be an equal and opposite reaction force exerted on the jet by the plate.
-
- In the **direction parallel to the plate**, the force exerted will depend upon the shear stress between the fluid and the surface of the plate.
 - For ideal fluid there would be no shear stress and hence no force parallel to the plate

Example:

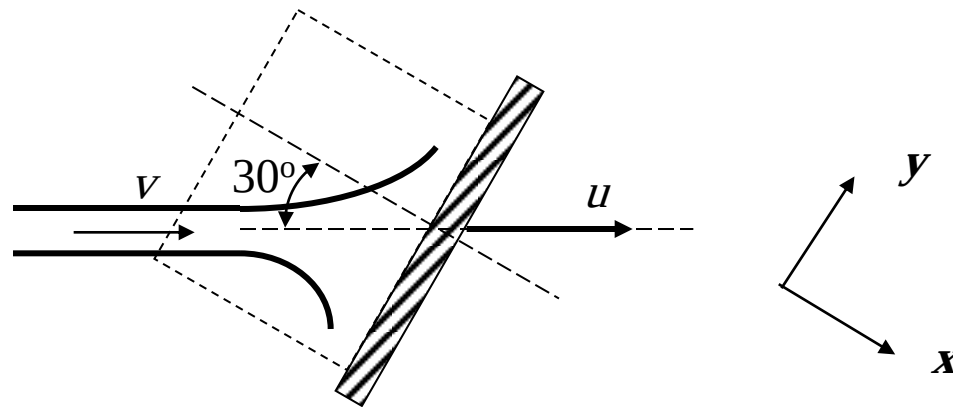
A flat plate is struck normally by a jet of water 50 mm in diameter with a velocity of 18 m/s. **Calculate:**

1. The force on the plate when it is stationary.
2. The force on the plate when it moves in the same direction as the jet with a velocity of 6m/s



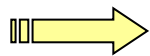
Example:

- A jet of water from a fixed nozzle has a diameter d of 25mm and strikes a flat plate at angle θ of 30° to the normal to the plate. The velocity of the jet v is 5m/s, and the surface of the plate can be assumed to be frictionless.
- Calculate the force exerted normal to the plate** (a) if the plate is stationary, (b) if the plate is moving with velocity u of 2m/s in the same direction as the jet.



- For each case the **control volume is fixed relative to the plate**.
- Since we wish to find the force exerted normal to the plate, the x direction is chosen perpendicular to the surface of the plate.
- Force exerted by fluid on the plate in x direction.

$$F_T = F_R + F_B + F_P = \dot{m}(v_{out} - v_{in})_x$$



$$R = -F_R = F_B + F_P - \dot{m}(v_{out} - v_{in})_x$$

- The **gravity force** (body force) F_B is negligible and if the fluid in the jet is assumed to be **atmospheric pressure** throughout, F_P is zero. **Thus:**

$$R = -F_R = -\dot{m}(v_{out} - v_{in})_x = \dot{m}(v_{in} - v_{out})_x$$

- Where $\dot{m}$ is the mass per unit time of the fluid **entering the C.V.**
- v_{out} and v_{in} are measured **relative** to the control volume, which is fixed relative to the plate.

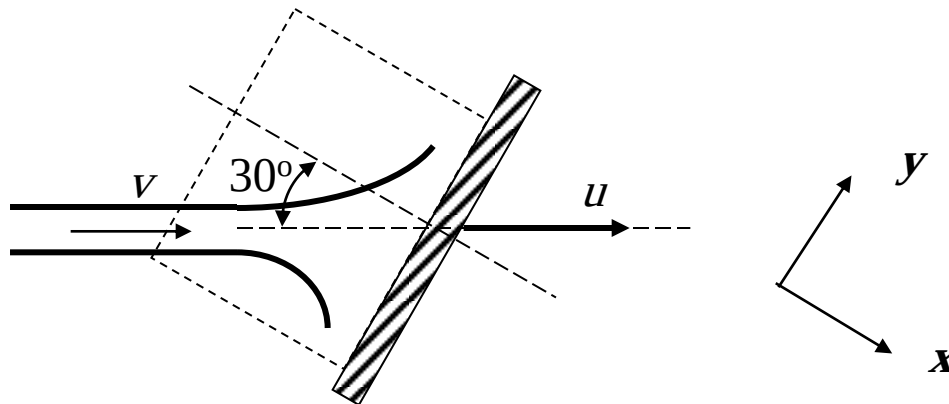
- (a) if the plate is stationary:

Mass per unit time of the fluid entering the control volume = mass per unit time leaving the nozzle = ρAv

- Initial component of velocity relative to plate in x direction = $v \cos \theta$
- Final component of velocity relative to plate in x direction = 0

⇒ $R_x = \dot{m}(v_{in} - v_{out})_x = \rho Av(v \cos \theta) = \rho Av^2 \cos \theta$

$$R_x = 1000 \times \left(\frac{\pi}{4} 0.025^2 \right) \times 5^2 \times \cos 30 = 10.63 N$$



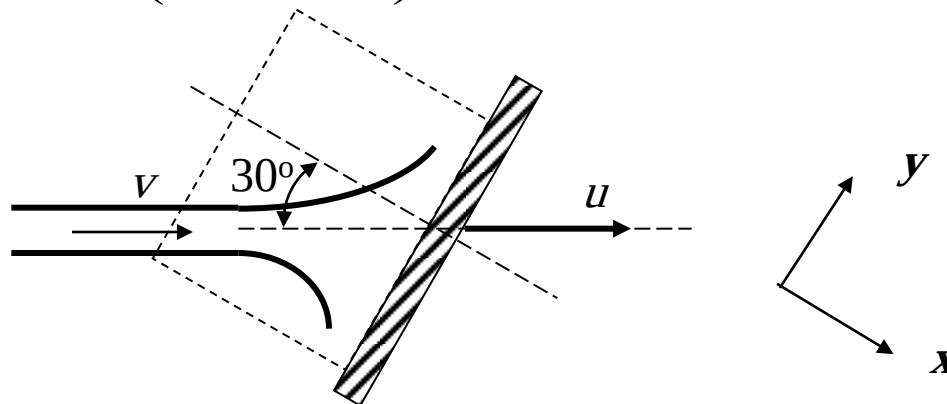
- (b) if the plate move:

$$\begin{aligned} \text{Mass per unit time of the fluid entering the control volume} &= \text{mass per unit time leaving the nozzle} - \text{mass per unit time required to extend jet} \\ &= \rho Av - \rho Au = \rho A(v - u) \end{aligned}$$

- Initial component of velocity relative to plate in x direction = $(v - u) \cos \theta$
- Final component of velocity relative to plate in x direction = 0

$$\Rightarrow R_x = \dot{m}(v_{in} - v_{out})_x = \rho A(v - u)(v - u) \cos \theta = \rho A(v - u)^2 \cos \theta$$

$$R_x = 1000 \times \left(\frac{\pi}{4} 0.025^2 \right) \times (5 - 2)^2 \times \cos 30 = 3.83 \text{ N}$$



Example: (Ex 5.2, page 121 Textbook)

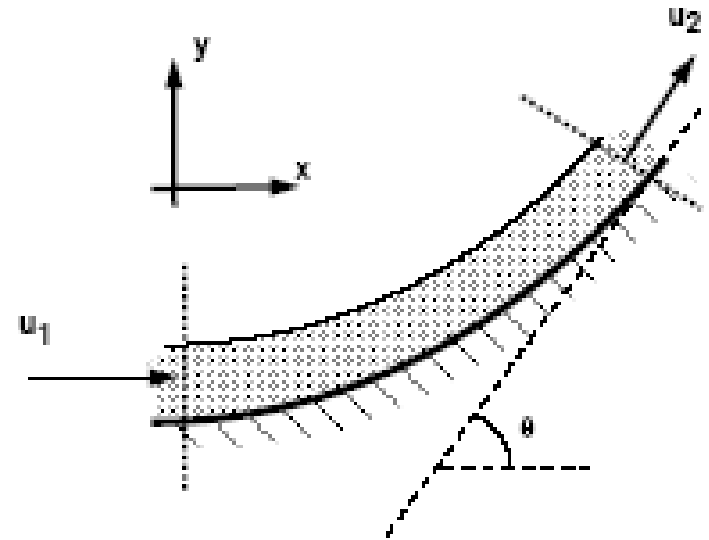
HW

A jet of water from a fixed nozzle has a diameter of 25 mm and strikes a flat plate inclined to the jet direction. The velocity of the jet is 5 m/s, and the surface of the plate may be assumed frictionless.

- (a) Indicate in tabular form the reduction in the force normal to the plate surface as the inclination of the plate to the jet varies from 90° to 0° .
- (b) Indicate in tabular form the force normal to the plate surface as the plate velocity changes from 2 m/s to -2 m/s in the direction of the jet, given that the plate is itself perpendicular to the approaching jet.

5.6 Force Due to Deflection of a jet by a Curved Vane

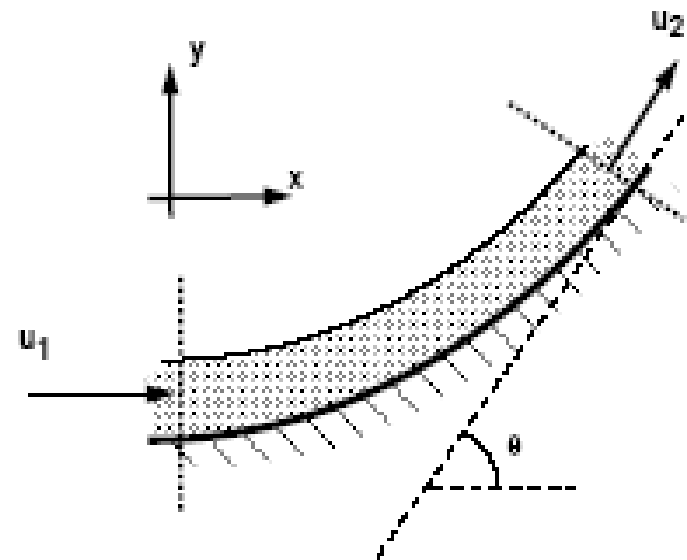
- Both velocity and momentum are **vector quantities**.
- Even if the **magnitude** of the velocity remains unchanged, a change in **direction** of a stream of fluid will give rise to a change of momentum.
- If the stream is deflected by a curved vane (entering and leaving tangentially without impact). A force will be exerted between the fluid and the surface of the vane to cause this change in momentum.



- It is usually convenient to calculate the components of this force **parallel and perpendicular** to the direction of the incoming stream.
- The resultant can be **combined** to give the magnitude of the resultant force which the vane exerts on the fluid, and **equal and opposite** reaction of the fluid on the vane.

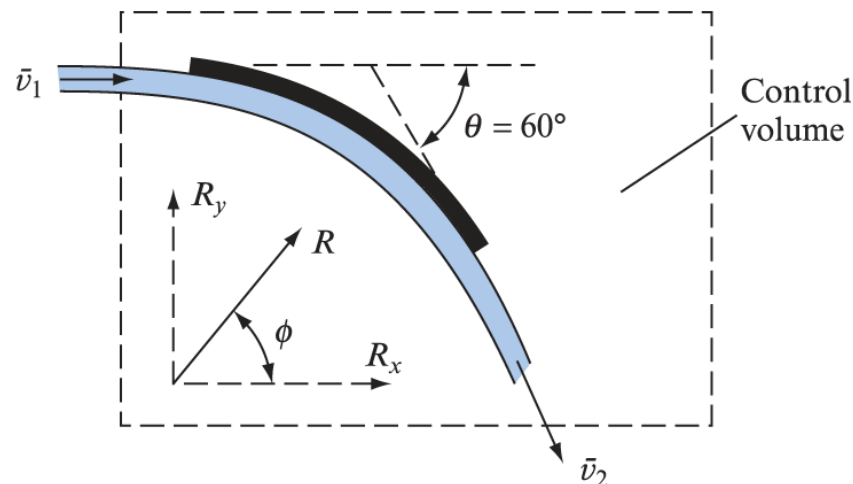
Note:

- The **pressure force** is zero as the pressure at both the inlet and the outlets are atmospheric.
- No **body forces** in the x-direction
- In the y-direction the body force acting is the weight of the fluid. **This is often small as the jet volume is small and sometimes ignored in analysis.**



Example: (Ex 5.3, page 123 Textbook)

A jet of water from a nozzle is deflected through an angle $\theta = 60^\circ$ from its original direction by a curved vane which enters tangentially without shock with mean velocity of 30 m/s and leaves with mean velocity of 25 m/s. If the discharge from the nozzle is 0.8 kg/s.



Calculate the magnitude and direction of the resultant force on the vane if the vane is stationary

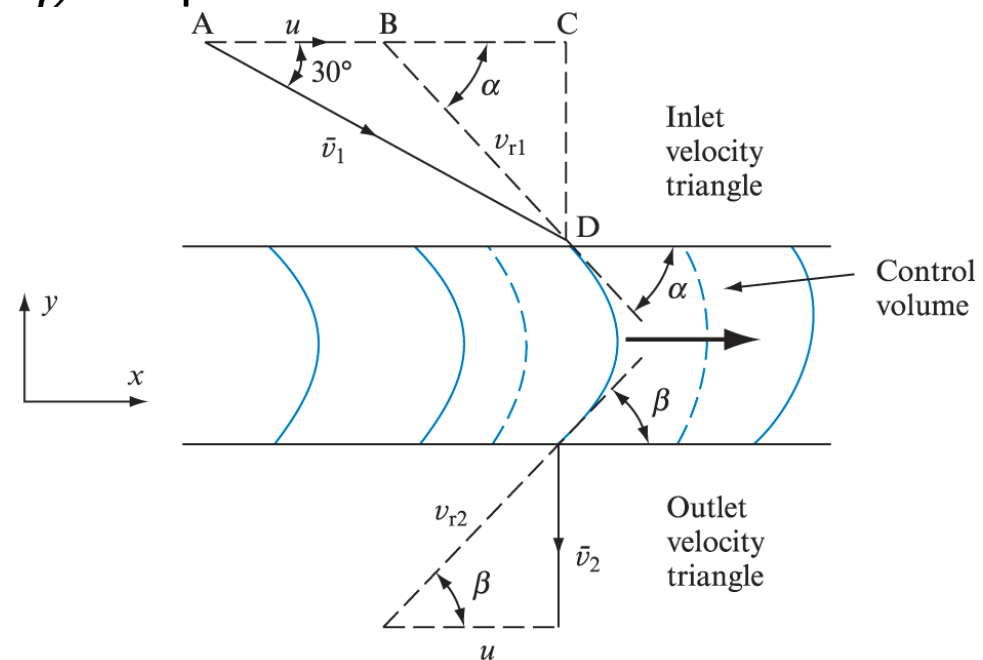
5.7 Force Exerted When a Jet is Deflected by a Moving Curved Vane

- If a jet of fluid is to be deflected by a moving curved vane without impact at the inlet to the vane, **the relation between the direction of the jet and the tangent to the curve of the vane at inlet** must be such that **the relative velocity of the fluid at inlet is tangential to the vane.**

Example: (Ex 5.4, page 125 Textbook)

A jet of water 100 mm in diameter leaves a nozzle with a mean velocity v_1 of 36 m/s and is deflected by a series of vanes moving with a velocity u of 15 m/s in a direction at 30° to the direction of the jet, so that it leaves the vane with an absolute mean velocity v_2 which is at right angles to the direction of motion of the vane. Owing to friction, the velocity of the fluid relative to the vane at outlet v_{r2} is equal to 0.85 of the relative velocity v_{r1} at inlet. Calculate:

- the inlet angle α and outlet angle β of the vane which will permit the fluid to enter and leave the moving vane tangentially without shock.
- the force exerted on the series of vanes in the direction of motion u .



Solution:

If the absolute velocity $\bar{v}_2$ is to be at right angles to the direction of motion, the vane must turn the fluid so that it leaves with a relative velocity $\bar{v}_{r2}$, which has a component velocity equal and opposite to u as shown in the outlet velocity triangle (Fig. 5.6).

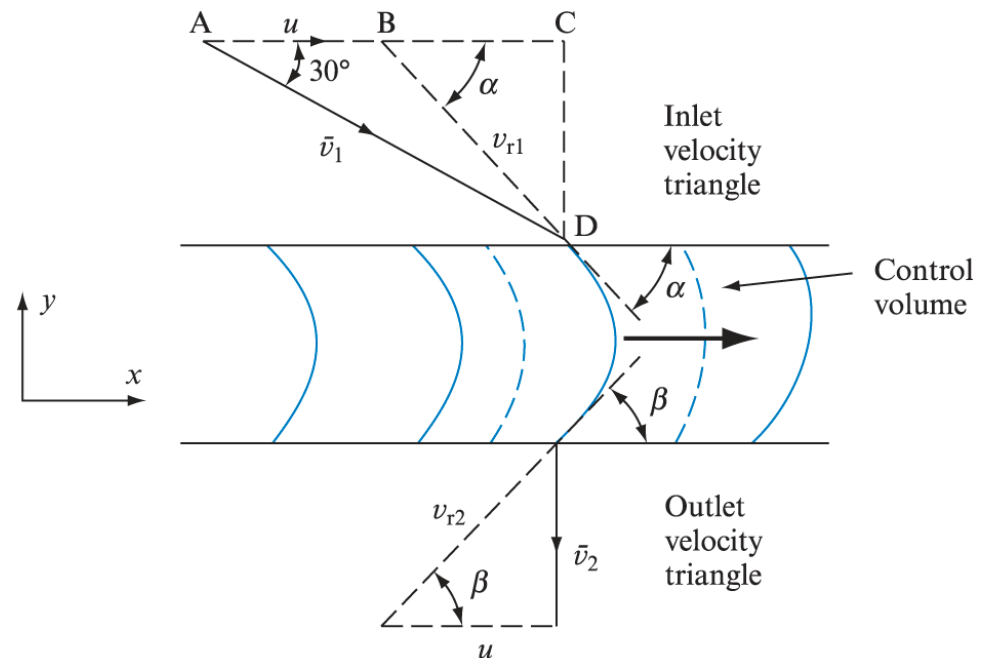
(a) To determine the inlet angle α , consider the inlet velocity triangle. The velocity of the fluid relative to the vane at inlet, $\bar{v}_{r1}$, must be tangential to the vane and make an angle α with the direction of motion,

$$\tan \alpha = CD/BC = \bar{v}_1 \sin 30^\circ / (\bar{v}_1 \cos 30^\circ - u).$$

Putting $\bar{v}_1 = 36 \text{ m s}^{-1}$ and $u = 15 \text{ m s}^{-1}$,

$$\tan \alpha = 36 \times 0.5 / (36 \times 0.866 - 15) = 1.113,$$

$$\alpha = 48^\circ 3'.$$



Solution:

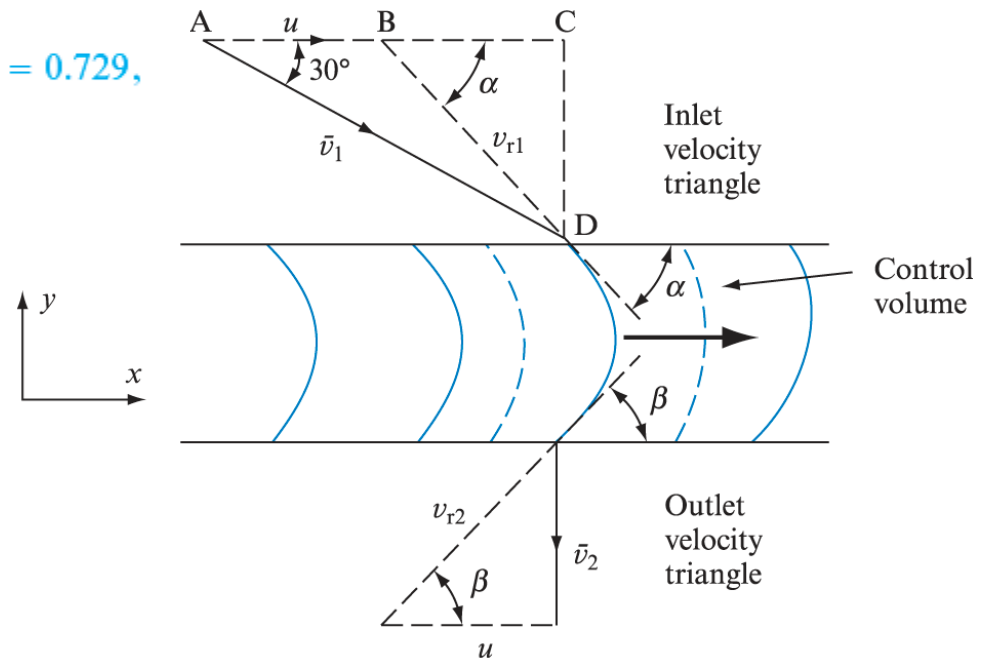
To determine the outlet angle β , if $\bar{v}_2$ has no component in the direction of motion, the outlet velocity triangle is right angled, $\cos \beta = u/\bar{v}_{r2}$, but $\bar{v}_{r2} = 0.85\bar{v}_{r1}$ and, from the inlet triangle,

$$\bar{v}_{r1} = CD/\sin \alpha = \bar{v}_1 \sin 30^\circ / \sin \alpha.$$

Therefore

$$\cos \beta = \frac{u \sin \alpha}{0.85 \bar{v}_1 \sin 30^\circ} = \frac{15 \times 0.744}{0.85 \times 36 \times 0.5} = 0.729,$$

$$\beta = 43^\circ 11'.$$



Solution:

(b) Since the jet strikes a series of vanes, perhaps mounted on the periphery of a wheel, so that as each vane moves on its place is taken by the next in the series, the average length of the jet does not alter and the whole flow from the nozzle of diameter d is deflected by the vanes.

Neglecting the force due to gravity and assuming a free jet that does not fill the space between the vanes completely, so that the pressure is constant everywhere, the component forces in the x and y directions (Fig. 5.6) can be found from equation (5.5) putting $R = -F_1$ and $F_2 = F_3 = 0$. In the direction of motion, which is the x direction,

$$R_x = \dot{m}(v_{in} - v_{out})_x$$

$$\text{Mass per unit time entering control volume} = \dot{m} = \text{Mass per unit time leaving nozzle}$$

$$= \rho(\pi/4)d^2\bar{v}_1,$$

$$v_{in} = \text{Component of } \bar{v}_1 \text{ in } x \text{ direction} = \bar{v}_1 \cos 30^\circ,$$

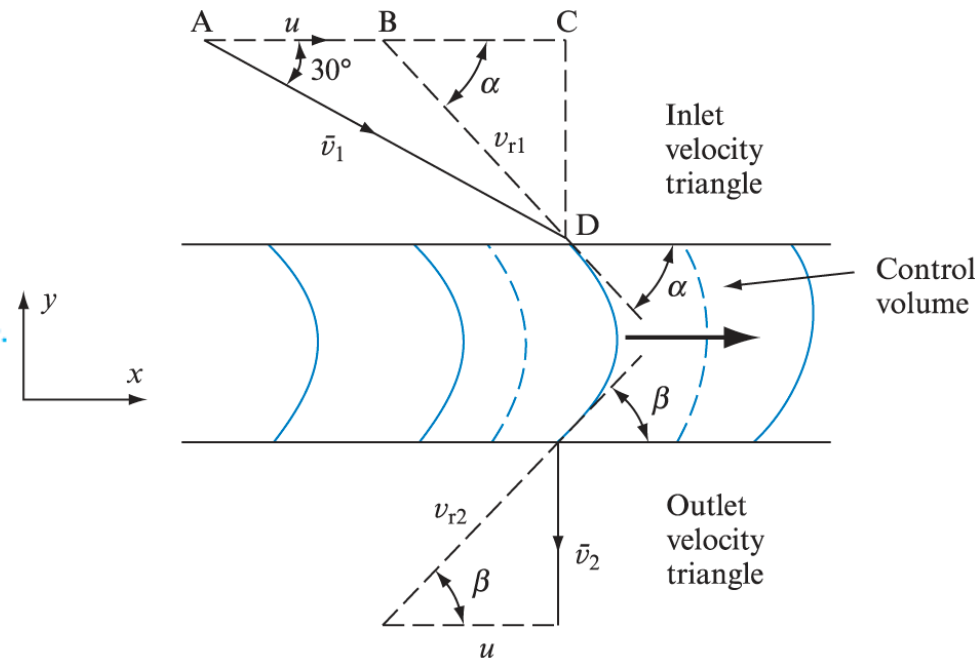
$$v_{out} = \text{Component of } \bar{v}_2 \text{ in } x \text{ direction} = \bar{v}_2 \cos 90^\circ = 0.$$

Substituting in (I),

$$\text{Force on vanes in direction of motion} = R_x = \rho(\pi/4)d^2\bar{v}_1 \times \bar{v}_1 \cos 30^\circ.$$

Putting in the numerical values,

$$\text{Force on vanes in direction of motion} = 1000 \times (\pi/4)(0.1)^2 \times 36 \times 36 \times 0.866 \text{ N} = \mathbf{8816 \text{ N}}.$$



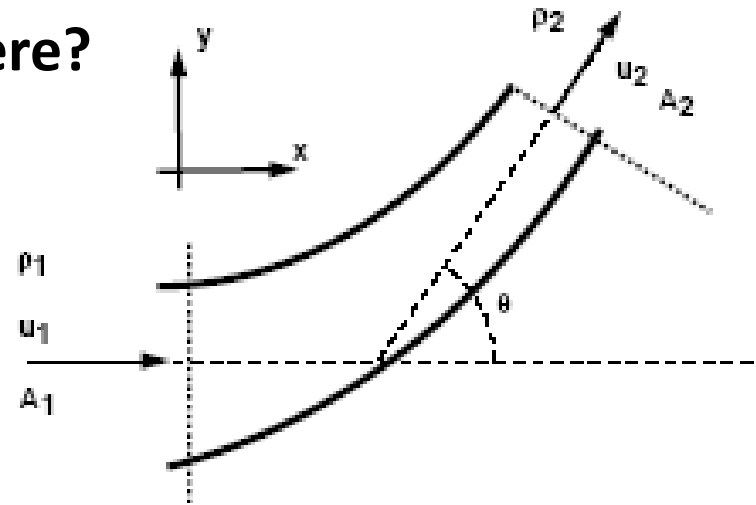
5.8 Force Exerted on a Pipe Bends and Closed Conduits

A force will act on the bend due to:

- The fluid changes its direction
- If the pipe tapers, there is a change in velocity magnitude.
- Do not forget pressure forces.

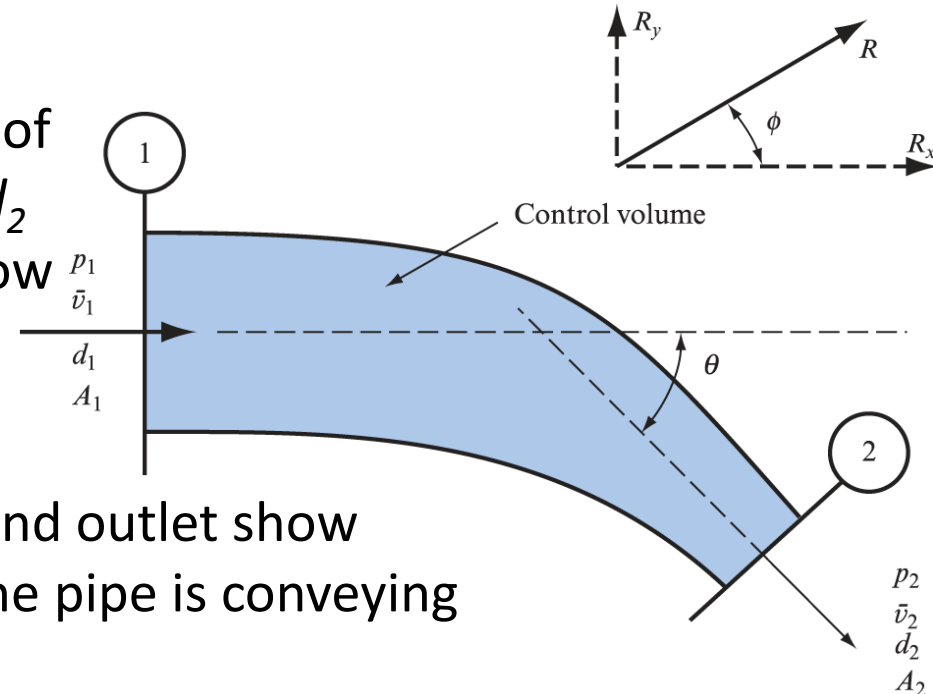
Why do we want to know the forces here?

- This force can be very large in the case of water supply pipes.
- If the bend is not fixed it will move and eventually break at the joints.
- We need to know how much force a support (thrust block) must withstand.



Example: (Ex 5.5, page 127 Textbook)

A pipe bend tapers from a diameter of d_1 of 500mm at inlet to a diameter d_2 of 250mm at outlet and turns the flow through an angle θ of 45° .



Measurements of pressure at inlet and outlet show that $p_1 = 40$ kPa and $p_2 = 23$ kPa. If the pipe is conveying oil ($\rho = 850$ kg/m³).

Calculate the magnitude and direction of the resultant force on the bend when the oil is flowing at the rate of 0.45m³/s.

Note: The bend is in horizontal plane

5.9 Reaction of a Jet

- Whenever the momentum of a stream of fluid increased in a given direction in passing from one section to another, there must be a net force acting on the fluid in that direction.
- By Newton's third law, there will be an equal and opposite force exerted by the fluid on the system.

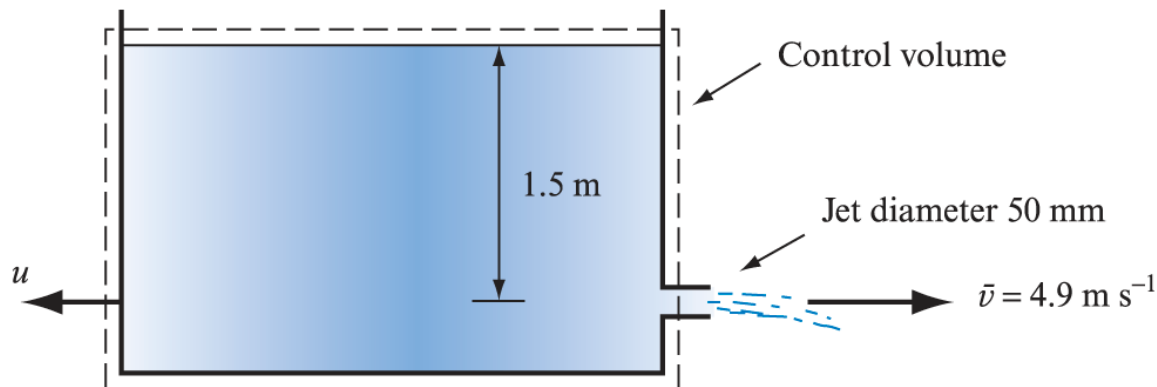
Typical example:

- Force on the nozzle at the outlet of a pipe. Anything holding the nozzle (e.g. a fireman) must be strong enough to withstand these forces.
- The reactive force exerted when the fluid is discharge in the form of high-velocity jet. (aircraft, rocket motors...)

Example: (Ex 5.6, page 129 Textbook)

A jet of water of diameter $d = 50$ mm issues with velocity of 4.9 m/s from a hole in the vertical side of an open tank which kept filled with water to a height of 1.5 m above the center of the hole. **Calculate the reaction of the jet on the tank and its contents when:**

1. It is stationary.
2. Its moving with a velocity $u = 1.2$ m/s in the opposite direction to the jet while the velocity of the jet relative to the tank remains unchanged.
3. In the latter case, what would be the work done per second.



Continue Rocket motor

- A rocket motor is, in principle, a simple form of engine in which the **thrust is developed as the result of the discharge of a high-velocity jet of gas produced by the combustion of the fuel and oxidizing agent.**
- Both the fuel and the oxidant are carried in the rocket and so it can operate even in outer space. It does not require atmospheric air, either for combustion or for the jet to push against;
- The thrust is entirely due to the reaction developed from the momentum per second discharged in the jet.

Example: (Ex 5.7, page 130 Textbook)

The mass of a rocket m_r is 150 000 kg and, when ready to launch, it carries a mass of fuel m_{f0} of 300 000 kg. The initial thrust of the rocket motor is 5 MN and fuel is consumed at a constant rate $\dot{m}$. The velocity $\bar{v}_r$ of the jet relative to the rocket is 3000 m/s.

Assuming that the flight is vertical, and neglecting air resistance, **find:**

- (a) the burning time.
 - (b) the speed of the rocket and the height above ground at the moment when all the fuel is burned.
 - (c) the maximum height that the rocket will reach.
- Assume that $\mathbf{g}$ is constant and equal to 9.81 m/s².

Solution:

(a) From equation (I), Example 5.6,

$$\text{Initial thrust, } T = \dot{m}\bar{v}_r,$$

$$\text{Rate of fuel consumption, } \dot{m} = T/\bar{v}_r$$

$$= 5 \times 10^6 / 3000 = 1667 \text{ kg s}^{-1},$$

$$\text{Initial mass of fuel, } m_{f0} = 300\,000 \text{ kg},$$

$$\text{Burning time} = m_{f0}/\dot{m} = 300\,000/1667 = \mathbf{180 \text{ s}}.$$

(b) If there is no air resistance, the forces acting on the rocket and the fuel which it contains during vertical flight are the thrust T acting upwards and the weight $(m_r + m_{ft})g$ acting downwards, where m_{ft} is the mass of the fuel in the rocket at time t .

From Newton's second law,

$$T - (m_r + m_{ft})g = (m_r + m_{ft})\frac{dv_t}{dt},$$

where v_t is the velocity of the rocket at time t .

$$\frac{dv_t}{dt} = \frac{T - (m_r + m_{ft})g}{m_r + m_{ft}}.$$

Since the fuel is being consumed at a rate $\dot{m}$,

$$\text{Mass of fuel at time } t, m_{ft} = m_{f0} - \dot{m}t.$$

Also, $T = \dot{m}\bar{v}_r$ and so

$$\frac{dv_t}{dt} = \frac{\dot{m}\bar{v}_r - (m_r + m_{f0} - \dot{m}t)g}{m_r + m_{f0} - \dot{m}t}.$$

Substituting numerical values,

$$\begin{aligned} \frac{dv_t}{dt} &= \frac{1667 \times 3000 - (150\,000 + 300\,000 - 1667t) \times 9.81}{150\,000 + 300\,000 - 1667t} \\ &= -9.81 + \frac{3000}{269.95 - t} \text{ m s}^{-2}. \end{aligned}$$

Solution:

Integrating,

$$v_t = -9.81t - 3000 \log_e(269.95 - t) + \text{constant}.$$

Putting $v_t = 0$ when $t = 0$, the value of the constant is $3000 \log_e 269.95$, giving

$$v_t = -9.81t - 3000 \log_e(1 - t/269.95). \quad (\text{I})$$

From (a), all the fuel will be burnt out when $t = 180$ s. Substituting in equation (I),

$$\begin{aligned} v_t &= -9.81 \times 180 - 3000 \log_e(1 - 180/269.95) \text{ m s}^{-1} \\ &= -1765.8 + 3296.9 = \mathbf{1531.2 \text{ m s}^{-1}}. \end{aligned}$$

The height at time $t = 180$ s is given by

$$\begin{aligned} Z_1 &= \int_0^{180} v_t \, dt = -9.81 \int_0^{180} t \, dt - 3000 \int_0^{180} \log_e(1 - t/269.95) \, dt \\ &= -(4.9t^2)_0^{180} + 3000\{269.95(1 - t/269.95)[\log_e(1 - t/269.95) - 1]\}_0^{180} \\ &= -158\,760 + 243\,451.9 = 84\,691.9 \text{ m} \\ &= \mathbf{84.692 \text{ km}}. \end{aligned}$$

(c) When the fuel is exhausted, the rocket will have reached an altitude of 84 692 m and will have kinetic energy $m_t v_t^2/2g$. It will, therefore, continue to rise a further distance Z_2 until this kinetic energy has been converted into an increase of potential energy.

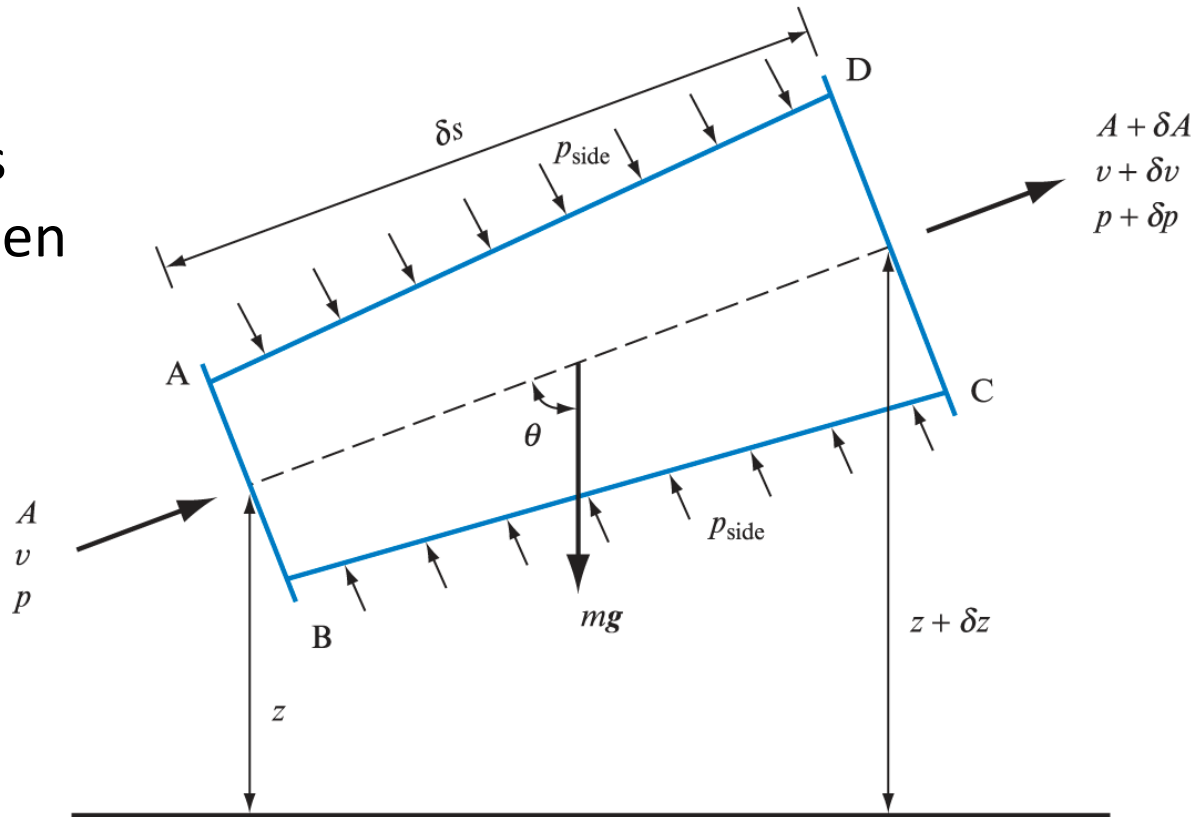
$$Z_2 = v_t^2/2g = 1531.2^2/(2 \times 9.81) = 119\,499 \text{ m},$$

$$\begin{aligned} \text{Maximum height reached} &= Z_1 + Z_2 = 84\,692 + 119\,499 \text{ m} \\ &= \mathbf{204.2 \text{ km}}. \end{aligned}$$

5.12 Euler's Equation of Motion along a Streamline

It is an equation shows the relationship between velocity, pressure, elevation and density along a streamline

Mass per unit time flowing = ρAv



Rate of increase of momentum from AB to CD (in direction of motion) = $\rho Av [(v + \delta v) - v] = \rho Av \delta v$

The force acting to produce this increase of momentum in the direction of motion are:

- Force due to p in direction of motion $= pA$
- Force due to $p+\delta p$ opposing motion $= (p+\delta p)(A+\delta A)$
- Force due to p_{side} producing a component in the direction of motion $= p_{side} \delta A$
- Force due to mg producing a component opposing motion $= mg \cos \theta$

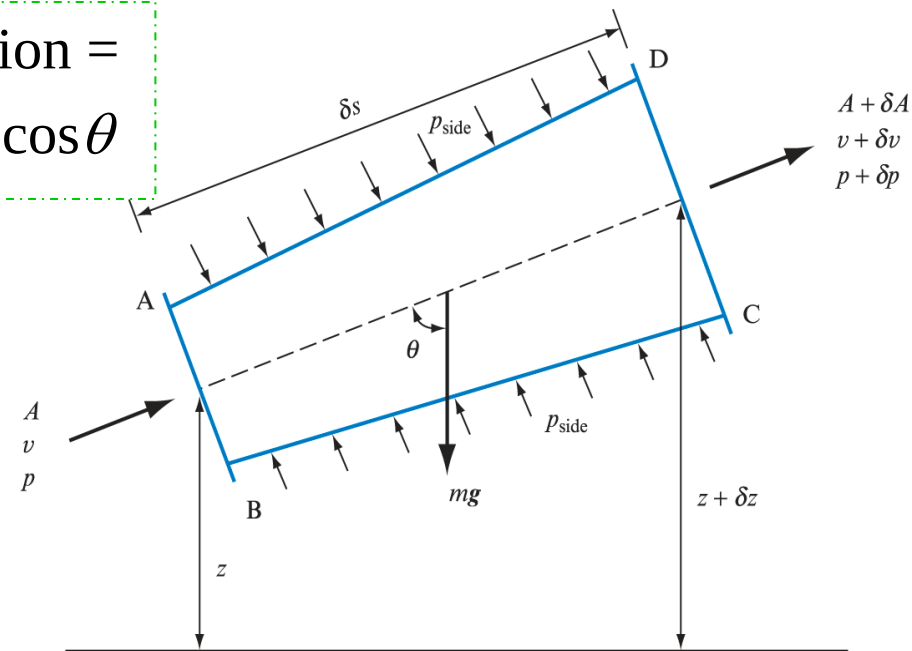
Resultant force in the direction of motion =

$$pA - (p+\delta p)(A+\delta A) + p_{side} \delta A - mg \cos \theta$$

$p_{side} = p + k\delta p$ where k is a fraction

$$mg = \rho g (\text{Volume}) = \rho g (A + 0.5\delta A) \delta s$$

$$\cos \theta = \delta z / \delta s$$



- Neglecting the product of small quantities:



$$\text{Resultant force in the direction of motion} = -A\delta p - \rho g A \delta z$$

- Applying Newton's second law:



$$\rho A v \delta v = -A\delta p -$$

$$\rho g A \delta z$$

- Dividing by : $\rho A \delta s$



$$\frac{1}{\rho} \frac{\delta p}{\delta s} + v \frac{\delta v}{\delta s} + g \frac{\delta z}{\delta s} = 0$$

- Or, in the limit as $\delta s \rightarrow 0$

$$\frac{1}{\rho} \frac{dp}{ds} + v \frac{dv}{ds} + g \frac{dz}{ds} = 0$$

This is known as ***Euler's equation***

Giving, in differential form, the relationship between velocity v , pressure p , elevation z and density ρ along a streamline for steady flow

$$\frac{1}{\rho} \frac{dp}{ds} + v \frac{dv}{ds} + g \frac{dz}{ds} = 0$$

- For incompressible flow, ρ is constant, then the integration of the above eq. along a streamline with respect to s , gives



$$\frac{p}{\rho} + \frac{v^2}{2} + gz = \text{constant}$$

- The terms represent energy per unit mass. Dividing by g



$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant} = H$$

**This is known as
*Bernoulli's equation***

- In which the terms represent energy per unit weight.

Bernoulli's equation states the relationship between velocity v , pressure p , elevation z for: steady flow of frictionless fluid of constant density.

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant} = H$$

Bernoulli's equation

- An alternative is:

$$p + \frac{1}{2}\rho v^2 + \rho g z = \text{constant}$$

In which terms represent energy per unit volume.

Bernoulli's equation can be written along a streamline between two points 1 and 2 as:

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$