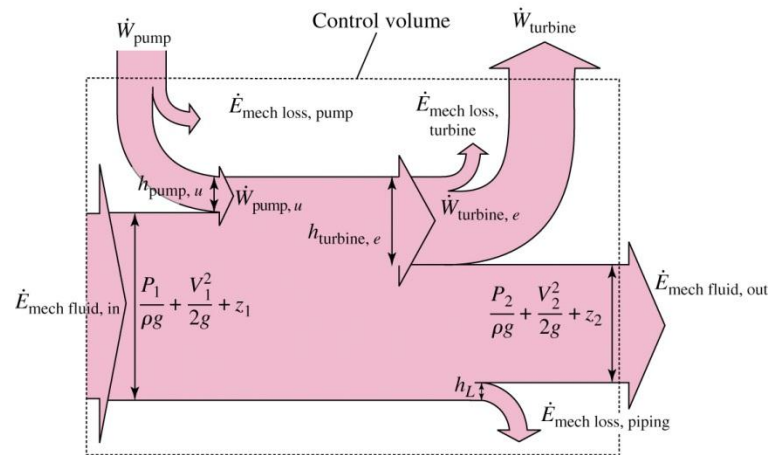


CHAPTER 6

The Energy Equation and its Applications



Dr. Khalil Mahmoud ALASTAL

Objectives of this Chapter:

- **Derive** the Bernoulli (energy) equation.
- **Demonstrate** practical uses of the Bernoulli and continuity equation in the analysis of flow.
- **Understand** the use of hydraulic and energy grade lines.
- **Apply** Bernoulli Equation to solve fluid mechanics problems (e.g. flow measurement).

6.1 Mechanical Energy of Flowing Fluid :

- Bernoulli's equation is one of the most important/useful equations in fluid mechanics.
- The Bernoulli equation is a statement of the principle of conservation of energy along a streamline.
- It can be written:

$$\frac{p}{\rho g} + \frac{V^2}{2g} + z = H = \text{constant}$$



Daniel Bernoulli
(1700-1782)

- These terms represent:

Pressure energy per unit weight	+	Kinetic energy per unit weight	+	Potential energy per unit weight	=	Total energy per unit weight
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$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = H = \text{constant}$$

- These term all have **units of length**.
- They are often referred to as the following:
 - pressure head = $\frac{p_1}{\rho g}$
 - velocity head = $\frac{V_1^2}{2g}$
 - potential head = z_1
 - total head = H

By the principle of conservation of energy the total *energy* in the system does not change, Thus the total *head* does not change.

Restrictions in application of Bernoulli's eq.:

Bernoulli's equation has some restrictions in its applicability:

- ☐ Flow is steady;
- ☐ Density is constant (which also means the fluid is incompressible);
- ☐ Friction losses are negligible.
- ☐ The equation relates the states at two points along a single streamline, (not conditions on two different streamlines).

All these conditions are impossible to satisfy at any instant in time! Fortunately for many real situations where the conditions are *approximately* satisfied, the equation gives very good results.

- As stated above, the Bernoulli equation applies to conditions along a streamline. We can apply it between two points, 1 and 2, on the streamline:

total energy per unit weight at 1 = total energy per unit weight at 2

or

Total head at 1 = Total head at 2

or

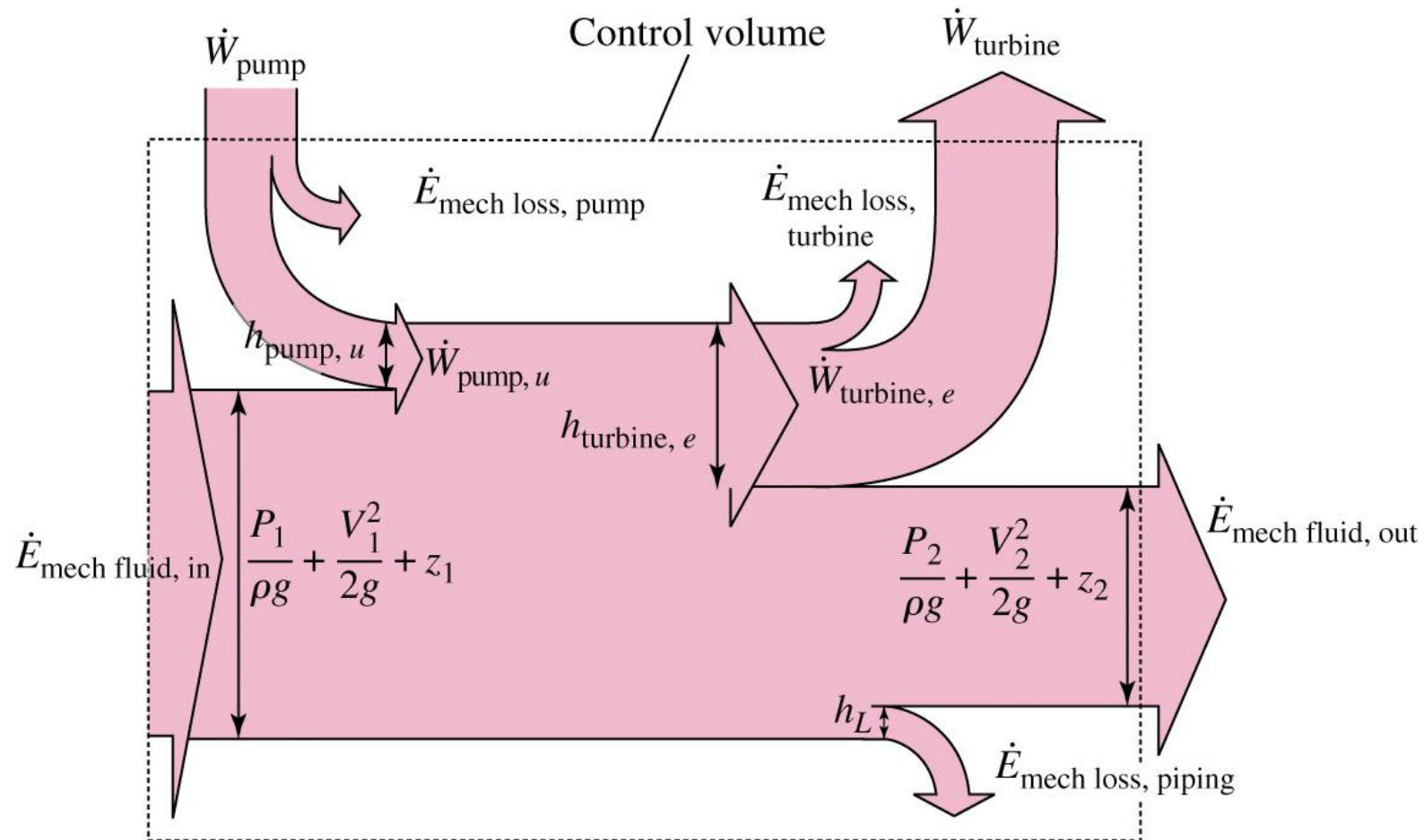
$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

- This equation assumes no energy losses (e.g. from friction) or energy gains (e.g. from a pump) along the streamline. It can be expanded to include these simply, by adding the appropriate energy terms:

Total energy per unit weight at 1	=	Total energy per unit weight at 2	+	Loss per unit weight	+	Work done per unit weight	-	Energy supplied per unit weight
--	----------	--	----------	-------------------------------------	----------	--	----------	--

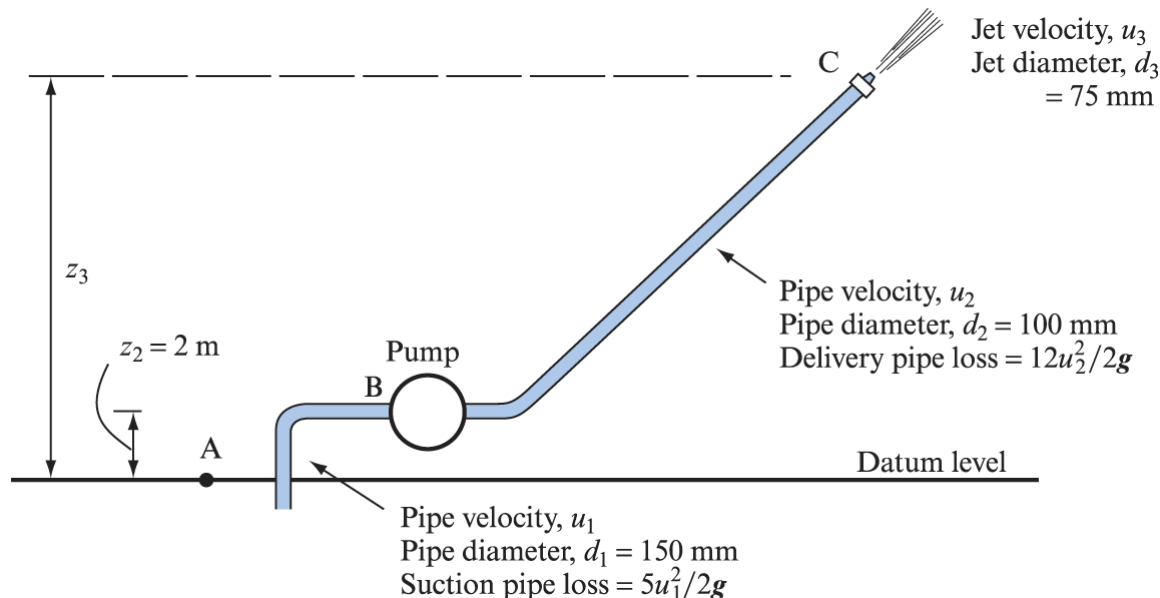
$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h + w - q$$



$$\frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 + h_{pump} = \frac{P_2}{\rho_2 g} + \frac{V_2^2}{2g} + z_2 + h_{turbine} + h_L$$

Example: (Ex 6.1, page 172 Textbook)

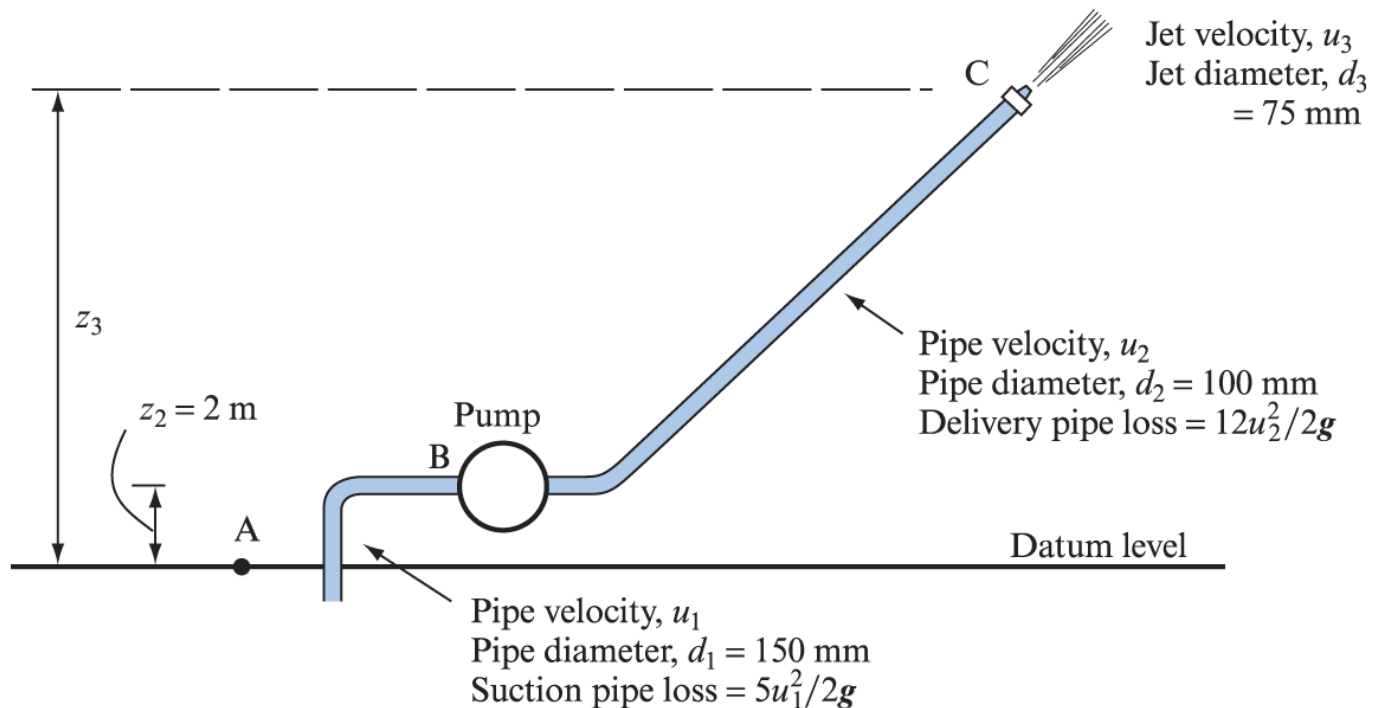
A fire engine pump develops a head of 50 m, i.e. it increases the energy per unit weight of the water passing through it by 50 N m N^{-1} . The pump draws water from a sump at A (Fig. 6.2) through a 150 mm diameter pipe in which there is a loss of energy per unit weight due to friction $h_1 = 5u_1^2/2g$ varying with the mean velocity u_1 in the pipe, and discharges it through a 75 mm nozzle at C, 30 m above the pump, at the end of a 100 mm diameter delivery pipe in which there is a loss of energy per unit weight $h_2 = 12u_2^2/2g$. Calculate (a) the velocity of the jet issuing from the nozzle at C and (b) the pressure in the suction pipe at the inlet to the pump at B.



Example: (Ex 6.1, page 172 Textbook)

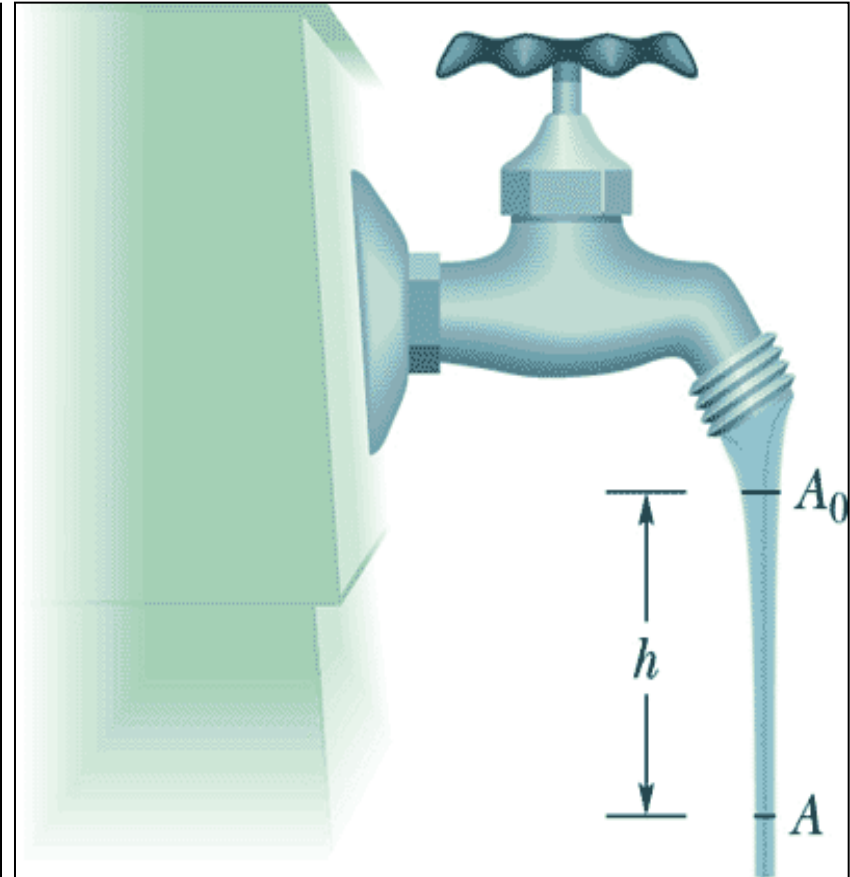
Calculate:

- the velocity of the jet issuing from the nozzle at C.
- the pressure in the suction pipe at the inlet to the pump.



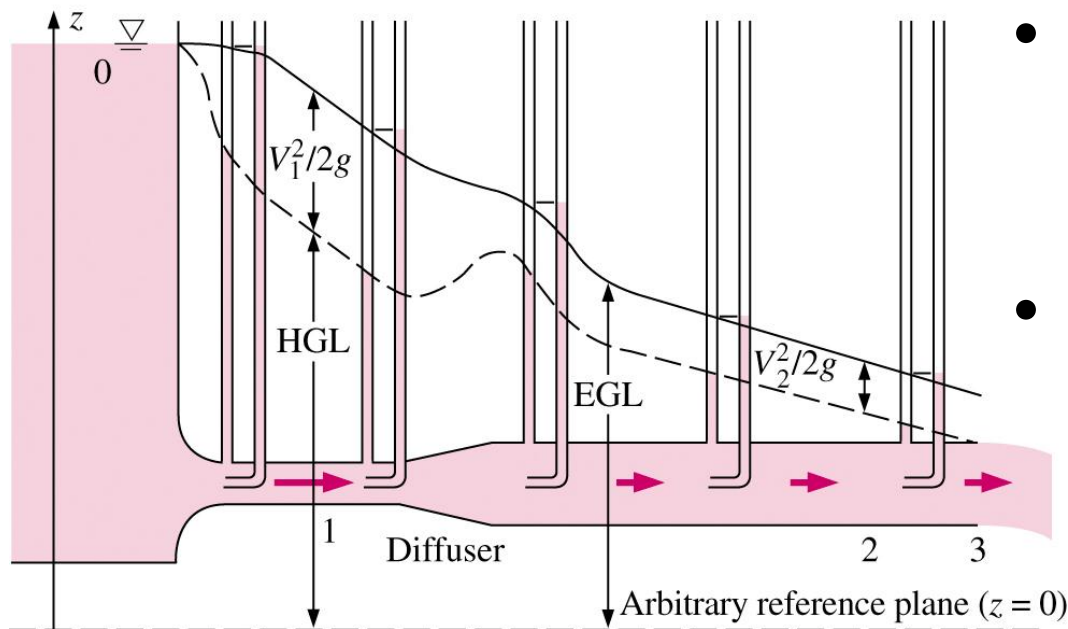
Example:

- The indicated cross-sectional areas are $A_0 = 12 \text{ cm}^2$ and $A = 0.35 \text{ cm}^2$. The two levels are separated by a vertical distance $h = 45 \text{ mm}$.
- **What is the volume flow rate from the tap ?**



6.5 Representation of Energy Changes in a Fluid System (HGL and EGL):

- It is often convenient to plot mechanical energy **graphically** using heights.



- Hydraulic Grade Line**

$$HGL = \frac{P}{\rho g} + z$$

- Energy Grade Line (or total energy)**

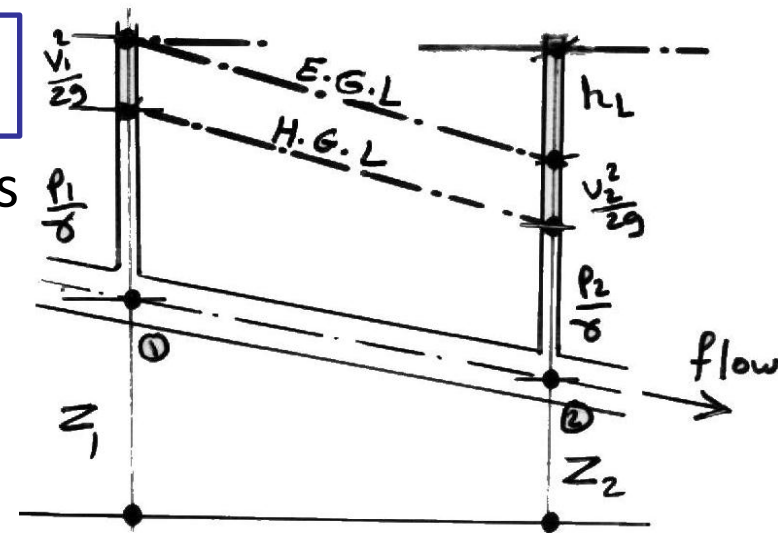
$$EGL = \frac{P}{\rho g} + \frac{V^2}{2g} + z$$

Hydraulic Gradient Line (H.G.L.):

- It is the line that joins all the points to which water would rise if piezometric tubes were inserted.
- or it is the line that connects the piezometric heads at all points ($p/\gamma + z$)

Energy Gradient Line (E.G.L.):

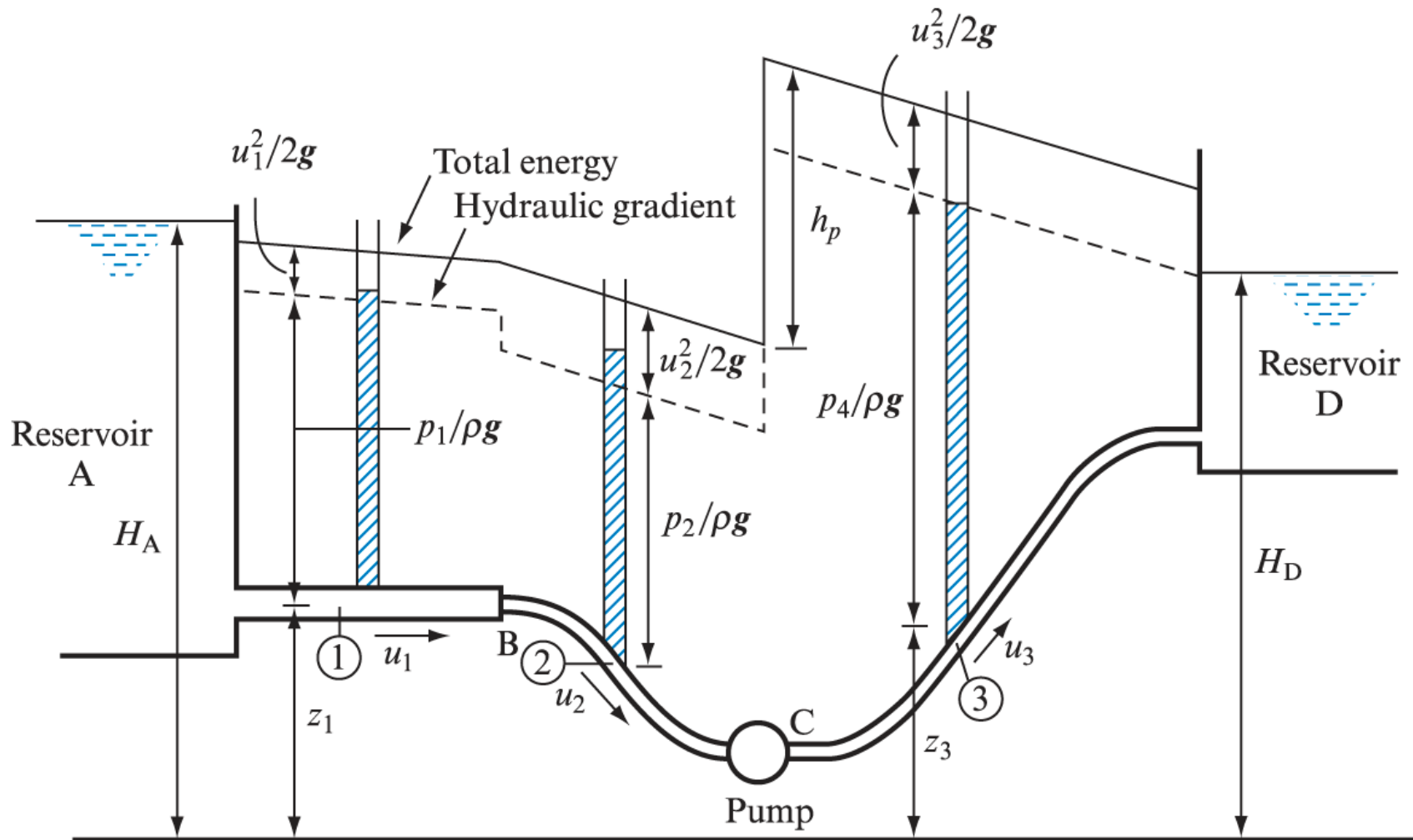
- It is the line that joins all the points that represent the sum of kinetic head and piezometric head ($V^2/2g$ above the HGL).



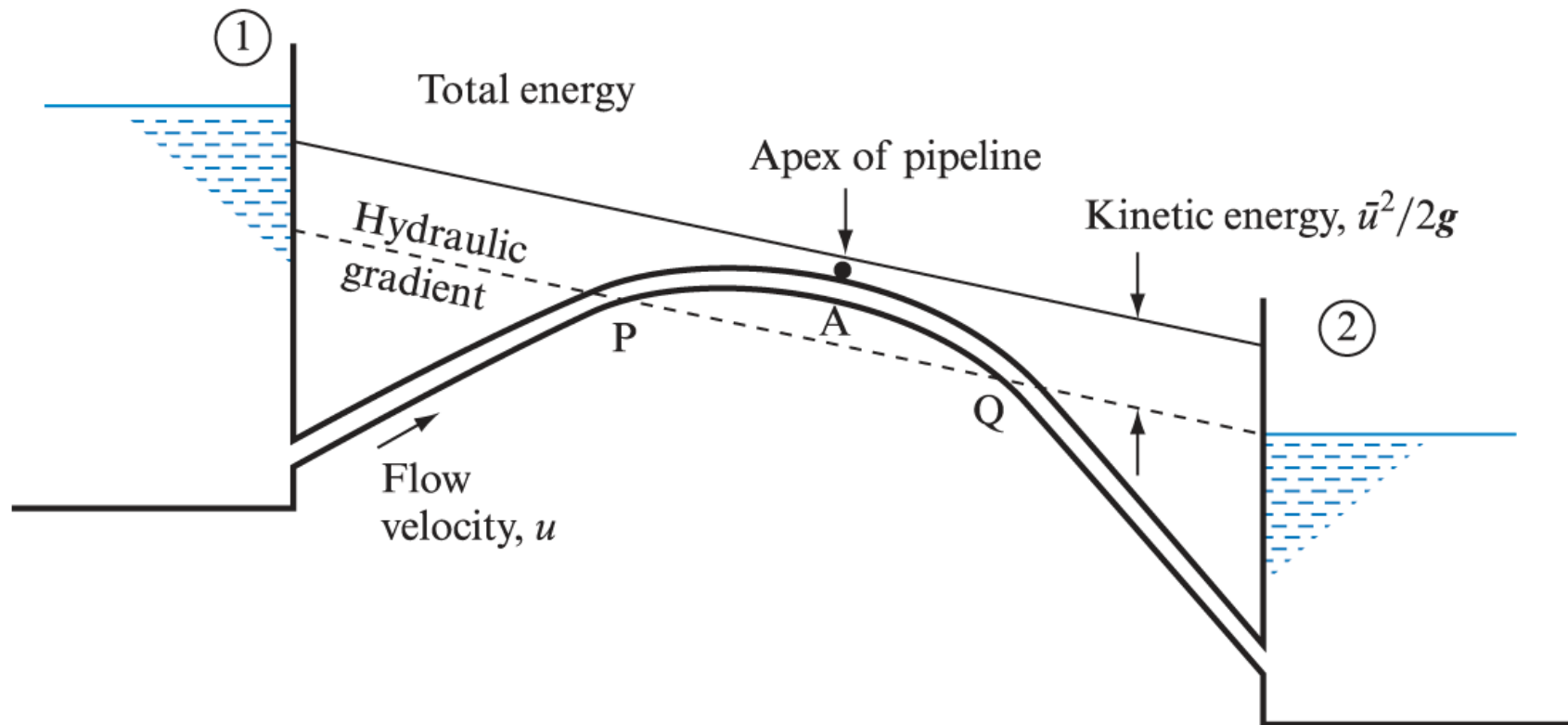
Note:

- The E.G.L. (total energy) falls due to friction losses (h_L).
- This loss can be, also, caused by any variations in the cross-section of the pipe such as enlargement, contraction, or because the presence of entrances or valves and so on ..

Example: (page 180 Textbook)



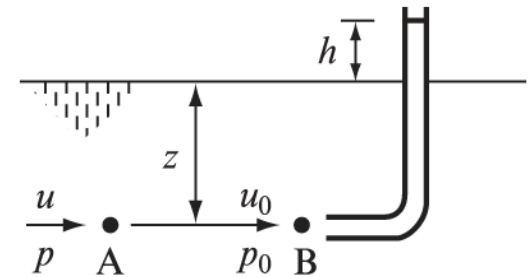
Example: (page 181 Textbook)



Applications of Bernoulli's Equation

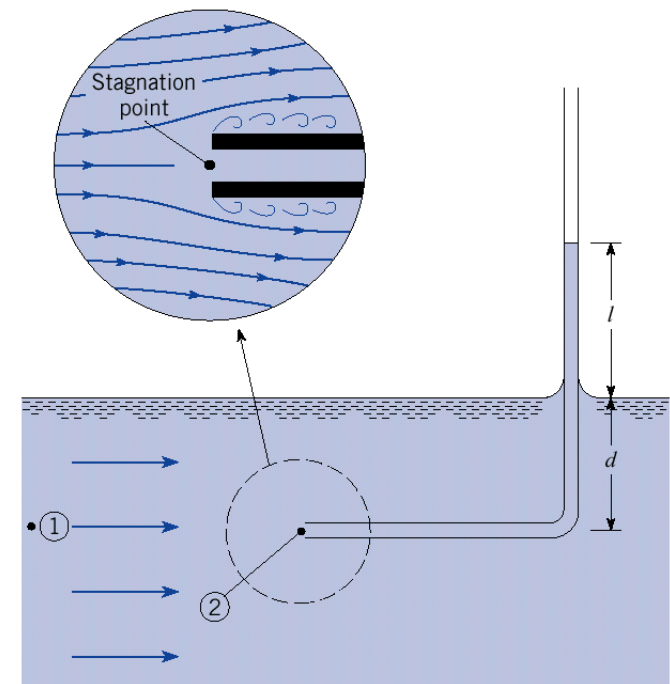
6.6 The Pitot Tube:

- The Pitot tube is used to measure the velocity of a stream.
- It consists of a simple L-shaped tube facing into the incoming flow.
- If the velocity of the stream at **A** is u , a particle moving from **A** to the mouth of the tube **B** will be brought to rest so that u_0 at **B** is zero.



Simple Pitot Tube

A point in a fluid stream where the velocity is reduced to zero is known as a *stagnation point*.
(Points B and 2)



- Apply Bernoulli's equation between points A and B :

Total head at A = Total head at B

$$\Rightarrow \frac{p}{\rho g} + \frac{u^2}{2g} = \frac{p_0}{\rho g} + \frac{u_0^2}{2g}$$

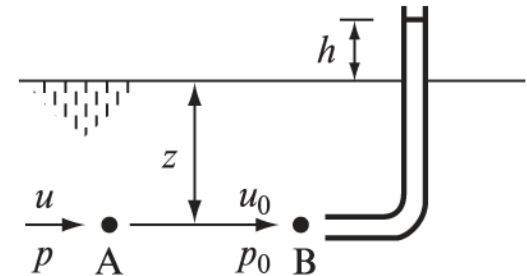
$$\Rightarrow \frac{p_0}{\rho g} = \frac{p}{\rho g} + \frac{u^2}{2g}$$

Thus, p_0 will be greater than p

$$\frac{p}{\rho g} = z \quad \text{and} \quad \frac{p}{\rho g} = z + h$$

$$\Rightarrow \frac{u^2}{2g} = \frac{p_0}{\rho g} - \frac{p}{\rho g} = \frac{p_0 - p}{\rho g} = h$$

$$\Rightarrow \text{Velocity at A} \quad u = \sqrt{2gh}$$



How we can use the Pitot tube in the pipe?:

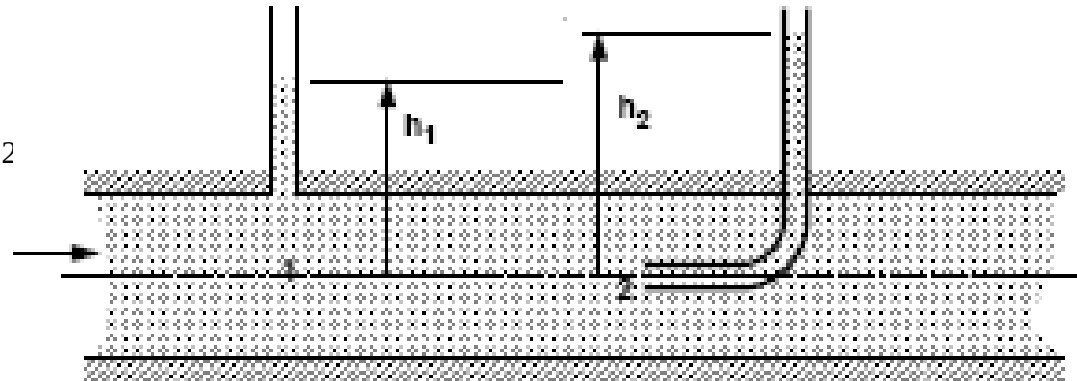
Method 1

- Two piezometers, **one as** normal and **one as** a Pitot tube within the pipe can be used in an arrangement shown below to measure velocity of flow in pipes.

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + z_2$$

$$\Rightarrow \frac{\rho g h_1}{\rho g} + \frac{u_1^2}{2g} = \frac{\rho g h_2}{\rho g}$$

$$\Rightarrow V = u_1 = \sqrt{2g(h_2 - h_1)}$$

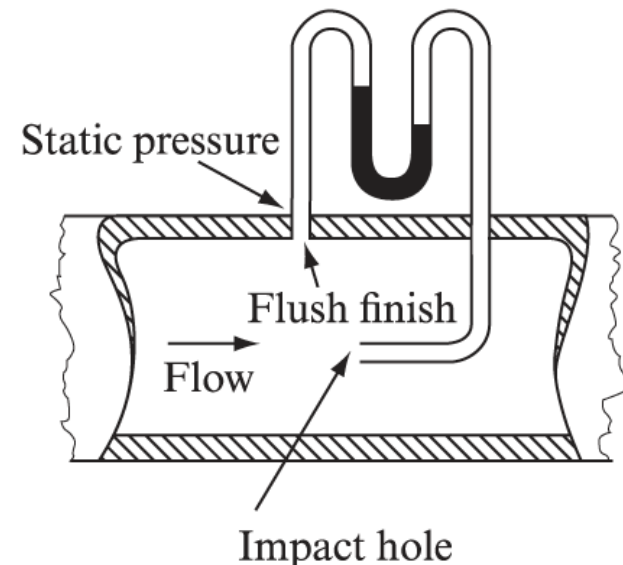


How we can use the Pitot tube in the pipe?:

Method 2

- Using a static pressure tapping in the pipe wall with a differential pressure gauge to measure the difference between the static pressure and the pressure at the impact hole

$$V = \text{????} (HW)$$

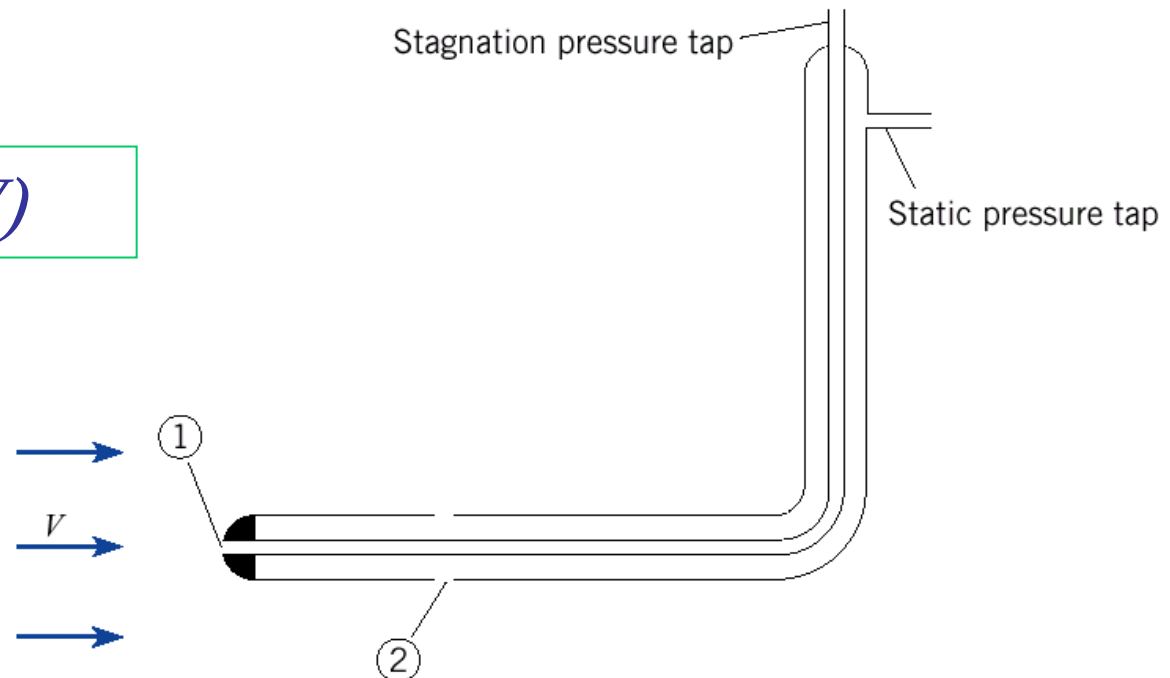


How we can use the Pitot tube in the pipe?:

Method 3

- Using combined Pitot static tube. In which the inner tube is used to measure the impact pressure while the outer sheath has holes in its surface to measure the static pressure

$$V = \text{???? (HW)}$$



Note:

- Theoretically: $u = \sqrt{2gh}$
- However, Pitot tubes **may require calibration**
- The true velocity is given by:

$$V = C\sqrt{2gh}$$

- Where C is the coefficient of the instrument

For example:

- $C = 1$ for Pitot static tube (when $Re > 3000$)

Disadvantages :

- The Pitot/Pitot-static tubes give velocities at points in the flow. It does not give the overall discharge of the stream, which is often what is wanted.
- It also has the drawback that it is liable to block easily, particularly if there is significant debris in the flow.

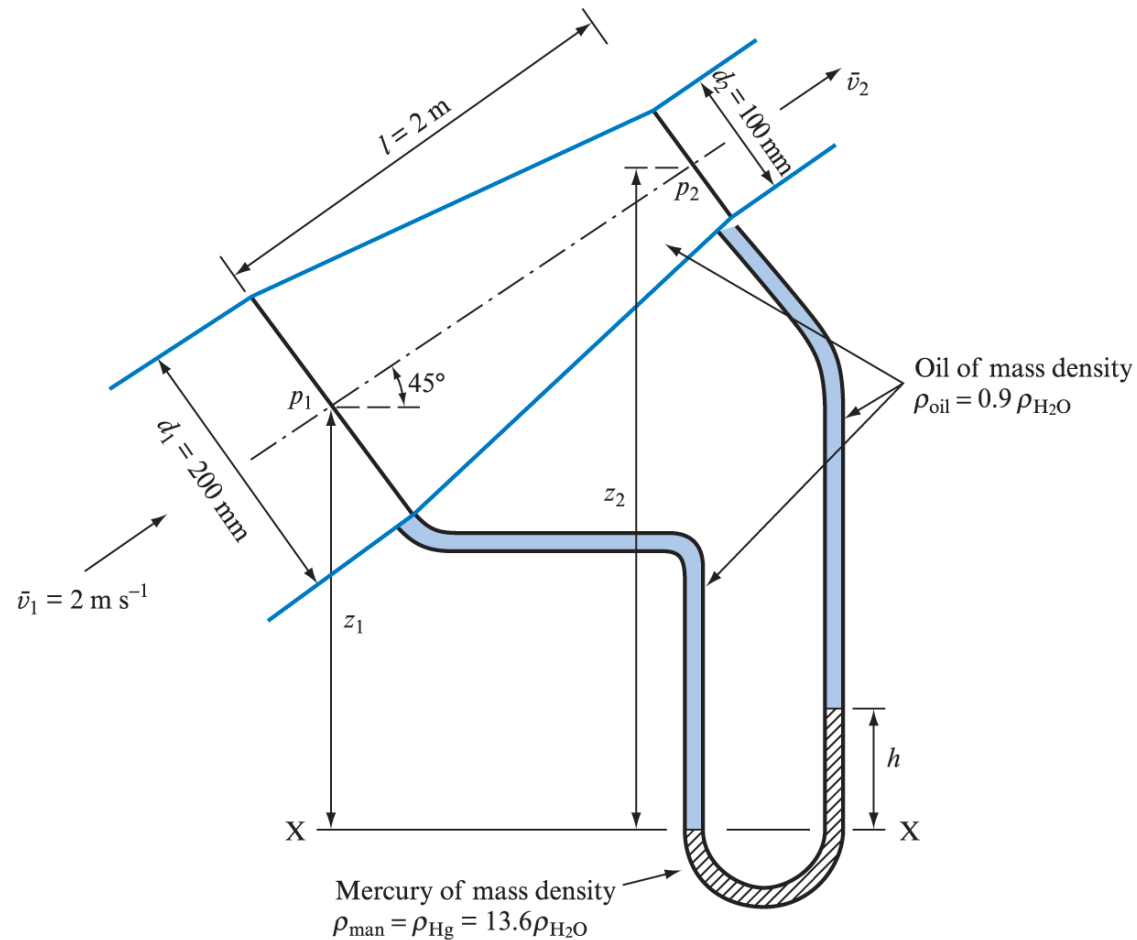
6.9 Changes of Pressure in a tapering pipe:

- Changes of velocity in a tapering pipe were determined by using the **continuity equation**.
- Changes of velocity will accompanied by a changed in pressure, modified by any changed in elevation or energy loss, which can be determined by the use of **Bernoulli's equation**.

Example: (Ex 6.2, page 185 Textbook)

Find:

- the pressure difference across the 2m length ignoring any losses of energy.
- the difference in level that would be shown on a mercury manometer connected across this length.



Solution:

- From continuity equation : $V_2 = 8\text{m/s}$
- Applying Bernoulli's equation between section 1 and 2:
(Ignoring losses)

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$


$$\rho g \left(\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 \right) = \rho g \left(\frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \right)$$


$$p_1 + \frac{1}{2} \rho V_1^2 + \rho g z_1 = p_2 + \frac{1}{2} \rho V_2^2 + \rho g z_2$$


$$p_1 - p_2 = \frac{1}{2} \rho_{oil} (V_2^2 - V_1^2) + \rho g (z_2 - z_1)$$

Substituting with V_1 , V_2 and observing that $z_2 - z_1 = 2\sin 45 = 1.41\text{m}$


$$p_1 - p_2 = 39484 \text{ N/m}^2$$

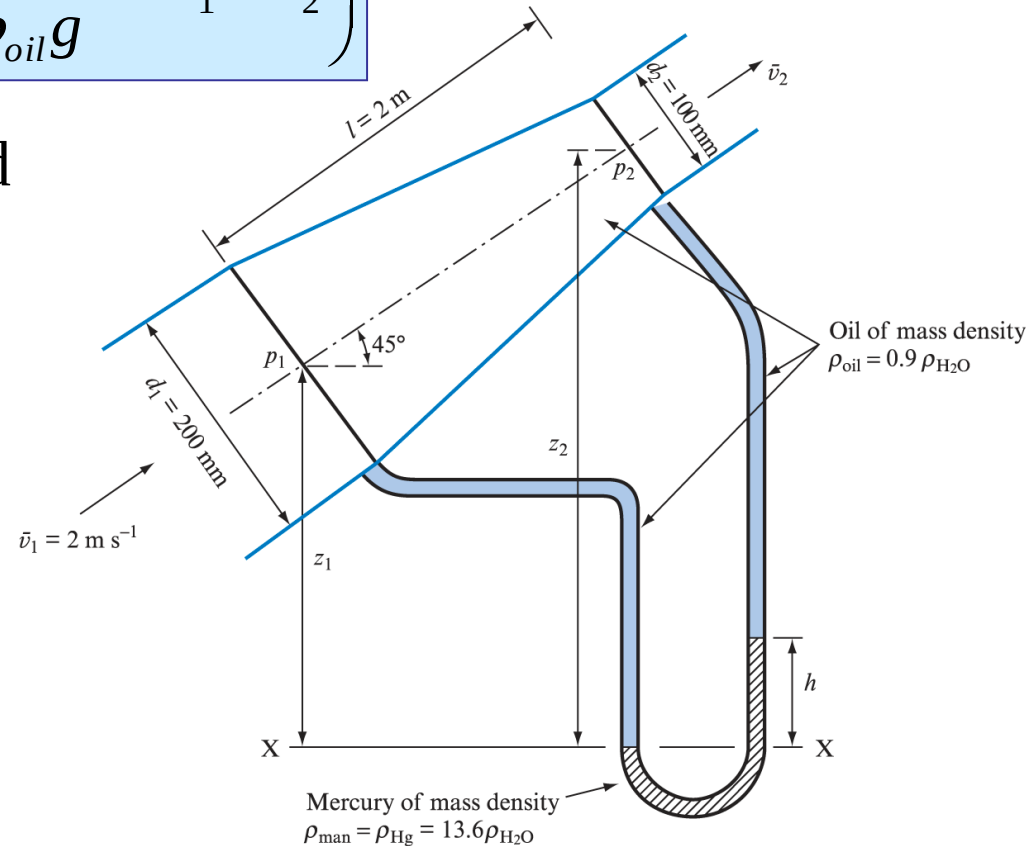
- For the manometer: The pressure at level XX is the same in each limb

$$p_1 + \rho_{oil}gz_1 = p_2 + \rho_{oil}g(z_2 - h) + \rho_{man}gh$$

$$\Rightarrow h = \left(\frac{\rho_{oil}}{\rho_{man} - \rho_{oil}} \right) \left(\frac{p_1 - p_2}{\rho_{oil}g} + z_1 - z_2 \right)$$

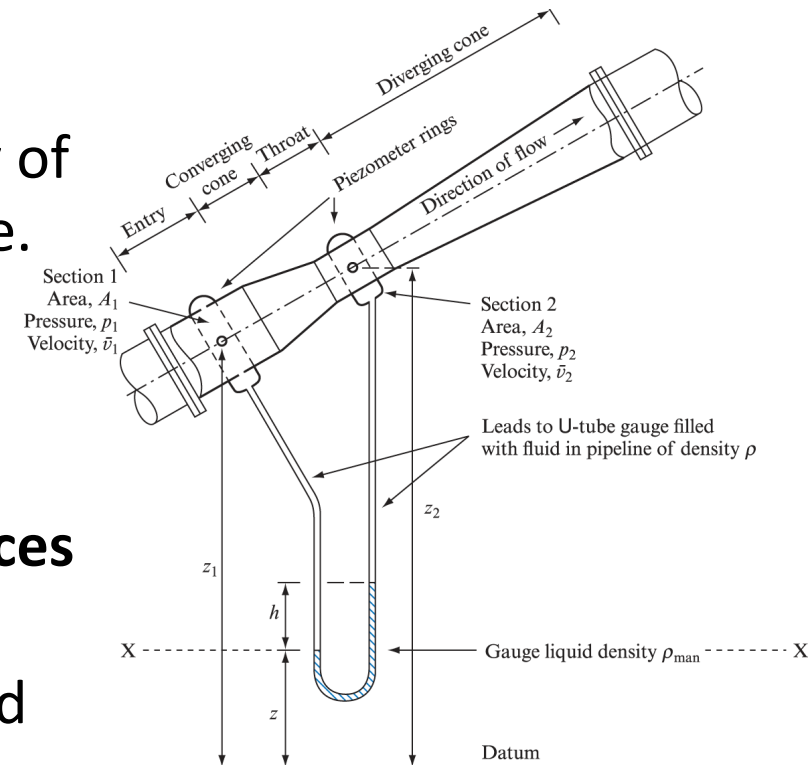
- Substituting with p_1 , p_2 and observing that $z_2 - z_1 = 2\sin 45 = 1.41\text{m}$

$$\Rightarrow h = 0.217\text{m}$$



6.10 Principle of the Venturi Meter:

- The Venturi meter is a device for **measuring discharge** in a pipe.
- It consists of a rapidly converging section, which increases the velocity of flow and hence reduces the pressure.
- It then returns to the original dimensions of the pipe by a gently diverging 'diffuser' section.
- **By measuring the pressure differences the discharge can be calculated.**
- This is a particularly accurate method of flow measurement as **energy losses are very small.**



- Applying Bernoulli equation between sections 1 and 2, we have: (assuming no losses)

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\Rightarrow V_2^2 - V_1^2 = 2g \left[\frac{p_1 - p_2}{\rho g} + (z_1 - z_2) \right]$$

- From continuity equation: $A_1 V_1 = A_2 V_2 \Rightarrow V_2 = \left(\frac{A_1}{A_2} \right) V_1$

$$\Rightarrow V_1^2 \left[\left(\frac{A_1}{A_2} \right)^2 - 1 \right] = 2g \left[\frac{p_1 - p_2}{\rho g} + (z_1 - z_2) \right]$$

$$\Rightarrow V_1 = \frac{A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g \left[\frac{p_1 - p_2}{\rho g} + (z_1 - z_2) \right]}$$

- Volume flow rate (Q):**

$$Q = A_1 V_1 = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g \left[\frac{p_1 - p_2}{\rho g} + (z_1 - z_2) \right]}$$



$$Q = A_1 V_1 = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g \left[\frac{p_1 - p_2}{\rho g} + (z_1 - z_2) \right]}$$

or

$$Q = \frac{A_1}{\sqrt{m^2 - 1}} \sqrt{2gH}$$

This is the **theoretical** discharge

Where: $H = \frac{p_1 - p_2}{\rho g} + (z_1 - z_2)$ and $m = \frac{A_1}{A_2}$

- The value of H can also be expressed **in terms of the manometer readings**



$$p_1 + \rho g(z_1 - z) = p_2 + \rho g(z_2 - z - h) + \rho_{man} g h$$



$$H = \frac{p_1 - p_2}{\rho g} + (z_1 - z_2) = h \left(\frac{\rho_{man}}{\rho} - 1 \right)$$



$$Q = \frac{A_1}{\sqrt{m^2 - 1}} \sqrt{2gh \left(\frac{\rho_{man}}{\rho} - 1 \right)}$$

This is also the theoretical discharge in terms of manometer readings

- In practice, some losses of energy between section 1 and 2 occurs.
- Therefore, we include a **coefficient of discharge** to get the actual discharge



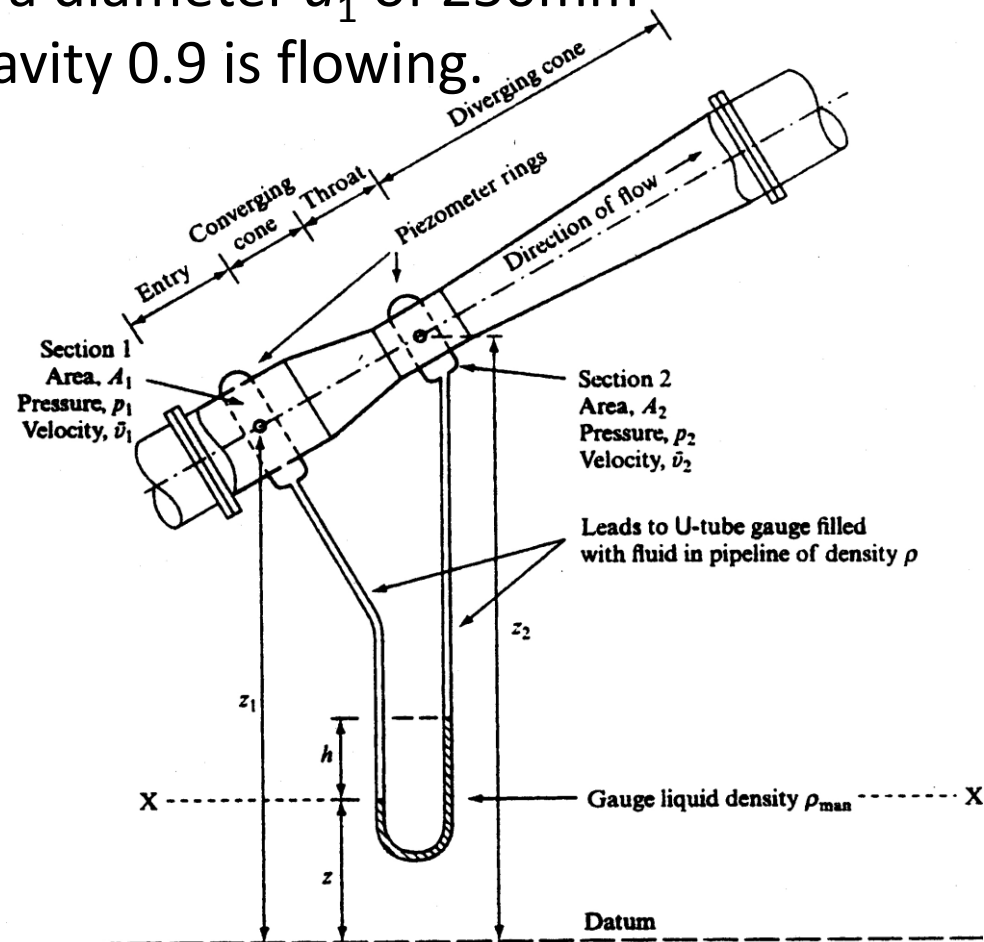
$$Q_{\text{actual}} = C_d \times Q_{\text{theoretical}}$$

Example: (Ex 6.3, page 189 Textbook)

A venturi meter having a throat diameter d_2 of 100mm is fitted into a pipeline which has a diameter d_1 of 250mm through which oil of specific gravity 0.9 is flowing.

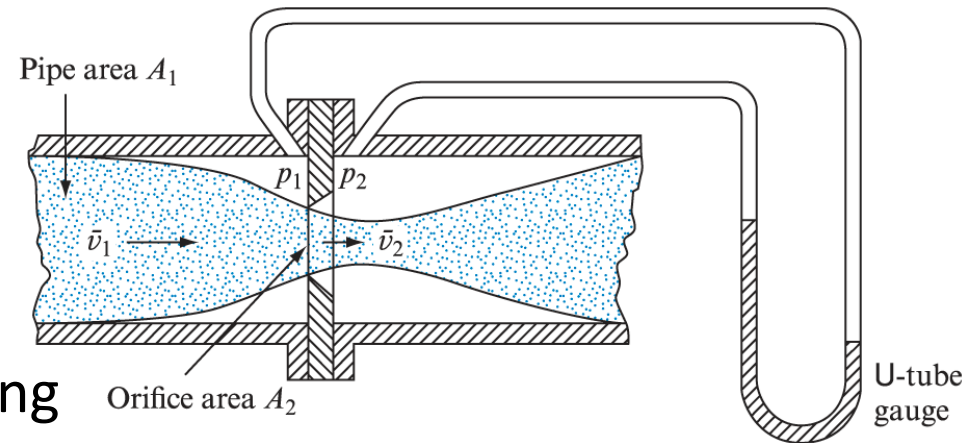
The pressure difference between the entry and throat tapings is measured by U-tube manometer, containing mercury of specific gravity 13.6.

If the difference of level of manometer is 0.63m, **calculate the theoretical discharge**



6.11 Pipe Orifices:

- A similar effect as the venturi meter can be achieved by inserting an orifice plate
- The orifice plate has an opening in it smaller than the internal pipe diameter



$$Q = \frac{A_1}{\sqrt{m^2 - 1}} \sqrt{2gH}$$

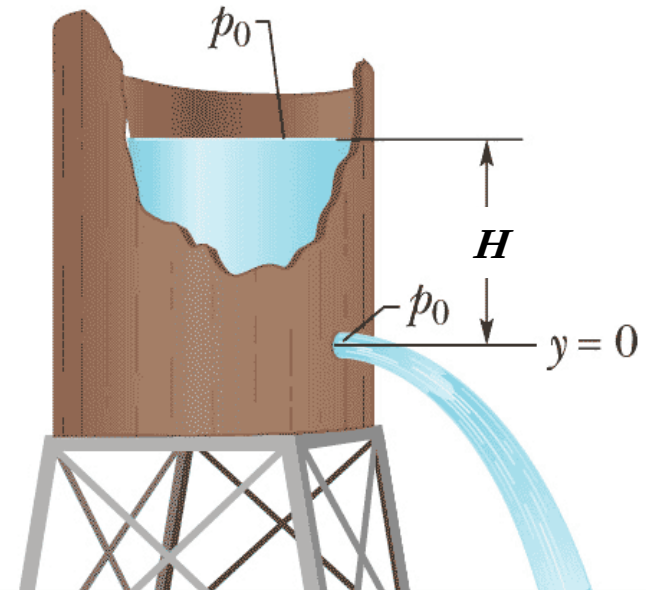
Where: $H = \frac{p_1 - p_2}{\rho g} + (z_1 - z_2)$ or $H = h \left(\frac{\rho_{man}}{\rho} - 1 \right)$

and $Q_{actual} = C_d \times Q_{theoretical}$

For Sharp-edged orifice $C_d = 0.65$

6.11 Theory of Small Orifices Discharging to Atmosphere

- An orifice is an opening in the side or base of a tank or reservoir through which fluid is discharge in the form of a jet.
- The discharge will depend upon the head of the fluid (H) above the level of the orifice.
- **The term small orifice means** that the diameter of the orifice is small compared with the head producing flow (it can be assumed that the head does not vary appreciably from point to point across the orifice).



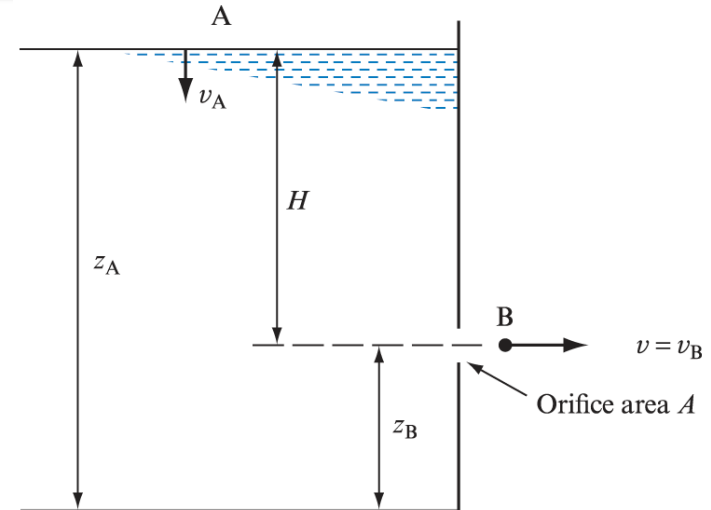
- Applying Bernoulli equation between sections A and B , we have: (assuming no losses)

$$\frac{p_A}{\rho g} + \frac{v_A^2}{2g} + z_A = \frac{p_B}{\rho g} + \frac{v_B^2}{2g} + z_B$$



$$\text{Velocity of jet} = v = \sqrt{2gH}$$

This result is known as Torricelli's Theorem.



- Theoretically**, if A is the cross sectional area of the orifice, then:

$$\text{Discharge} = Q_{\text{Theoretical}} = A\sqrt{2gH}$$

- The actual discharge**, is given by:

$$Q_{\text{Actual}} = C_d Q_{\text{Theoretical}} = C_d A\sqrt{2gH}$$

- Where:** C_d is the coefficient of discharge

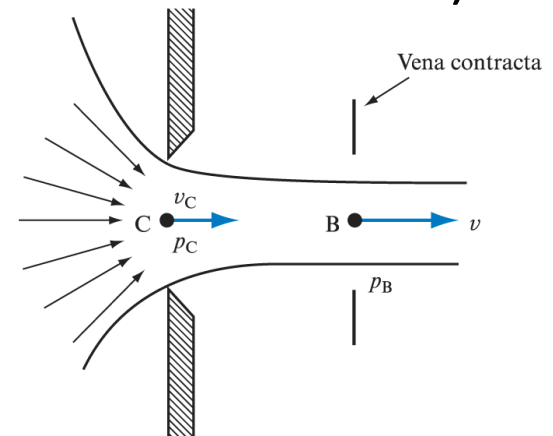
- **Two reasons for the difference between theoretical and actual discharges.**
- **FIRST:** the velocity of jet is less than the velocity calculated because there is ***losses of energy*** between A and B.

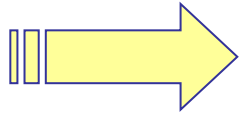
$$\text{Actual velocity at B} = C_v \times v = C_v \sqrt{2gH}$$

- **Where** C_v is the coefficient of velocity
- **SECOND:** The streamlines at the orifice contract reducing the area of flow. (This contraction is called the *vena contracta*.)

$$\text{Actual area of jet at B} = C_c \times A$$

- **Where** C_c is the coefficient of contraction





$$\begin{aligned}
 \text{Actual discharge} &= \text{Actual area at B} \times \text{Actual velocity at B} \\
 &= C_c A \times C_v \sqrt{2gH} \\
 &= C_c \times C_v A \sqrt{2gH}
 \end{aligned}$$

Note that: $C_d = C_c \times C_v$

These values are determined ***experimentally***, where:

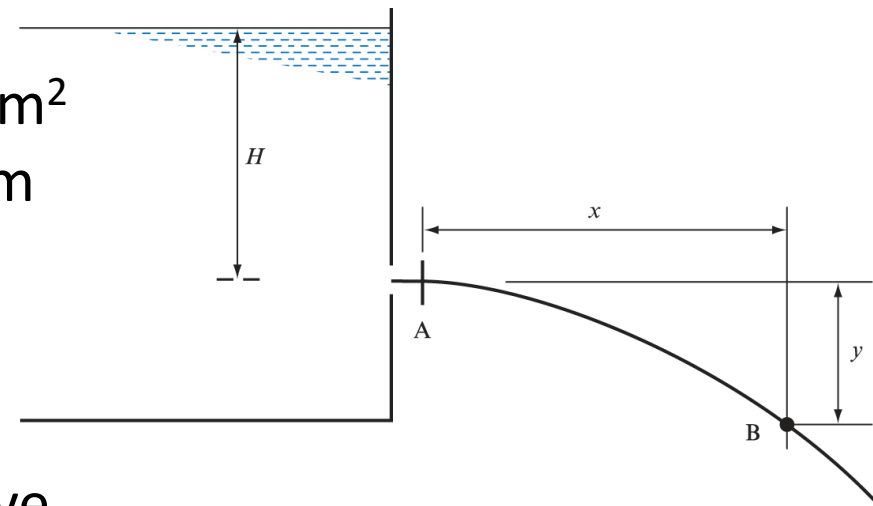
$$C_d = \frac{\text{Actual measured discharge}}{\text{Theoretical discharge}}$$

$$C_c = \frac{\text{Area of jet at vena contracta}}{\text{Area of orifice}}$$

$$C_v = \frac{\text{Velocity at vena contracta}}{\text{Theoretical velocity}}$$

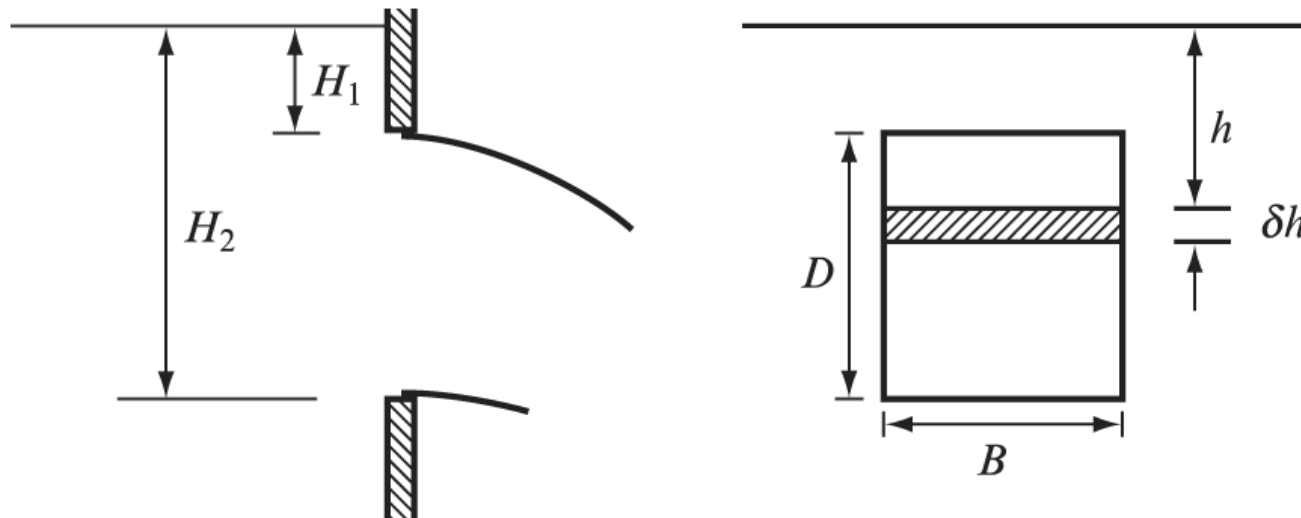
Example: (Ex 6.4, page 192 Textbook)

- A jet of water discharge horizontally into the atmosphere from an orifice as shown. Drive an expression for the actual velocity v of a jet at the vena contracta if the jet falls a distance y vertically in a horizontal distance x , measured from the vena contracta. If the head of water above the orifice is H , determine the coefficient of velocity.
- If the orifice has an area of 650 mm^2 and the jet falls a distance $y = 0.5 \text{ m}$ in a horizontal distance $x = 1.5 \text{ m}$.
- Calculate C_c , C_v , C_d . Given that the volume flow rate of flow is $0.117 \text{ m}^3/\text{min}$ and the head H above the orifice is 1.2 m



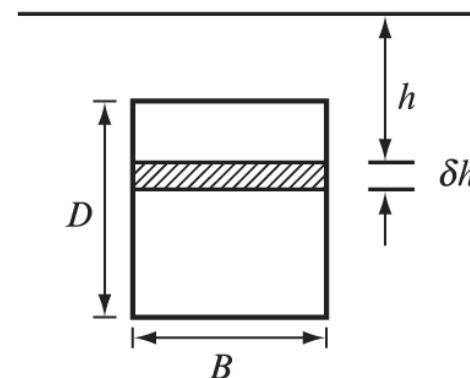
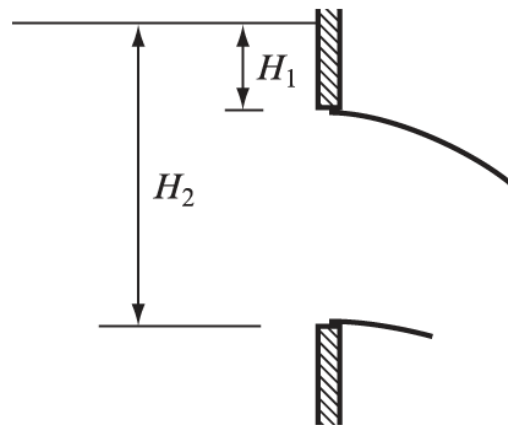
6.14 Theory of Large Orifices :

- It is an orifice with large vertical height.
- So that the head producing flow is substantially less at the top of the opening than at the bottom (and so do the velocity)
- The method adopted is to calculate the flow **through a thin horizontal strip** and then **integrate** from top to bottom to obtain the *theoretical discharge*



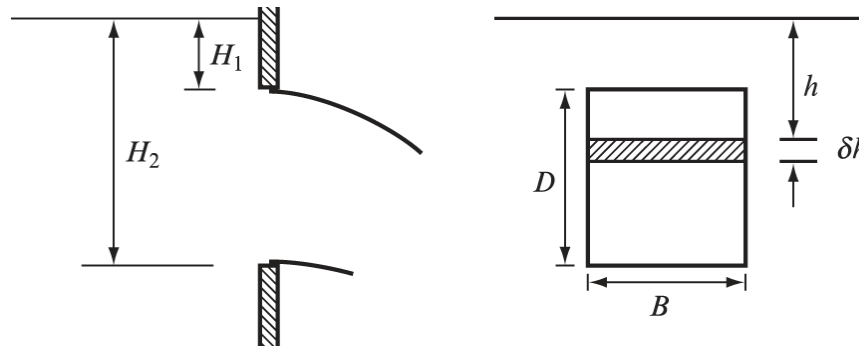
Example: (Ex 6.5, page 194 Textbook)

- A reservoir discharges through a rectangular sluice gate of width B and height D . the top and bottom of the opening are at depths H_1 and H_2 below the free surface.
- Derive an expression for the theoretical discharge through the opening.
 - If $H_1 = 0.4\text{m}$ and $B = 0.7\text{m}$ and $D = 1.5\text{m}$, find $Q_{\text{theoretical}}$.
 - What would be the percentage of error if the opening treated as a small orifice



Derivation :

- Consider a horizontal strip of height dh at a depth h below the free surface



Area of strip = $B \times dh$

Velocity through the strip = $\sqrt{2gh}$

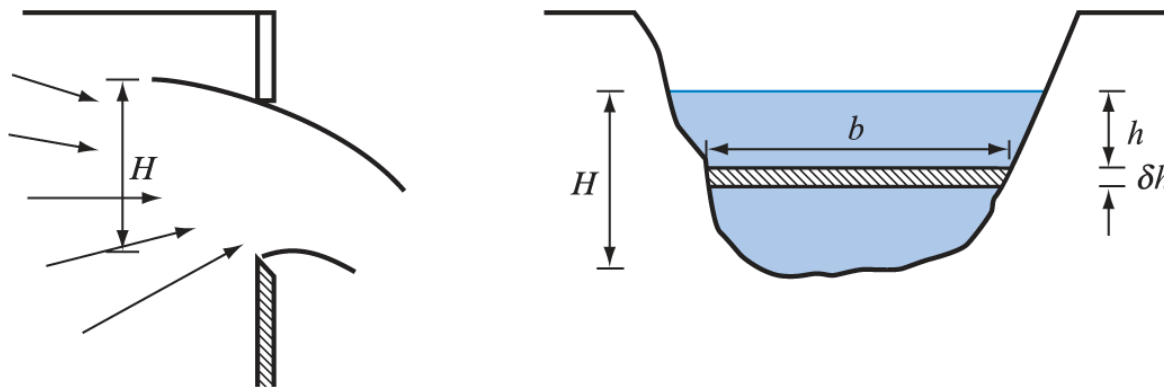
Discharge through the strip = $dQ = V \, dA = B\sqrt{2gh}dh$

⇒ $Q = B\sqrt{2g} \int_{H_1}^{H_2} h^{1/2} dh$

⇒ $Q = \frac{2}{3} B\sqrt{2g} (H_2^{3/2} - H_1^{3/2})$

6.15 Elementary Theory of Notches & Weirs:

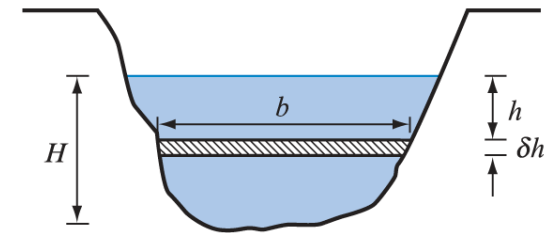
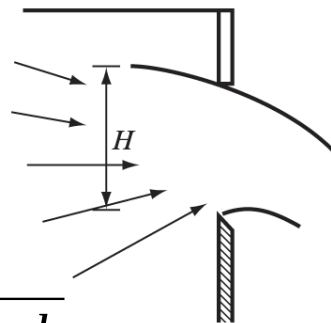
- A notch is an opening in the side of a tank or reservoir which extends above the surface of the liquid. (Large orifice with no upper edge)
- It is usually a device for measuring discharge.



- A weir is a notch on a larger scale - usually found in rivers.
- It is used as both a flow measuring device and a device to raise water levels.

A General Weir Equation (As in large orifice)

- To determine an expression for the theoretical flow through a notch we will consider a horizontal strip of width b and depth h below the free surface, as shown:



Area of strip $= b \times dh$

Velocity through the strip $= \sqrt{2gh}$

Discharge through the strip $= dQ = AV = B\sqrt{2gh} \, dh$

⇒ $Q_{\text{theoretical}} = \sqrt{2g} \int_0^H b h^{1/2} dh$

- Before the integration of the above equation, b must be expressed in terms of h

Rectangular Notch

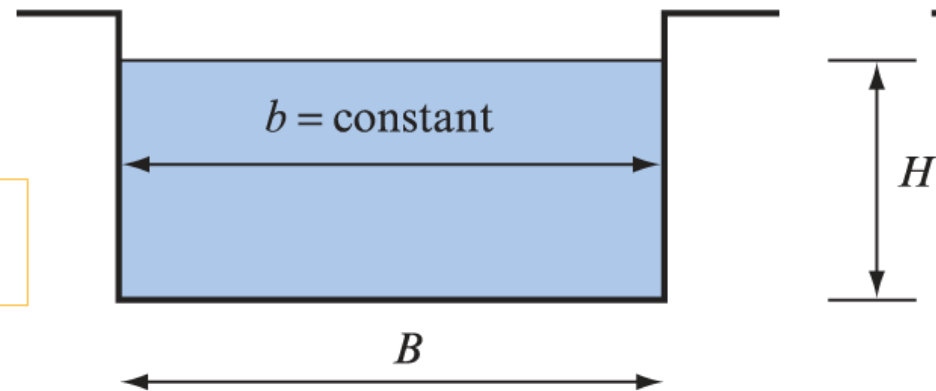
- For a rectangular notch the width does not change with depth so there is no relationship between b and depth h . We have the equation
- Put $b = \text{constant} = B$

$$Q_{\text{theoretical}} = \sqrt{2g} \int_0^H b h^{1/2} dh$$

$$\Rightarrow Q_{\text{theoretical}} = B\sqrt{2g} \int_0^H h^{1/2} dh$$

$$\Rightarrow Q_{\text{theoretical}} = \frac{2}{3} B\sqrt{2g} H^{3/2}$$

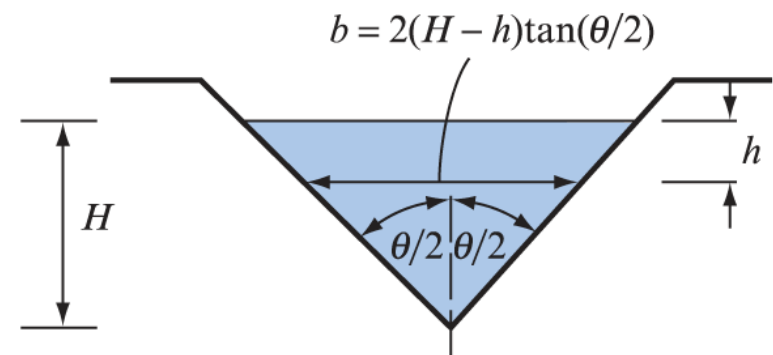
$$Q_{\text{Actual}} = C_d Q_{\text{Theoretical}}$$



Vee Notch

- For the “V” notch the relationship between width and depth is dependent on the angle of the V notch (θ).
- Put $b = 2(H-h) \tan(\theta/2)$

$$Q_{\text{theoretical}} = \sqrt{2g} \int_0^H b h^{1/2} dh$$



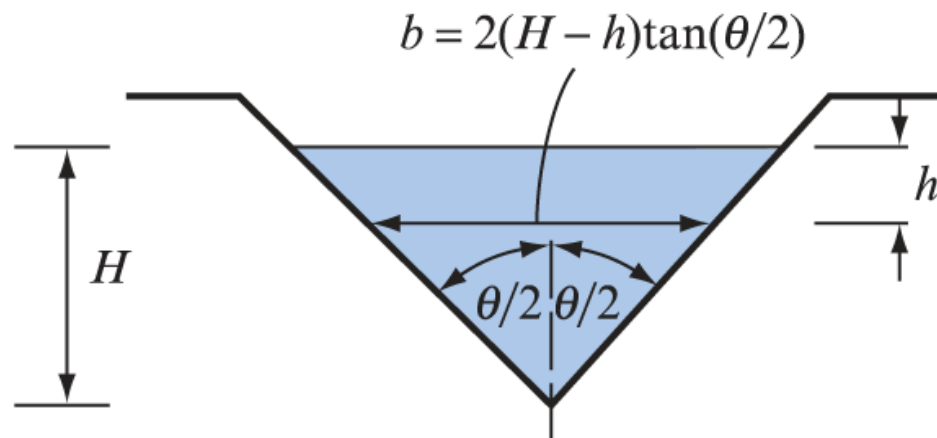
$$\Rightarrow Q_{\text{theoretical}} = 2\sqrt{2g} \tan\left(\frac{\theta}{2}\right) \int_0^H (H-h) h^{1/2} dh$$

$$\Rightarrow Q_{\text{theoretical}} = \frac{8}{15} \sqrt{2g} \tan\left(\frac{\theta}{2}\right) H^{5/2}$$

$$Q_{\text{Actual}} = C_d Q_{\text{Theoretical}}$$

Example: (Ex 6.6, page 196 Textbook)

- It is proposed to use a notch to measure the flow of water from a reservoir and it is estimated that the error in measuring the head above the bottom of the notch could be 1.5mm.
- For a discharge of $0.28\text{m}^3/\text{s}$, determine the percentage error which may occur, using right triangular notch with a coefficient of discharge of 0.6



6.16 The Power of a Stream of Fluid :

- A stream of fluid can do work as a result of its pressure p , velocity v and elevation z .
- Remember that the total energy per unit weight H of the fluid is given by:

$$\text{Energy per unit weight} = \frac{p}{\rho g} + \frac{V^2}{2g} + z$$

- The power of the stream can be calculated as:

$$\text{Power} = \text{Energy per unit Time}$$

$$\Rightarrow \text{Power} = \frac{\text{Weight}}{\text{Unit time}} \times \frac{\text{Energy}}{\text{unit weight}}$$

$$\Rightarrow \text{Power} = \rho g Q H = \rho g Q \left(\frac{p}{\rho g} + \frac{V^2}{2g} + z \right)$$

Example: (Ex 6.8, page 199 Textbook)

Water is drawn from a reservoir, in which the water level is 240m above datum, at rate of $0.13\text{m}^3/\text{s}$ the outlet of the pipeline is at datum level and is fitted with a nozzle to produce a high speed jet to drive a turbine of Pelton wheel type. If the velocity of jet is 66m/s , **calculate:**

1. The power of the jet.
2. The power supplied from the reservoir
3. The head used to overcome losses.
4. The efficiency of the pipeline and nozzle in transmitted power.

Example:

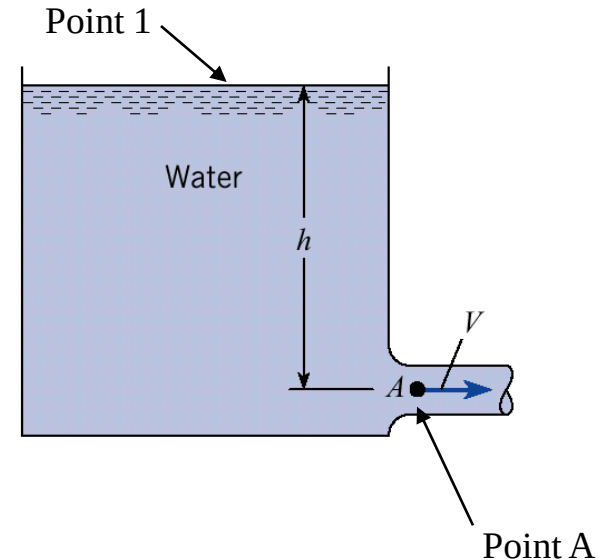
- **Given:** Velocity in outlet pipe from reservoir is 6 m/s and $h = 15$ m.
- **Find:** Pressure at A.
- **Solution:** Bernoulli equation

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_A}{\gamma} + z_A + \frac{V_A^2}{2g}$$

$$\frac{0}{\gamma} + h + \frac{0}{2g} = \frac{p_A}{\gamma} + 0 + \frac{V_A^2}{2g}$$

$$p_A = \gamma \left(h - \frac{V_A^2}{2g} \right) = 9810 \left(15 - \frac{18}{9.81} \right)$$

$$p_A = 129.2 \text{ kPa}$$



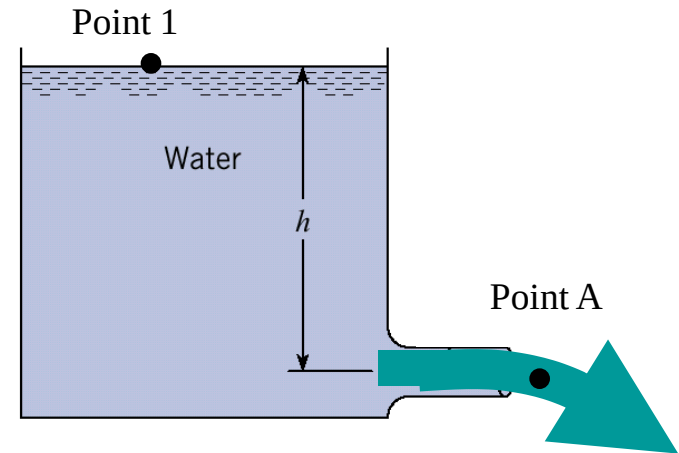
Example:

- **Given:** $D=30$ in, $d=1$ in, $h=4$ ft
- **Find:** V_A
- **Solution:** Bernoulli equation

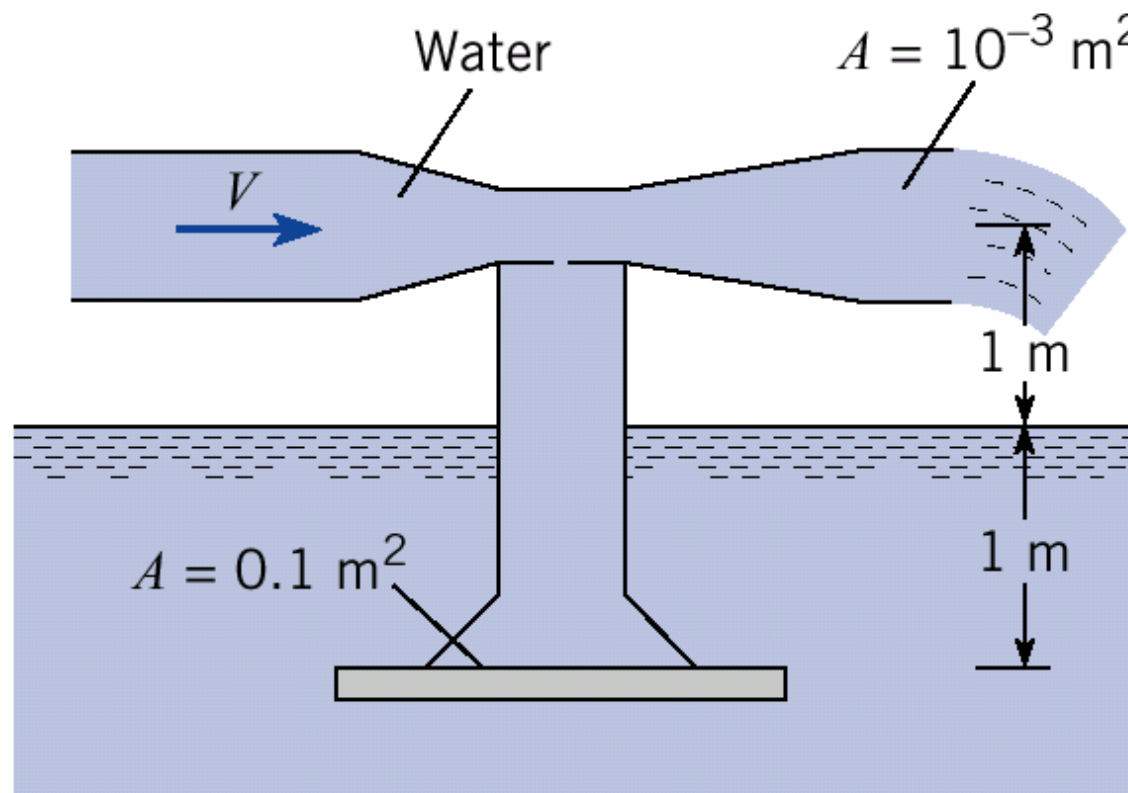
$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_A}{\gamma} + z_A + \frac{V_A^2}{2g}$$

$$\frac{0}{\gamma} + h + \frac{0}{2g} = \frac{0}{\gamma} + 0 + \frac{V_A^2}{2g}$$

$$\begin{aligned} V_A &= \sqrt{2gh} \\ &= 16 \text{ ft/s} \end{aligned}$$



Example:



Example:

- **Given:** Velocity in circular duct = 100 ft/s, air density = 0.075 lbm/ft³.
- **Find:** Pressure change between circular and square section.
- **Solution:** Continuity equation

$$V_c A_c = V_s A_s$$

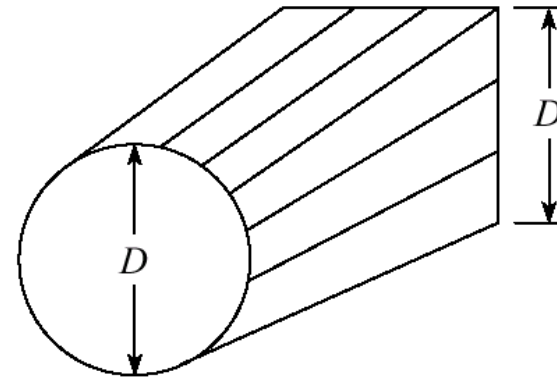
$$100\left(\frac{\pi}{4} D^2\right) = V_s D^2$$

$$V_s = 100\left(\frac{\pi}{4}\right) = 78.54 \text{ ft/s}$$

- Bernoulli equation

$$\frac{p_c}{\gamma} + z_c + \frac{V_c^2}{2g} = \frac{p_s}{\gamma} + z_s + \frac{V_s^2}{2g}$$

$$p_c - p_s = \frac{\rho}{2} (V_s^2 - V_c^2)$$



Air conditioning ($\sim 60^\circ\text{F}$)

$$\begin{aligned} p_c - p_s &= \frac{0.075 \text{ lbm/ft}^3}{2 * 32.2 \text{ lbm/slug}} (78.54^2 - 100^2) \\ &= -4.46 \text{ lbf/ft}^2 \end{aligned}$$

