

Chapter 2

Analysis of Statically Determinate Structures



Idealized Structure

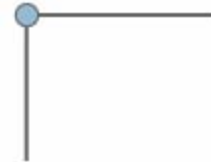
- *In real sense exact analysis of a structure can never be carried out.*

Support connection

- *Pin support and pin connection*



pin support

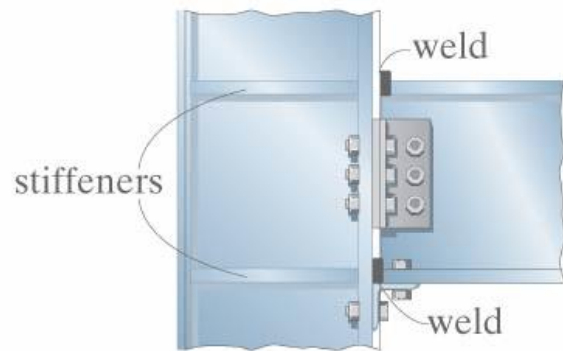


pin-connected joint

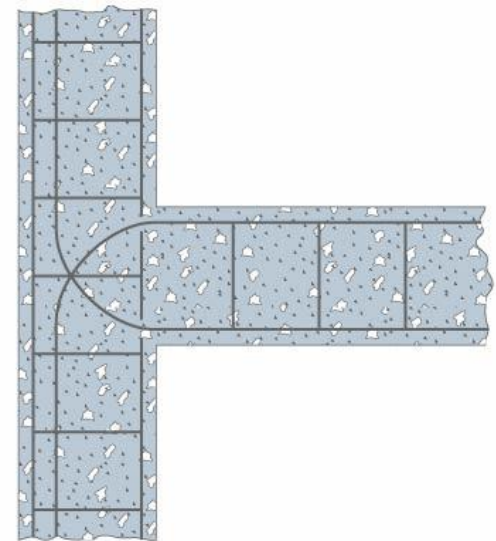


typical “pin-supported” connection (metal)

- *Fixed support & fixed connection*

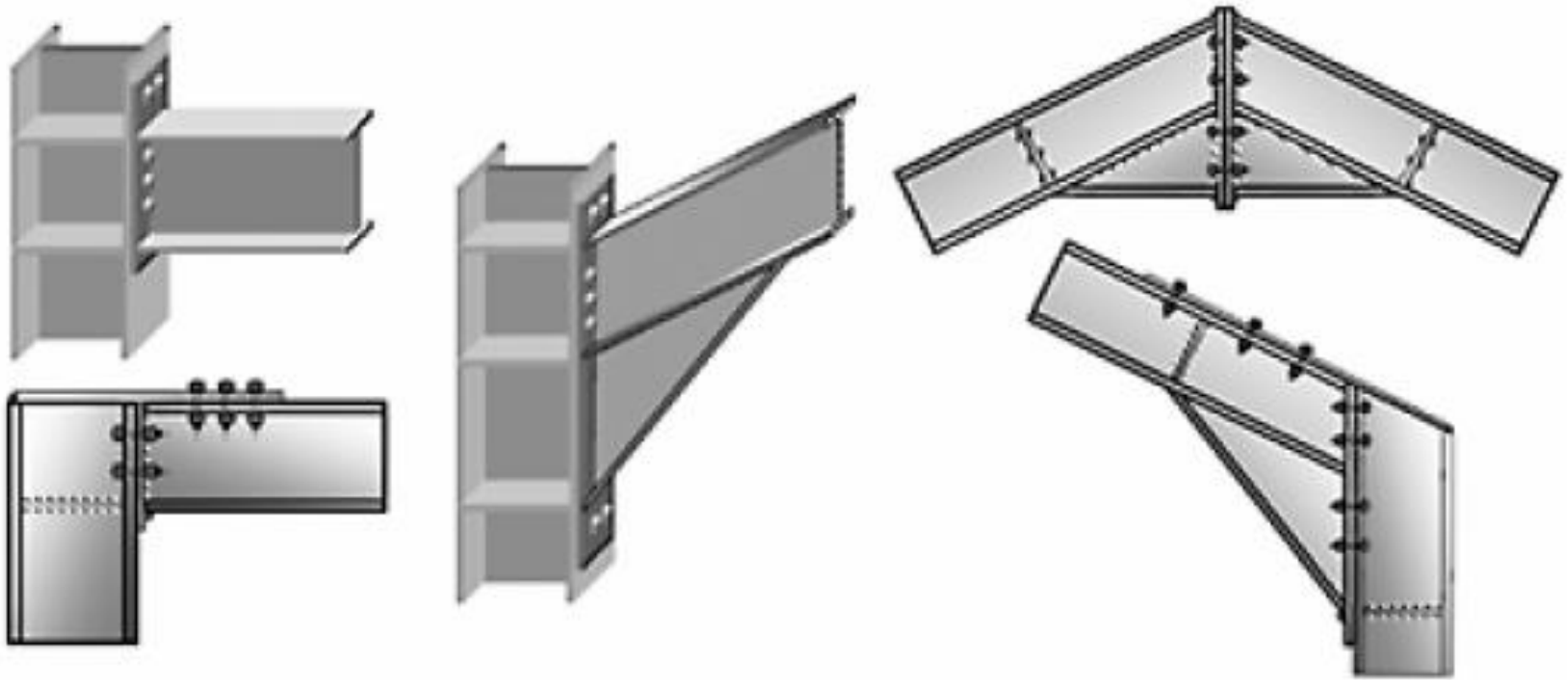


typical “fixed-sustained” connection (metal)



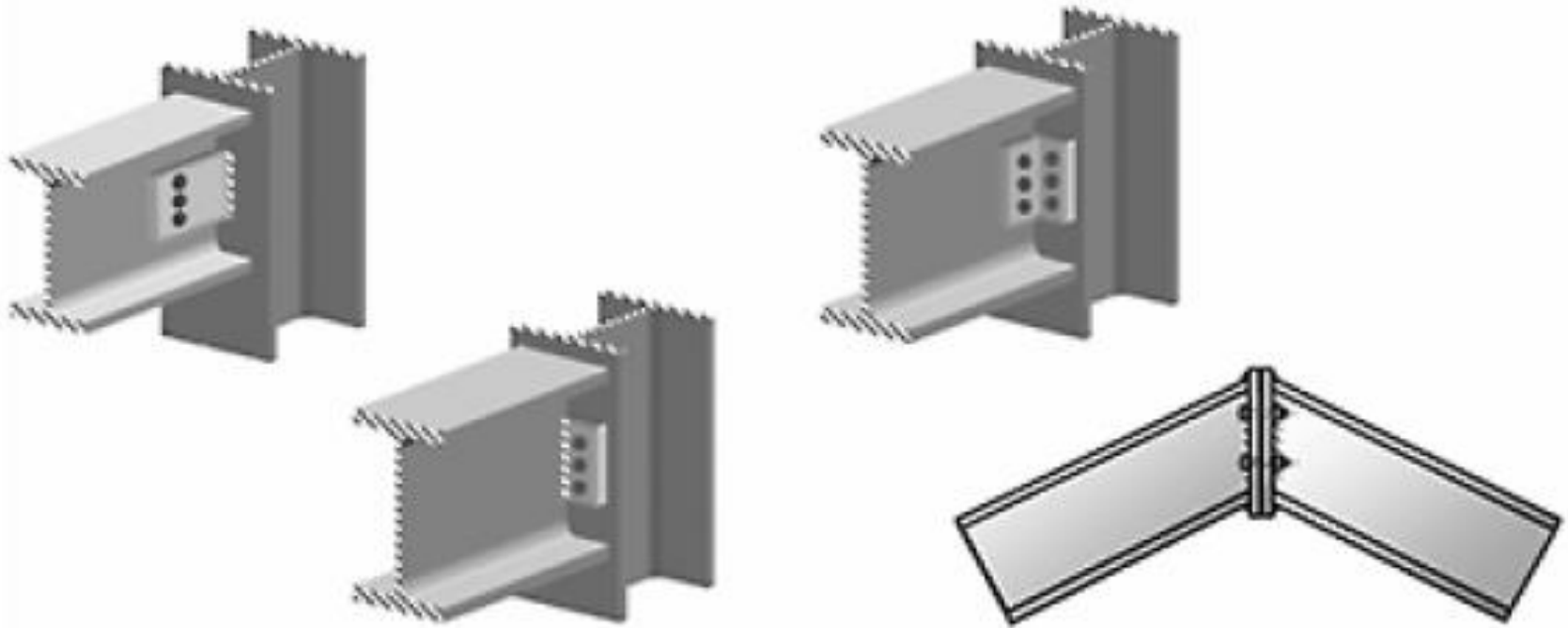
typical “fixed-sustained” connection (concrete)

- *Typical fixed connection*



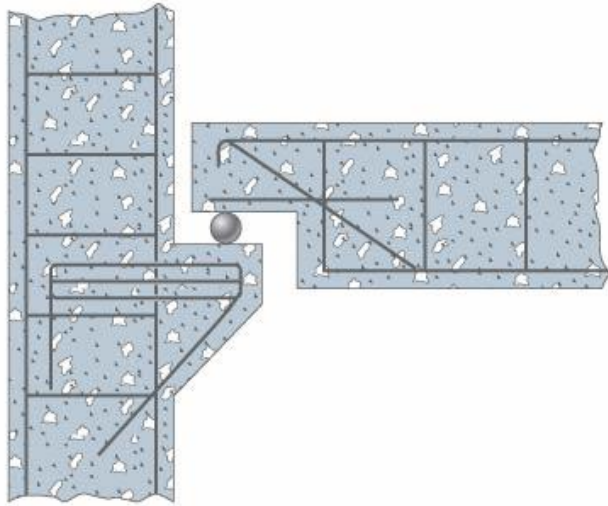
(a) Typical moment connections between members

- *Typical pin connection*



(b) Typical simple connections between members

- *Roller support*



typical “roller-supported” connection (concrete)





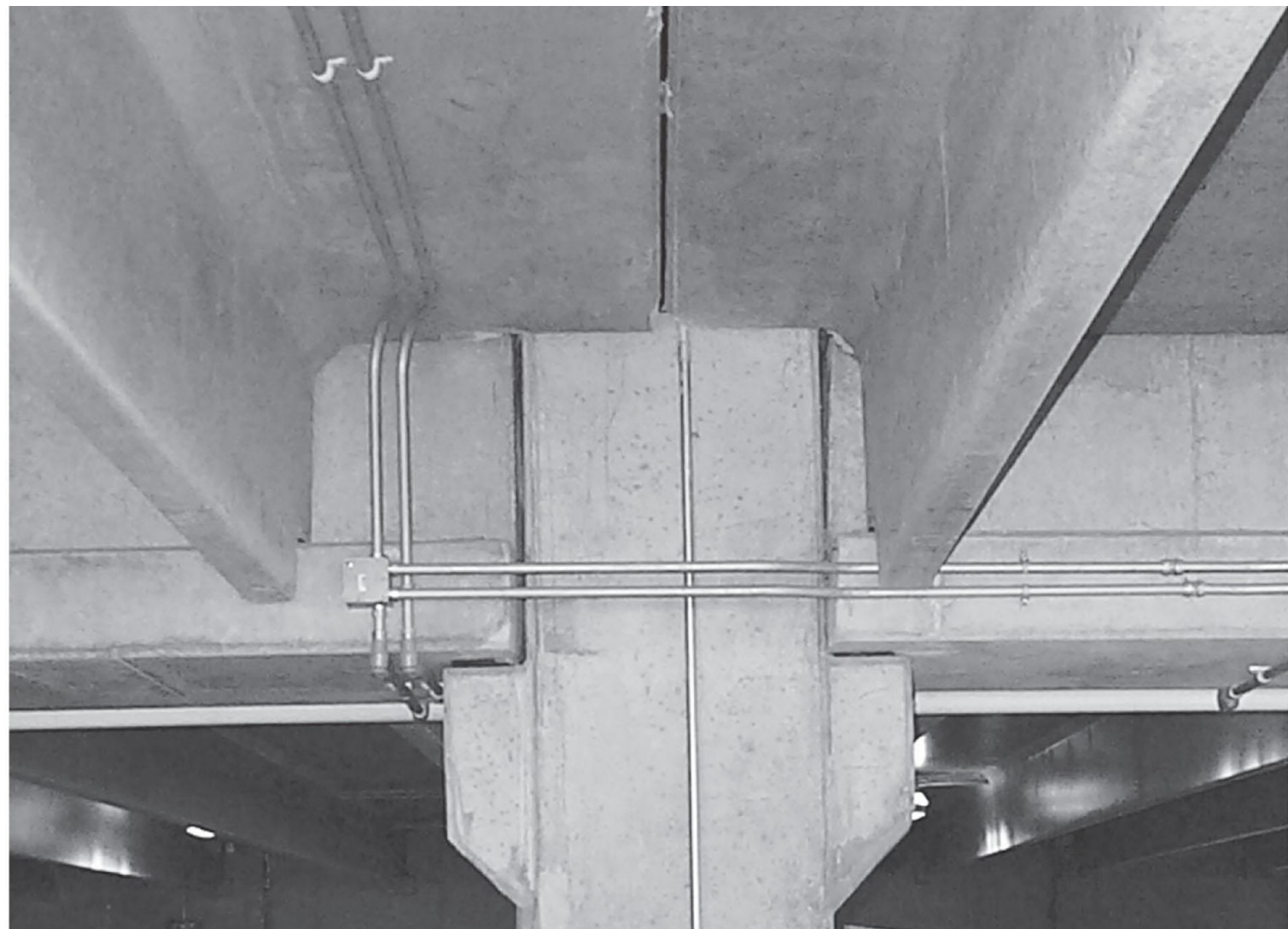
The short link is used to connect the two girders of the highway bridge and allow for thermal expansion of the deck.

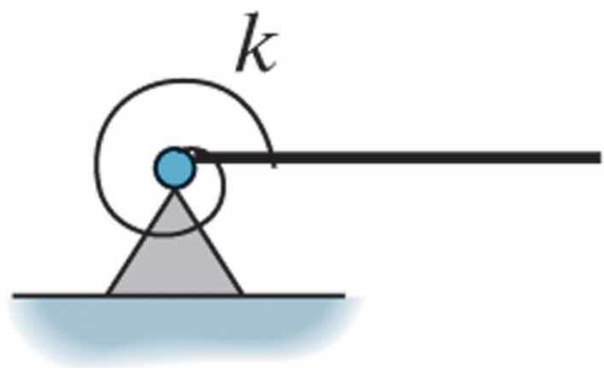


Steel pin support.

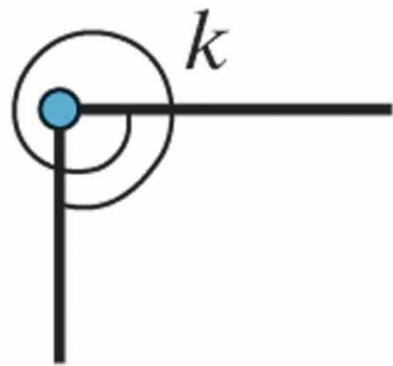


Notice that the deck of this concrete bridge is made so that one section can be considered roller supported on the other section.





torsional spring support



torsional spring joint

(c)

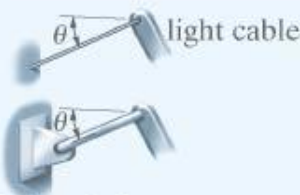
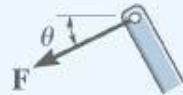
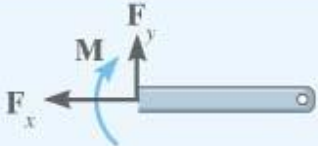




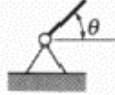
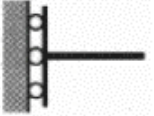
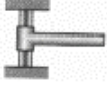
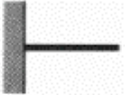
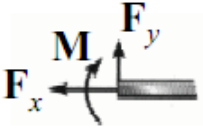




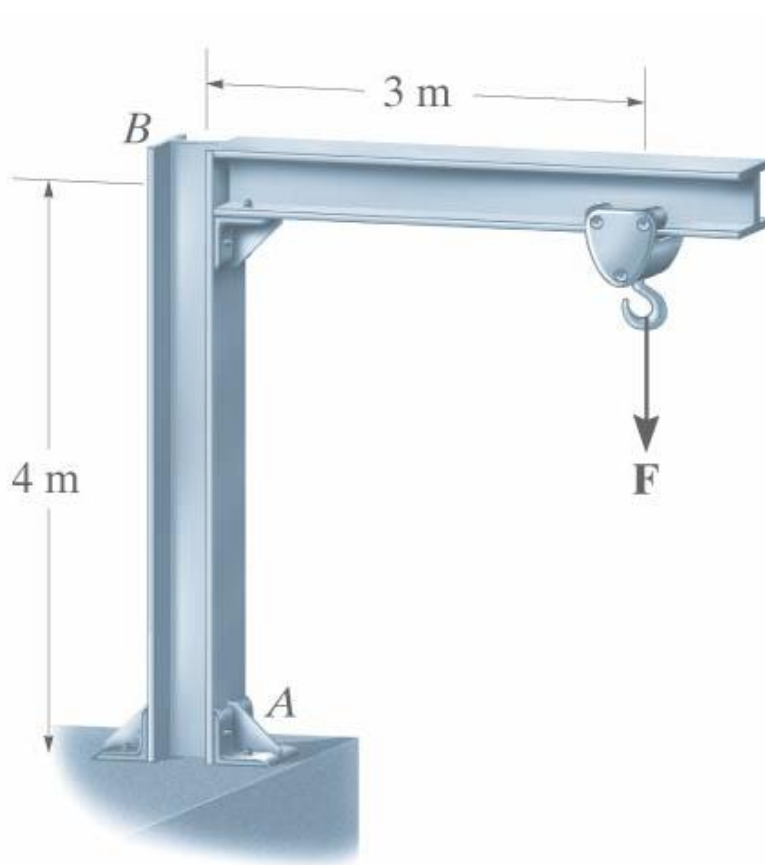
Support for coplanar structures

Type of Connection	Idealized Symbol	Reaction	Number of Unknowns
(1)  light cable weightless link			One unknown. The reaction is a force that acts in the direction of the cable or link.
(2)  rollers rocker			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(5)  smooth pin or hinge			Two unknowns. The reactions are two force components.
(7)  fixed support			Three unknowns. The reactions are the moment and the two force components.

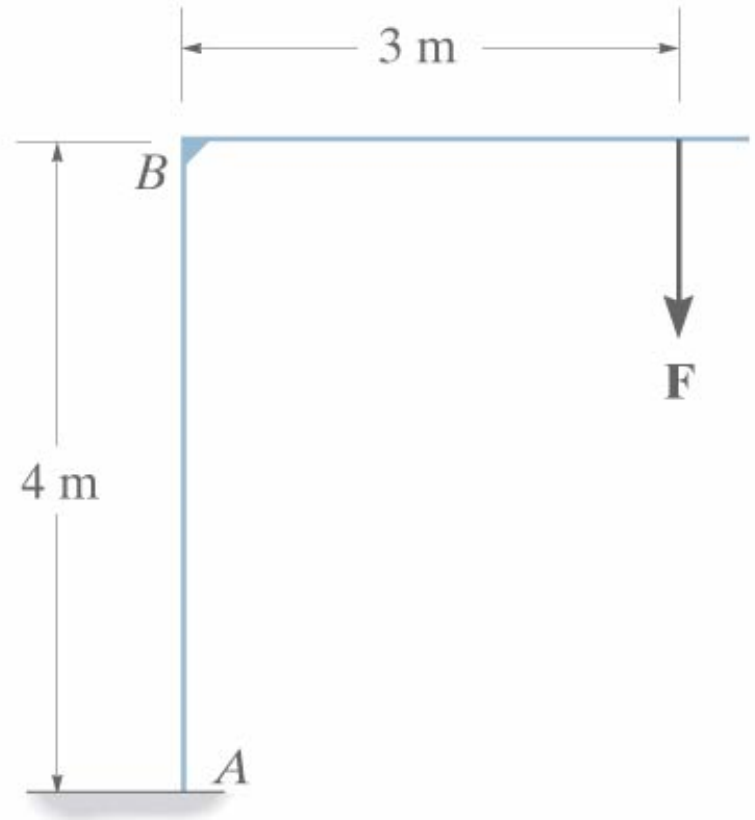
Support for coplanar structures

Type of Connection	Idealized Symbol	Reaction	Number of Unknowns
(5)	 		Two unknowns. The reactions are two force components.
(6)	  		Two unknowns. The reactions are a force and moment.
(7)	 		Three unknowns. The reactions are the moment and the two force components.

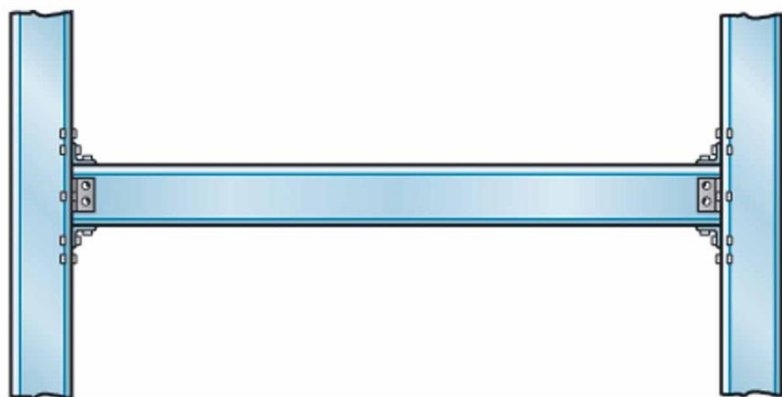
Idealized Structure



actual structure



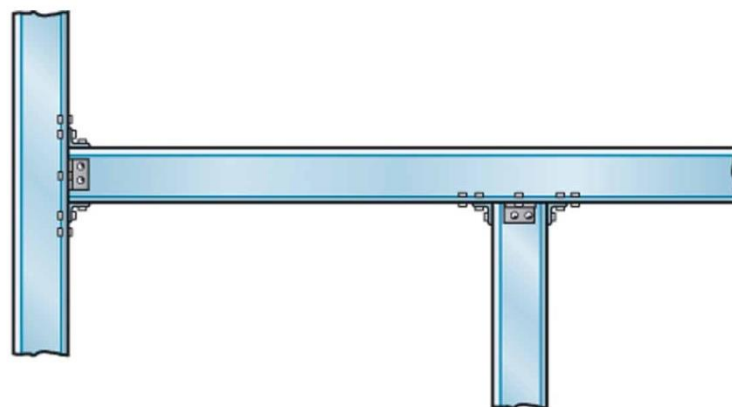
idealized structure



fixed-connected beam



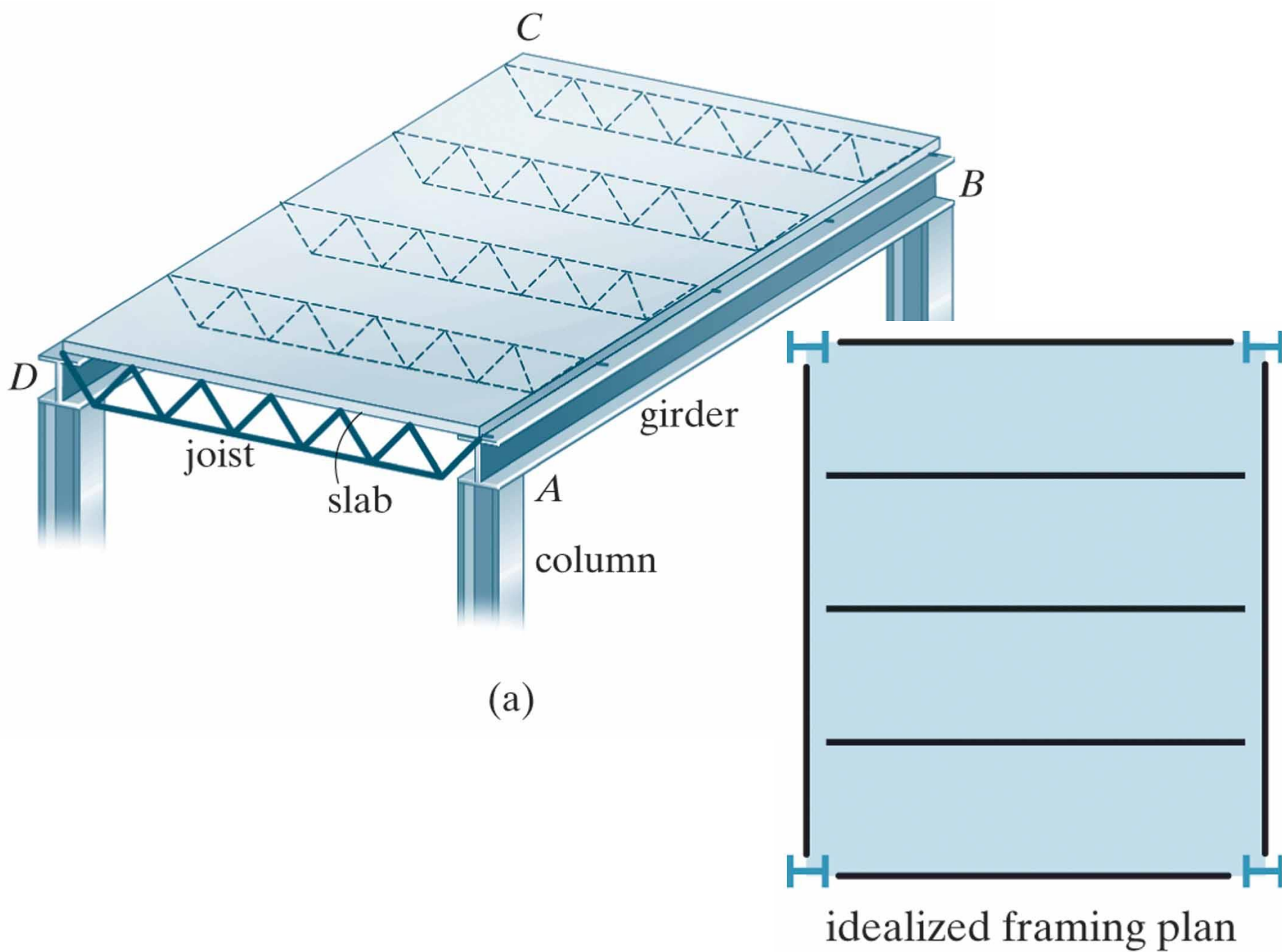
idealized beam

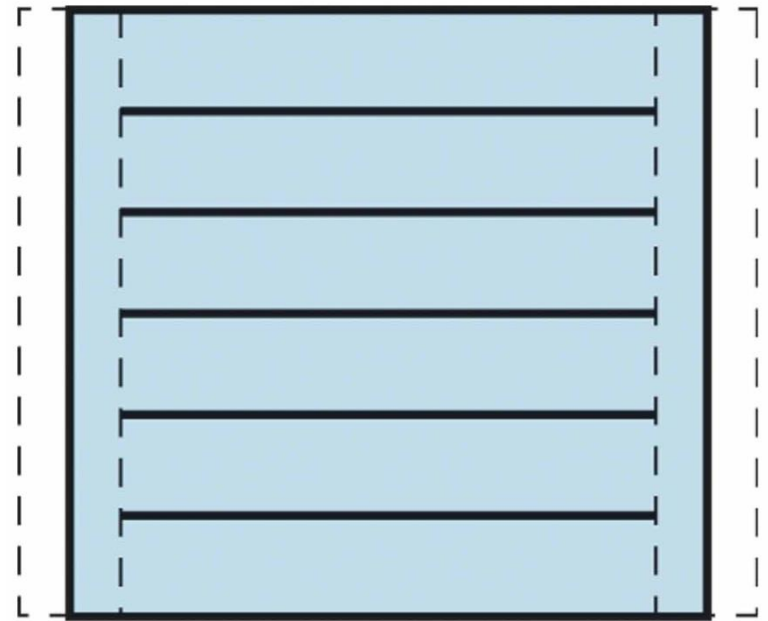
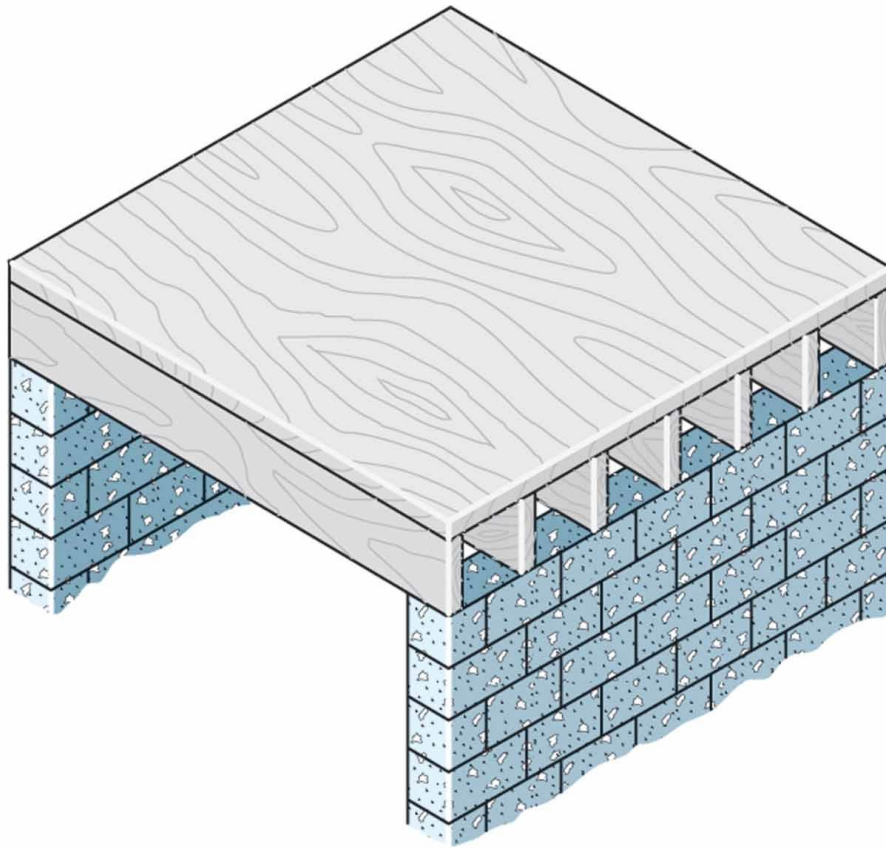


fixed-connected overhanging beam



idealized beam



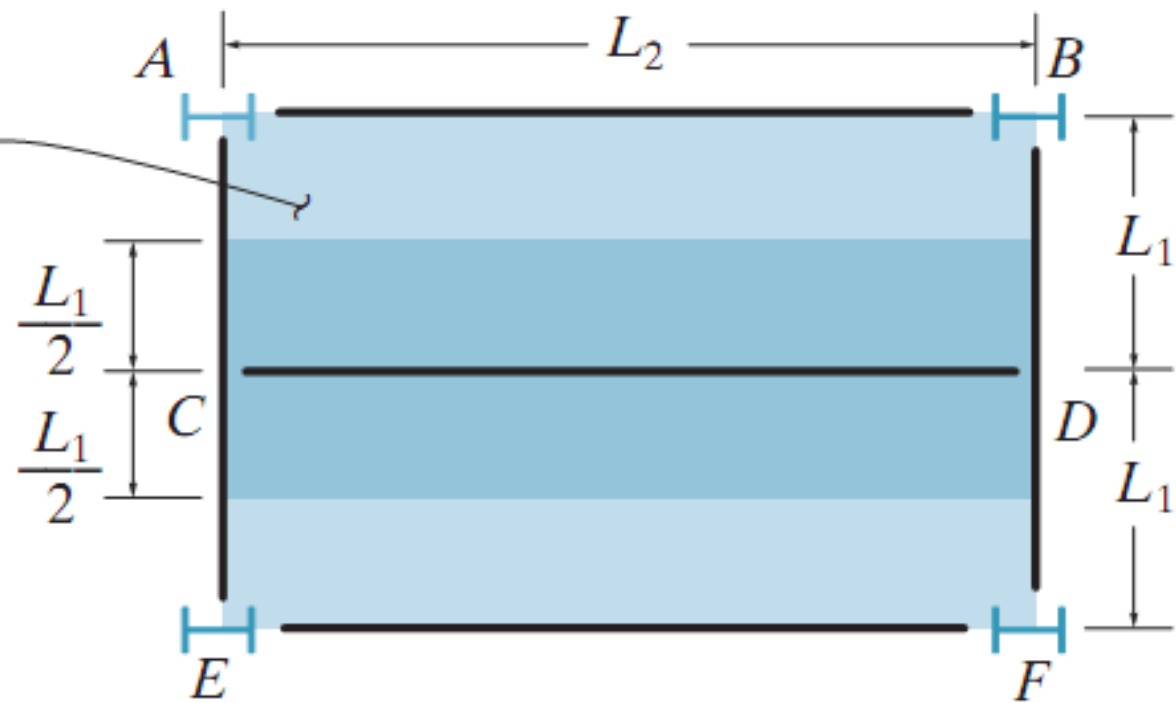


idealized framing plan

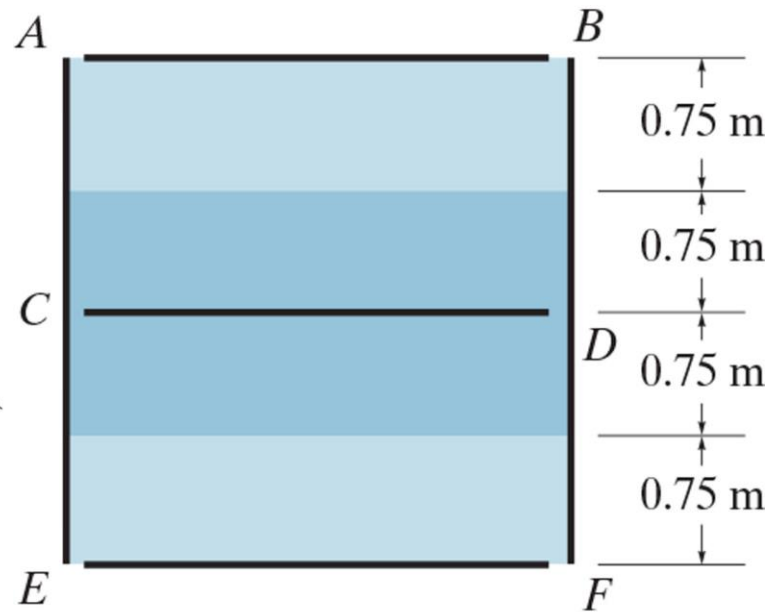
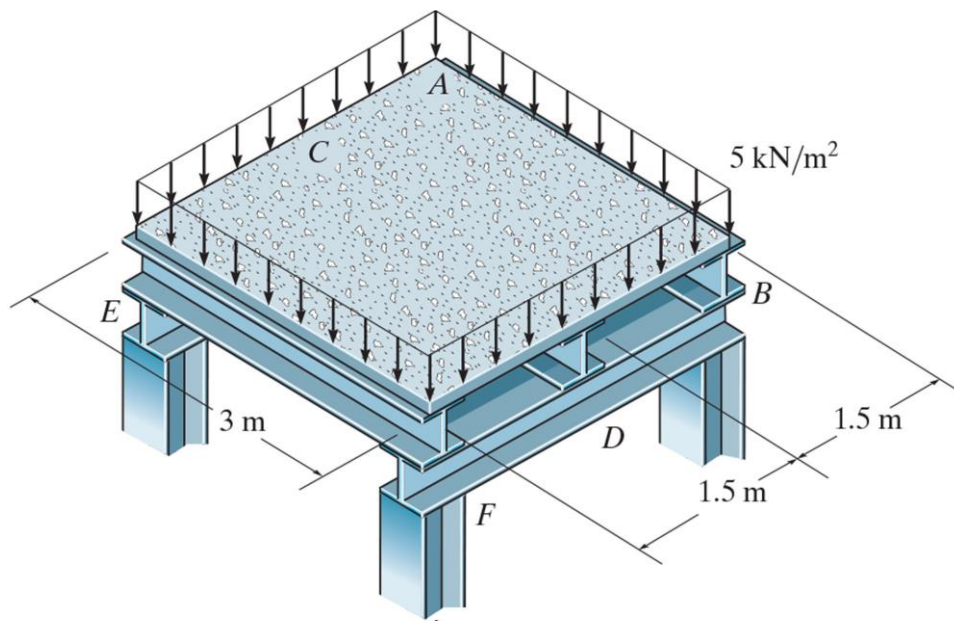




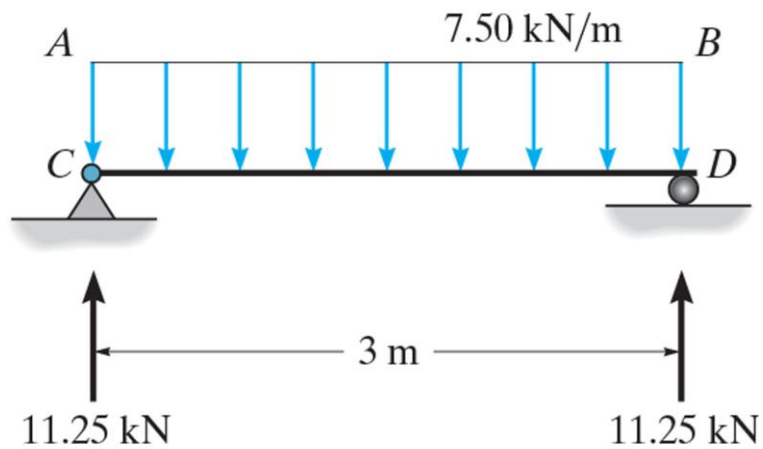
concrete slab is
reinforced in
two directions,
poured on plane
forms



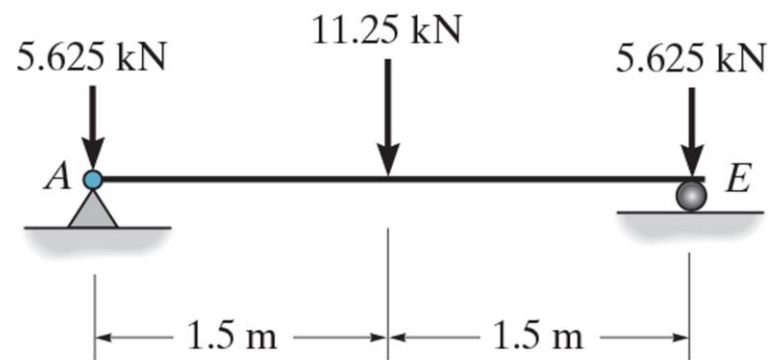
idealized framing plan
for one-way slab action
requires $L_2/L_1 > 2$



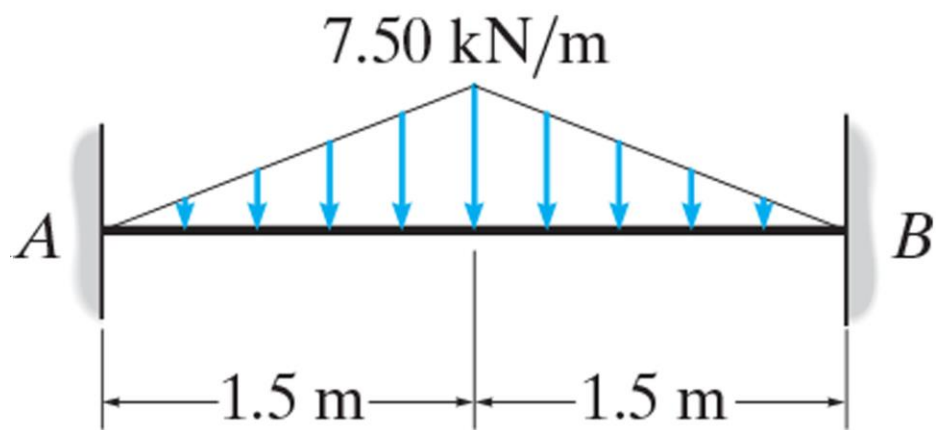
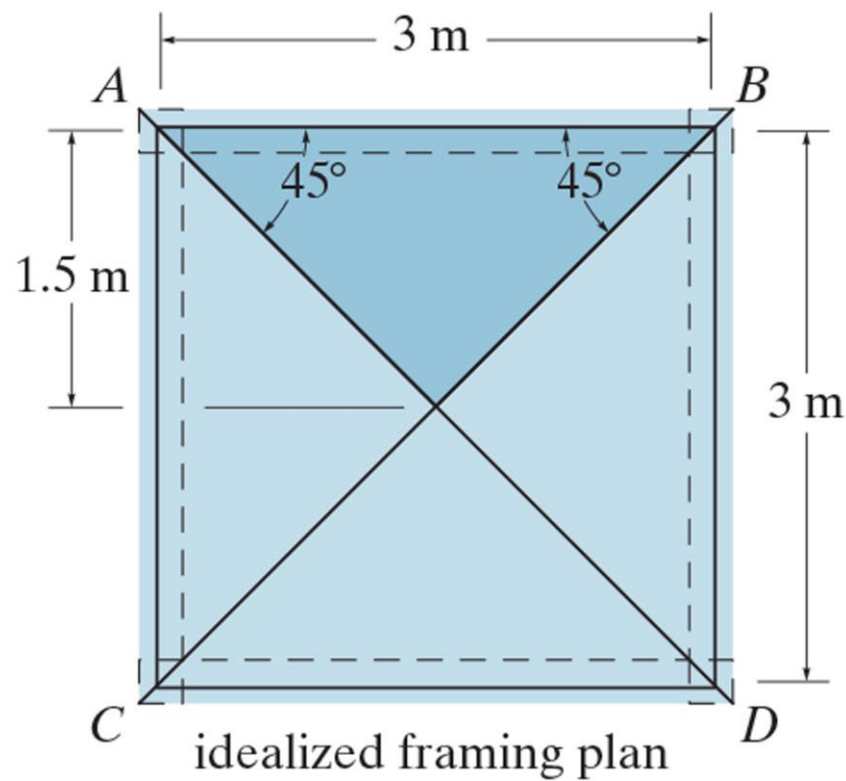
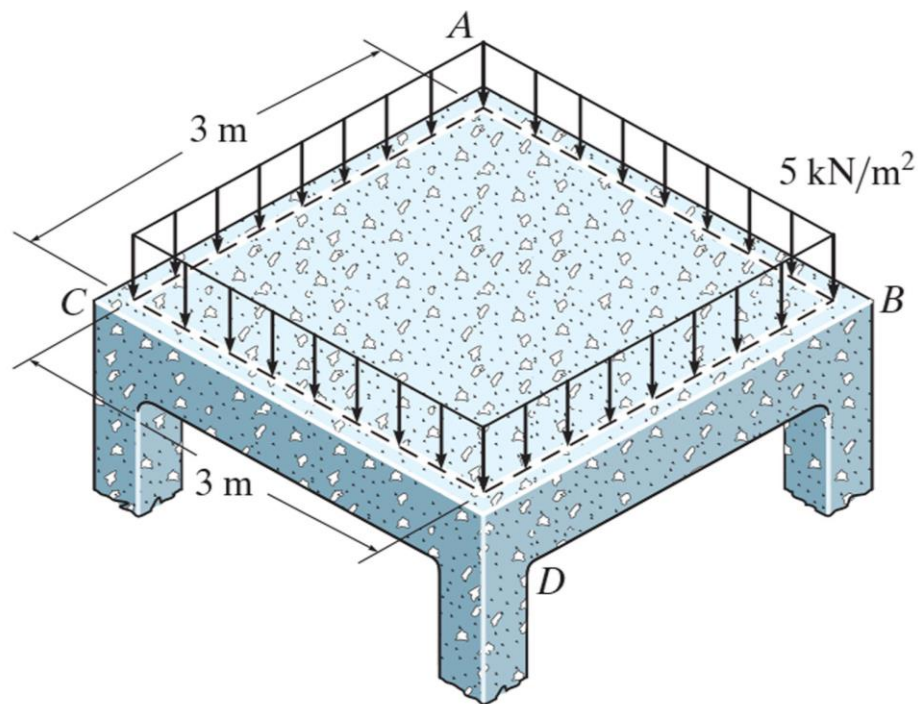
idealized framing plan



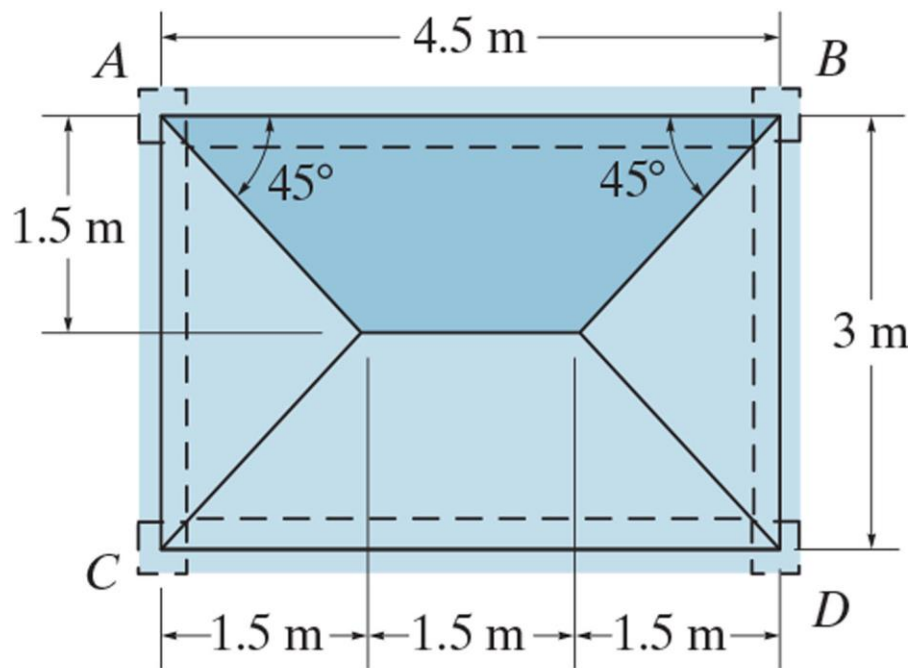
idealized beam



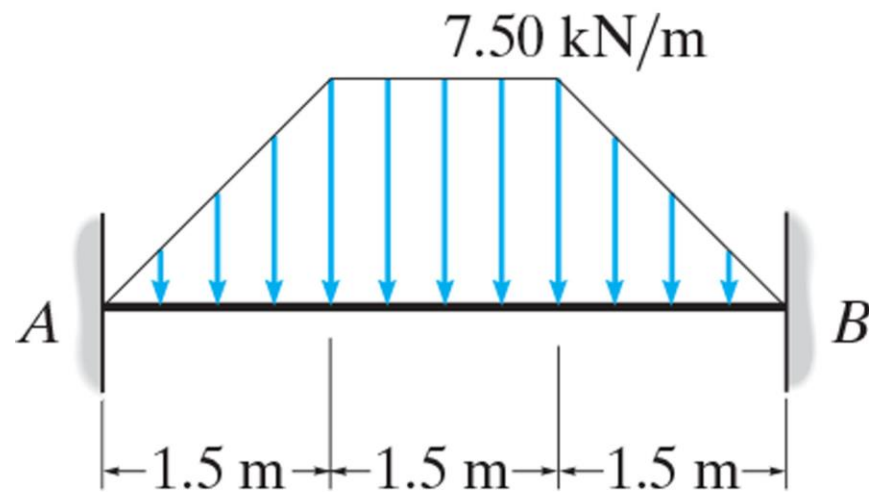
idealized girder



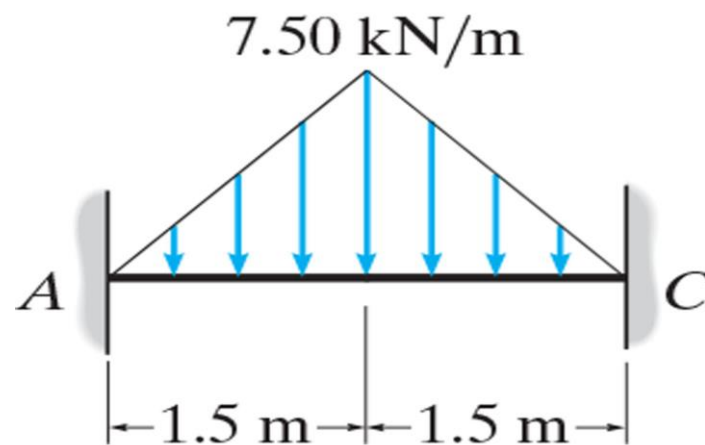
idealized beam



idealized framing plan



idealized beam



idealized beam

EXAMPLE 2-1

The floor of a classroom is supported by the bar joists shown in Fig. 2-15a. Each joist is 15 ft long and they are spaced 2.5 ft on centers. The floor itself is made from lightweight concrete that is 4 in. thick. Neglect the weight of the joists and the corrugated metal deck, and determine the load that acts along each joist.

Solution

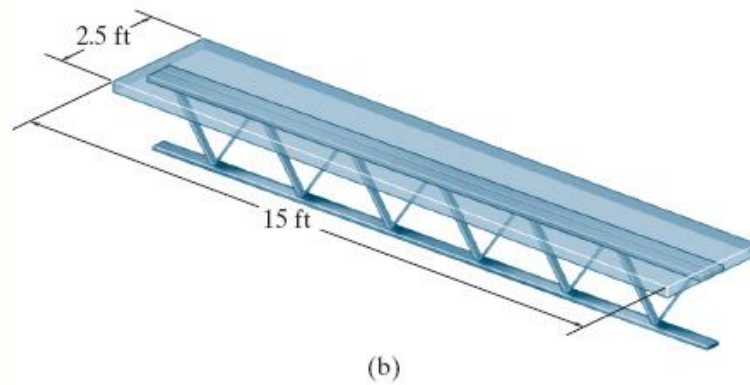
The dead load on the floor is due to the weight of the concrete slab. From Table 1-3 for 4 in. of lightweight concrete it is $(4)(8 \text{ lb/ft}^2) = 32 \text{ lb/ft}^2$. From Table 1-4, the live load for a classroom is 40 lb/ft^2 . Thus the total floor load is $32 \text{ lb/ft}^2 + 40 \text{ lb/ft}^2 = 72 \text{ lb/ft}^2$. For the floor system, $L_1 = 2.5 \text{ ft}$ and $L_2 = 15 \text{ ft}$. Since $L_2/L_1 > 1.5$ the concrete slab is treated as a one-way slab. The tributary area for each joist is shown in Fig. 2-15b. Therefore the uniform load along its length is

$$w = 72 \text{ lb/ft}^2(2.5 \text{ ft}) = 180 \text{ lb/ft}$$

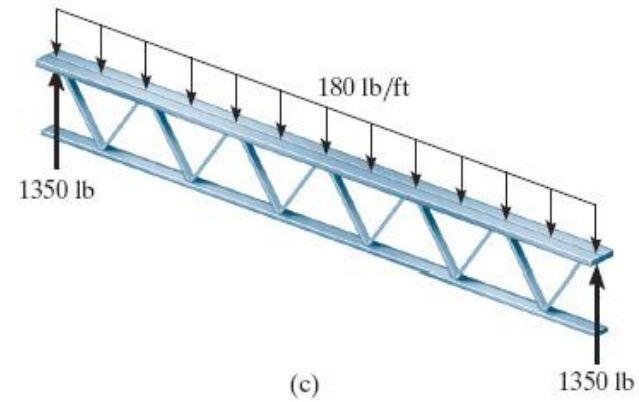
This loading and the end reactions on each joist are shown in Fig. 2-15c.



(a)



(b)



(c)

Fig. 2-15

Example 2.1

EXAMPLE 2.1

The floor of a classroom is to be supported by the bar joists shown in the photo. Each joist is 4.5 m long and they are spaced 0.75 m apart. The floor itself is to be made from lightweight concrete that is 100 mm thick. Neglect the weight of the joists and the corrugated metal deck, and determine the load that acts along each joist.

SOLUTION

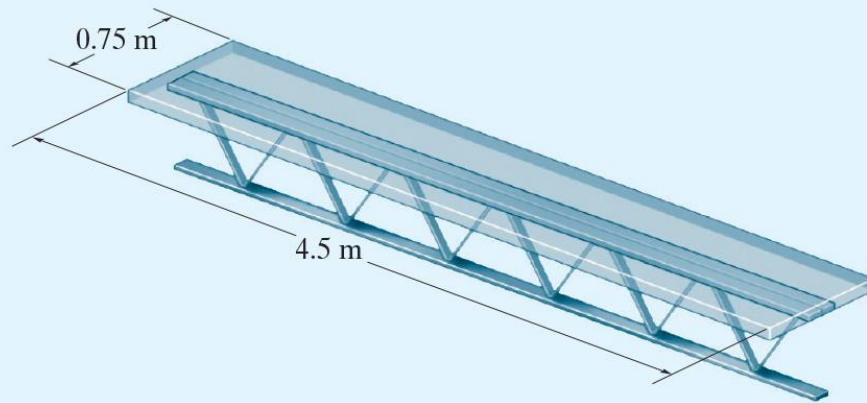
The dead load on the floor is due to the weight of the concrete slab. From Table 1.3 for 100 mm of lightweight concrete it is $(100 \text{ mm})(0.015 \text{ kN/m}^2 \cdot \text{mm}) = 1.50 \text{ kN/m}^2$. From Table 1.4, the live load for a classroom is 1.92 kN/m^2 . Thus the total floor load is $1.50 \text{ kN/m}^2 + 1.92 \text{ kN/m}^2 = 3.42 \text{ kN/m}^2$. For the floor system, where $L_2 > L_1$, we have $L_2 = 4.5 \text{ m}$ and $L_1 = 0.75 \text{ m}$. Since $L_2/L_1 > 2$ the concrete slab is treated as a one-way slab. The tributary area for each joist is shown in Fig. 2-15a. Therefore the uniform load along its length is

$$w = (3.42 \text{ kN/m}^2)(0.75 \text{ m}) = 2.565 \text{ kN/m}$$

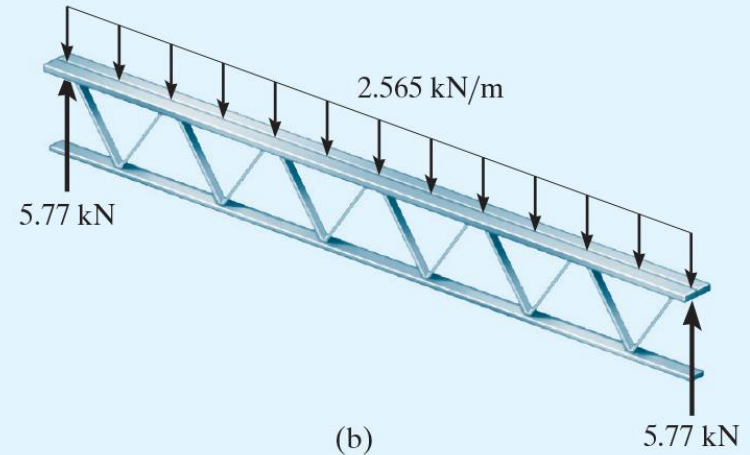


Example 2.1

This loading and the end reactions on each joist are shown in Fig. 2-15b.



(a)

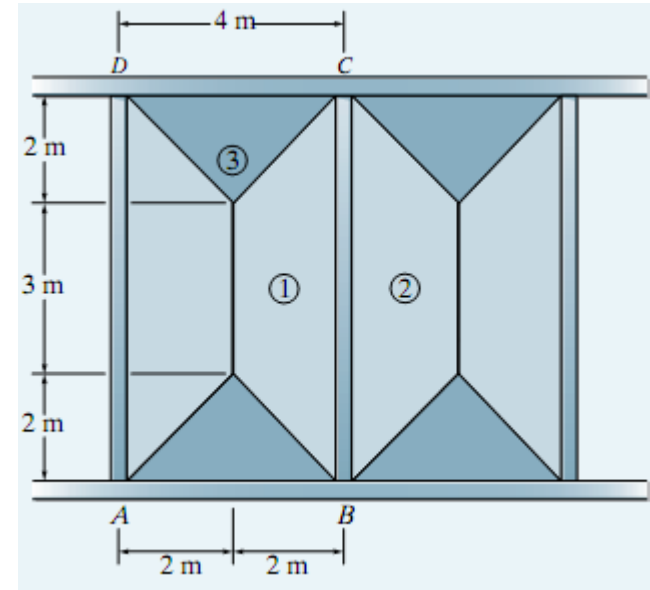
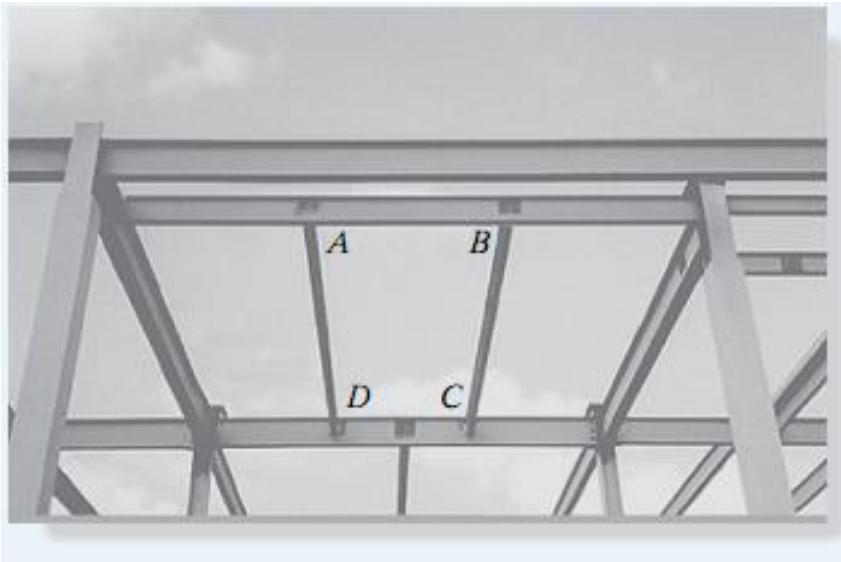


(b)

Fig. 2-15

Example 2.2

The flat roof of the steel-frame building shown in the photo is intended to support a total load of 2 kN/m^2 over its surface. Determine the roof load within region ABCD that is transmitted to beams BC and DC.

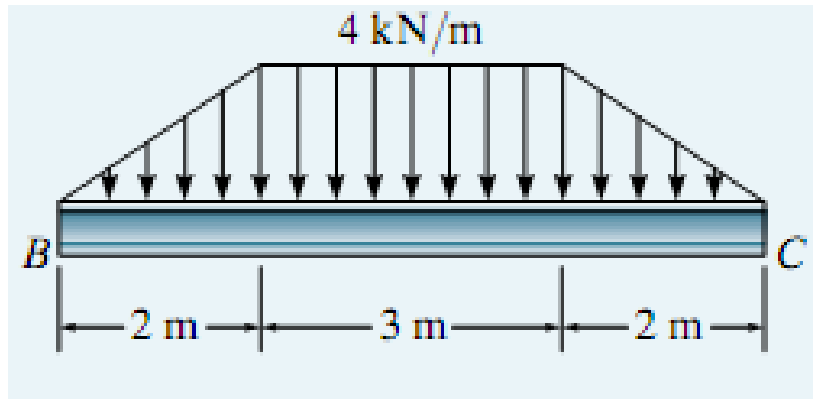


$$L_2 = 7\text{m and } L_1 = 4\text{m}$$

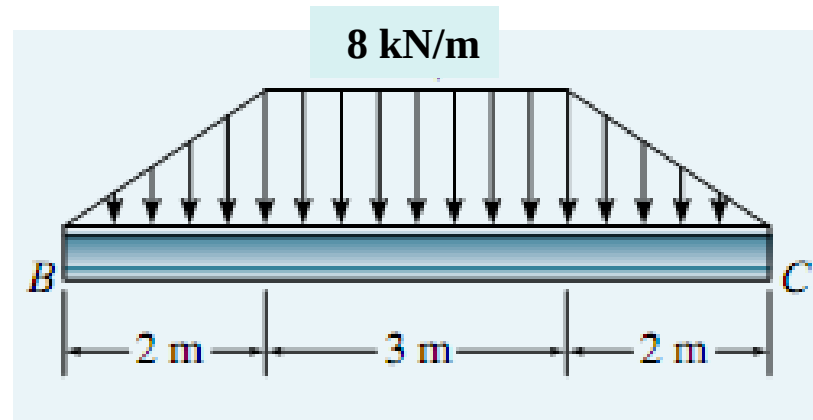
$$L_2 / L_1 = 7 / 4 = 1.75 < 2 \Rightarrow \text{Two way slab action}$$

$$\text{The peak intensity} = (2\text{kN} / \text{m}^2)(2) = 4\text{kN} / \text{m}$$

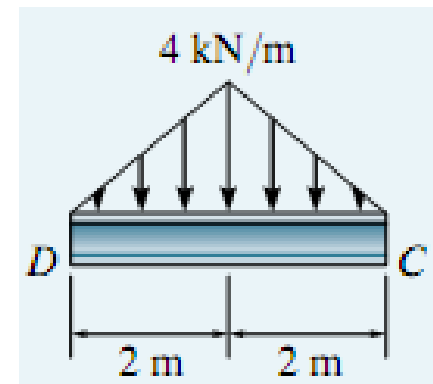
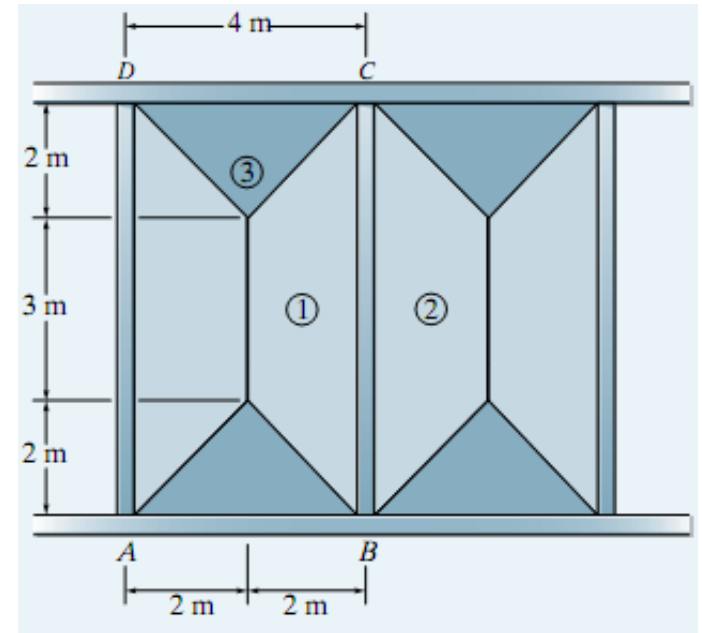
Example 2.2



Loading on Beam BC from Area 1



Loading on Beam BC from Area 1 & 2



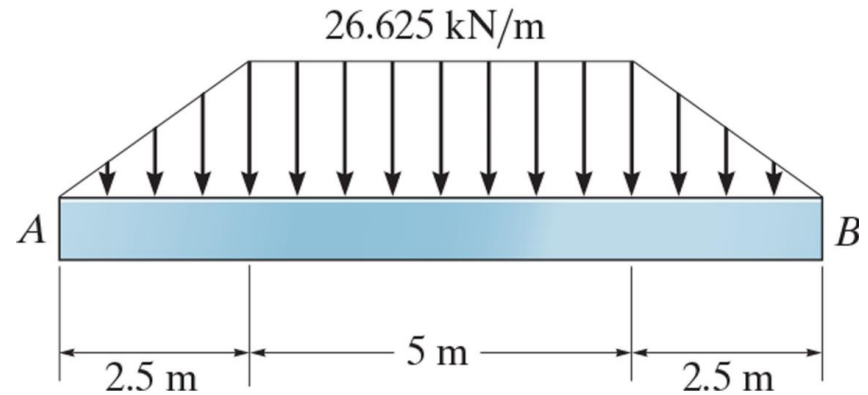
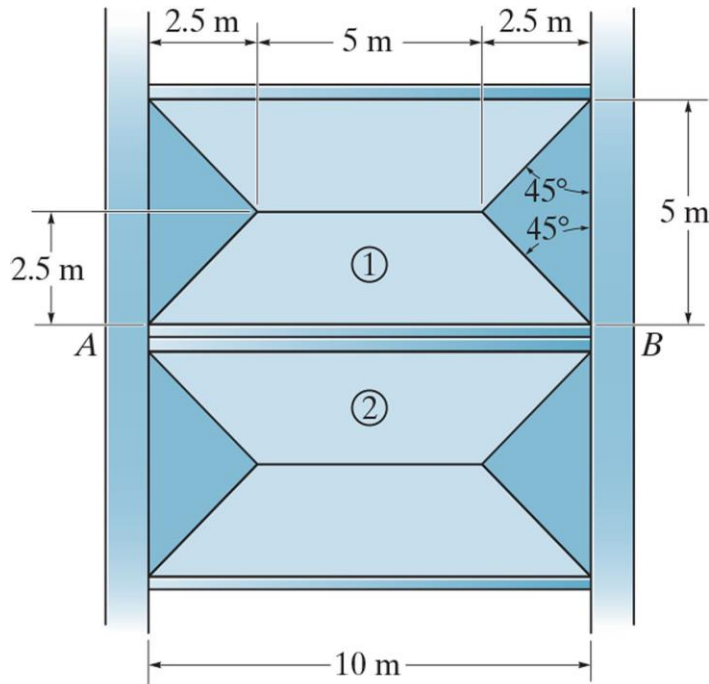
Loading on Beam DC from Area 3

Example 2.3

The concrete girders shown in the photo of the passenger car parking garage span 10m and are 5 m on center. If the floor slab is 125 mm. thick and made of reinforced stone concrete, and the specified live load is 2.5 kN/m^2 , determine the distributed load the floor system transmits to each interior girder.



Example 2.3



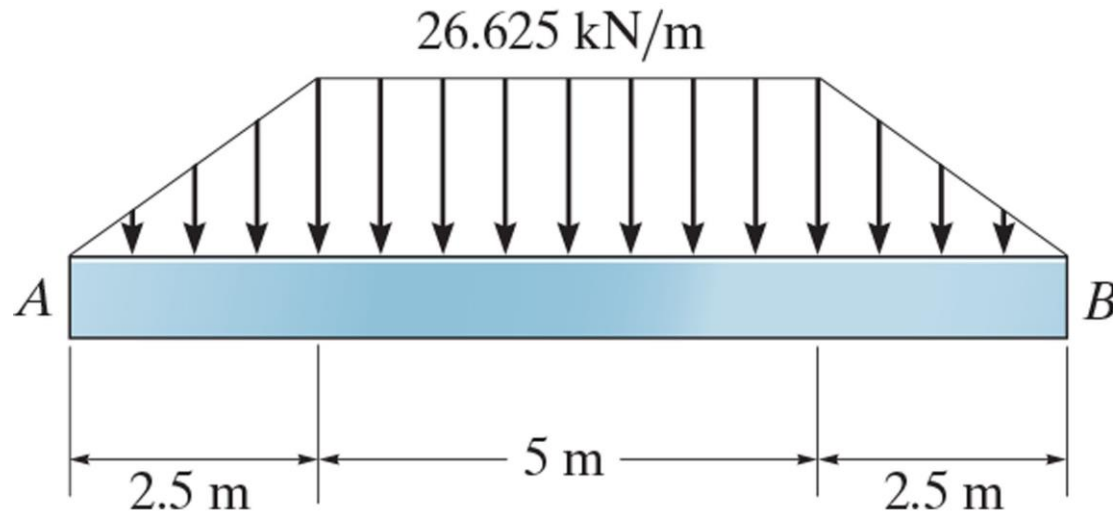
$L_2=10\text{m}$ and $L_1=5\text{m}$, so that $L_2/L_1=2$ We have a two-way slab.

From Table 1–2, for reinforced stone concrete, the specific weight of the concrete is 22.6 kN/m³.

Thus the design floor loading is

$$p = (22.6 \text{ kN/m}^3)(0.125\text{m}) + 2.5 \text{ kN/m}^2 = 5.325 \text{ lN/m}^2$$

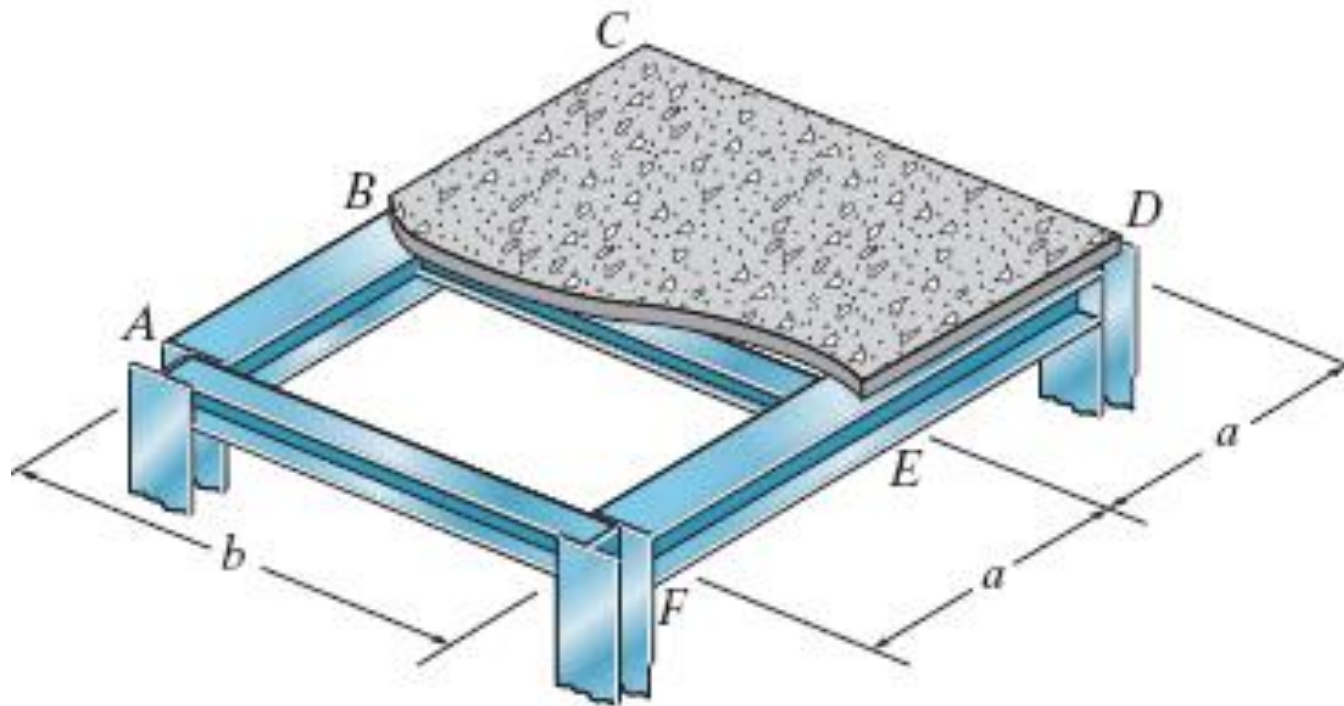
Example 2.3



A trapezoidal distributed loading is transmitted to each interior girder AB from each of its sides. The maximum intensity of each of these distributed loadings is $(5.325 \text{ kN/m}^2)(2.5\text{m})=13.3125 \text{ kN/m}$, so that on the girder this intensity becomes , $2(13.3125 \text{ kN/m})= 26.625 \text{ kN/m}$

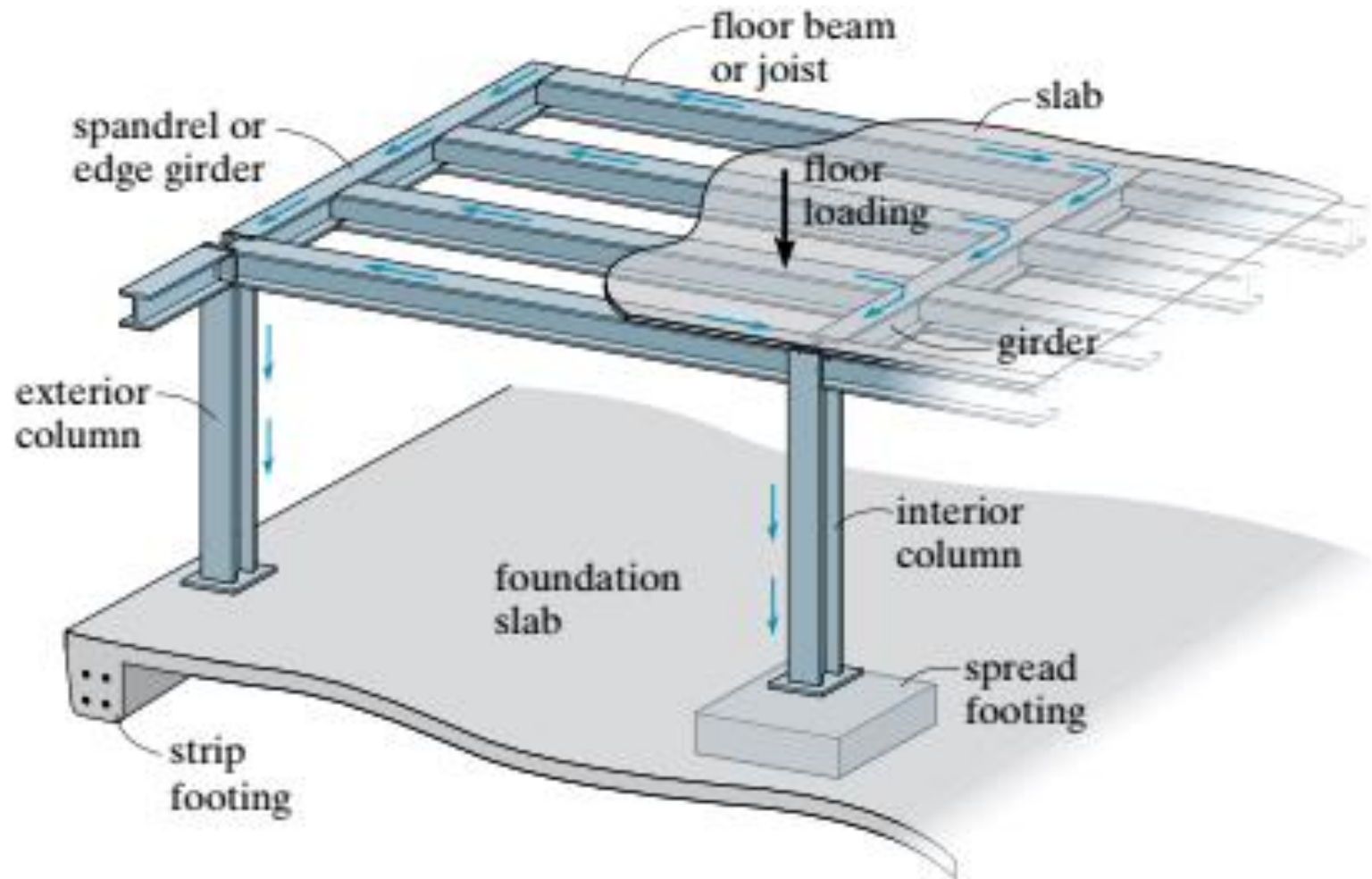
Note: *For design, consideration should also be given to the weight of the girder.*

Problem



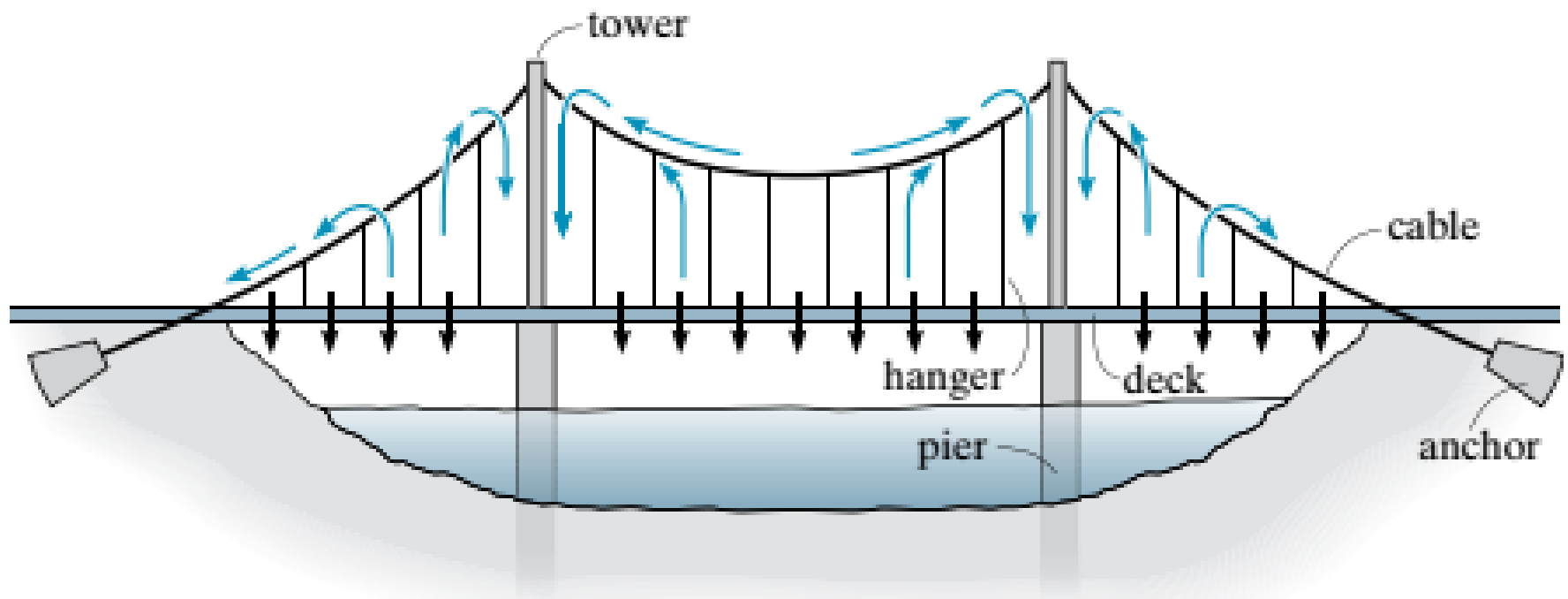
Probs. 2-1/2-2

Load Path



(a)

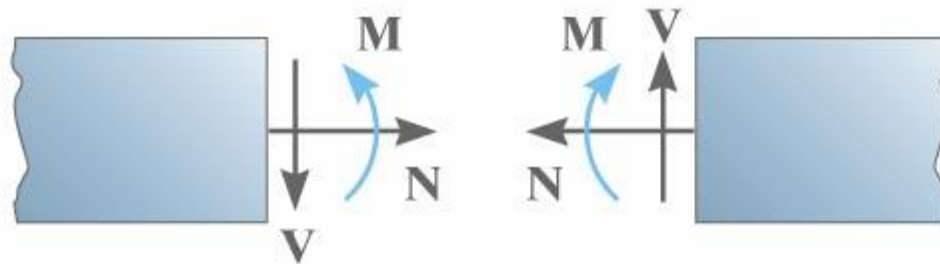
Load Path



Equation of Equilibrium

- In x-y plane*
$$\sum F_x = 0$$
$$\sum F_y = 0$$
$$\sum M_o = 0$$

Internal loading

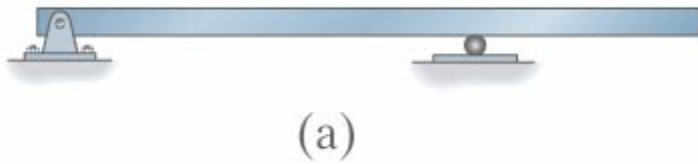


internal loadings

Determinacy & Stability

- **Determinacy**: when all the forces in structure can be determined from equilibrium equation, the structure is referred to as *statically determinate*. Structure having more unknown forces than available equilibrium equations called *statically indeterminate*
- If n is number of structure parts & r is number of unknown forces:
 $r = 3n$, statically determinate
 $r > 3n$, statically indeterminate

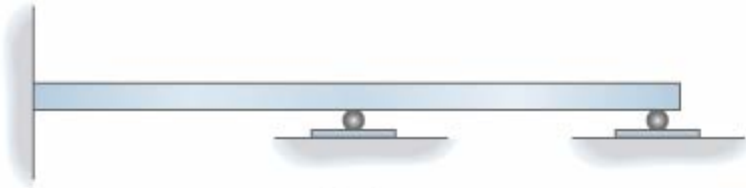
Classify determinate & indeterminate structure



$$r = 3$$

$$n = 1$$

$$3 = 3(1) \quad \text{Statically Determinate}$$



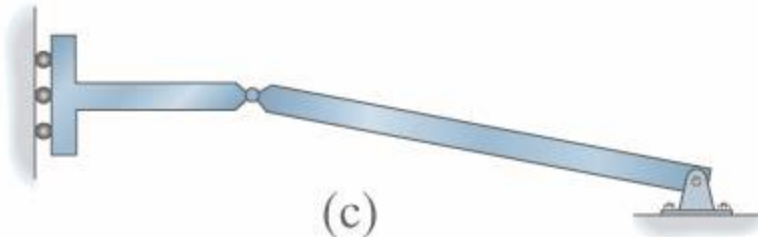
(b)

$$r = 5$$

$$n = 1$$

$5 > 3(1)$ Statically indeterminate

2^{nd} degree

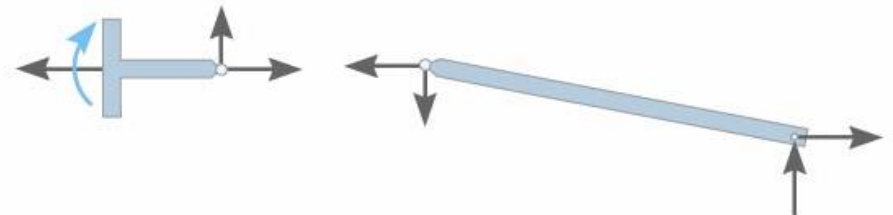


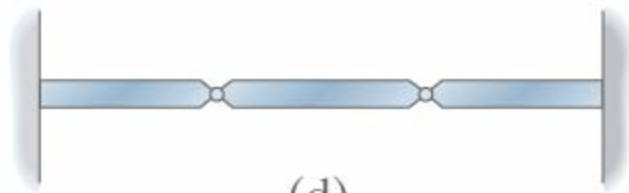
(c)

$$r = 6$$

$$n = 2$$

$6 = 3(2)$ Statically determinate





(d)

$$r = 10$$

$$n = 3$$

$10 > 3(3)$ Statically indeterminate

1st degree

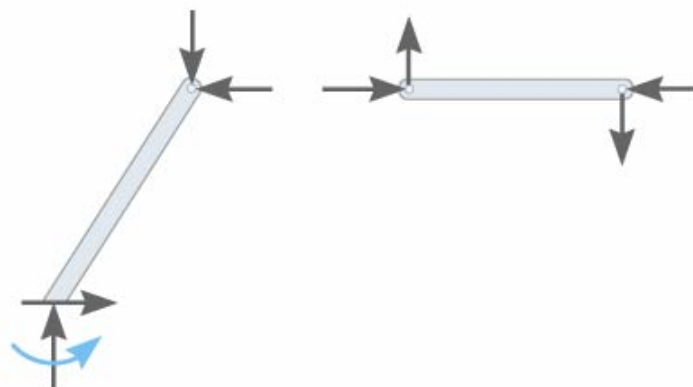


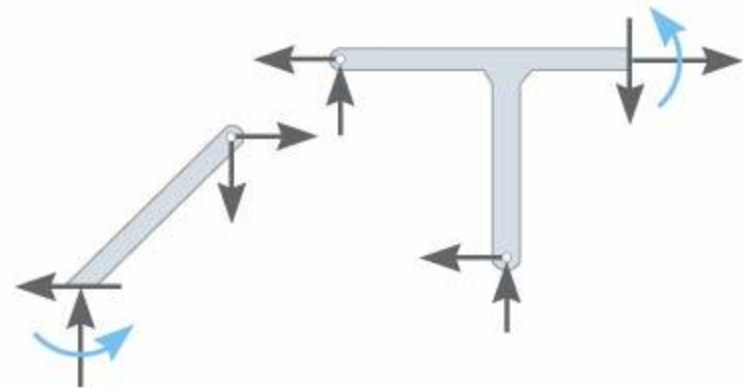
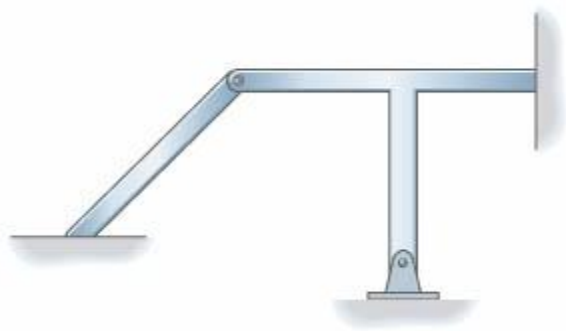
$$r = 7$$

$$n = 2$$

$7 > 3(2)$ Statically indeterminate,

1st degree



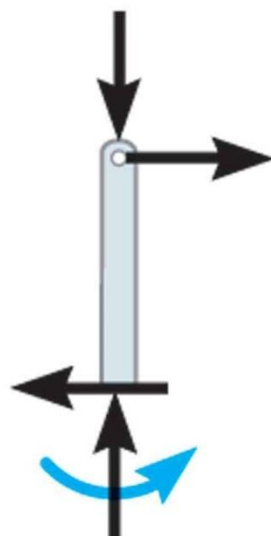
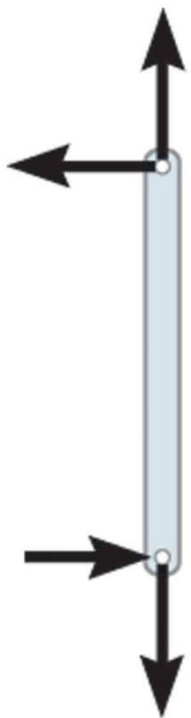
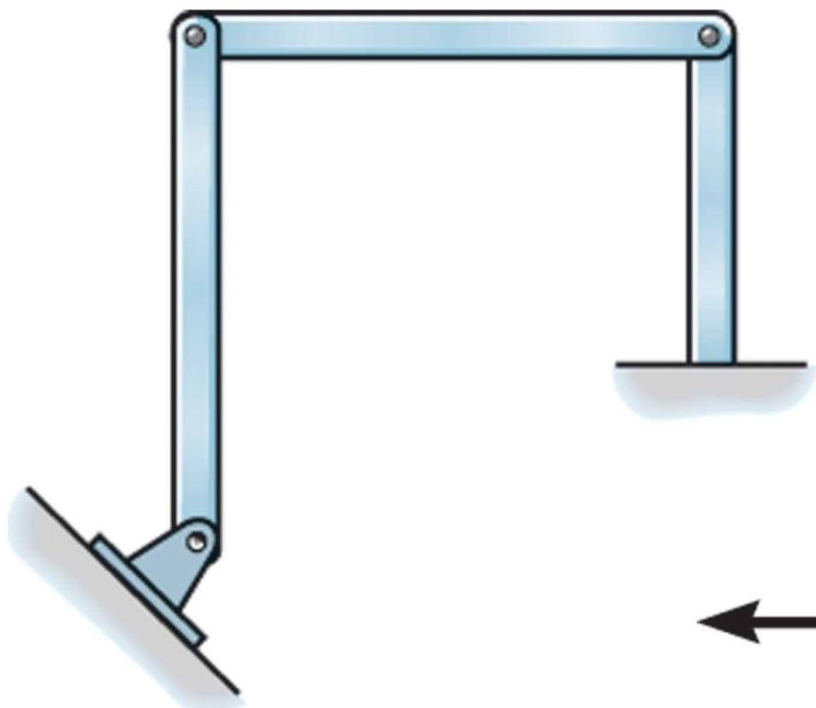


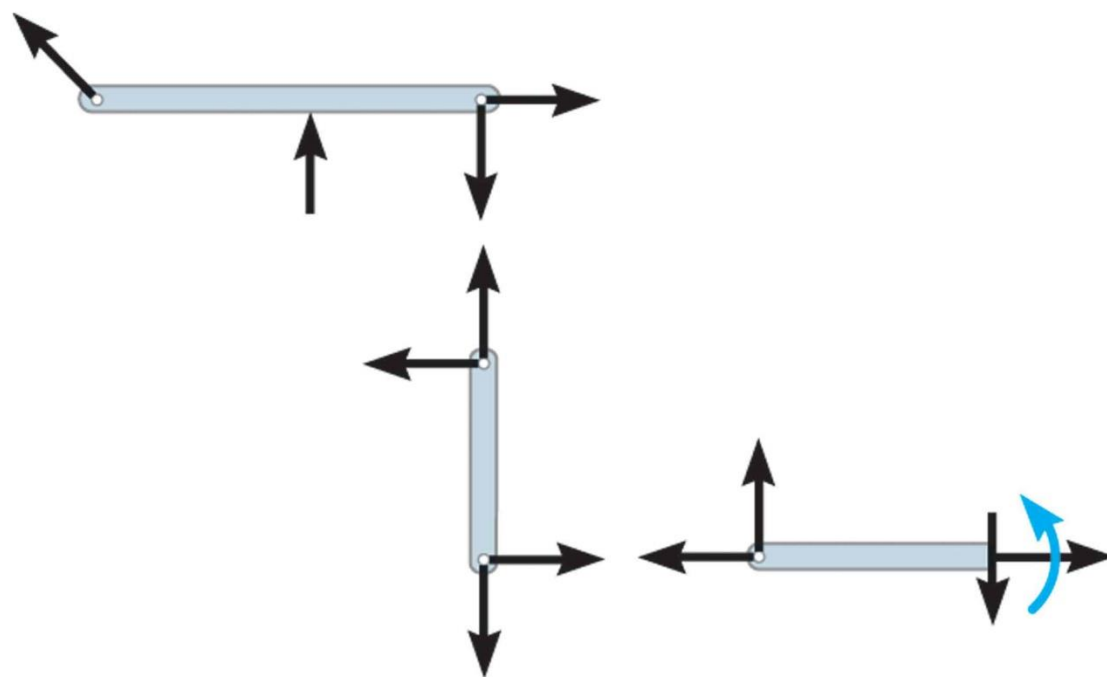
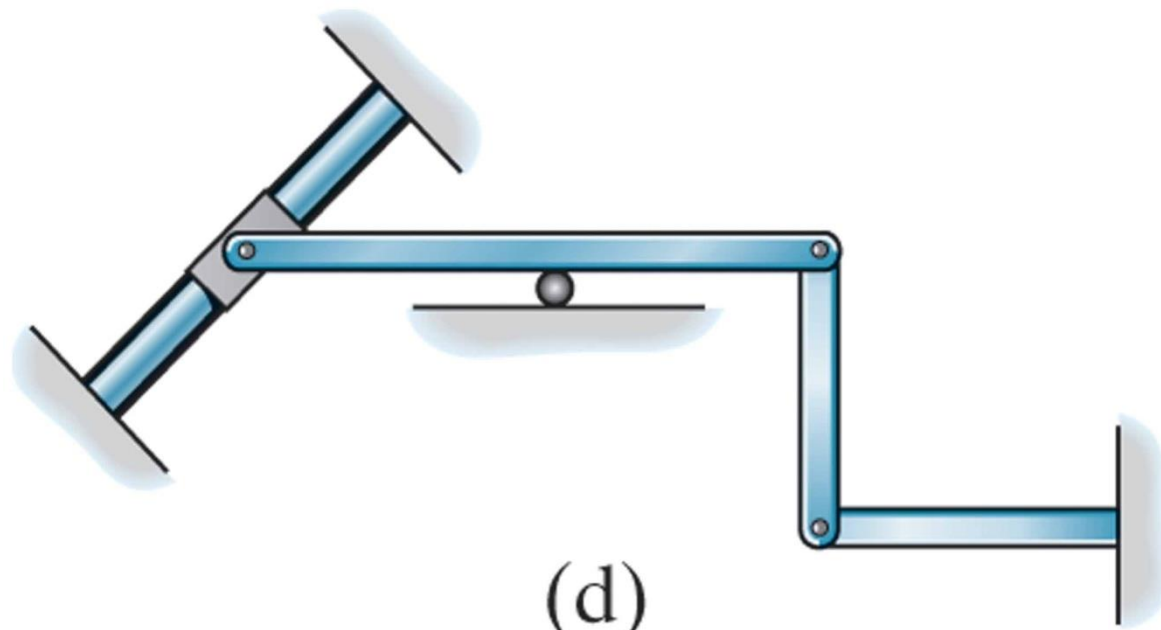
$$r = 10$$

$$n = 2$$

$10 > 3(2)$ Statically indeterminate

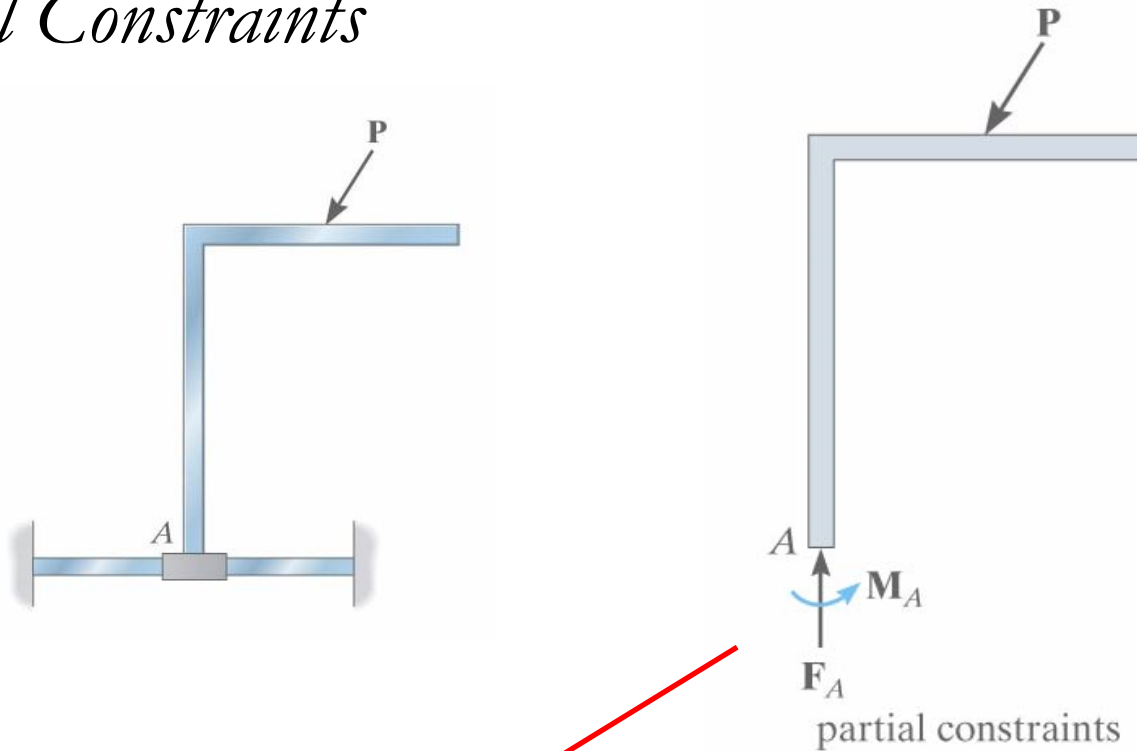
4th degree





- Stability

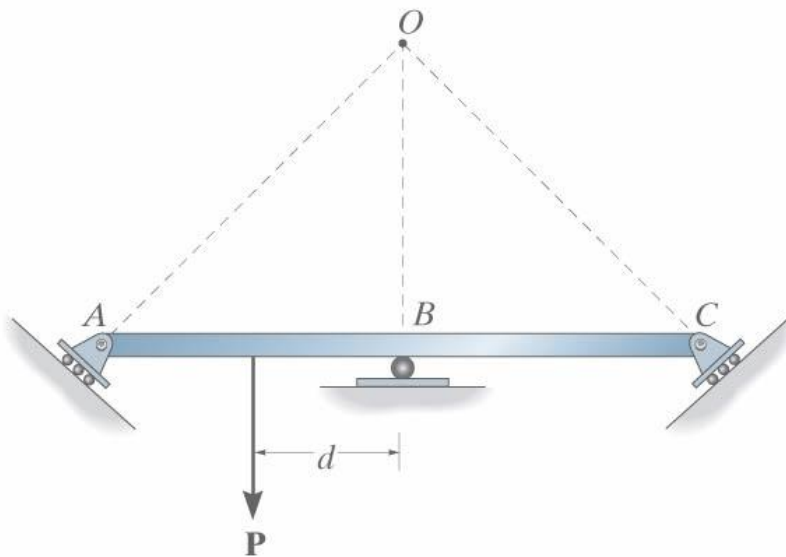
Partial Constraints



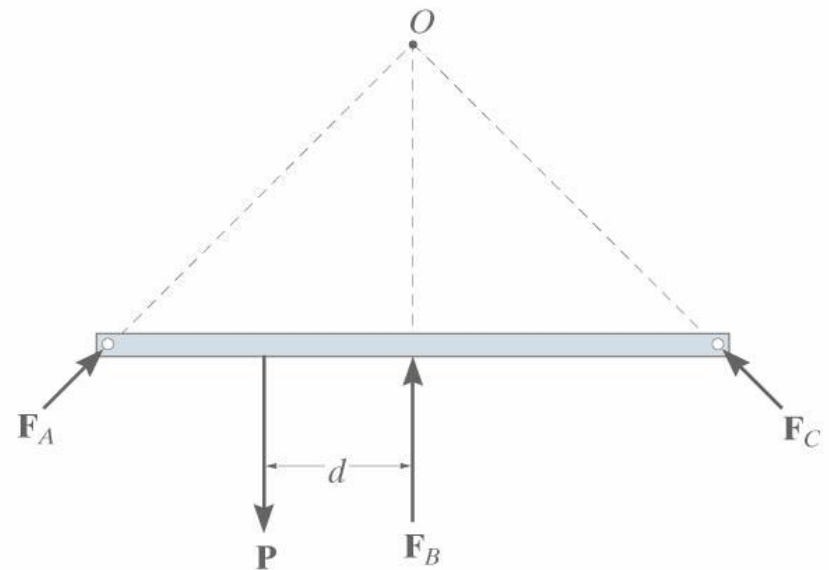
$\sum F_x = 0$ will not be satisfied $\Rightarrow$ unstable with this loading case

- *Improper Constraints*

This can occur if all the support reactions are concurrent at a point.

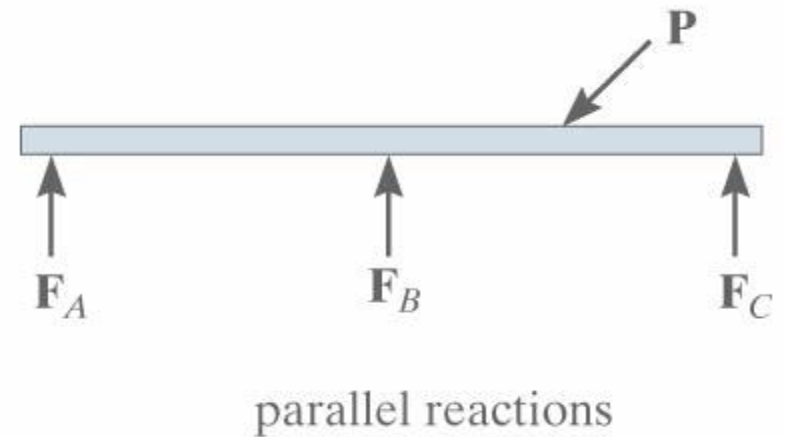
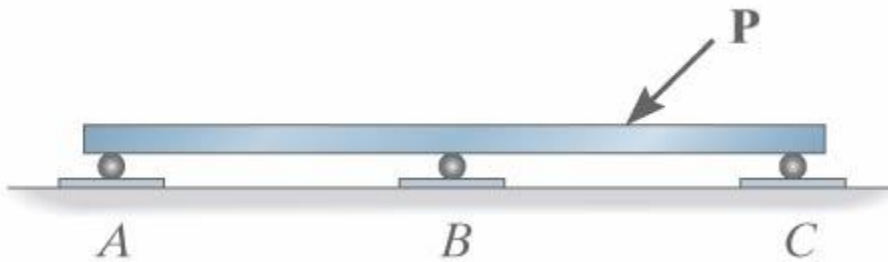


$$P \times d \neq 0$$



concurrent reactions

- This can occur also when the reactive forces are all parallel

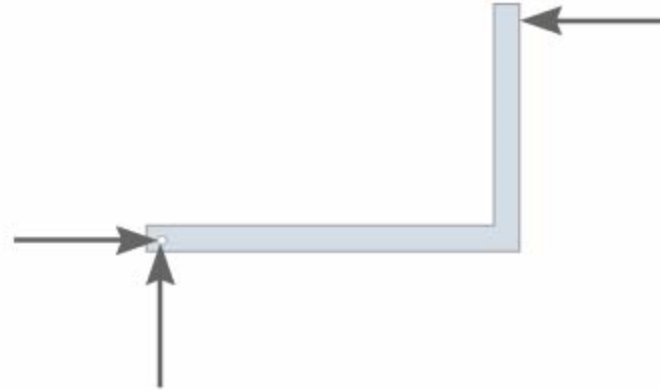
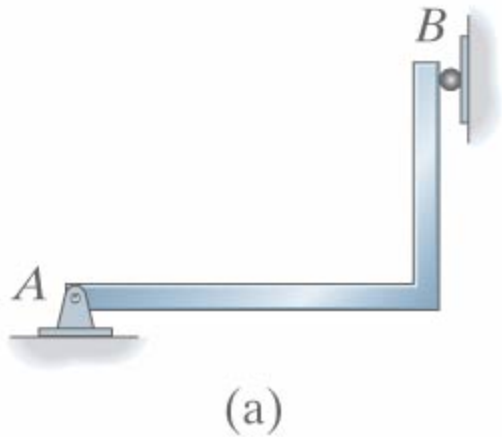


In General

$r < 3n$, Then the structure is Unstable

$r \geq 3n$, Also, Unstable if member reactions are concurrent or parallel or some of the components form a collapsible mechanism

Classify The structure Stable or Unstable



$$r = 3$$

$$n = 1$$

$$3 = 3(1) \quad \text{no special cases} \quad \Rightarrow \text{Stable}$$

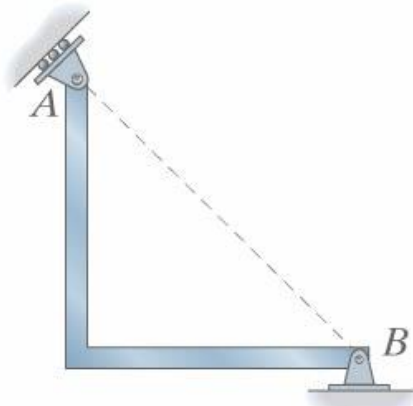
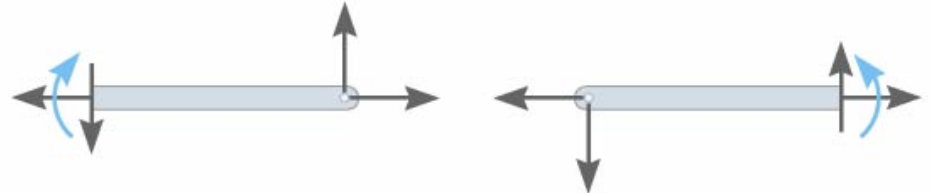


(b)

$$r = 8$$

$$n = 2$$

$8 > 3(2)$ *no special cases* $\Rightarrow$ Stable

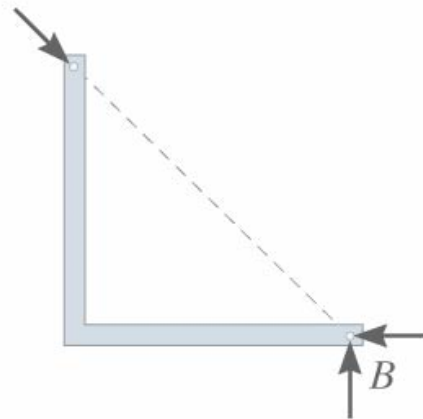


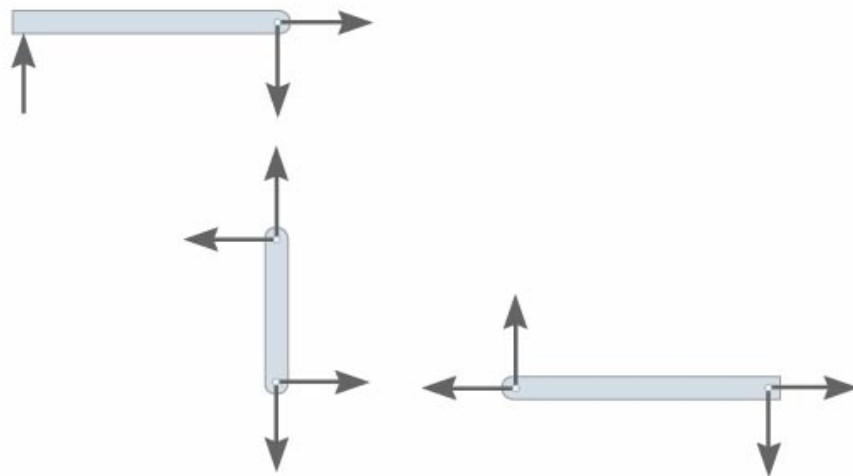
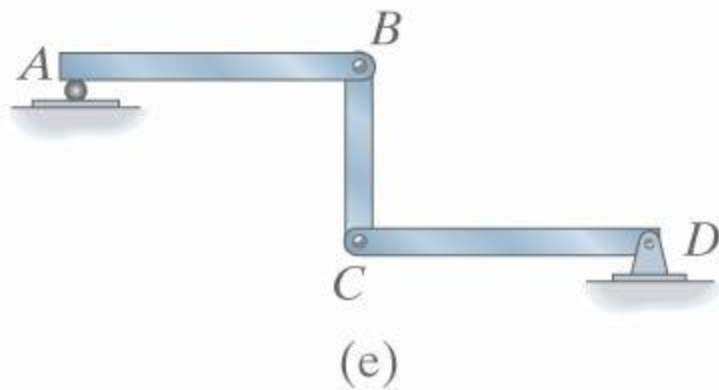
(c)

$$r = 3$$

$$n = 1$$

$3 = 3(1)$ the three reactions are concurrent at B $\Rightarrow$ Unstable





$$r = 7$$

$$n = 3$$

$$7 < 3(3) \Rightarrow \text{Unsable}$$

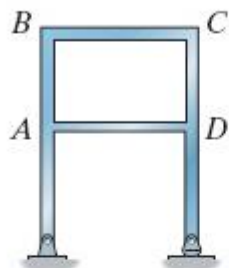
EXAMPLE 2-5

Classify each of the frames shown in Fig. 2–20*a* and 2–20*b* as statically determinate or statically indeterminate. If statically indeterminate, report the number of degrees of indeterminacy. The frames are subjected to external loadings that are assumed to be known and can act anywhere on the frames.

Solution

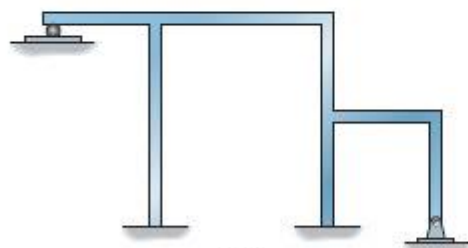
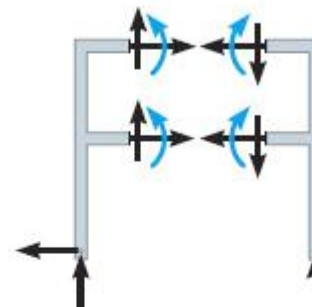
Unlike the beams and pin-connected structures of the previous examples, frame structures consist of members that are connected together by rigid joints. Sometimes the members form internal loops as in Fig. 2–20*a*. Here $ABCD$ forms a closed loop. In order to classify these structures, it is necessary to use the method of sections and “cut” the loop apart. The free-body diagrams of the sectioned parts are drawn and the frame can then be classified. Notice that only *one section* through the loop is required, since once the unknowns at the section are determined, the internal forces at any point in the members can then be found using the method of sections and the equations of equilibrium. The frame in Fig. 2–20*b* has no closed loops and so it does not need to be sectioned between the supports when finding its determinacy. The resulting classifications are indicated in Fig. 2–20*a* and 2–20*b*.

EXAMPLE 2-5 (Continued)



(a)

$r = 9, n = 2, 9 > 6,$
Statically indeterminate to the
third degree *Ans.*



(b)

(This frame has no closed loops.)

$r = 9, n = 1, 9 > 3,$
Statically indeterminate to the
sixth degree *Ans.*

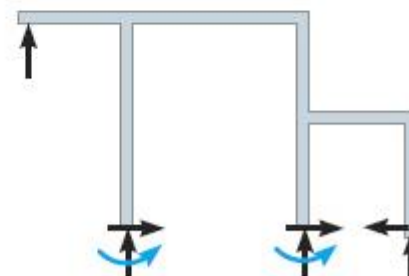


Fig. 2-20

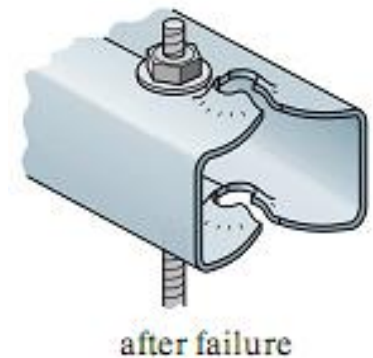
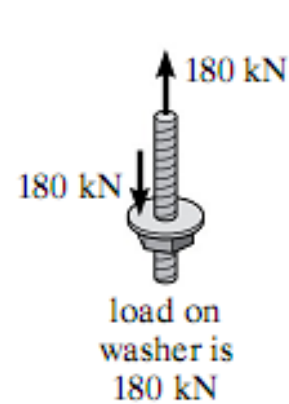
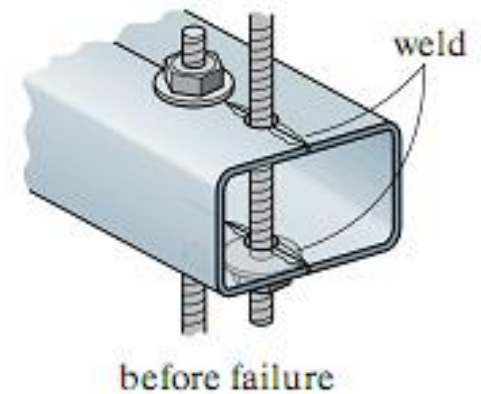
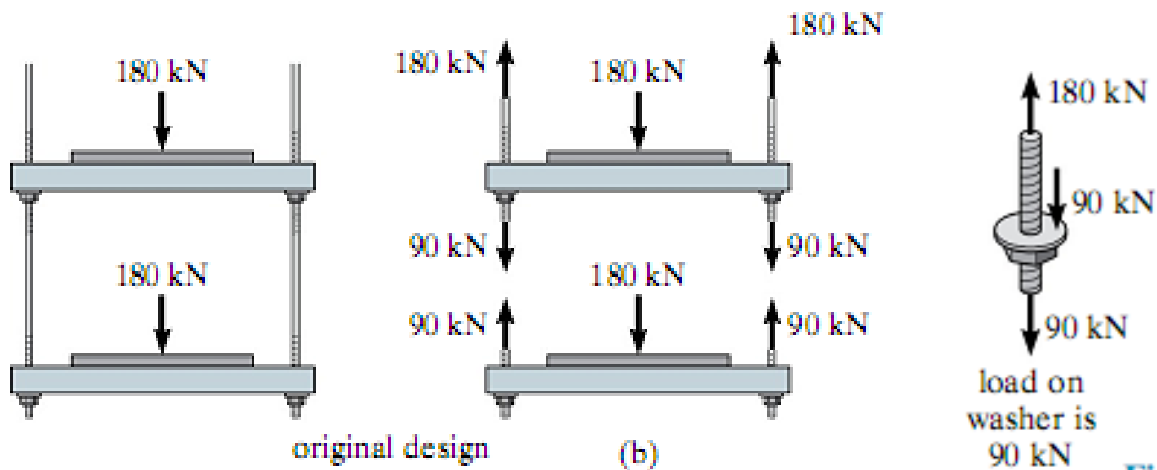
Application of Equilibrium Equation

Application of Equation of Equilibrium

Procedure Steps

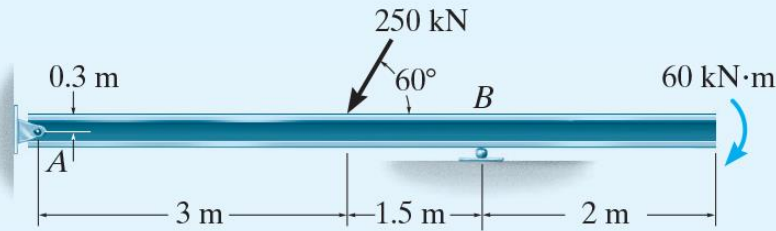
1. If not given, establish a suitable x - y coordinate system.
2. Draw a free body diagram (FBD) of the object under analysis.
3. Apply the three equations of equilibrium to solve for the unknowns.

How Important is the Free-Body Diagram?

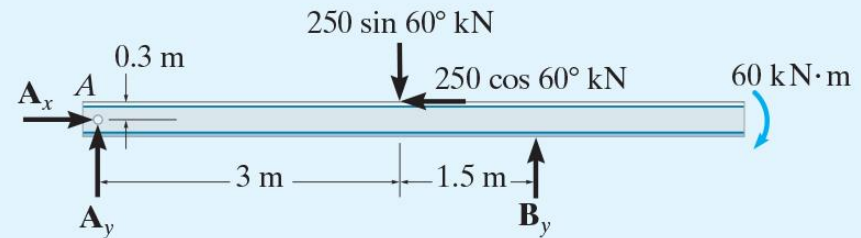


EXAMPLE 2.8

Determine the reactions on the beam shown in Fig. 2–30*a*.



(a)



(b)

Fig. 2–30

SOLUTION

Free-Body Diagram. As shown in Fig. 2–30*b*, the 250-kN force is resolved into x and y components. Also, the 2-m dimension line is not needed since a couple moment is a *free vector* and can therefore act anywhere on the beam for the purpose of calculating the reactions.

Equations of Equilibrium. Applying Eqs. 2–2 in a sequence, using previously calculated results, we have

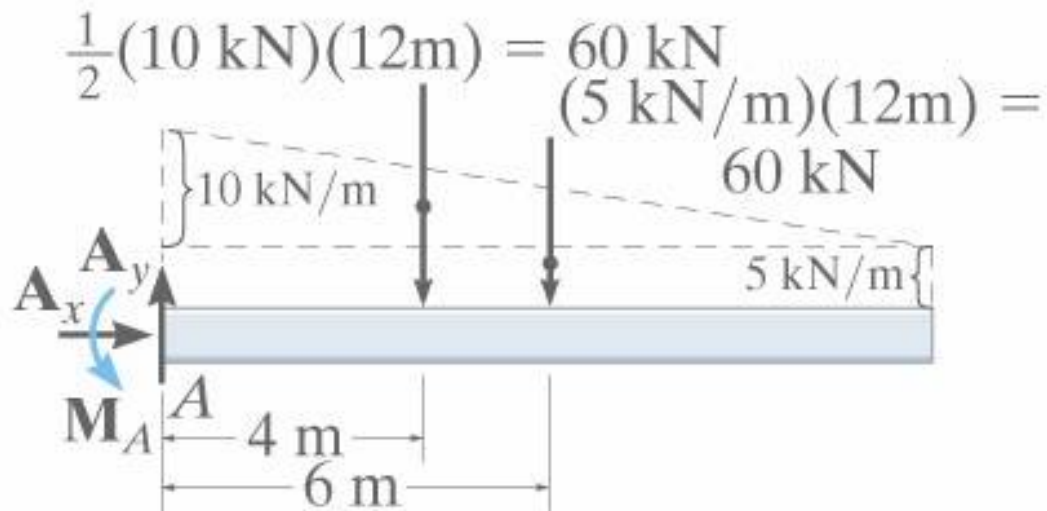
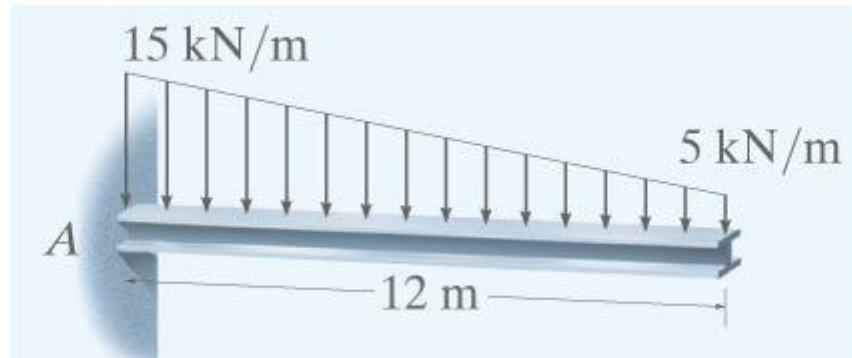
$$\rightarrow \Sigma F_x = 0; \quad A_x - 250 \cos 60^\circ = 0 \qquad A_x = 125 \text{ kN} \qquad \text{Ans.}$$

$$\downarrow + \Sigma M_A = 0; \quad -250 \sin 60^\circ(3) + 250 \cos 60^\circ(0.3) + B_y(4.5) - 60 = 0 \qquad B_y = 149.33 \text{ kN} = 149 \text{ kN} \qquad \text{Ans.}$$

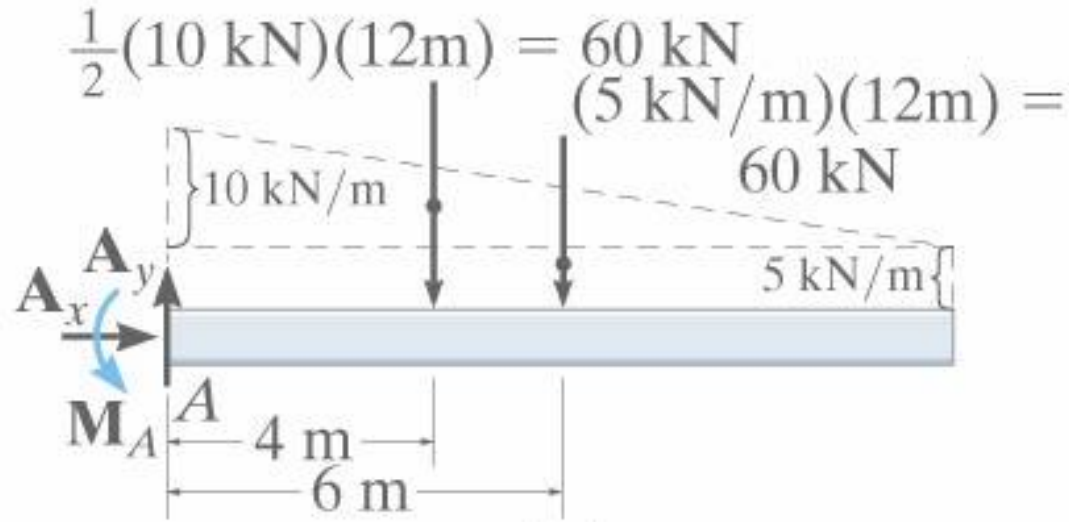
$$+ \uparrow \Sigma F_y = 0; \quad -250 \sin 60^\circ + 149.33 + A_y = 0 \qquad A_y = 67.2 \text{ kN} \qquad \text{Ans.}$$

Example 2.9

Determine the Reactions



Example 2.9



$$\sum F_x = 0 \Rightarrow A_x = 0$$

$$\sum F_y = 0 \Rightarrow A_y - 60 - 60 = 0$$

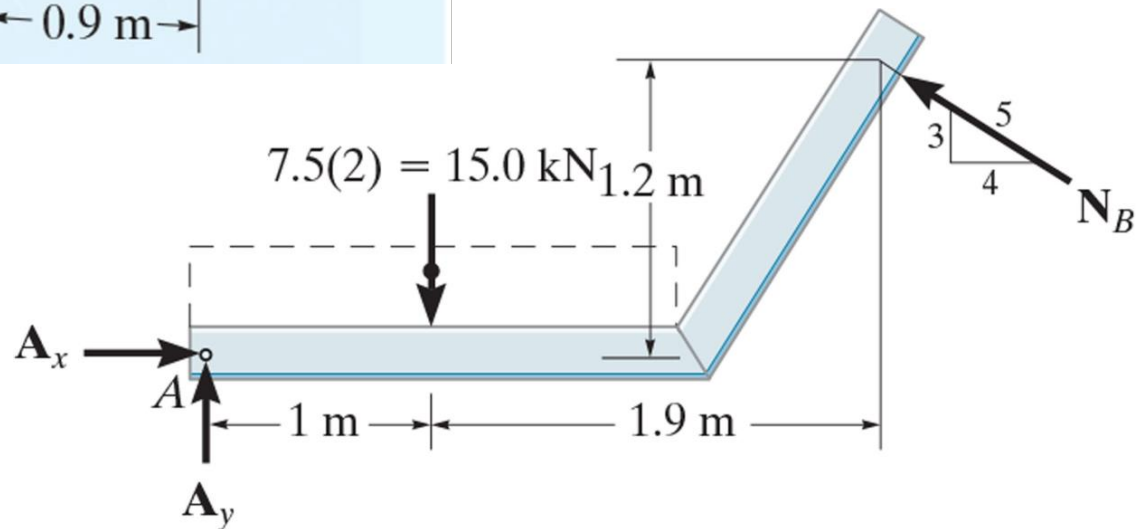
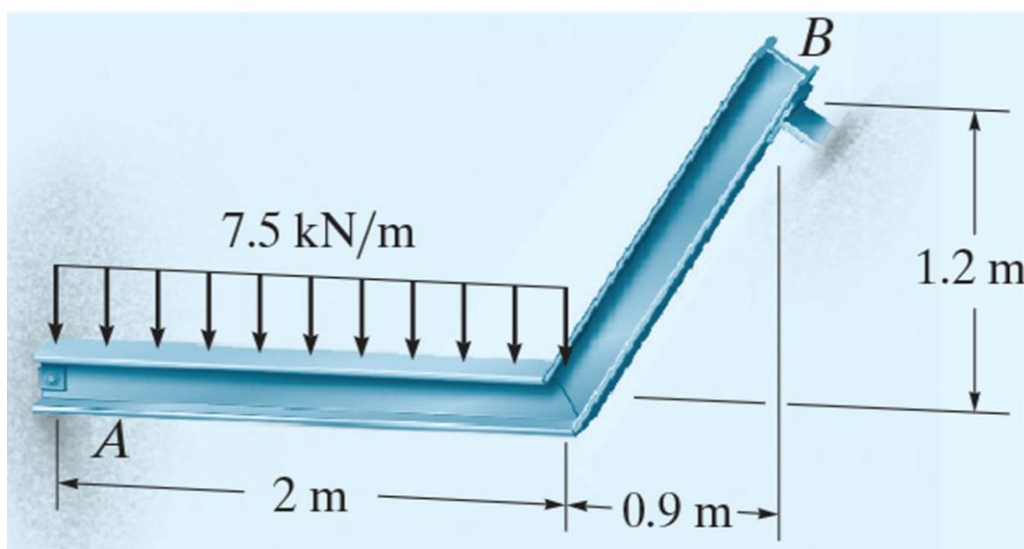
$$A_y = 120 \text{ kN}$$

$$\sum M_A = 0 \Rightarrow -60(4) - 60(6) + M_A = 0$$

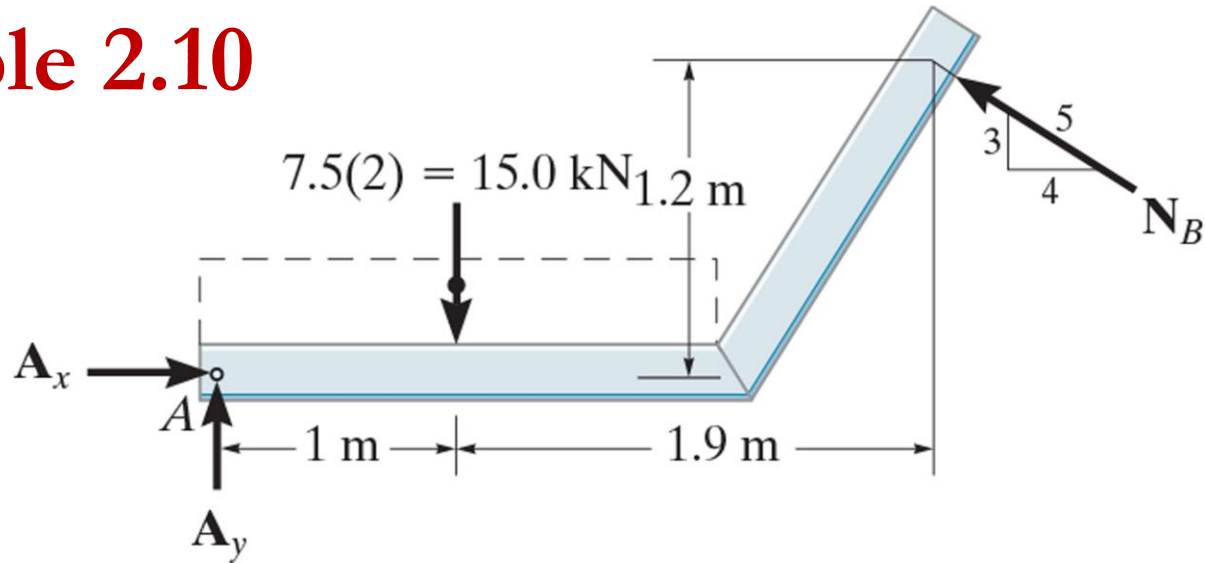
$$M_A = 600 \text{ kN} \cdot \text{m}$$

Example 2.10

Determine the Reactions



Example 2.10



$$\sum M_A = 0 \Rightarrow -15(1.0) + \frac{4}{5} N_B (1.2) + \frac{3}{5} N_B (2.9) = 0$$

$$N_B = 5.556 \text{ kN}$$

$$\sum F_x = 0 \Rightarrow A_x - \frac{4}{5} (5.556) = 0.0$$

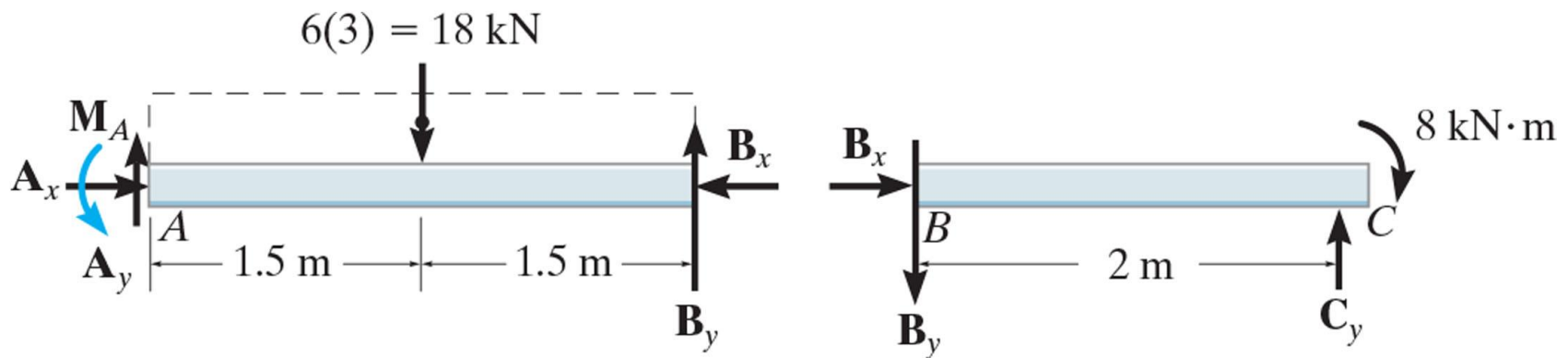
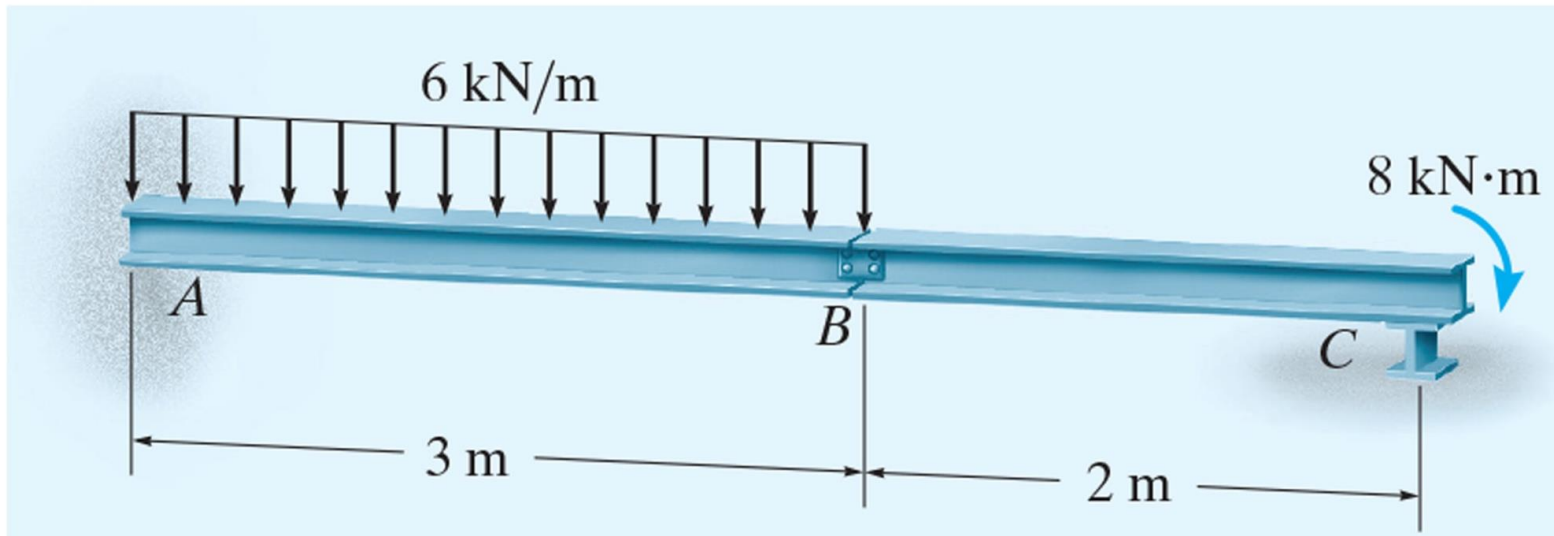
$$A_x = 4.44 \text{ kN}$$

$$\sum F_y = 0 \Rightarrow A_y - 15.0 + \frac{3}{5} (5.556) = 0$$

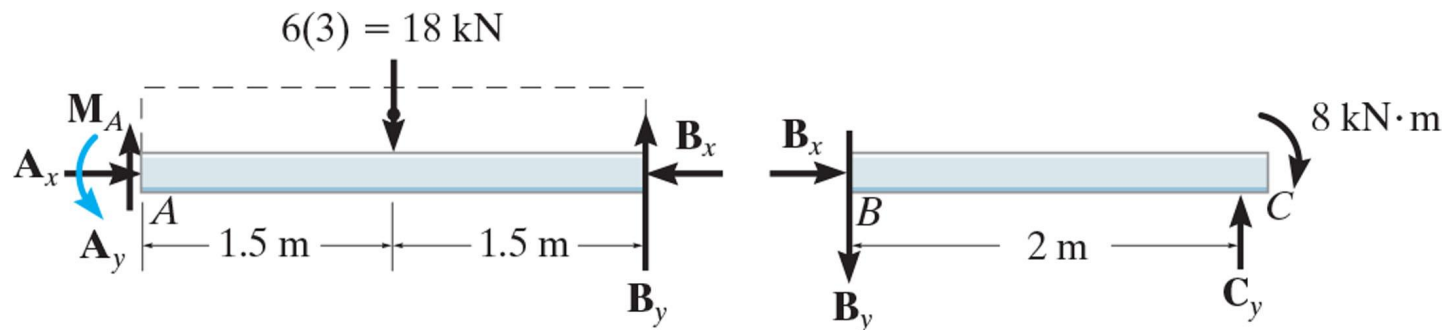
$$A_y = 11.67 \text{ kN}$$

Example 2.11

Determine the Reactions



Example 2.11



Segment *BC*:

$$\curvearrowleft + \Sigma M_C = 0; \quad -8 + B_y(2) = 0 \quad B_y = 4.00 \text{ kN} \quad \text{Ans.}$$

$$+ \uparrow \Sigma F_y = 0; \quad -4.00 + C_y = 0 \quad C_y = 4.00 \text{ kN} \quad \text{Ans.}$$

$$\rightarrow \Sigma F_x = 0; \quad B_x = 0 \quad \text{Ans.}$$

Segment *AB*:

$$\curvearrowleft + \Sigma M_A = 0; \quad M_A - 18(1.5) + 4.00(3) = 0$$

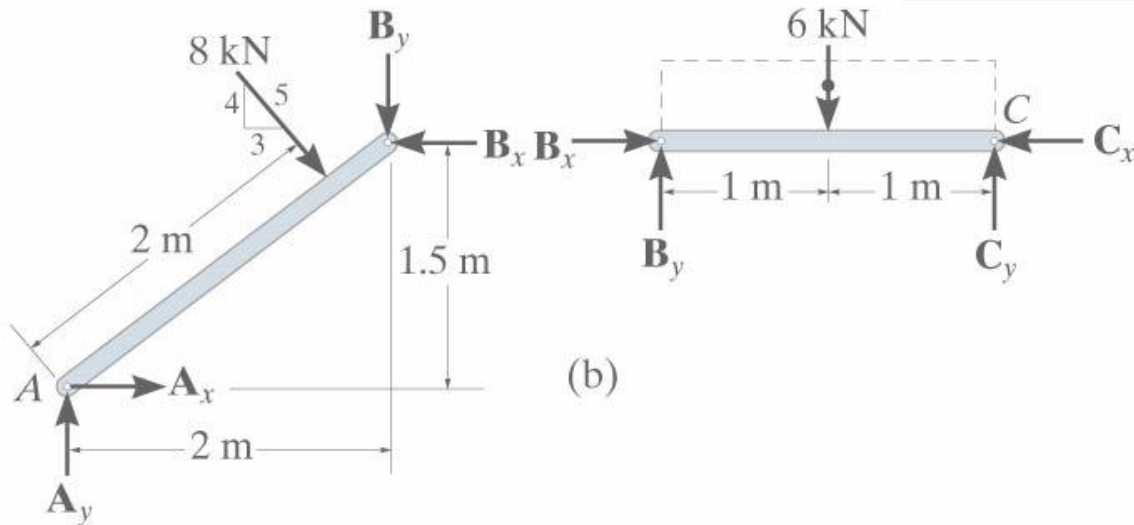
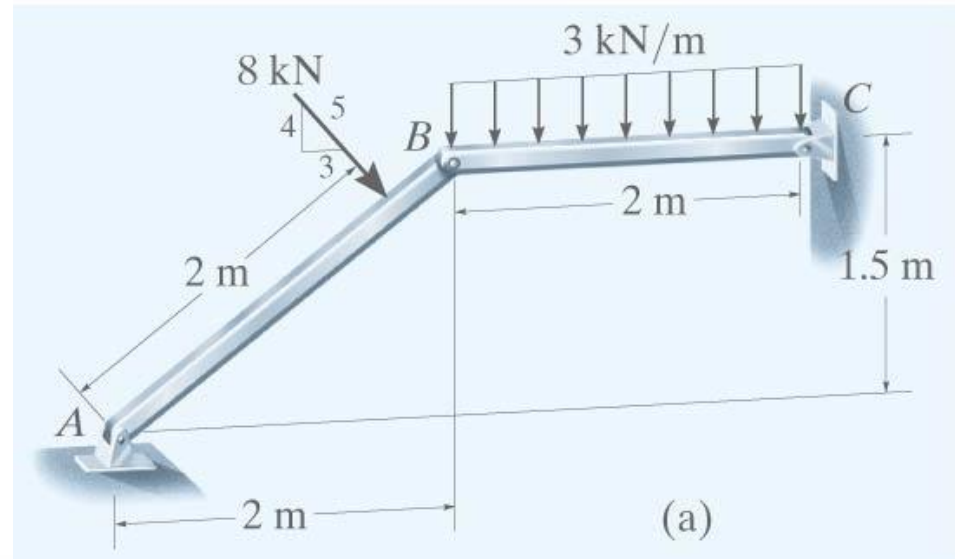
$$M_A = 15.0 \text{ kN} \cdot \text{m} \quad \text{Ans.}$$

$$+ \uparrow \Sigma F_y = 0; \quad A_y - 18 + 4.00 = 0 \quad A_y = 14.0 \text{ kN} \quad \text{Ans.}$$

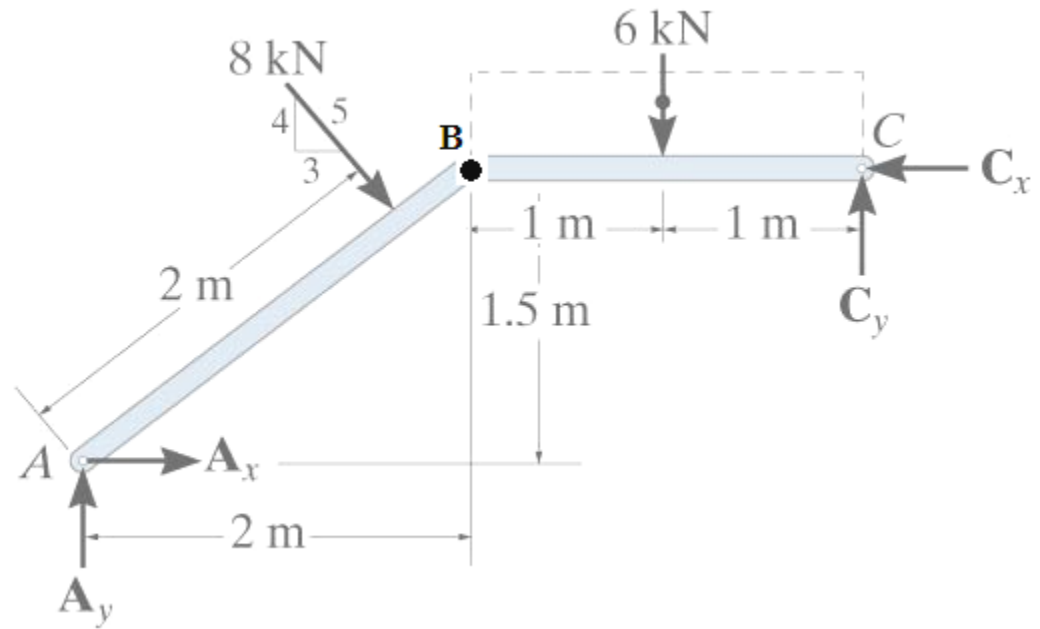
$$\rightarrow \Sigma F_x = 0; \quad A_x - 0 = 0 \quad A_x = 0 \quad \text{Ans.}$$

Example 2.12

Determine the Reactions



Example 2.12



$$\sum M_{B \text{ (Right)}} = 0 \Rightarrow 2C_y - 6(1) = 0$$

$$C_y = 3 \text{ kN}$$

$$\sum M_A = 0 \Rightarrow C_x(1.5) + 3(4) - 6(3) - 8(2) = 0$$

$$C_x = 14.7 \text{ kN}$$

$$\sum F_x = 0 \Rightarrow A_x - 14.7 + 8\left(\frac{3}{5}\right) = 0$$

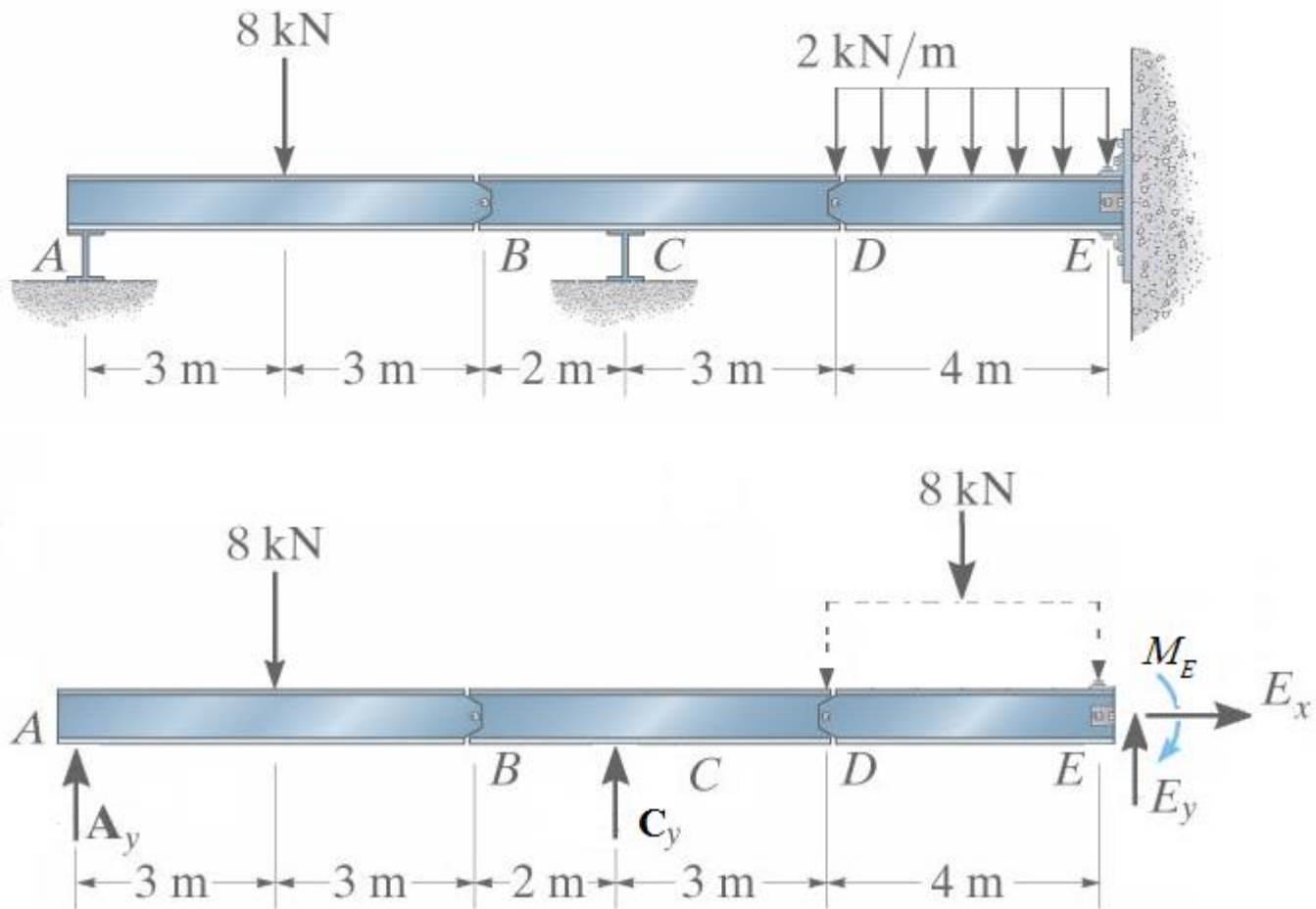
$$A_x = 9.87 \text{ kN}$$

$$\sum F_y = 0 \Rightarrow A_y + 3 - 6 - 8\left(\frac{4}{5}\right) = 0$$

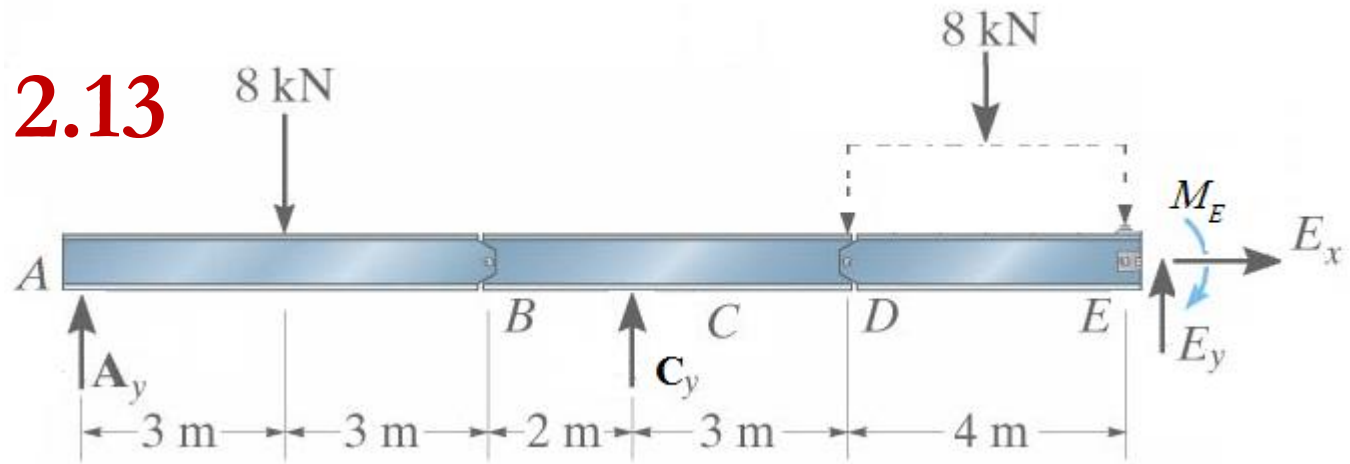
$$A_y = 9.4 \text{ kN}$$

Example 2.13

Determine the Reactions



Example 2.13



$$\sum M_{B(\text{Left})} = 0 \Rightarrow -6A_y + 8(3) = 0$$

$$A_y = 4 \text{ kN}$$

$$\sum M_{D(\text{Left})} = 0 \Rightarrow -3C_y - 4(11) + 8(8) = 0$$

$$C_y = 6.67 \text{ kN}$$

$$\sum F_x = 0 \Rightarrow E_x = 0$$

$$\sum F_y = 0 \Rightarrow E_y - 8 - 8 + 4 + 6.67 = 0$$

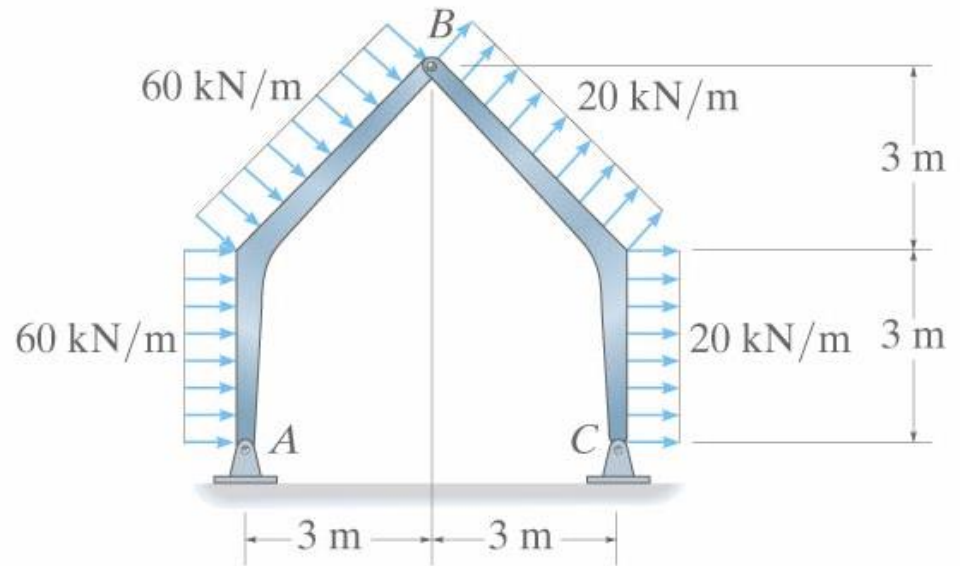
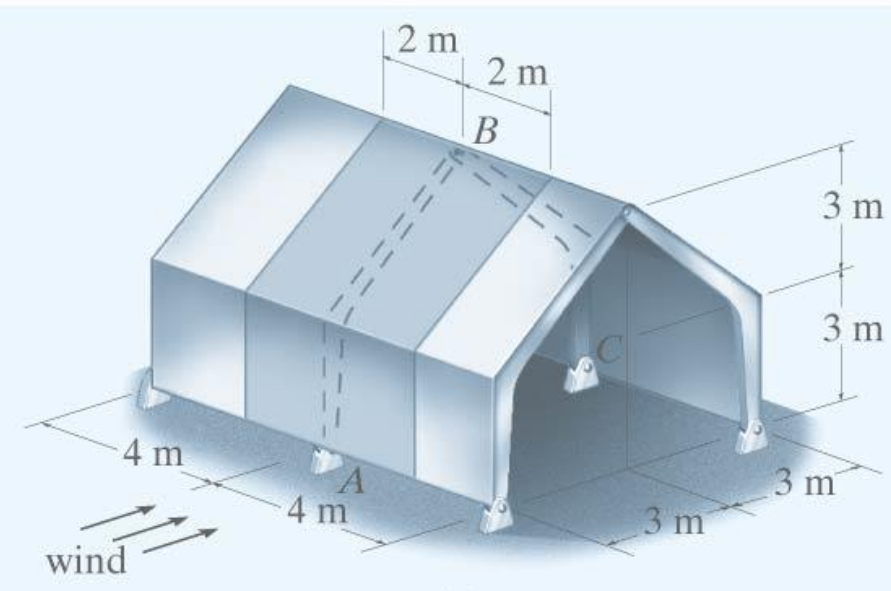
$$E_y = 5.33 \text{ kN}$$

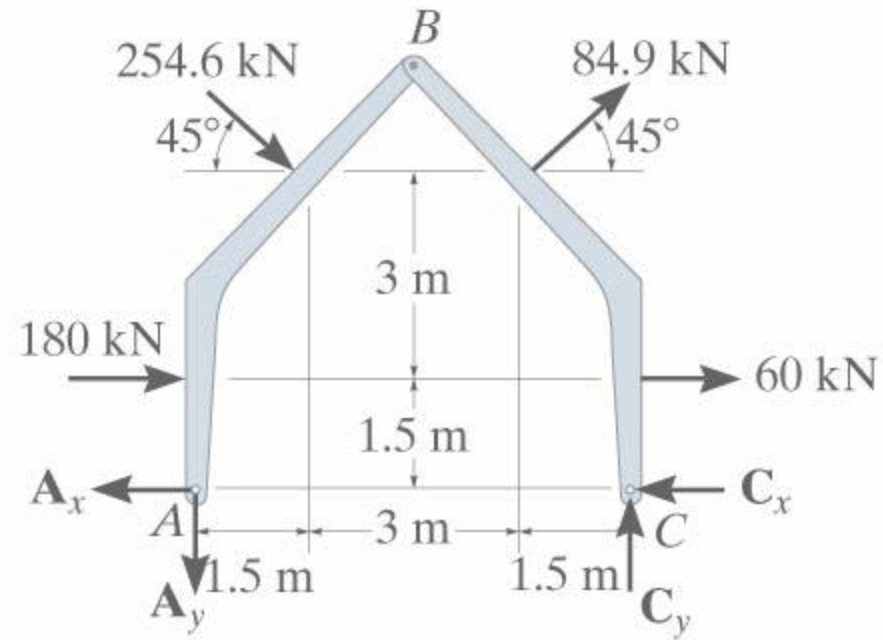
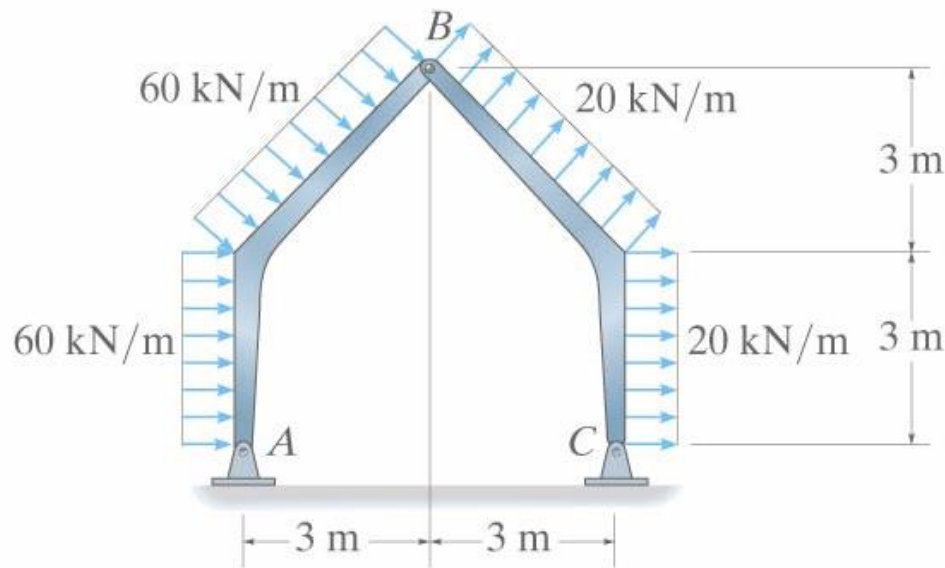
$$\sum M_{D(\text{right})} = 0 \Rightarrow -8(2) + 5.33(4) - M_E = 0$$

$$M_E = 5.33 \text{ kN.m}$$

Example 2.14

Determine the Reactions





$$\sum M_A = 0 \Rightarrow C_y(6) - 180(1.5) - 60(1.5) - 254.6 \cos 45(4.5) - 254.6 \sin 45(1.5) - 84.9 \cos 45(4.5) + 84.9 \cos 45(4.5) = 0$$

$$C_y = 240 \text{ kN}$$

$$\sum M_{B(\text{Right})} = 0 \Rightarrow -C_x(6) + 240(3) + 60(4.5) + 84.9\left(\frac{3\sqrt{2}}{2}\right) = 0$$

$$C_x = 195 \text{ kN}$$

$$\sum F_x = 0 \Rightarrow -A_x - 195 + 180 + 60 + 254.6 \cos 45 + 84.9 \cos 45 = 0$$

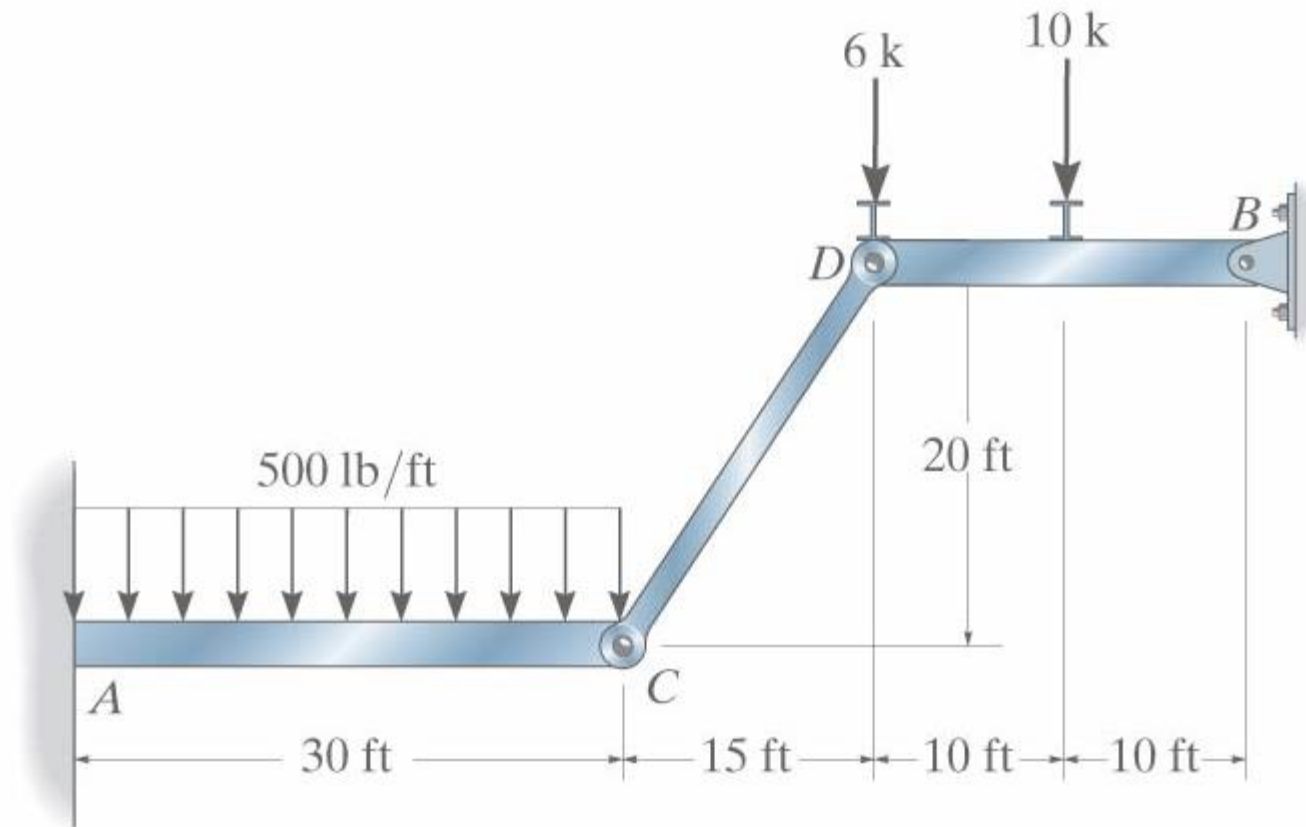
$$A_x = 285 \text{ kN}$$

$$\sum F_y = 0 \Rightarrow -A_y + 240 - 254.6 \sin 45 + 84.9 \sin 45 = 0$$

$$A_y = 120 \text{ kN}$$

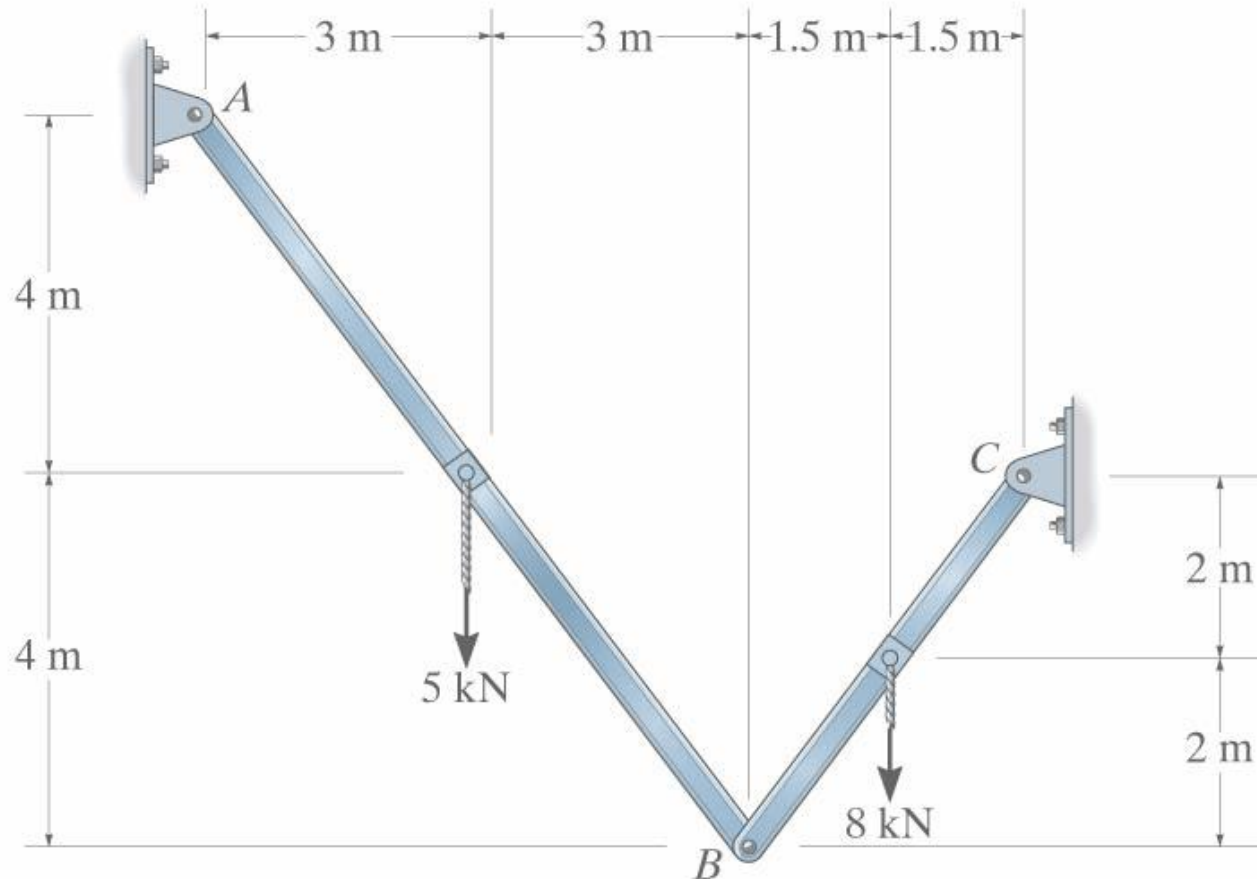
Problem 1

Determine the Reactions



Problem 2

Determine the Reactions



Problem 3

Determine the Reactions

$$\sum M_{\text{hinge}} = 0$$

$$C_y (3) - (10 \times 3)(1.5) = 0$$

$$C_y = 15 \text{ kN}$$

$$\sum M_A = 0$$

$$B_x (3) - (10 \times 12)(6) + (15)(12) = 0$$

$$B_x = 180 \text{ kN}$$

$$\sum F_y = 0$$

$$A_y - (10 \times 12) + 15 = 0$$

$$A_y = 105 \text{ kN}$$

