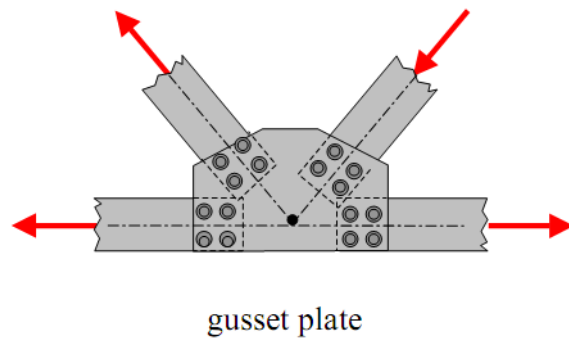
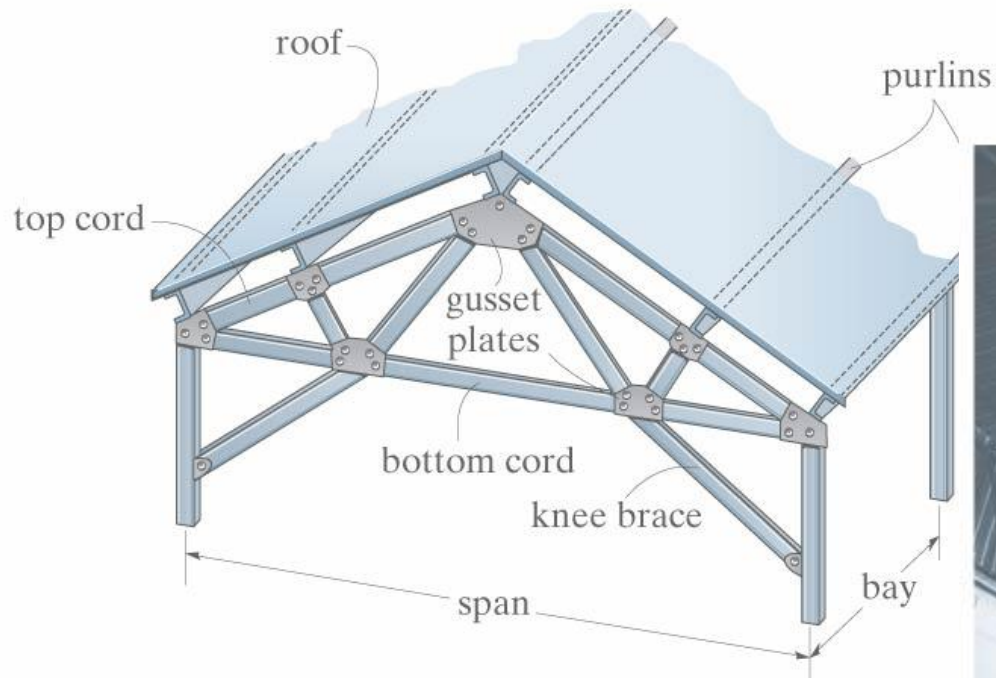


Chapter 3

Trusses



Trusses in Building



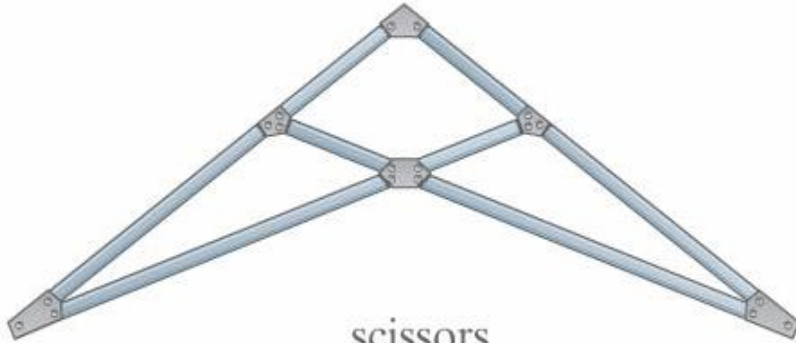
Trusses in Building



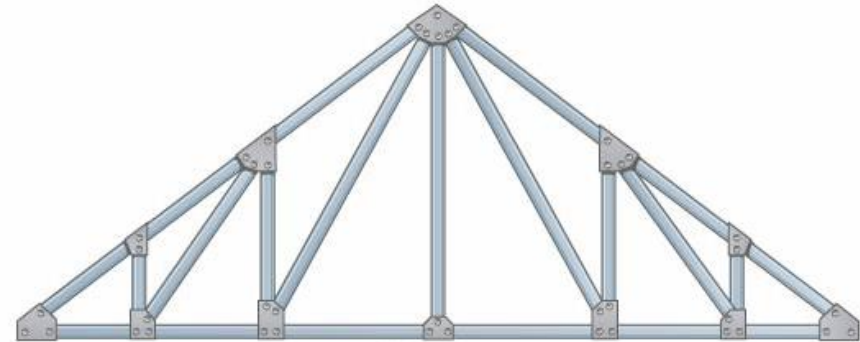
Gusset plate used to connect eight members



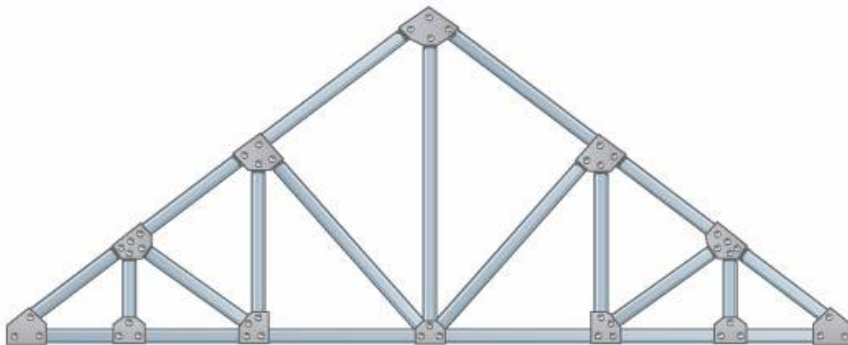
Trusses Types for Building



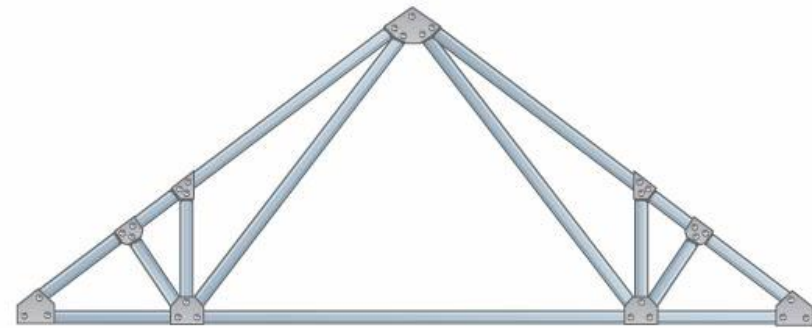
scissors
(a)



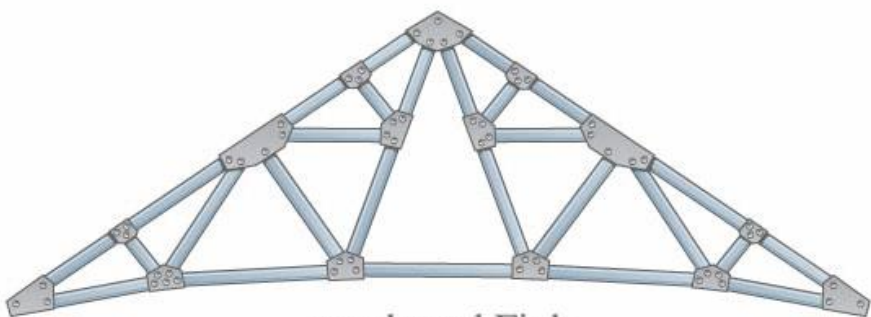
Howe
(b)



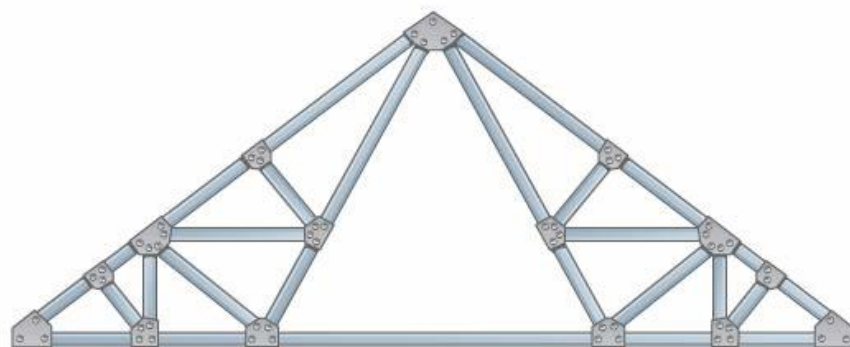
Pratt
(c)



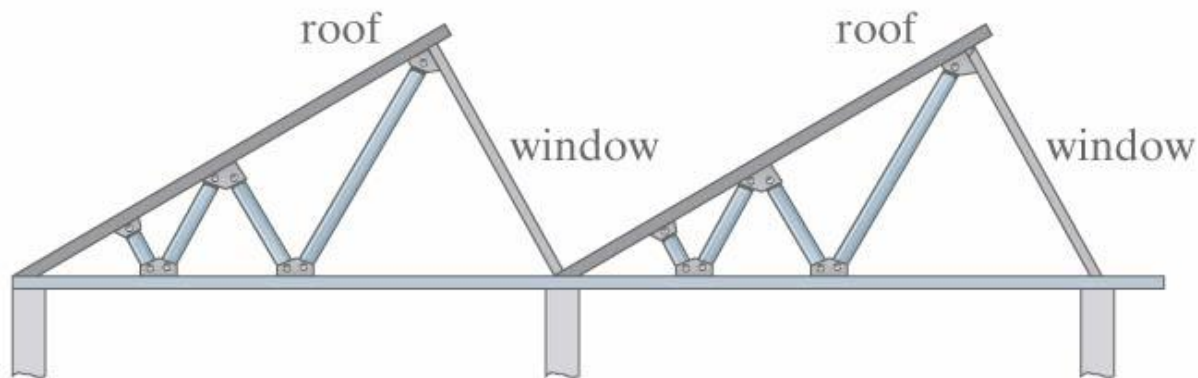
fan
(d)



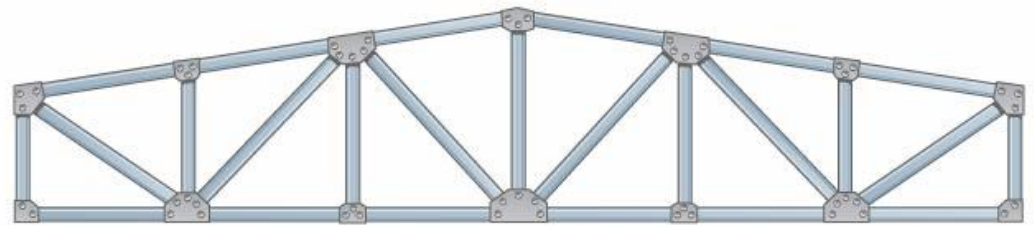
cambered Fink
(f)



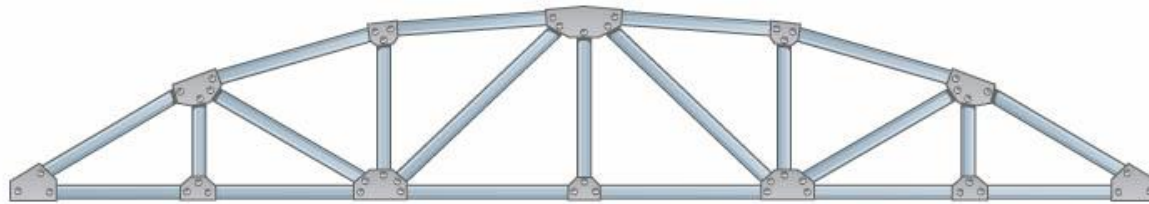
Fink
(e)



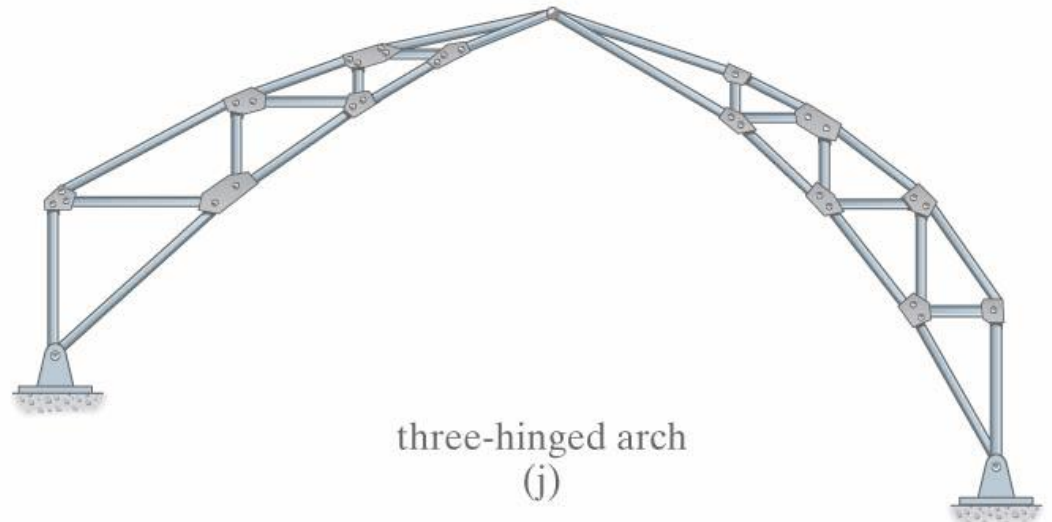
sawtooth
(h)



Warren
(g)

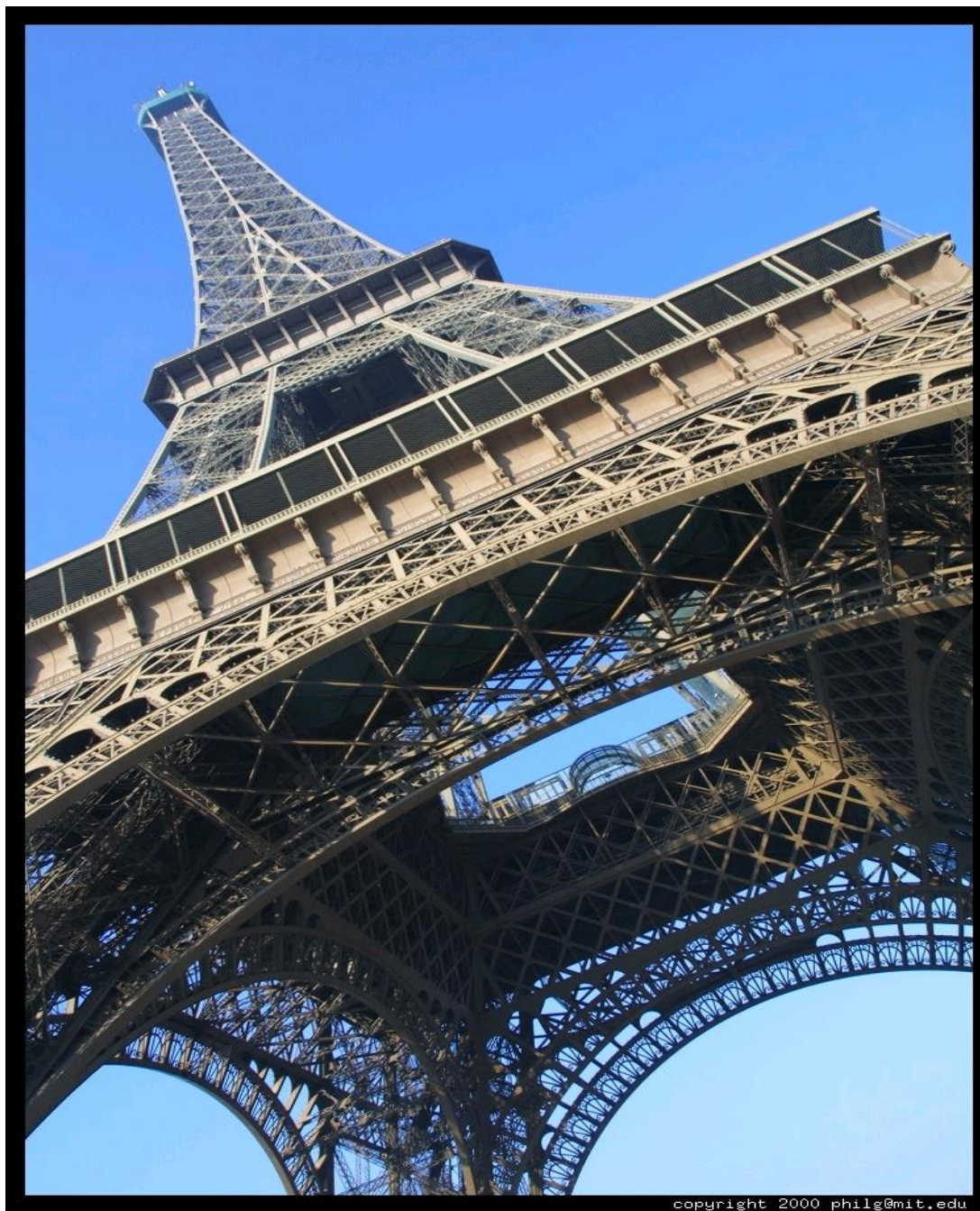


bowstring
(i)

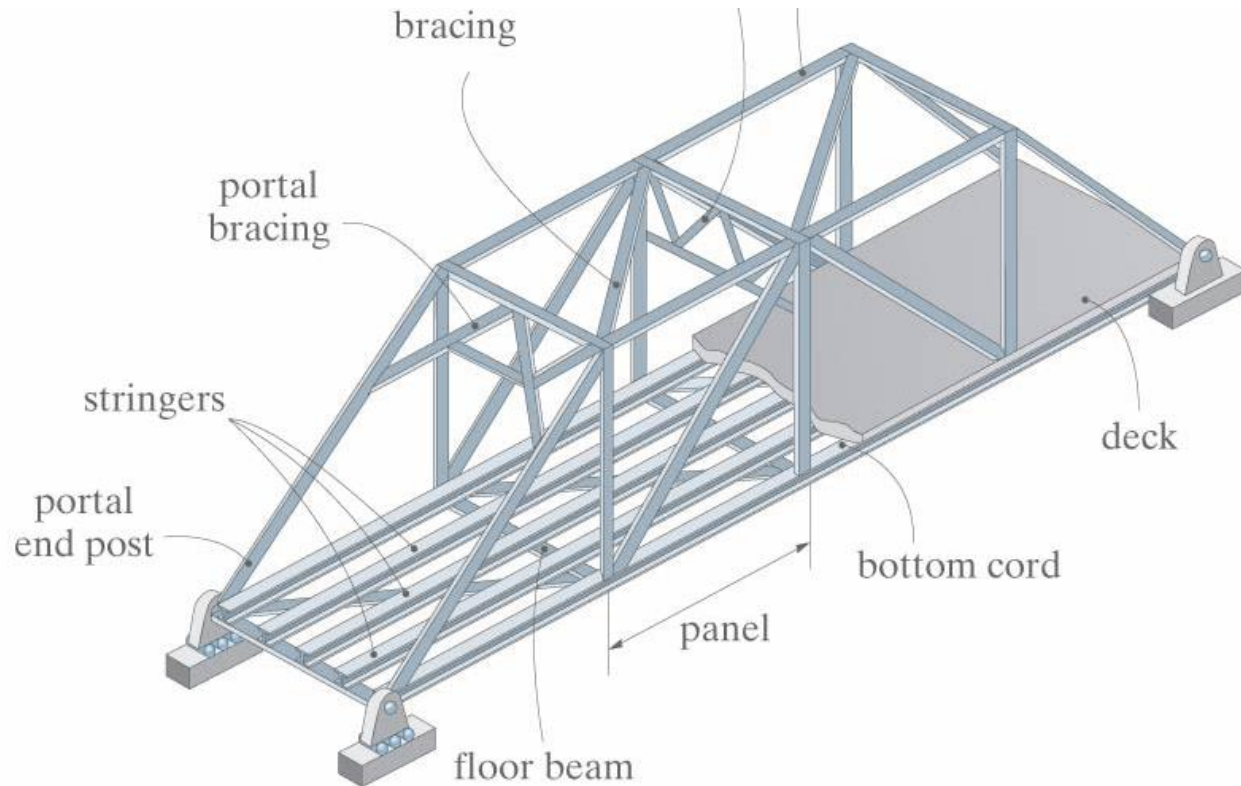


three-hinged arch
(j)





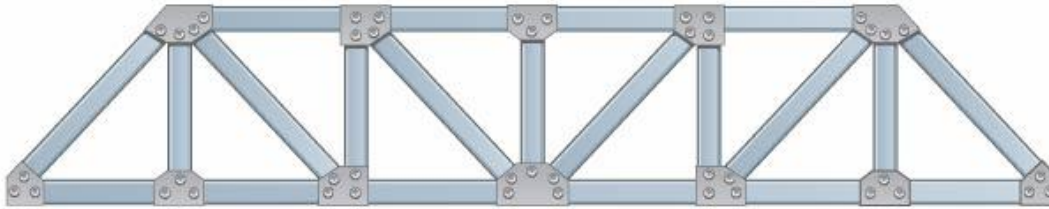
Truss Bridge



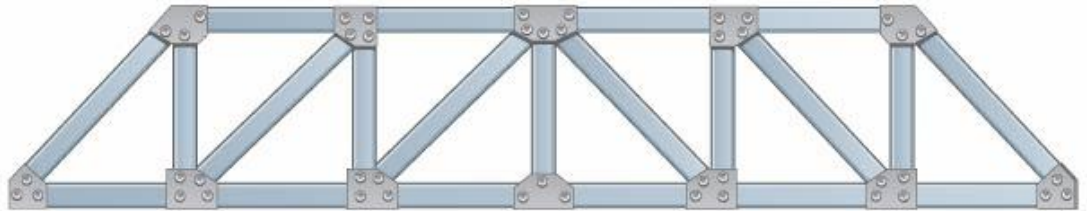
Truss Bridge



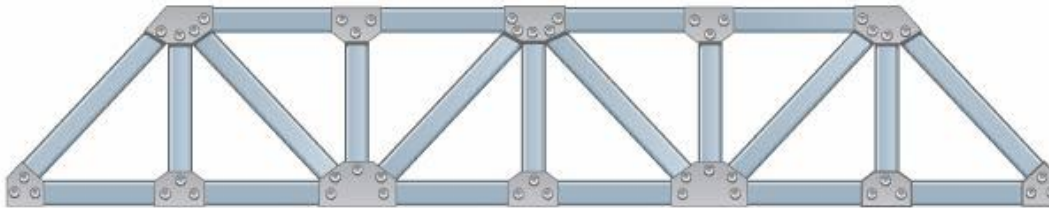
Truss Bridge Types



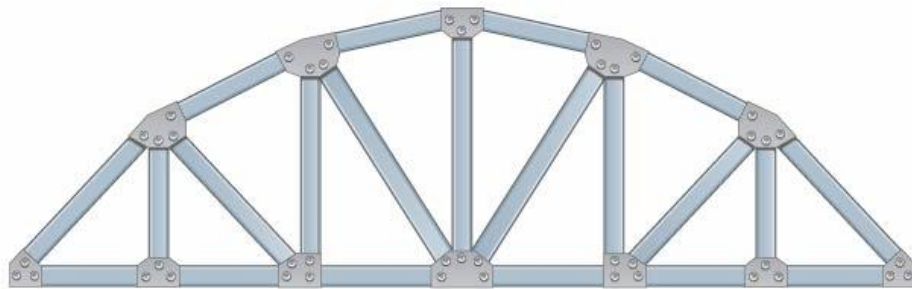
Pratt
(a)



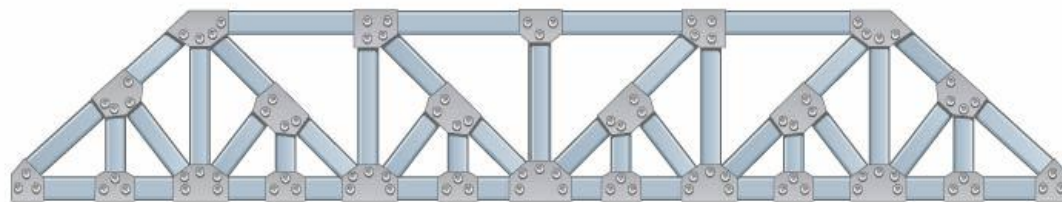
Howe
(b)



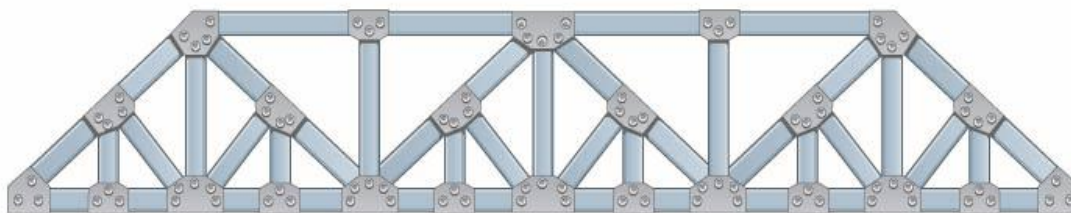
Warren (with verticals)
(c)



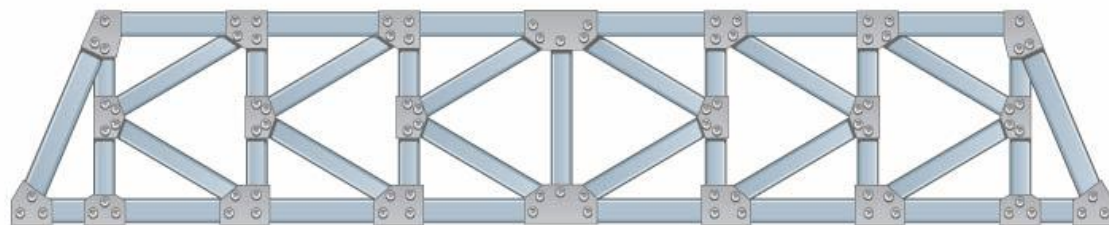
Parker
(d)



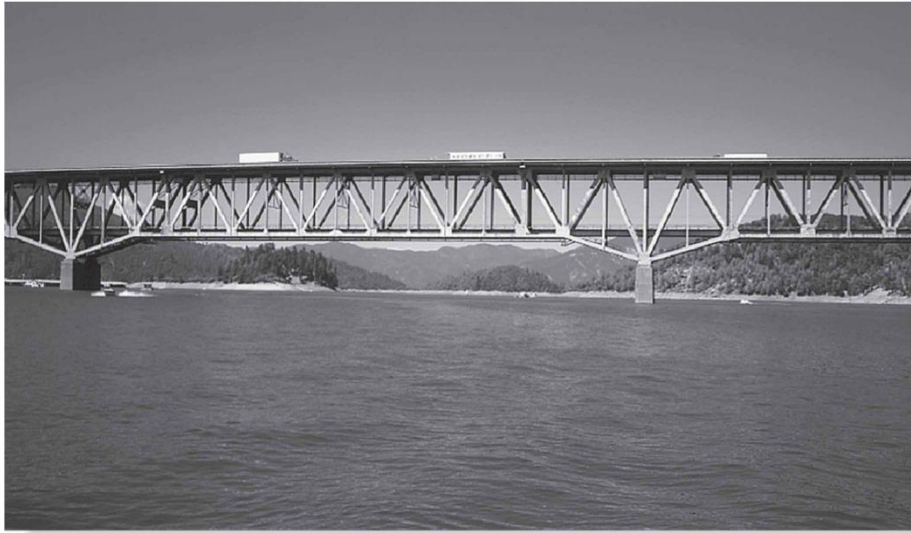
Baltimore
(e)



subdivided Warren
(f)



K-truss
(g)



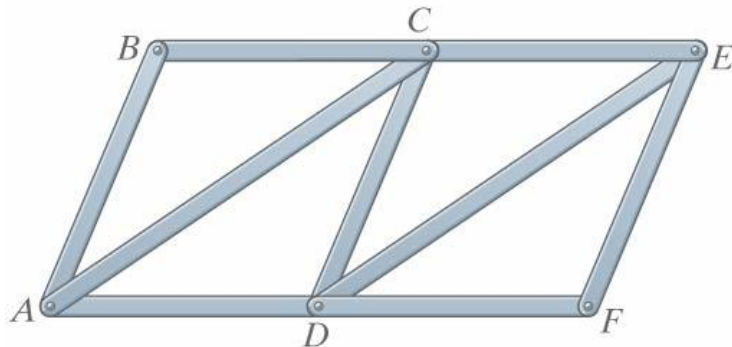
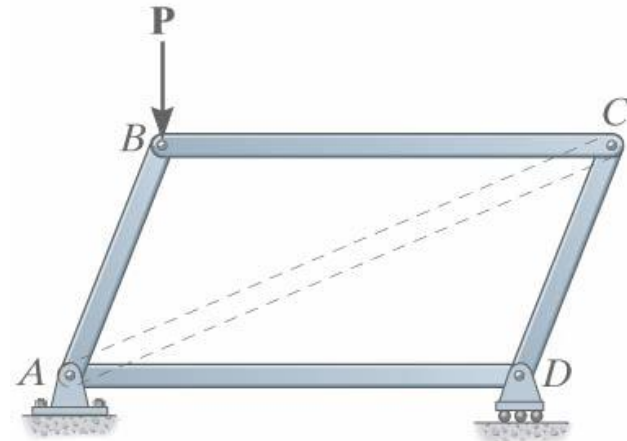
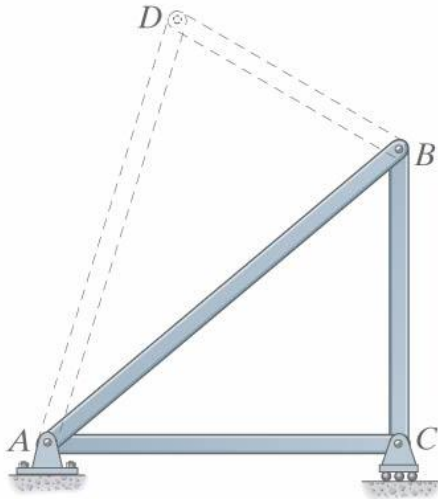


Assumptions for Design

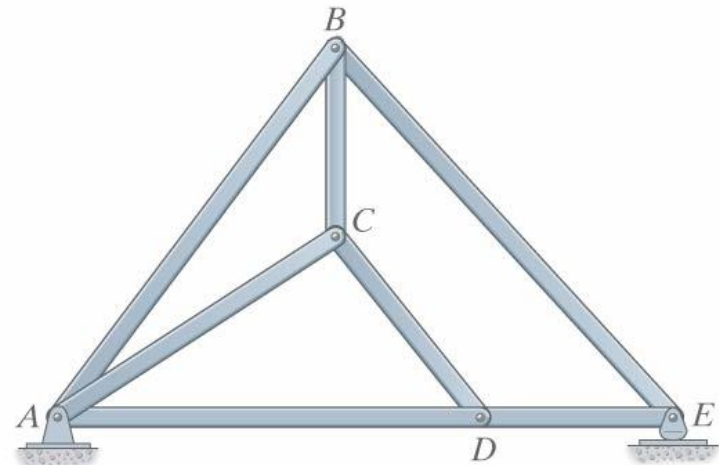
- All members are connected at both ends by smooth frictionless pins.
- All loads are applied at joints (member weight is negligible)
- Centroids of all joint members coincide at the joint.
- All members are straight.
- All load conditions satisfy Hooke's law.

Classification of coplanar Truss

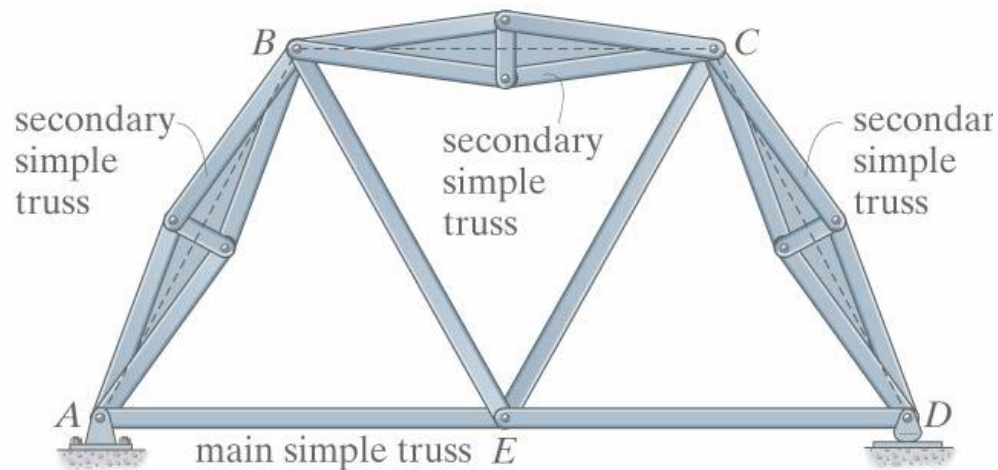
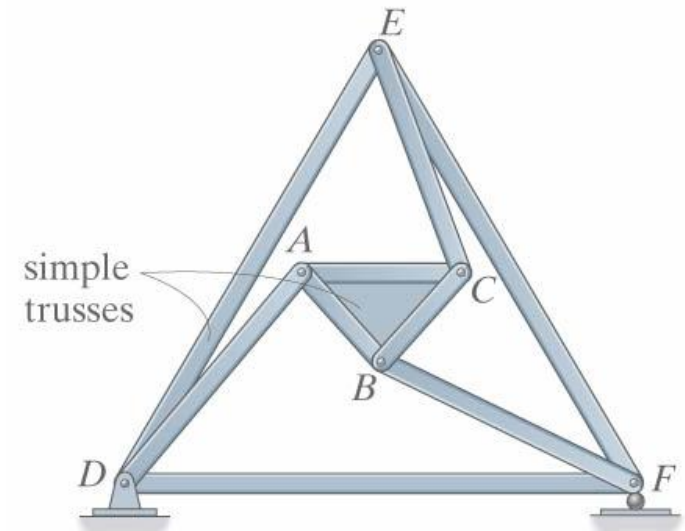
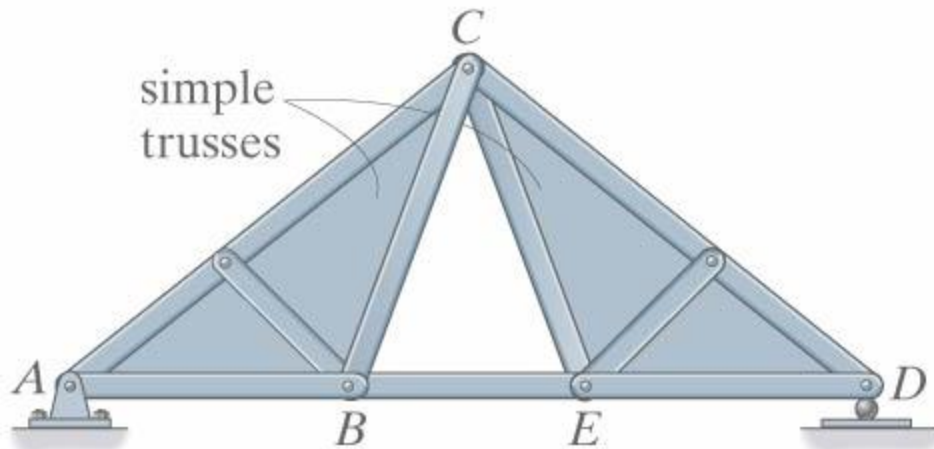
Simple Truss



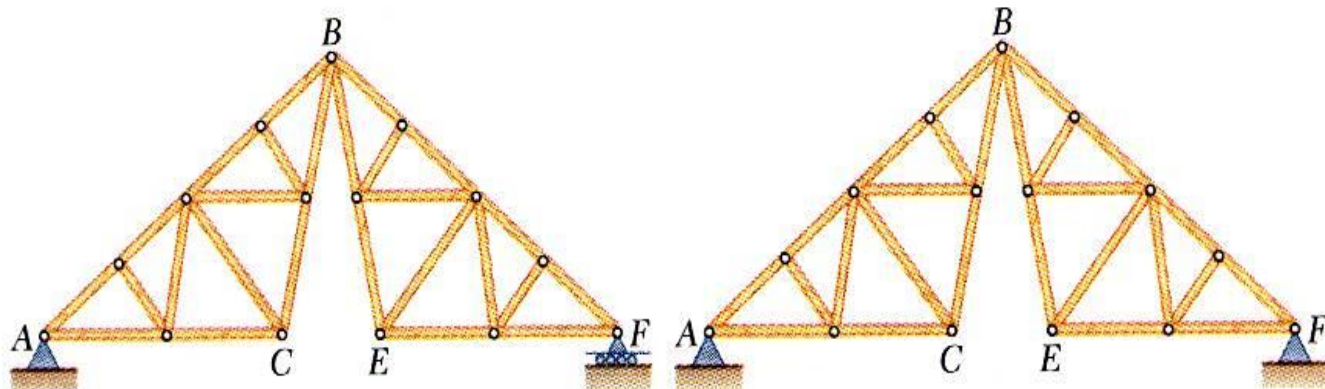
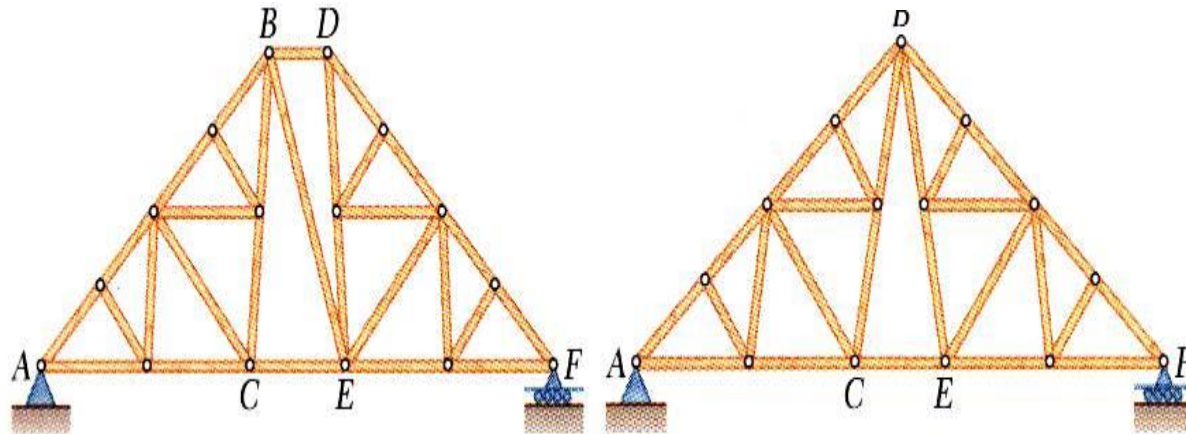
simple truss



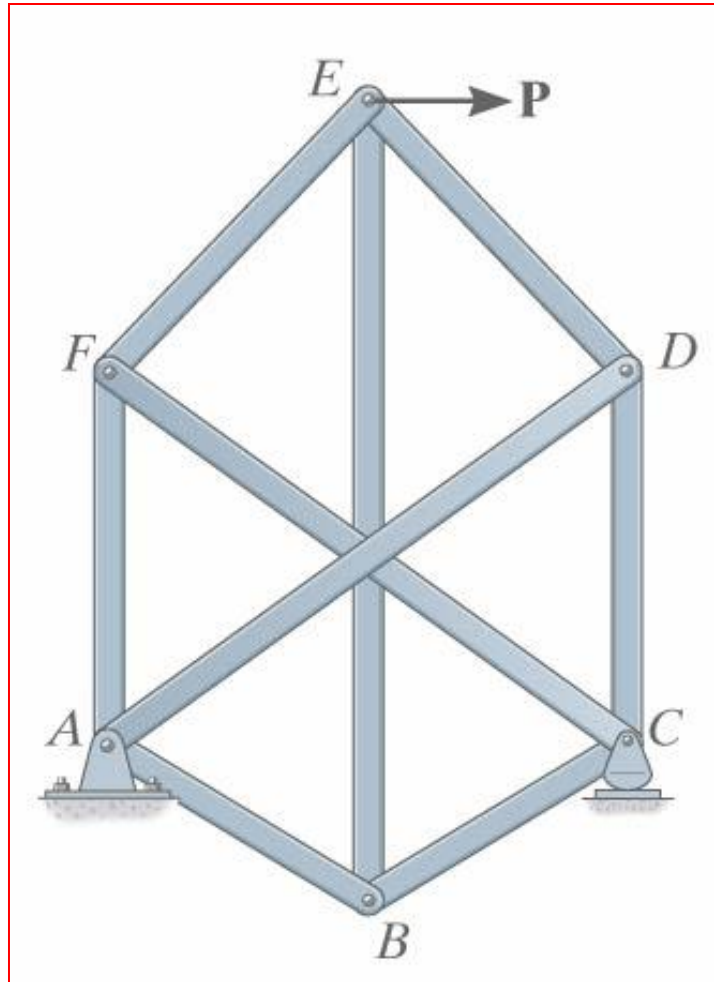
Compound Truss



Stability of Compound Truss



Complex Truss



Determinacy

For plane truss

If $b+r = 2j$ statically determinate

If $b+r > 2j$ statically indeterminate

Where

$b = \text{number of bars}$

$r = \text{number of external support reaction}$

$j = \text{number of joints}$

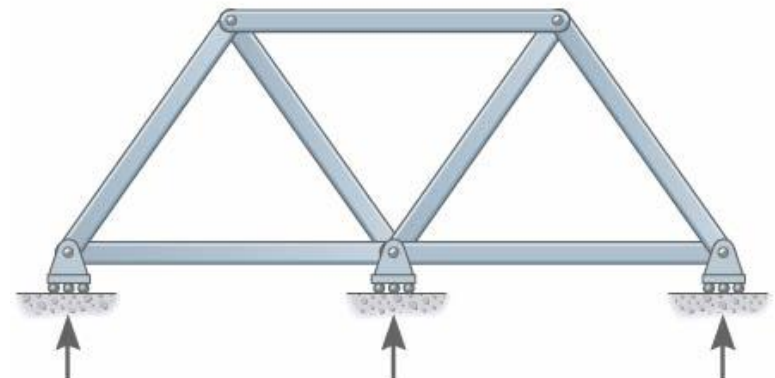
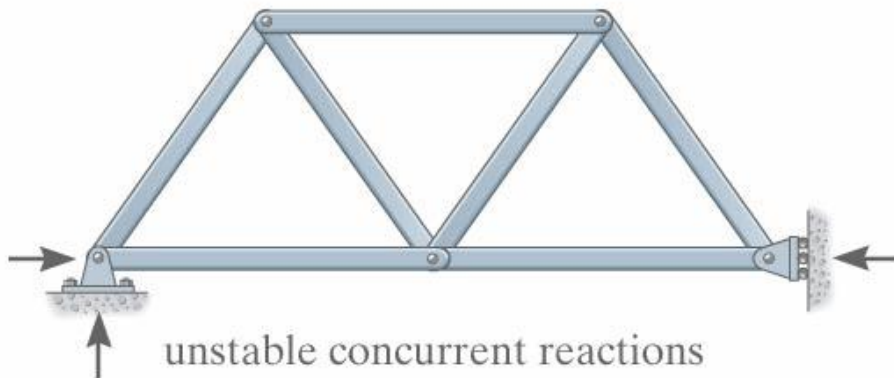
Stability

For plane truss

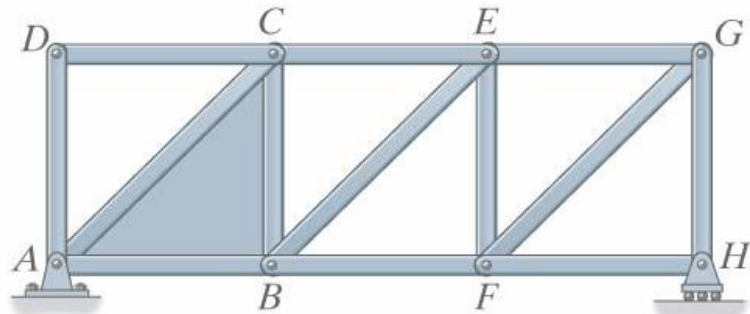
If $b+r < 2j$ unstable

*The truss can be statically determinate or indeterminate ($b+r \geq 2j$) but **unstable** in the following cases:*

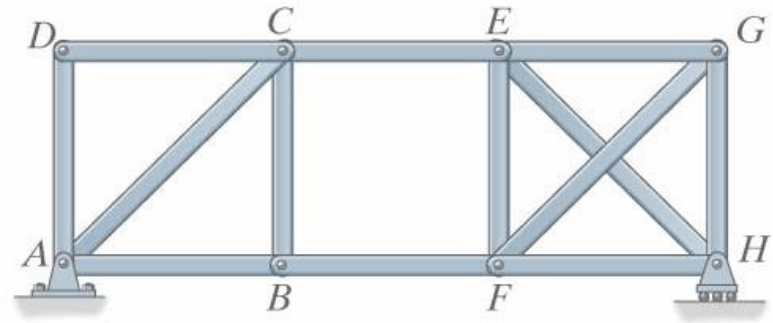
External Stability: *truss is externally unstable if all of its reactions are concurrent or parallel*



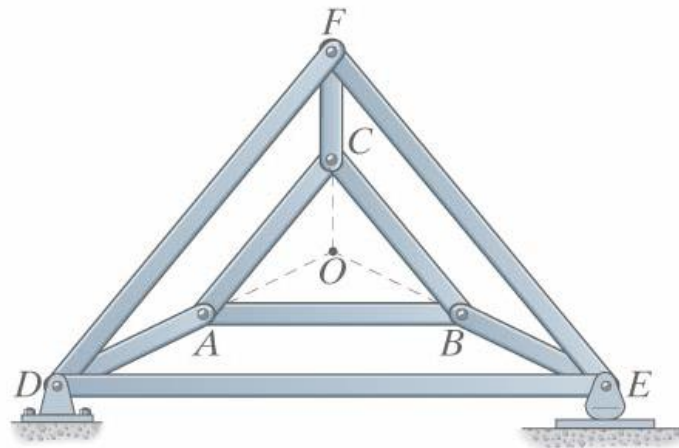
Internal stability:



Internally stable

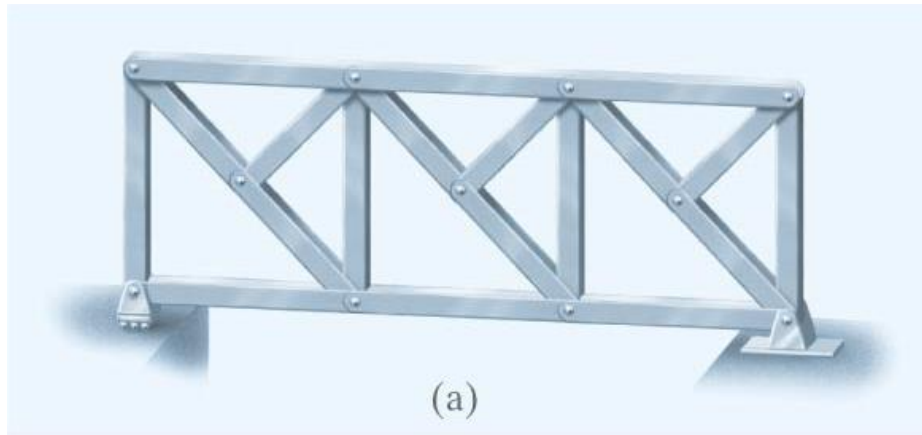


Internally unstable



Internally unstable

Classify each of the truss as stable , unstable, statically determinate or indeterminate.

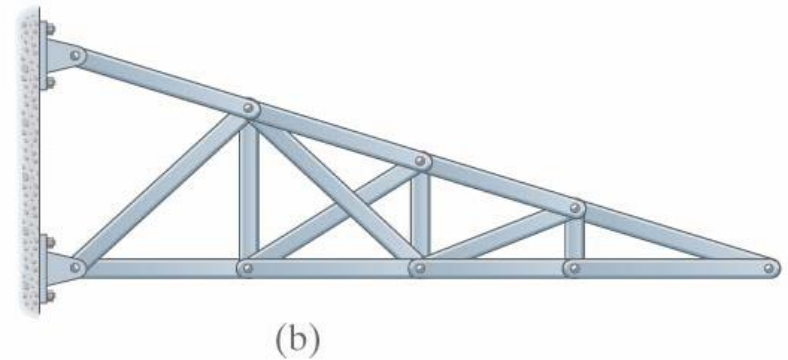


$$b = 19 \dots\dots r = 3 \dots\dots j = 11$$

$$b + r = 22$$

$$2j = 2(11) = 22$$

Statically determinate & Stable

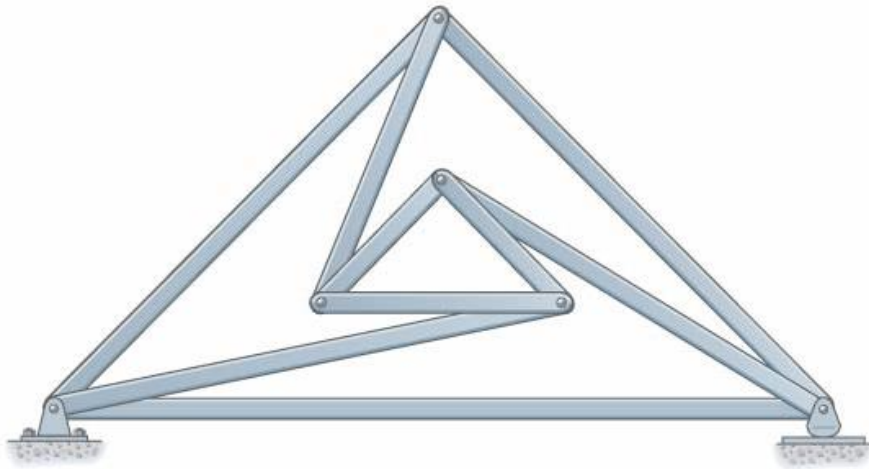


$$b = 15 \dots\dots r = 4 \dots\dots j = 9$$

$$b + r = 19$$

$$2j = 2(9) = 18$$

Statically indeterminate & Stable



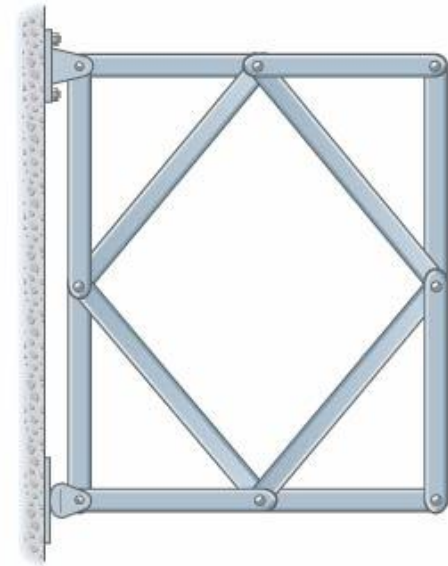
(c)

$$b = 9 \quad \dots \quad r = 3 \quad \dots \quad j = 6$$

$$b + r = 12$$

$$2j = 2(6) = 12$$

Statically determinate & Stable



(d)

$$b = 12 \quad \dots \quad r = 3 \quad \dots \quad j = 8$$

$$b + r = 15$$

$$2j = 2(8) = 16$$

Unstable

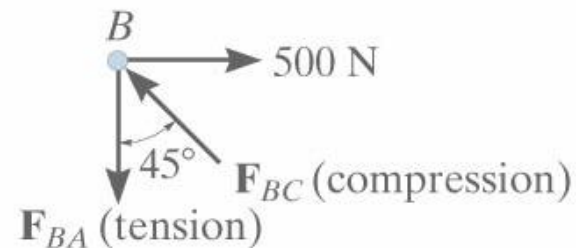
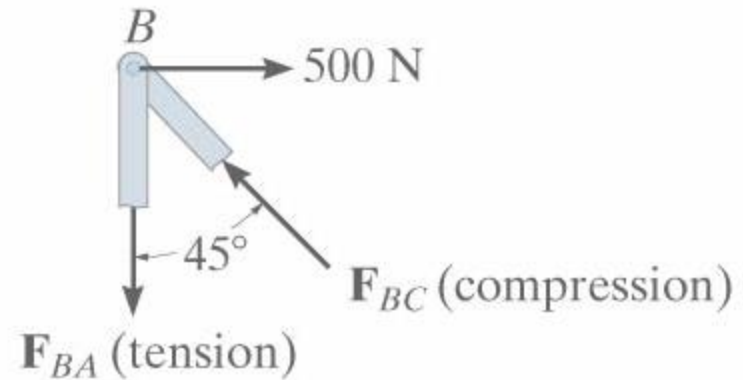
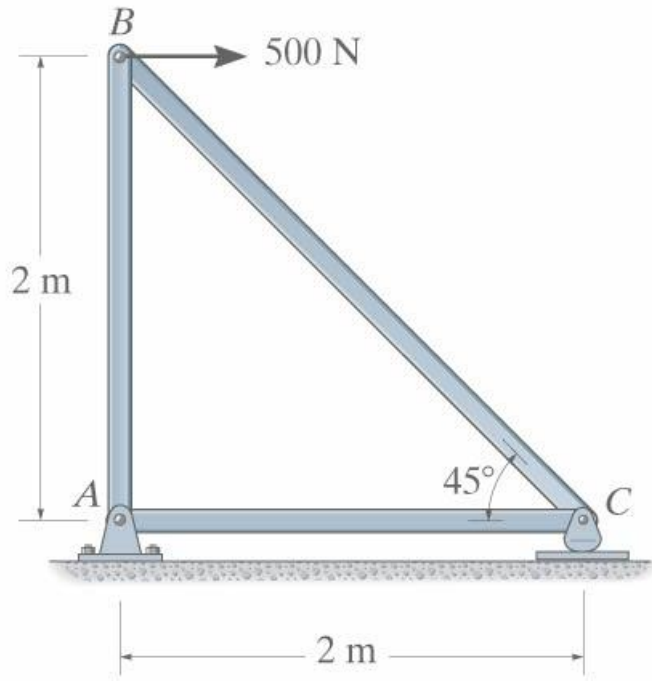
The Method of Joints

STEPS FOR ANALYSIS

1. If the *support reactions are not given*, draw a *FBD* of the entire truss and determine all the support reactions using the equations of equilibrium.
2. Draw the *free-body diagram of a joint with one or two unknowns*. Assume that all unknown member forces act in tension (pulling the pin) unless you can determine by inspection that the forces are compression loads.
3. Apply the scalar equations of equilibrium, $\sum F_X = 0$ and $\sum F_Y = 0$, to determine the unknown(s). If the answer is positive, then the assumed direction (tension) is correct, otherwise it is in the opposite direction (compression).
4. Repeat steps 2 and 3 at each joint in succession until all the required forces are determined.

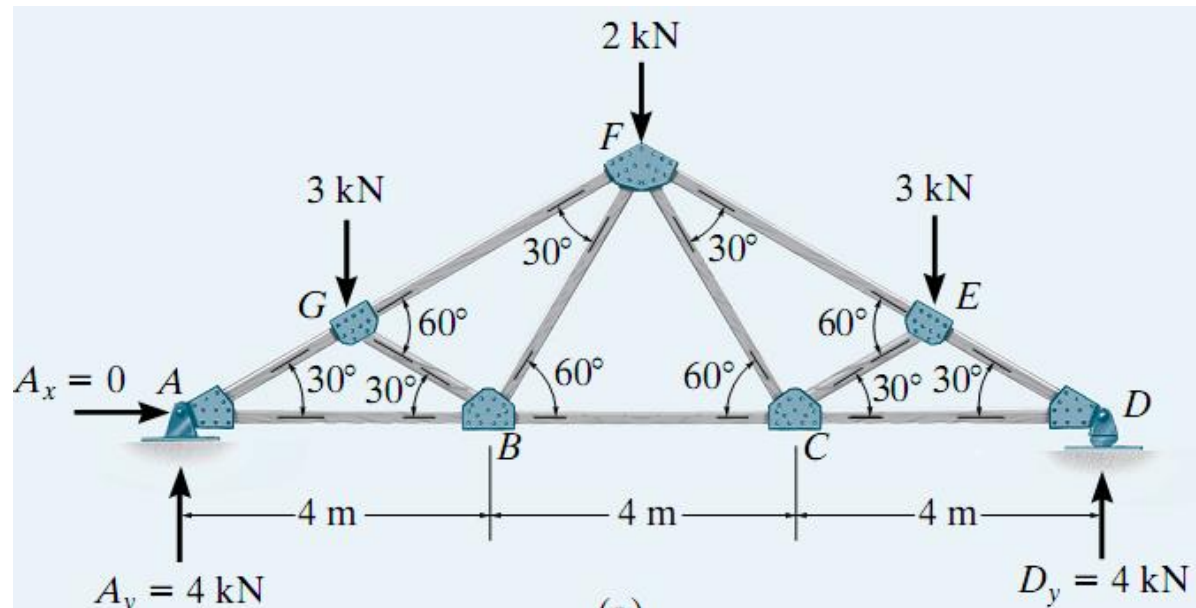
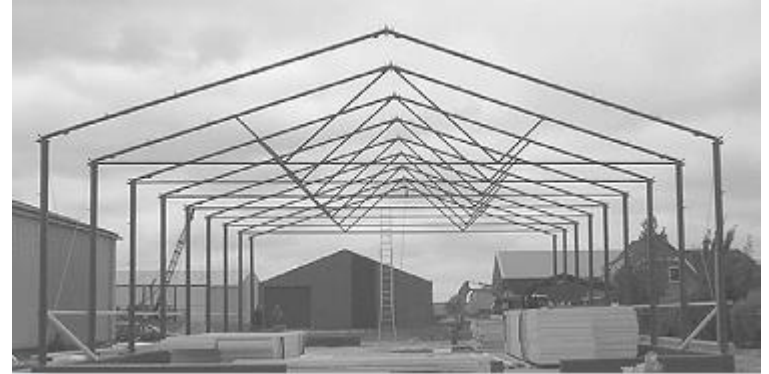
The Method of Joints

Example 1



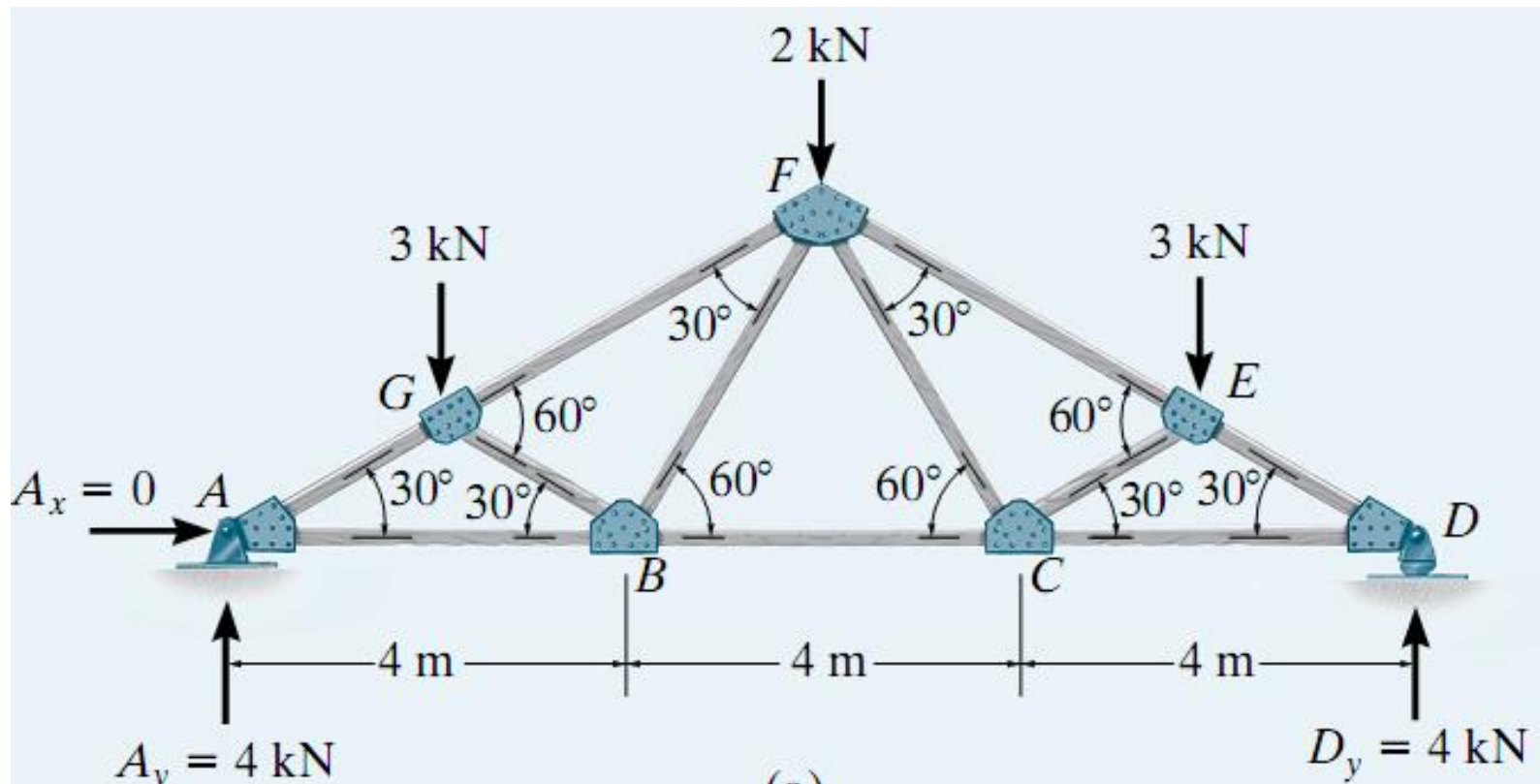
Example 2

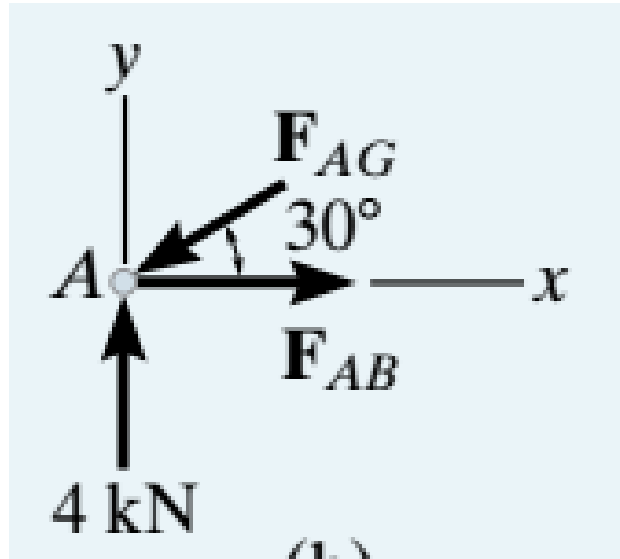
Solve the following truss



Example 2

Solve the following truss



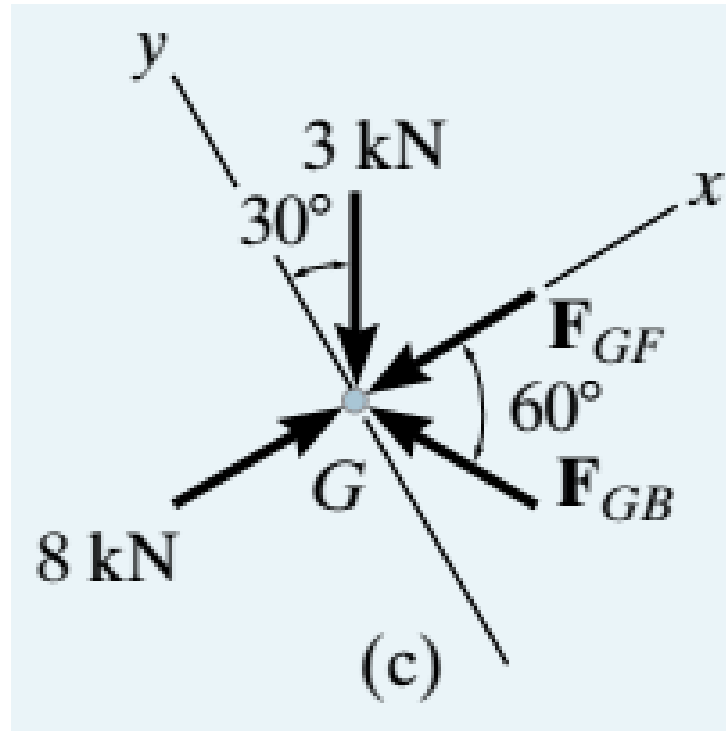


$$\sum F_y = 0;$$

$$4 - F_{AG} \sin 30 = 0 \Rightarrow F_{AG} = 8 \text{ kN} \quad \text{C}$$

$$\sum F_x = 0;$$

$$F_{AB} - 8 \cos 30 = 0 \Rightarrow F_{AB} = 6.93 \text{ kN} \quad \text{T}$$

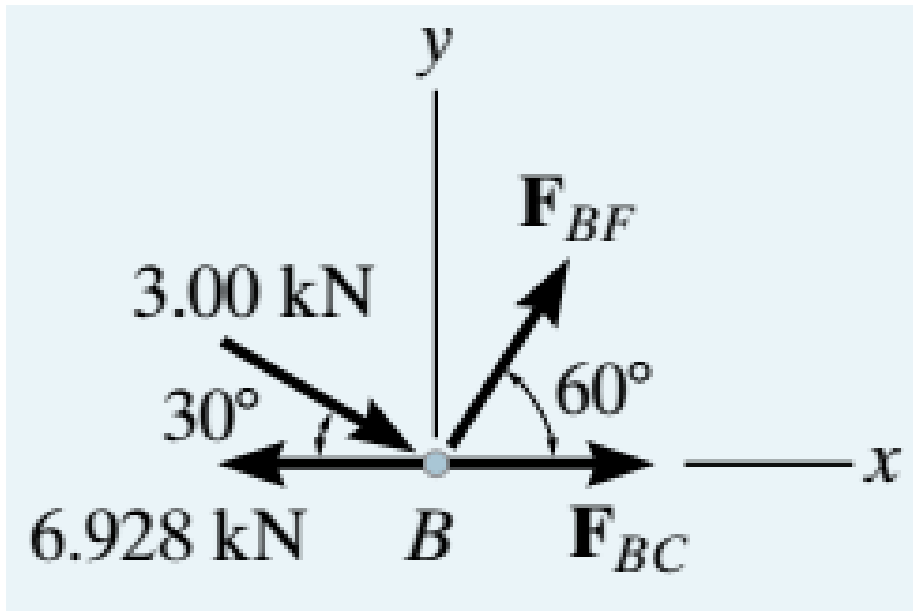


$$\sum F_y = 0;$$

$$F_{GB} \cos(30) - 3 \cos(30) = 0 \Rightarrow F_{GB} = 3.0 \text{ kN C}$$

$$\sum F_x = 0;$$

$$8 - 3 \sin(30) - 3 \sin(30) - F_{GF} = 0 \Rightarrow F_{GF} = 5.0 \text{ kN C}$$



$$\sum F_y = 0;$$

$$F_{BF} \sin(60) - 3.0 \sin(30) = 0 \Rightarrow F_{BF} = 1.73 \text{ kN} \quad \text{T}$$

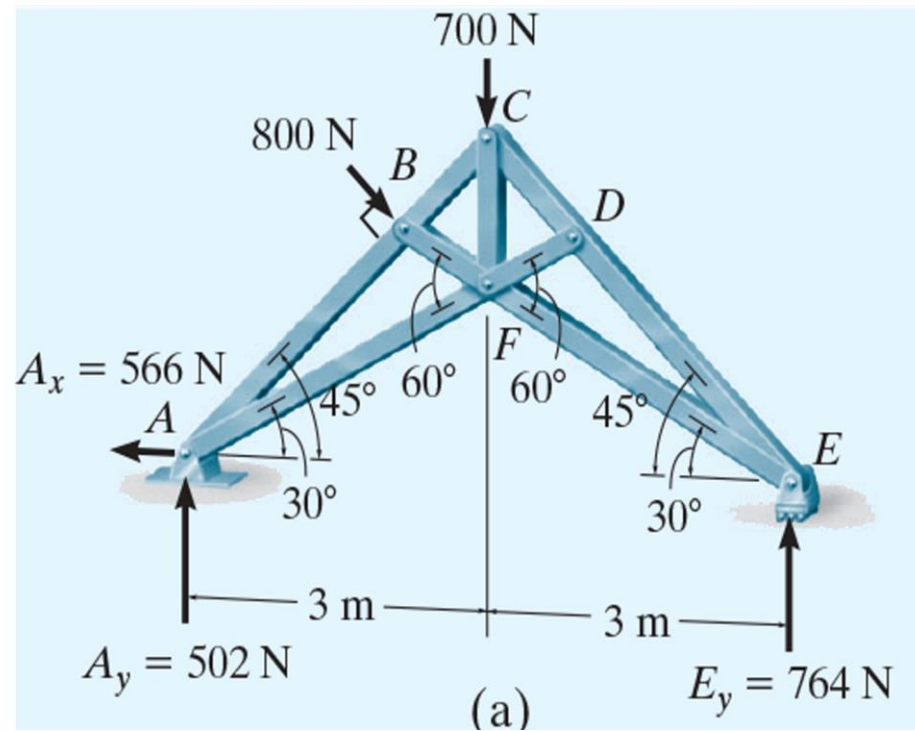
$$\sum F_x = 0;$$

$$F_{BC} + 1.73 \cos(60) + 3.0 \cos(30) - 6.93 = 0$$

$$\Rightarrow F_{BC} = 4.46 \text{ kN} \quad \text{T}$$

Example 3

Solve the following truss



Example 3

Joint E, Fig. 3-21b. Note that simultaneous solution of equations is avoided by the x, y axes orientation.

$$+\nearrow \Sigma F_y = 0; \quad 764 \cos 30^\circ - F_{ED} \sin 15^\circ = 0$$

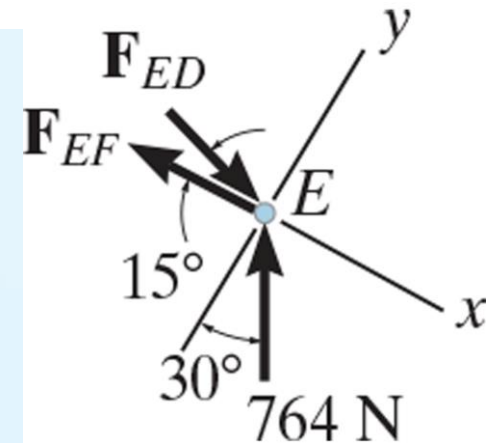
$$F_{ED} = 2557 \text{ N (C)}$$

Ans.

$$+\searrow \Sigma F_x = 0; \quad 2557 \cos 15^\circ - F_{EF} - 764 \sin 30^\circ = 0$$

$$F_{EF} = 2088 \text{ N (T)}$$

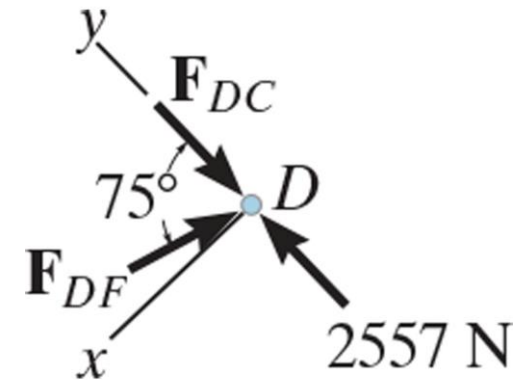
Ans.



Joint D, Fig. 3-21c.

$$+\swarrow \Sigma F_x = 0; \quad -F_{DF} \sin 75^\circ = 0 \quad F_{DF} = 0$$

$$+\nwarrow \Sigma F_y = 0; \quad -F_{DC} + 2557 = 0 \quad F_{DC} = 2557 \text{ N (C)}$$



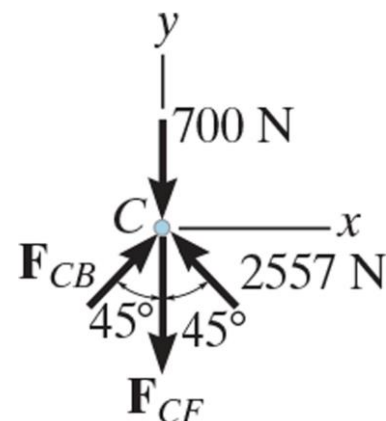
Joint C, Fig. 3-21d.

$$\rightarrow \Sigma F_x = 0; \quad F_{CB} \sin 45^\circ - 2557 \sin 45^\circ = 0$$

$$F_{CB} = 2557 \text{ N (C)}$$

$$+\uparrow \Sigma F_y = 0; \quad -F_{CF} - 700 + 2(2557) \cos 45^\circ = 0$$

$$F_{CF} = 2916 \text{ N (T)}$$

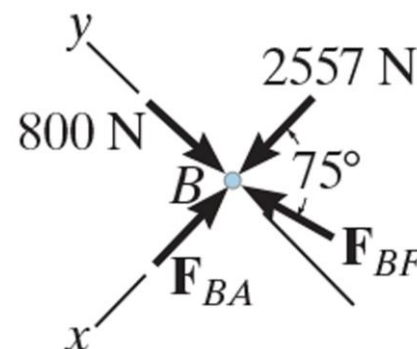


Joint B, Fig. 3-21e.

$$+\nearrow \Sigma F_y = 0; \quad F_{BF} \sin 75^\circ - 800 = 0 \quad F_{BF} = 828 \text{ N (C)}$$

$$+\swarrow \Sigma F_x = 0; \quad 2557 + 828 \cos 75^\circ - F_{BA} = 0$$

$$F_{BA} = 2771 \text{ N (C)}$$

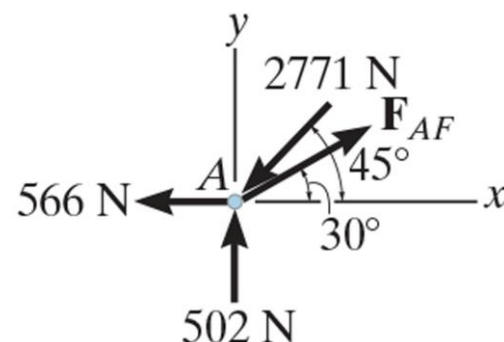


Joint A, Fig. 3-21f.

$$\rightarrow \Sigma F_x = 0; \quad F_{AF} \cos 30^\circ - 2771 \cos 45^\circ - 566 = 0$$

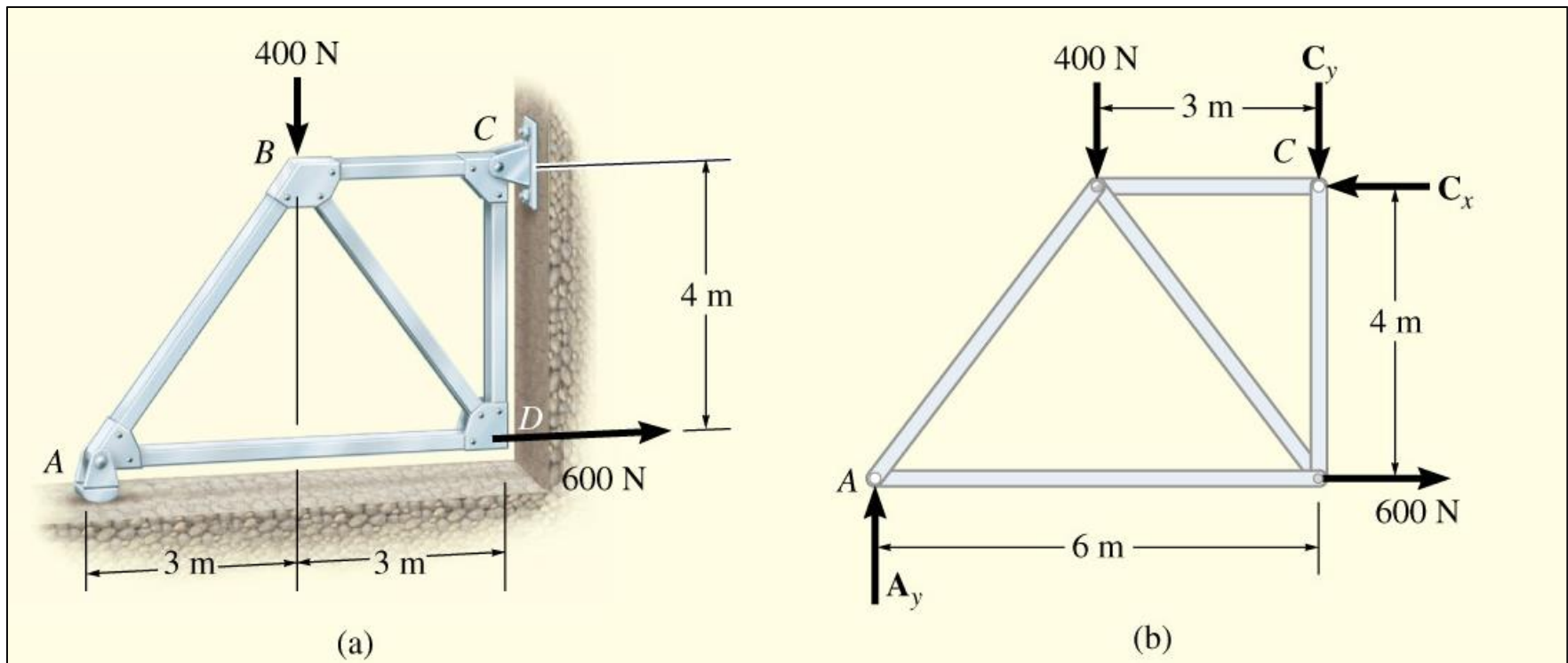
$$F_{AF} = 2916 \text{ N (T)}$$

$$+\uparrow \Sigma F_y = 0; \quad 502 - 2771 \sin 45^\circ + 2916 \sin 30^\circ = 0$$



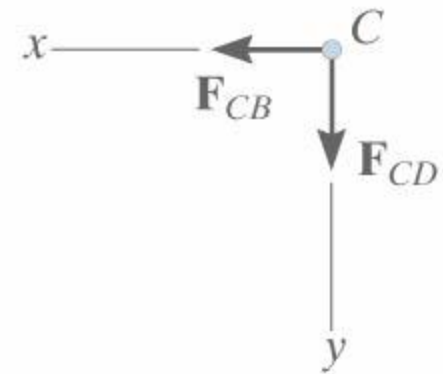
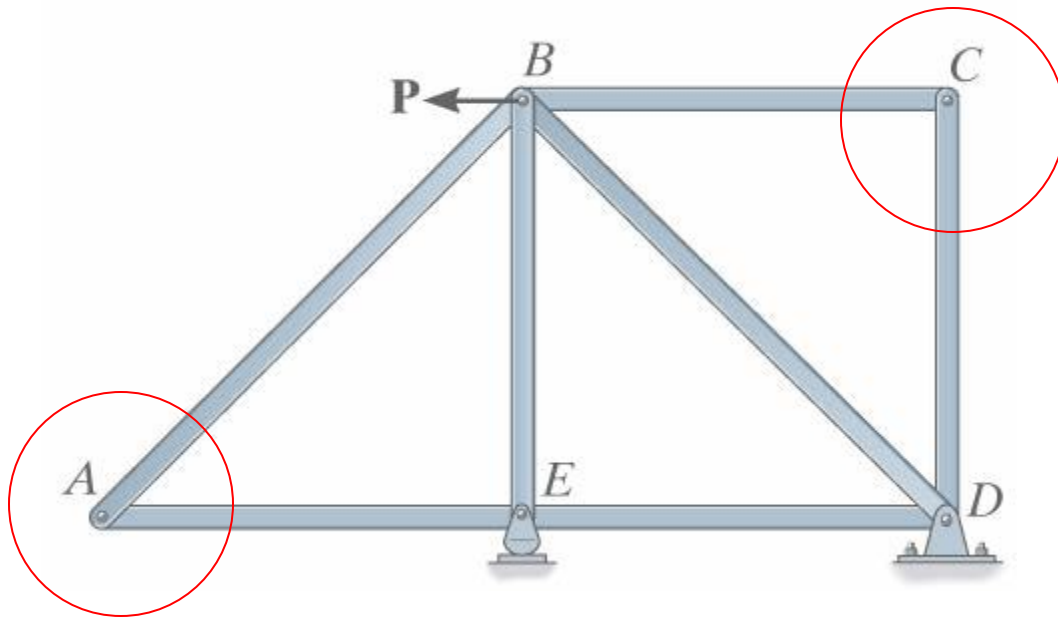
Problem 1

Determine the force in each member



Zero Force Member

1- If a joint has only **two non-collinear members** and there is **no external load or support reaction** at that joint, then those two members are zero-force members

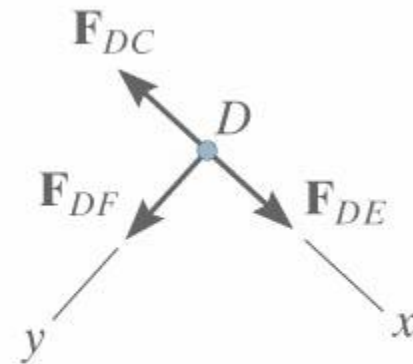
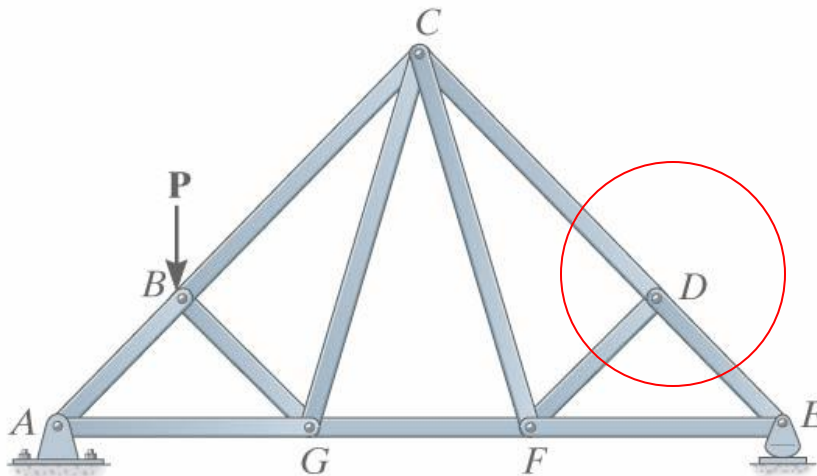


$$\overset{+}{\leftarrow} \Sigma F_x = 0; F_{CB} = 0$$

$$+\downarrow \Sigma F_y = 0; F_{CD} = 0$$

Zero Force Member

2- If **three members** form a truss joint for which **two of the members are collinear** and there is **no external load or reaction** at that joint, then the **third non-collinear member** is a zero force member.

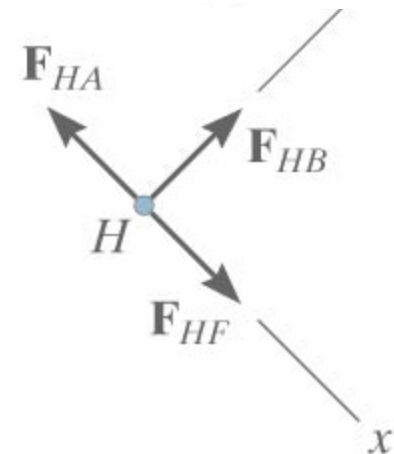
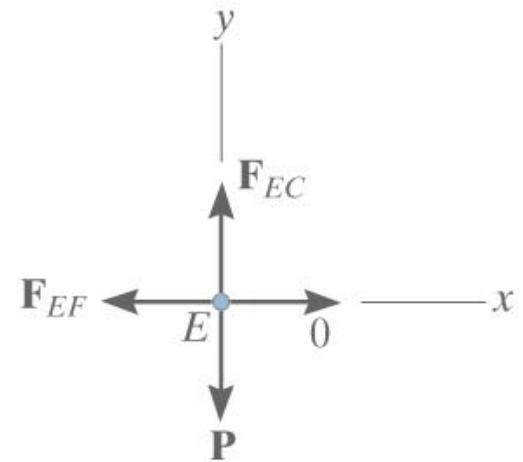
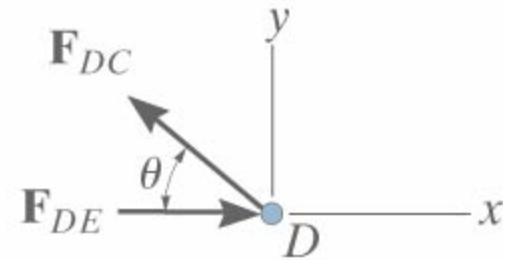
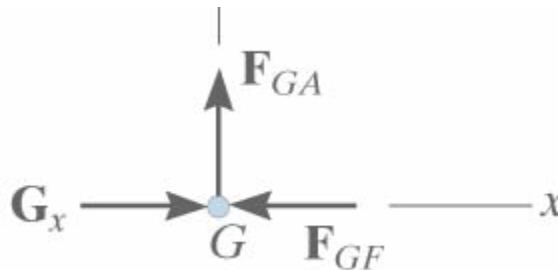
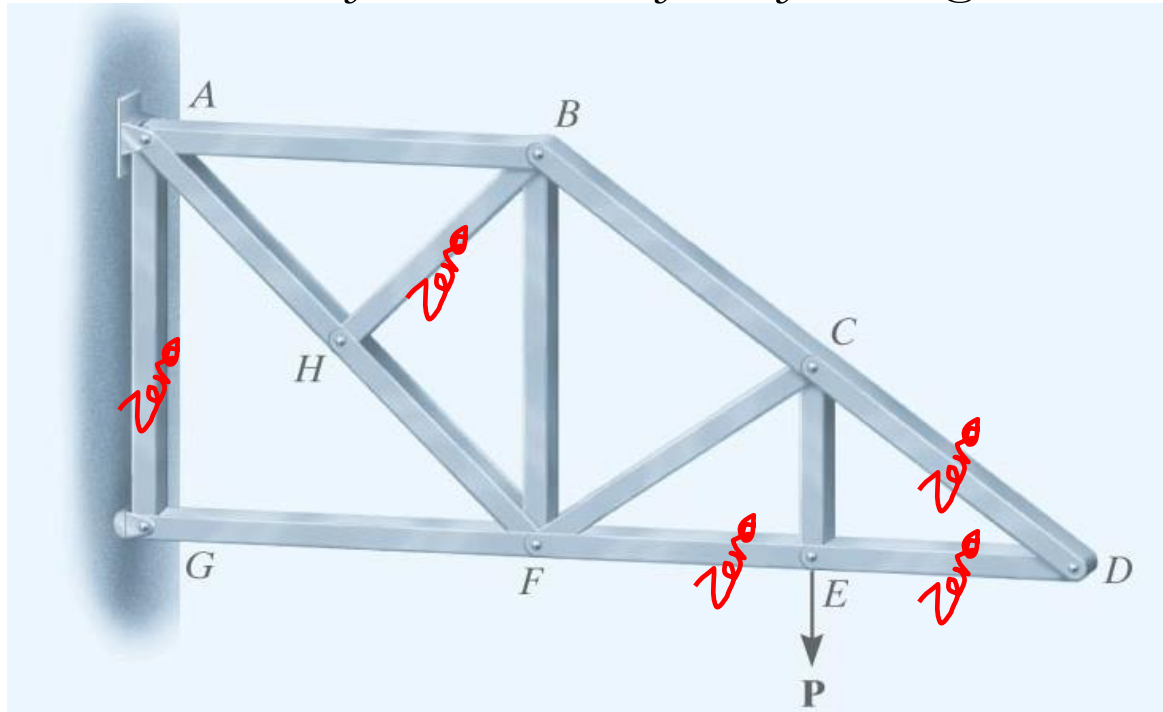


(b)

$$+\curvearrowright \sum F_y = 0; F_{DF} = 0$$

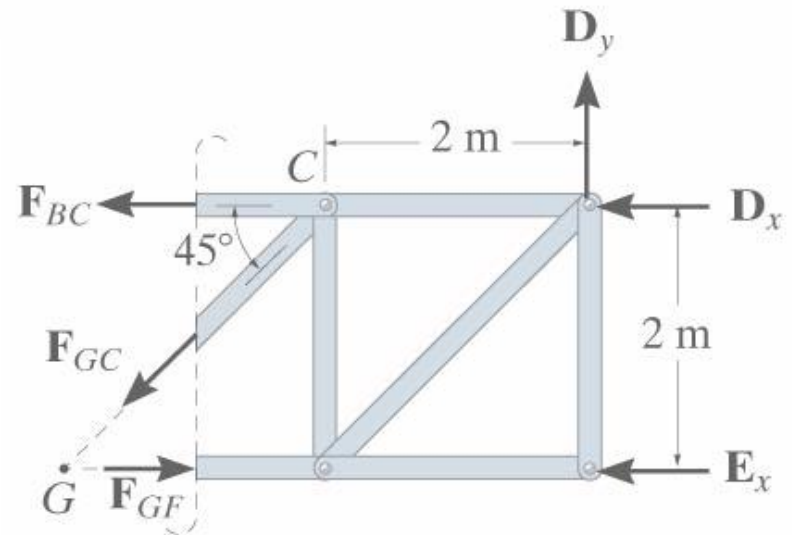
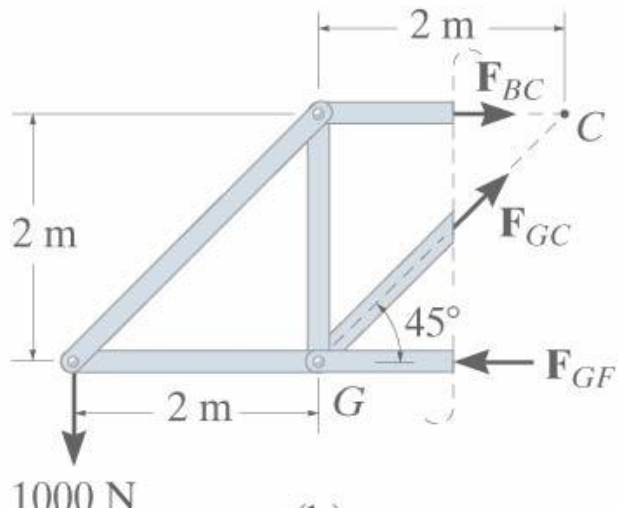
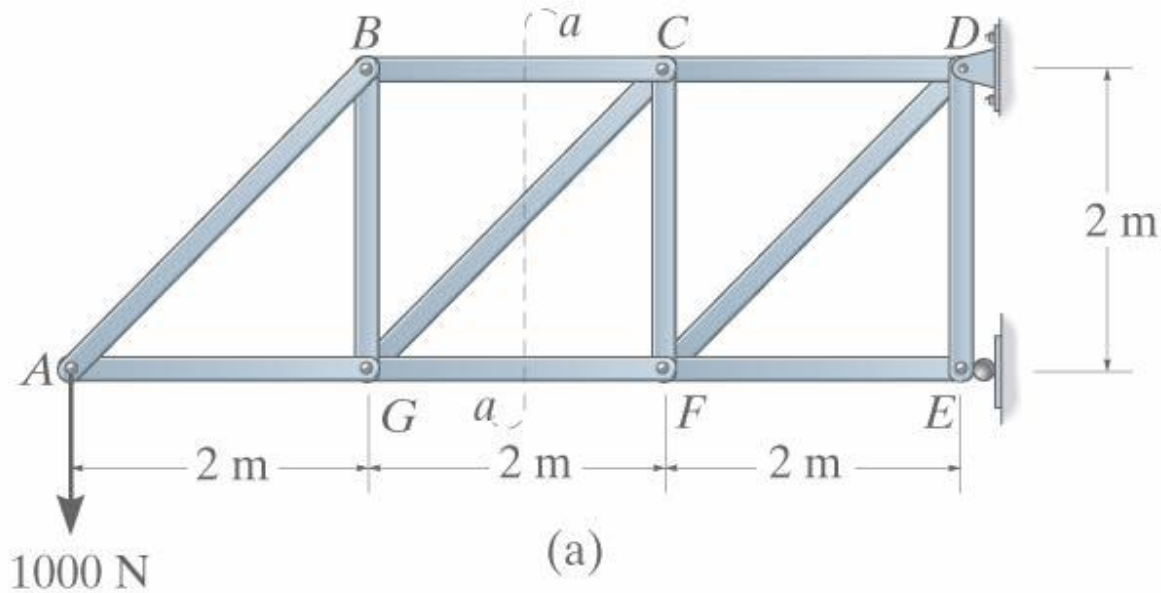
Problem 2

Find Zero force member of the following truss



Method of Section

The Method of Section



The Method of Section

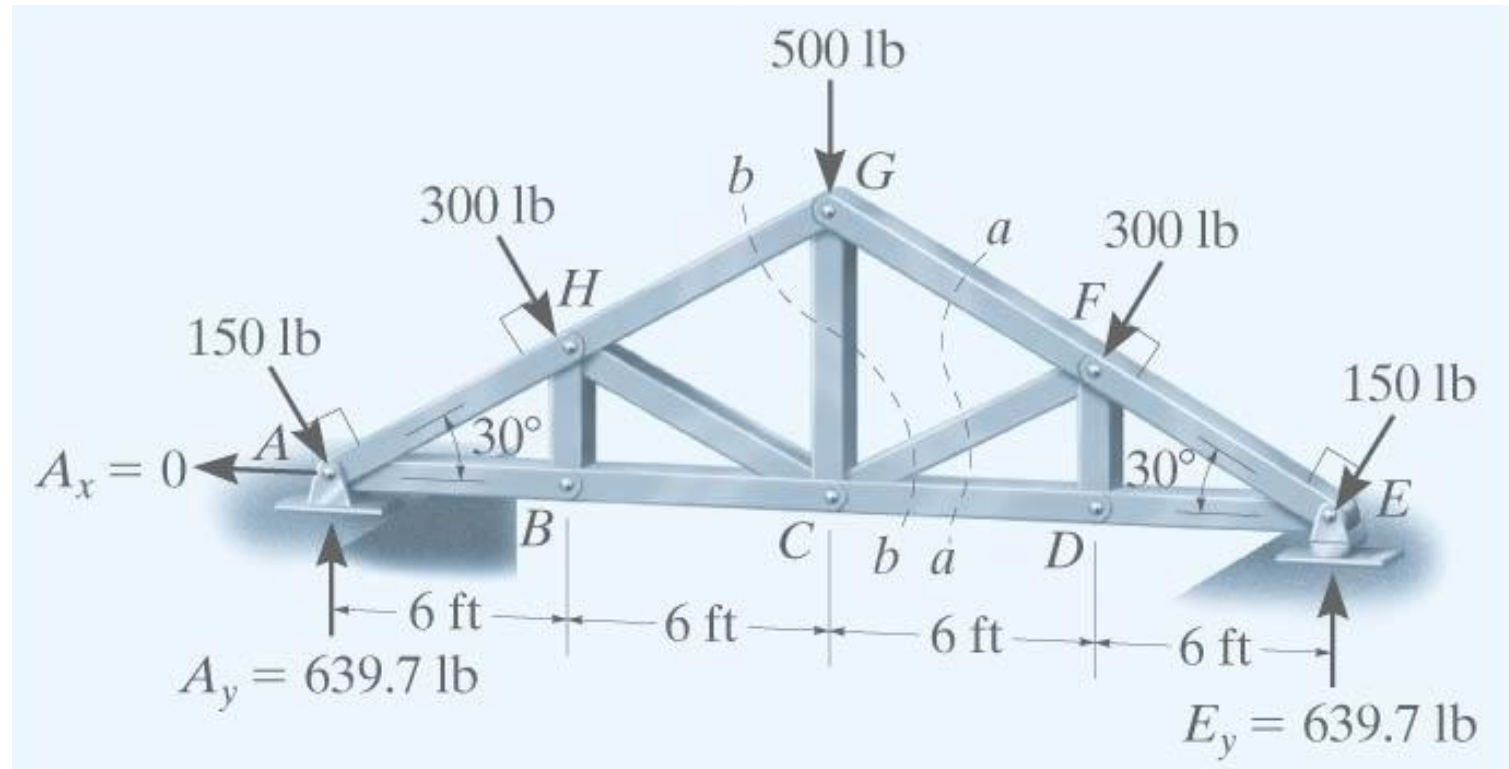
STEPS FOR ANALYSIS

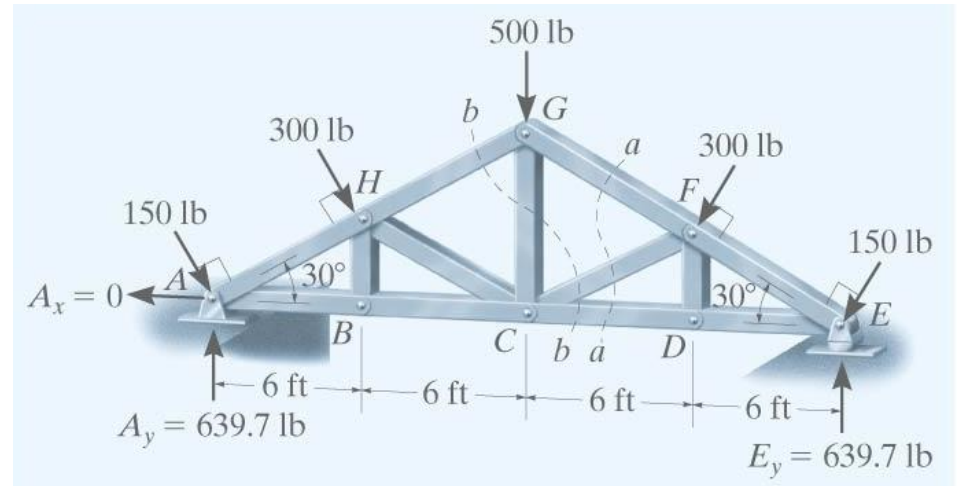
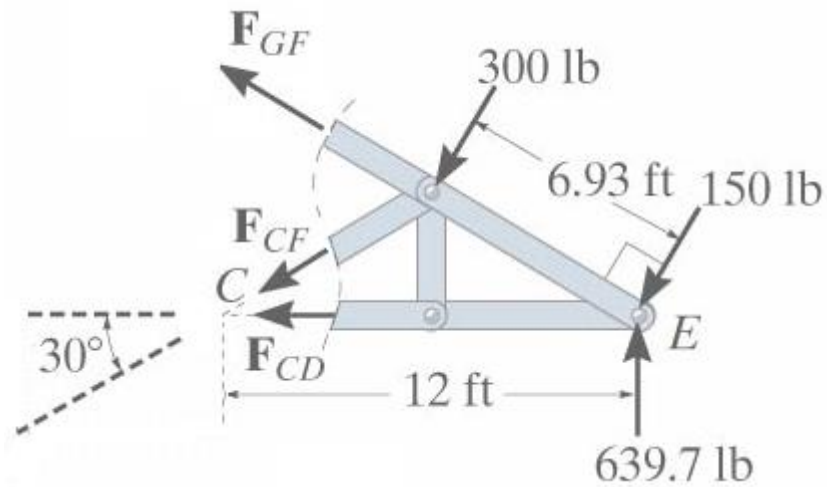
1. Decide how you need to “cut” the truss. This is based on:
a) where you need to determine forces, and, b) where the total number of unknowns does not exceed three (in general).
2. Decide which side of the cut truss will be easier to work with (minimize the number of reactions you have to find).
3. If required, determine the necessary support reactions by drawing the FBD of the entire truss and applying the E. of E.

4. Draw the FBD of the selected part of the cut truss. We need to **indicate the unknown forces** at the cut members. Initially we assume **all the members are in tension**, as we did when using the method of joints. Upon solving, if the answer is positive, the member is in tension as per our assumption. If the answer is negative, the member must be in compression. *(Please note that you can also assume forces to be either tension or compression by inspection as was done in the previous example above.)*
5. Apply the equations of equilibrium (EofE) to the selected cut section of the truss to solve for the unknown member forces. Please note that in most cases it is possible to write one equation to solve for one unknown directly.

Example 4

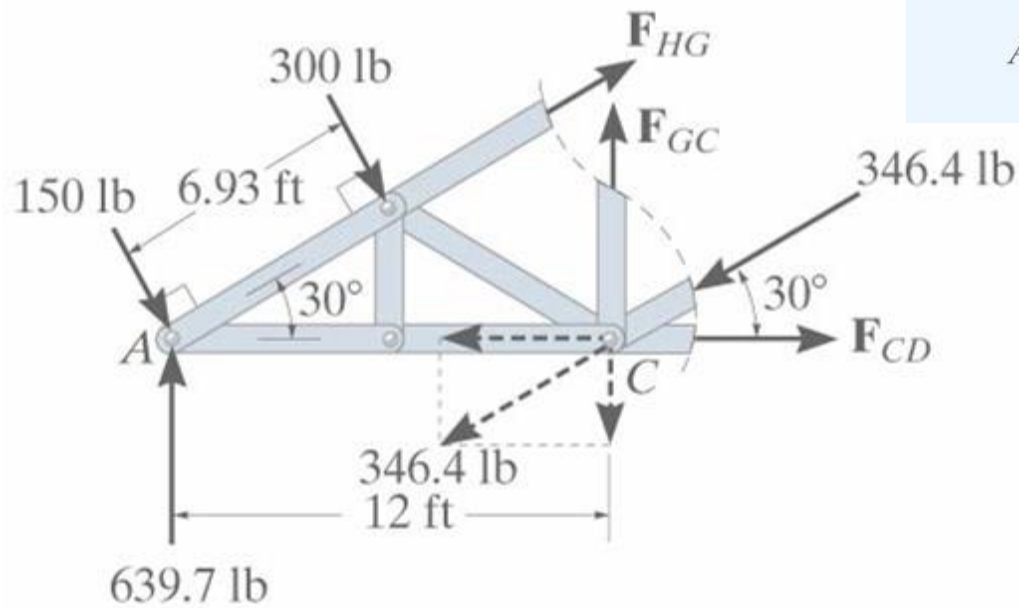
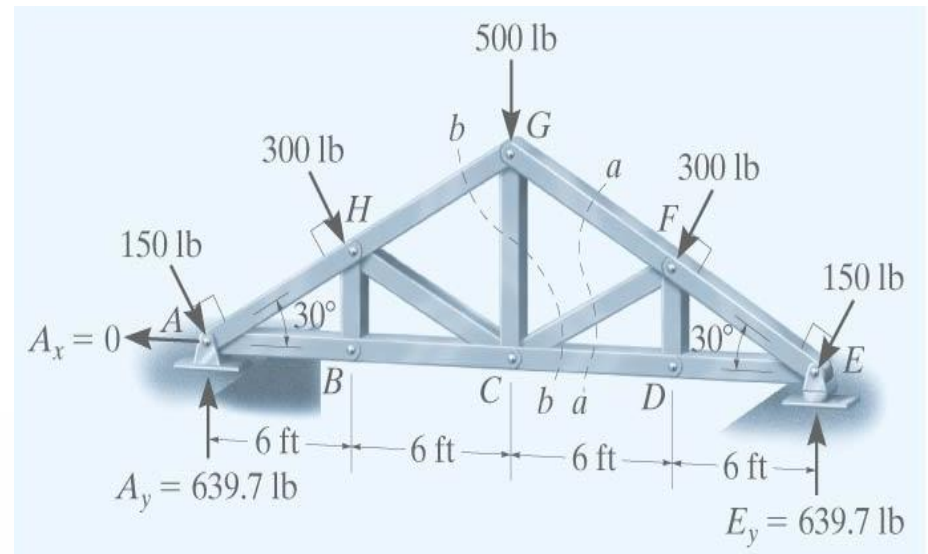
Solve the CF & GC members in the truss





$$\sum M_E = 0;$$

$$F_{CF} \sin 30(12) + 300(6.93) = 0 \Rightarrow F_{CF} = -346.4 \text{ lb}$$

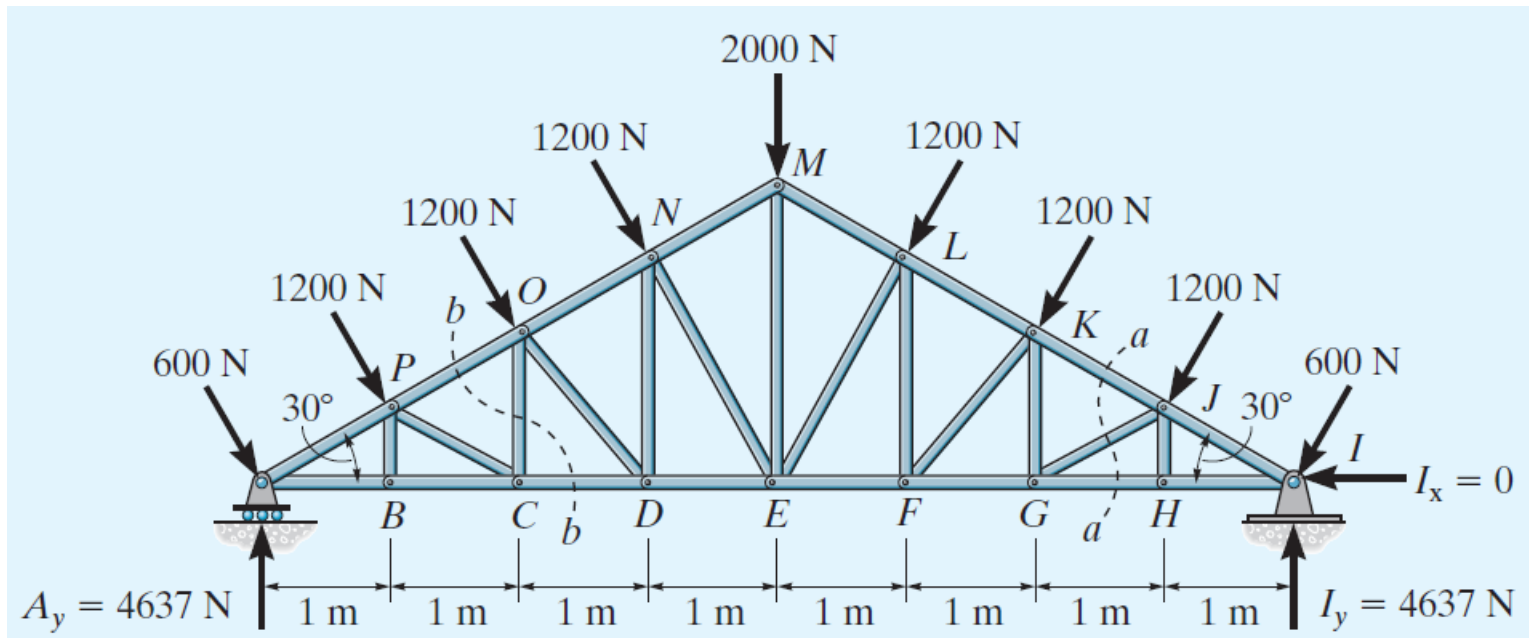


$$\sum M_A = 0;$$

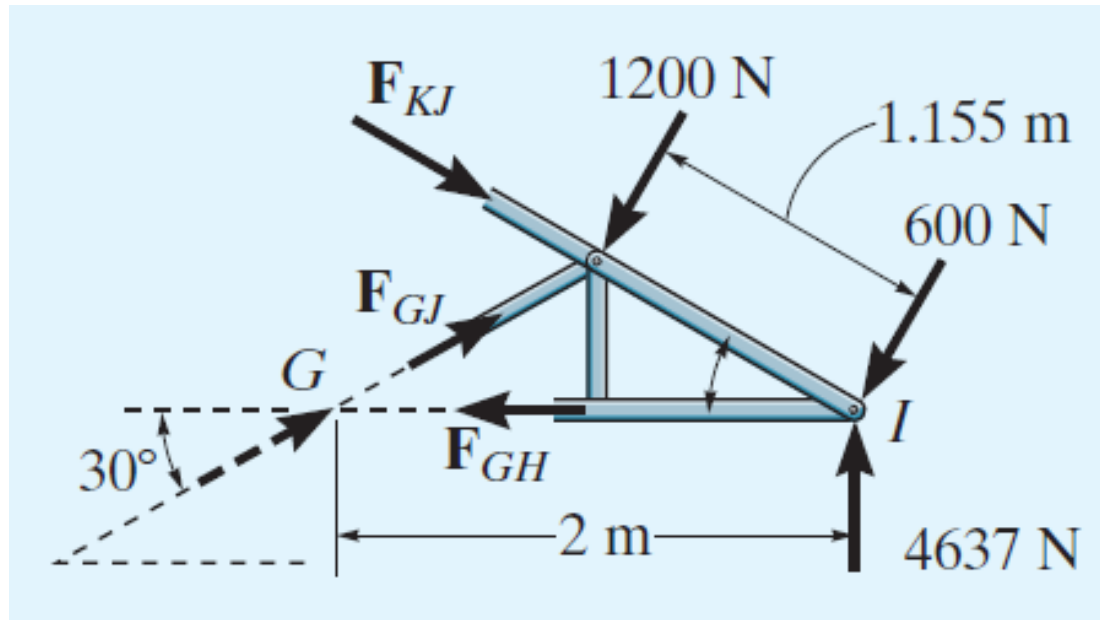
$$F_{GC} (12) - 300(6.93) - 346.4 \sin 30 (12) = 0 \Rightarrow F_{GC} = 346.4 \text{ lb} \quad \text{T}$$

Example 5

Determine the force in members **GJ** and **CO** of the roof truss shown.

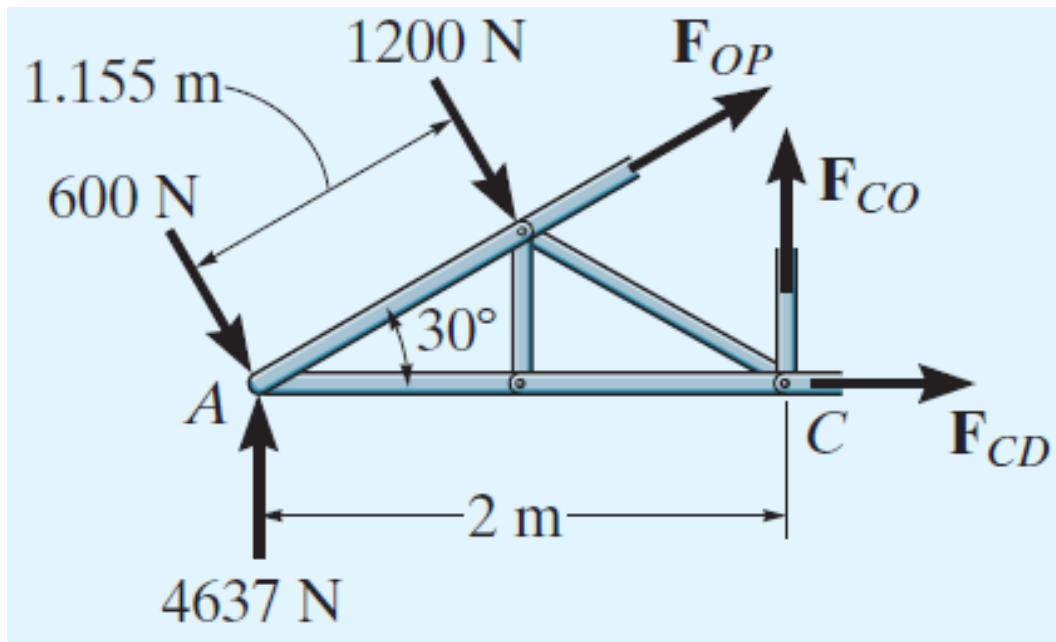


Example 5



$$\downarrow + \Sigma M_I = 0; \quad -F_{GJ} \sin 30^\circ (2) + 1200(1.155) = 0$$
$$F_{GJ} = 1386\text{ N (C)}$$

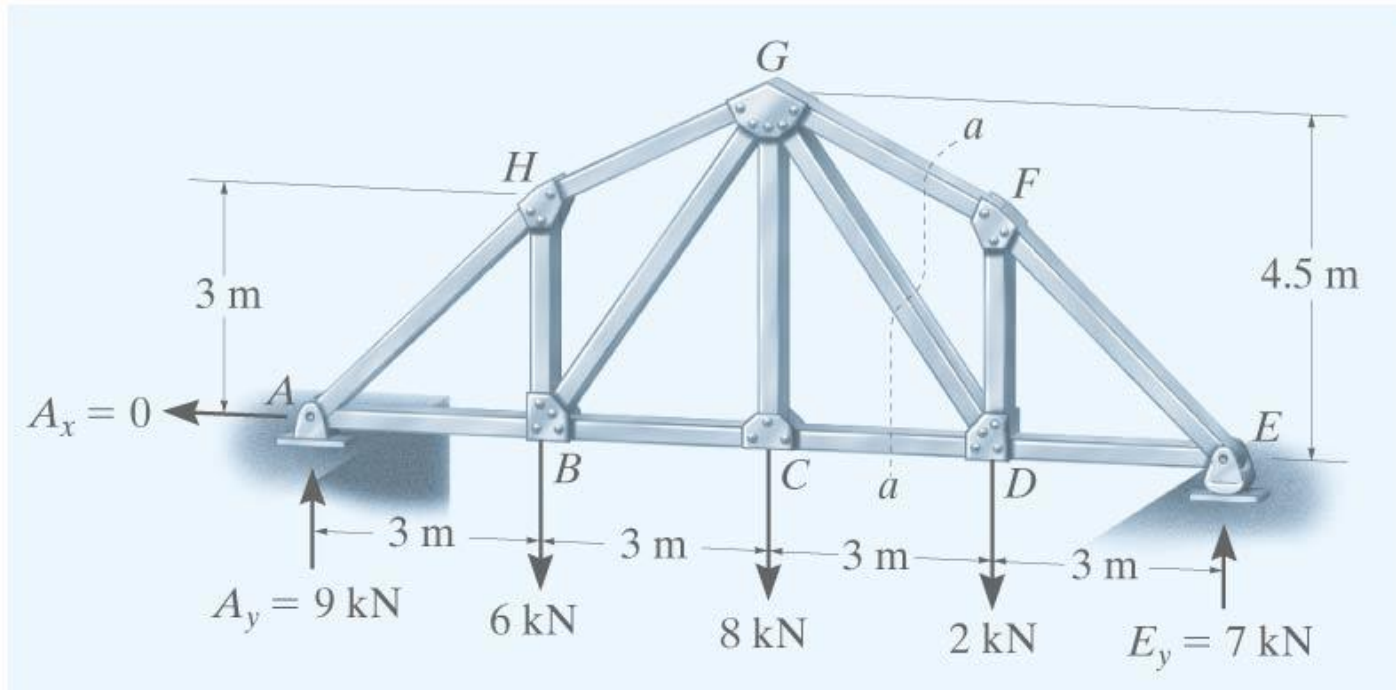
Example 6

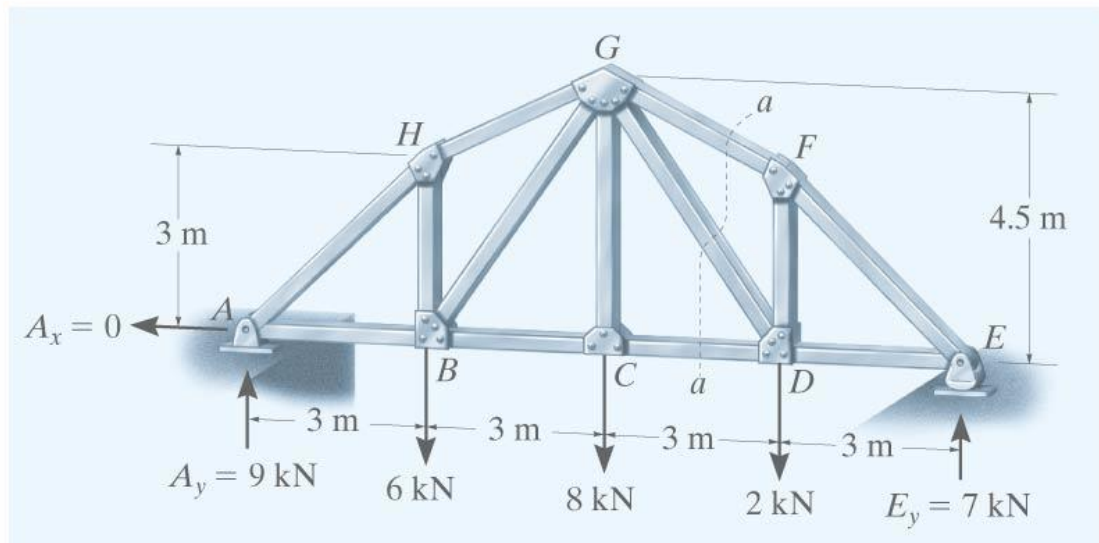
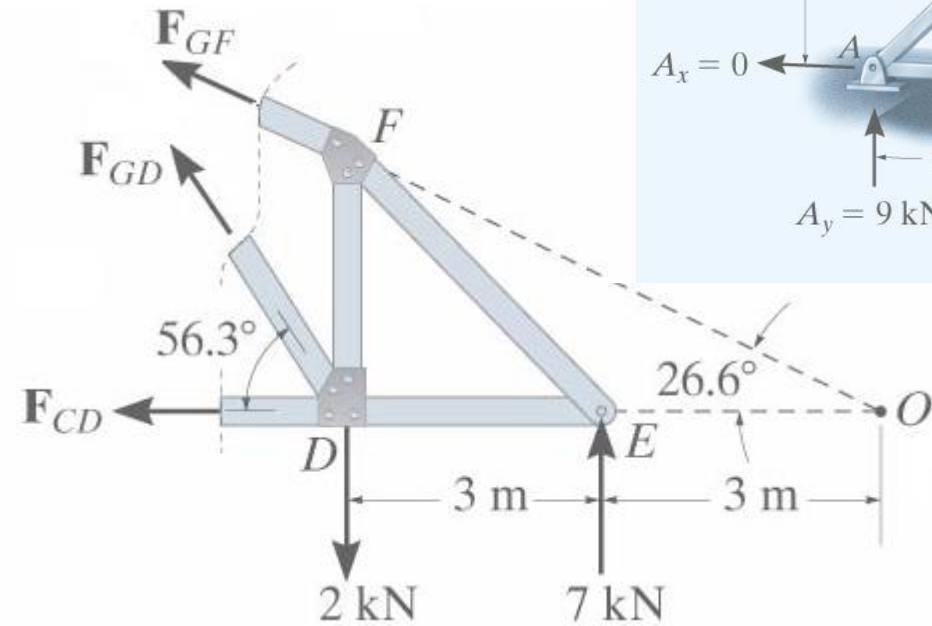


$$\begin{aligned}\downarrow + \Sigma M_A &= 0; & -1200(1.155) + F_{CO}(2) &= 0 \\ & & F_{CO} &= 693 \text{ N (T)}\end{aligned}$$

Example 6

Solve the GF & GD members in the truss



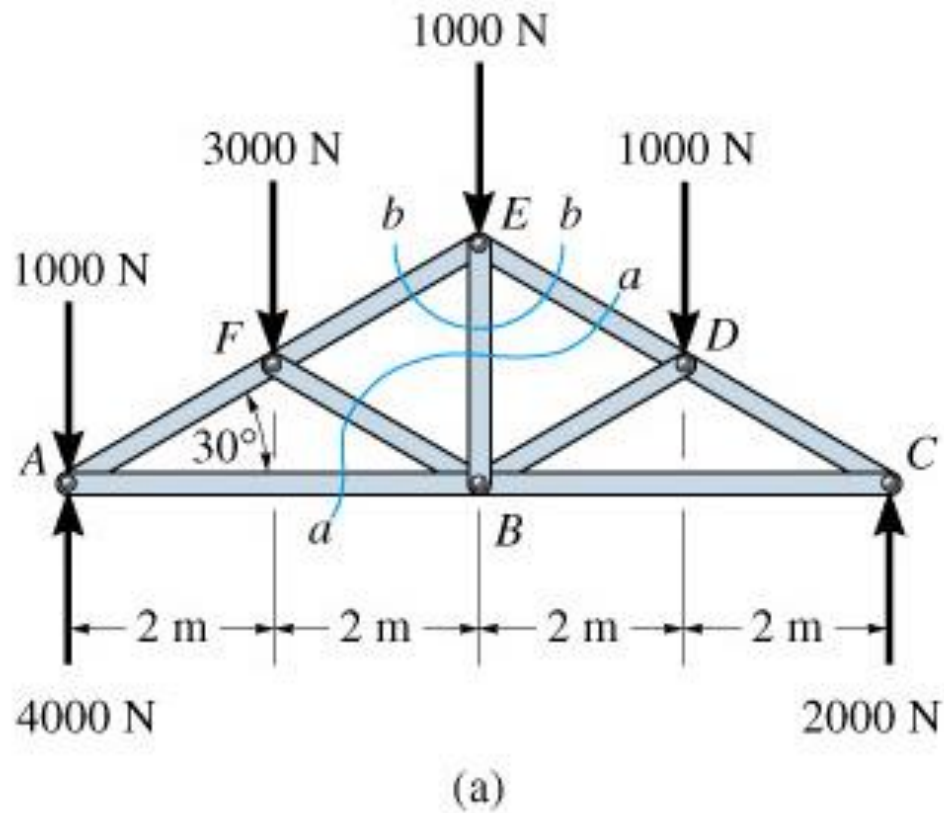


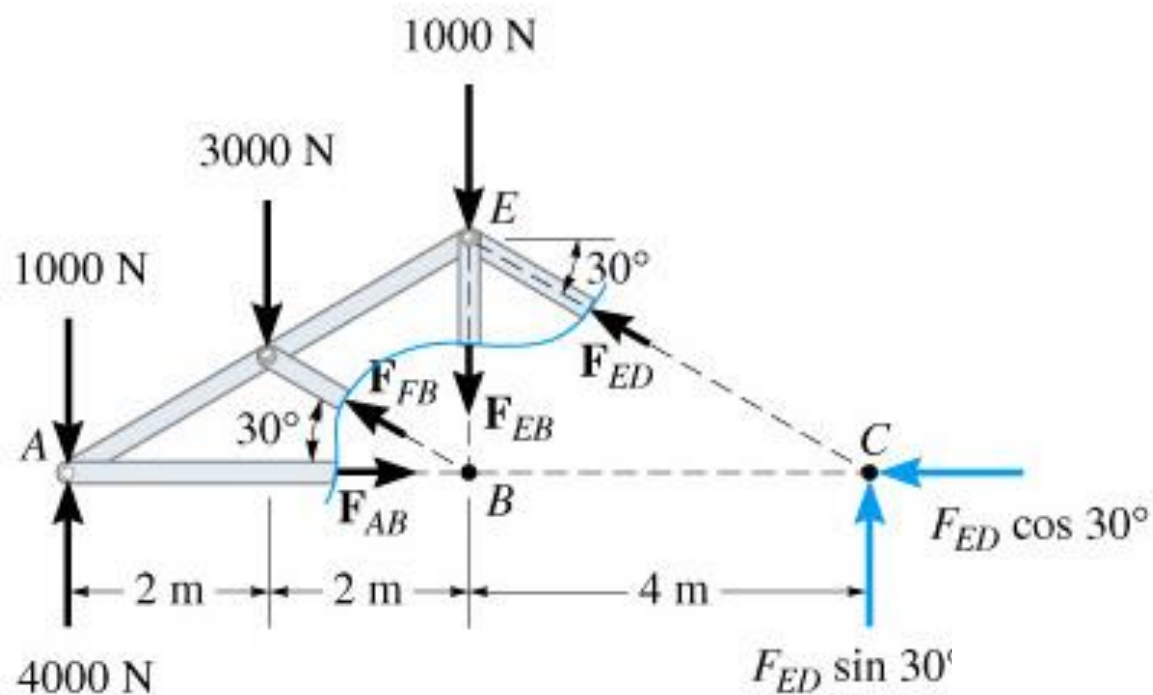
$$\sum M_D = 0 \rightarrow F_{GF} \cos 26.6(3) + 7(3) = 0 \Rightarrow F_{GF} = -7.83 \text{ kN} \quad \text{C}$$

$$\sum M_O = 0 \rightarrow -F_{GD} \sin 56.3(6) - 7(3) + 2(6) = 0 \Rightarrow F_{GF} = -1.8 \text{ kN} \quad \text{C}$$

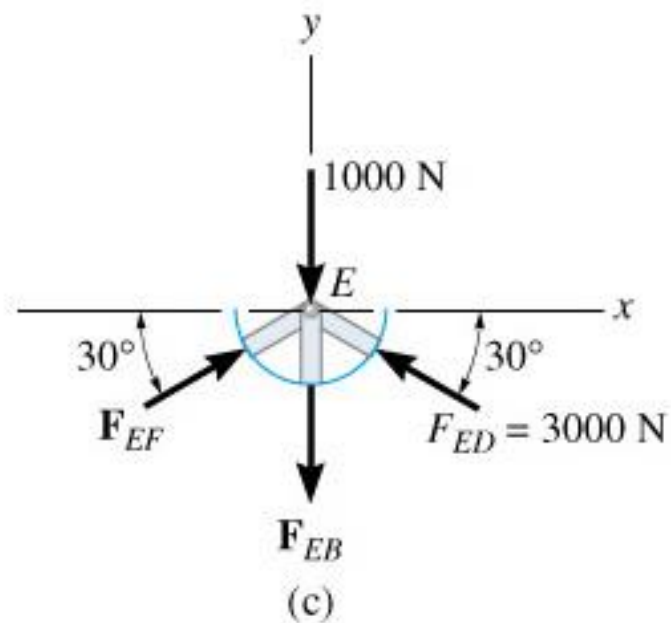
Example 7

Solve the ED & EB members in the truss





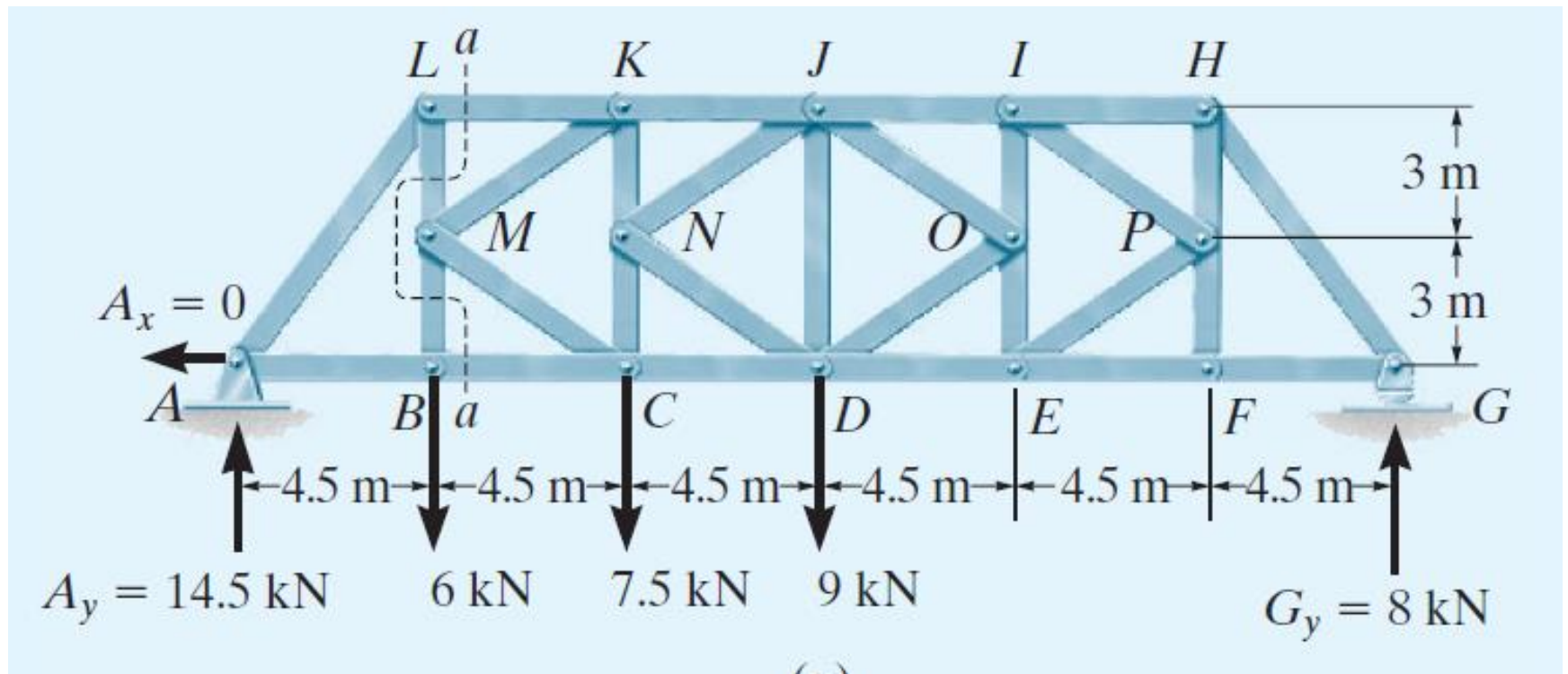
(b)

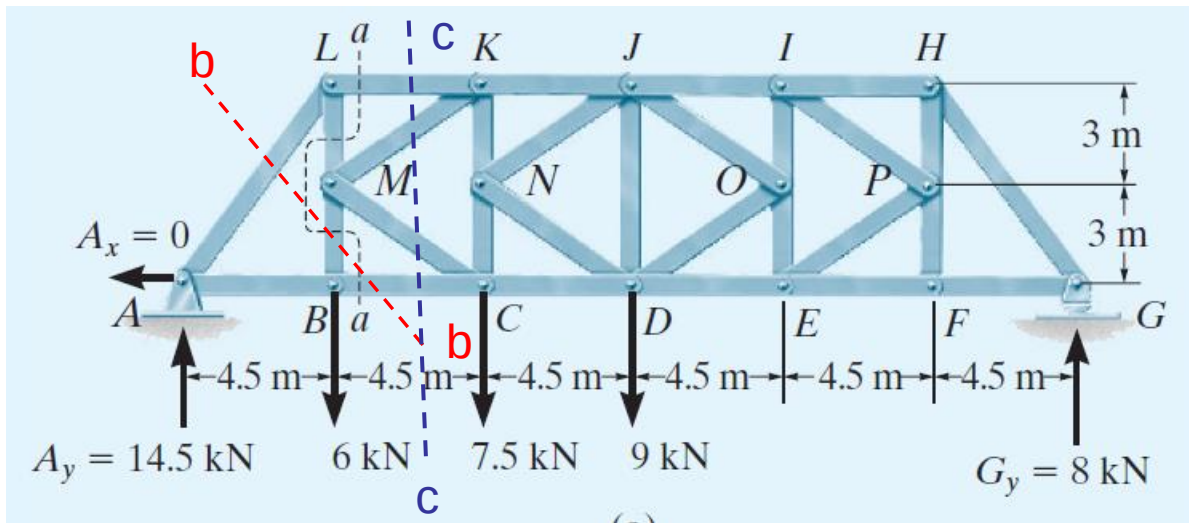


(c)

Example 8

Solve the BC & MC members in the K-truss





Section b-b (Left)

$$\sum M_L = 0 \rightarrow 14.5(4.5) + F_{BC}(6) = 0 \Rightarrow F_{BC} = 10.875 \text{ kN} \quad \text{T}$$

Section c-c (Right)

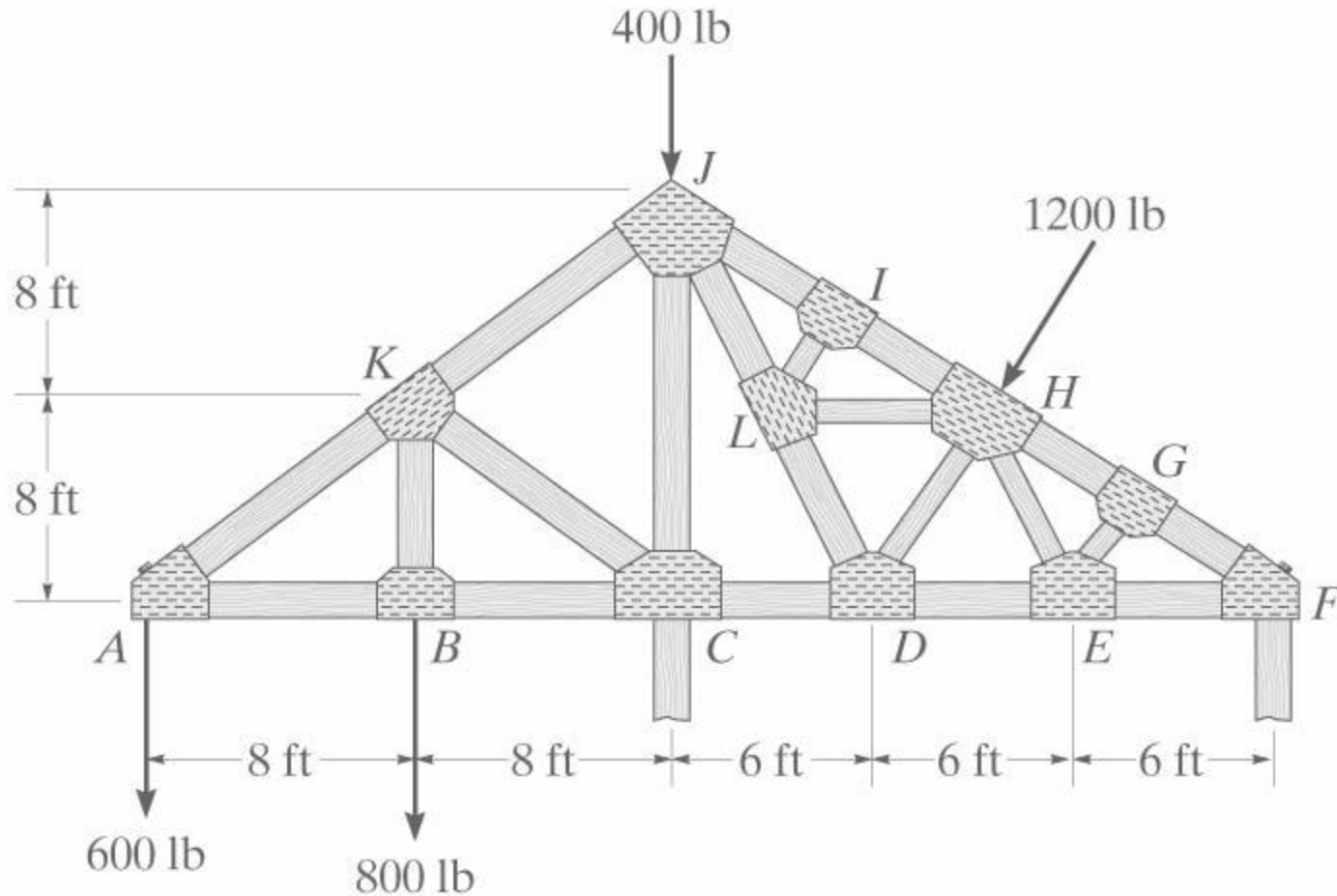
$$\sum M_K = 0 \rightarrow 10.875(6.0) + 9(4.5) - 8(18) + \left(F_{MC} \cos\left(\tan^{-1}\left(\frac{3}{4.5}\right)\right) \right)(6) = 0$$

$$\sum M_K = -38.25 + (F_{MC} \cos(33.69))(6) = -38.25 + (0.8321 F_{MC})(6) = 0$$

$$\Rightarrow F_{MC} = 7.66 \text{ kN} \quad \text{T}$$

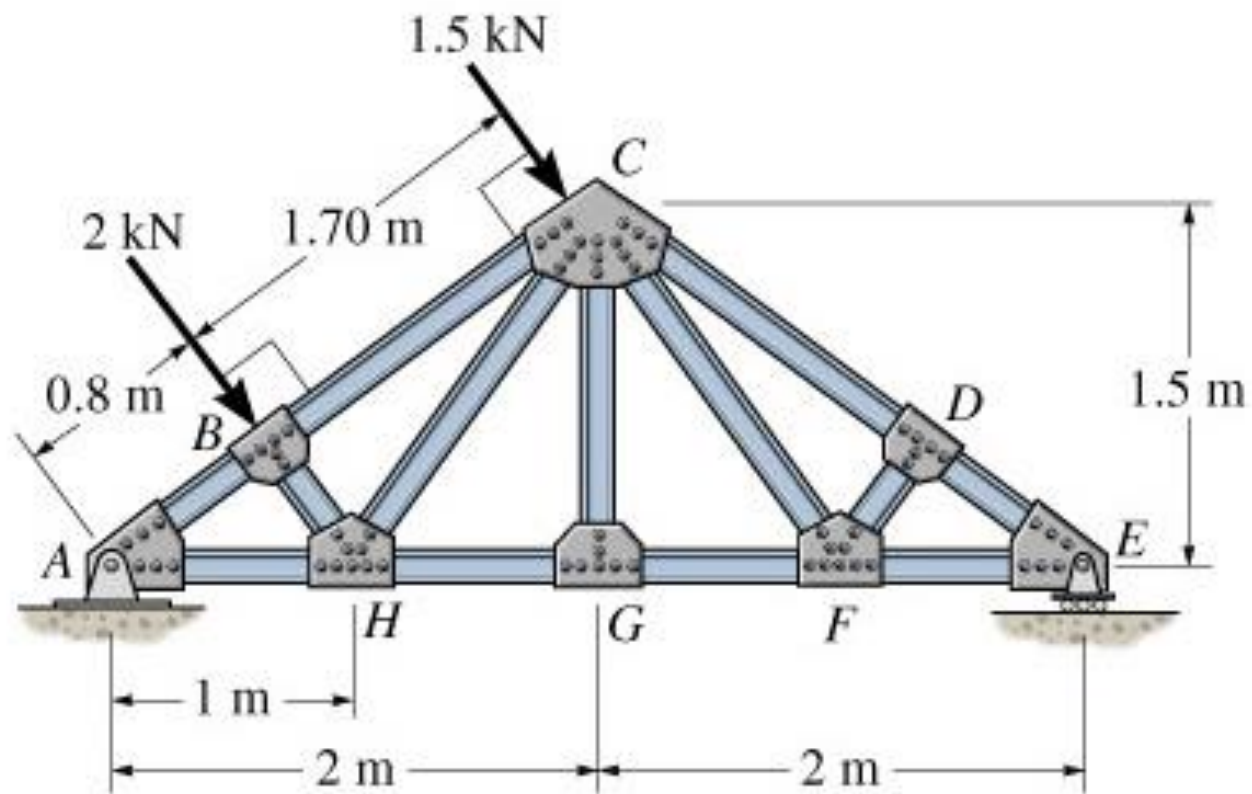
Problem 2

Zero Force Members



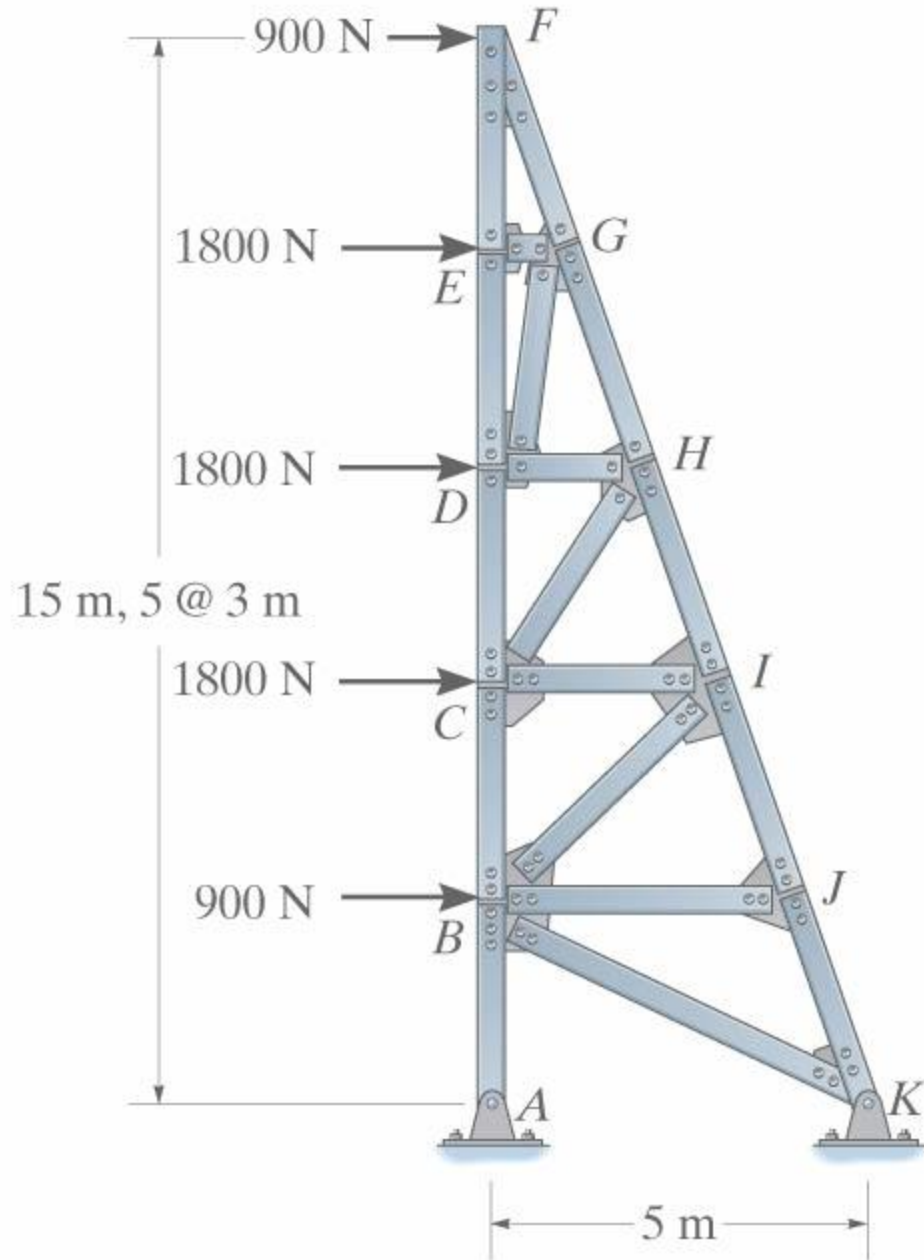
Problem 3

Solve All members



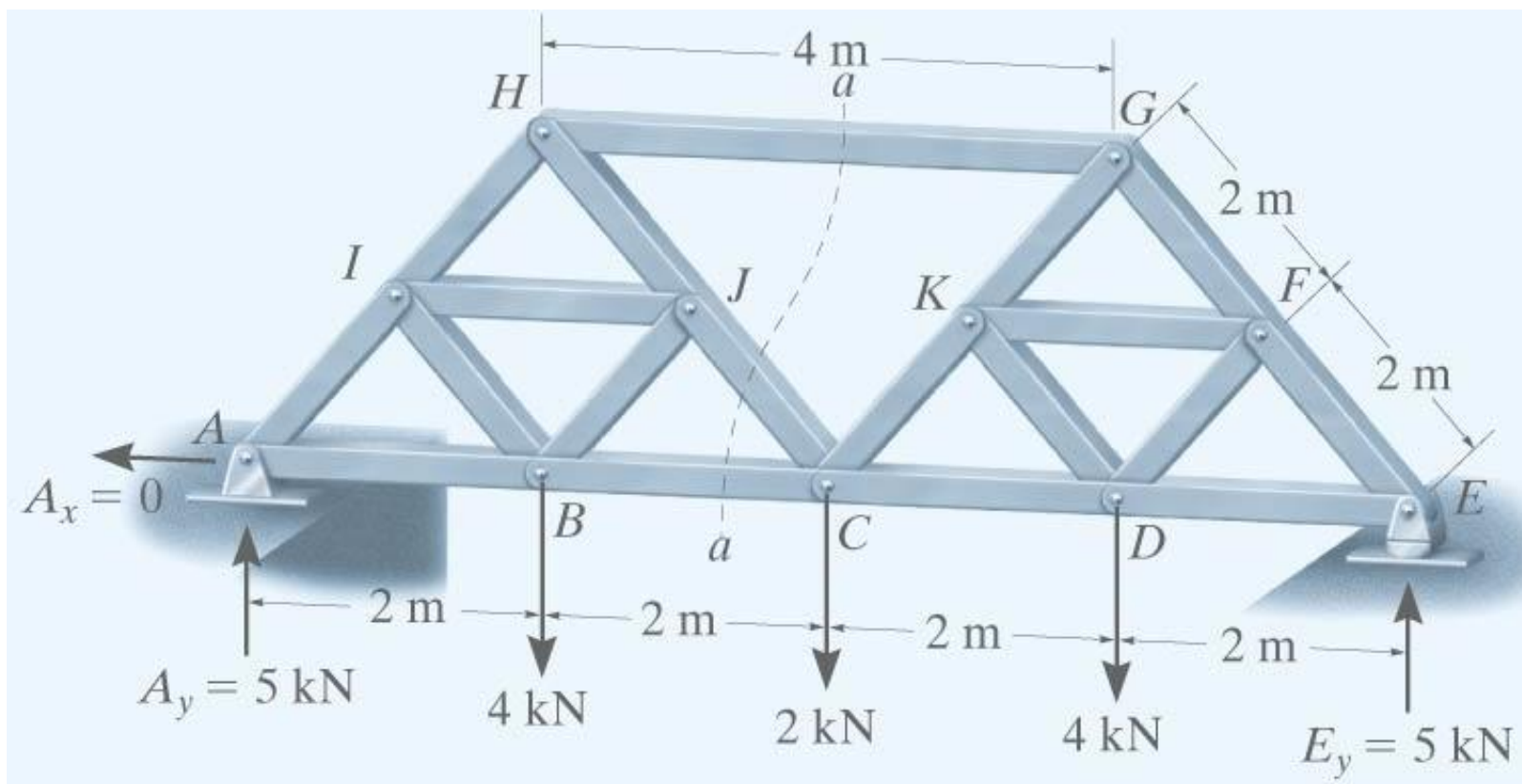
Problem 4

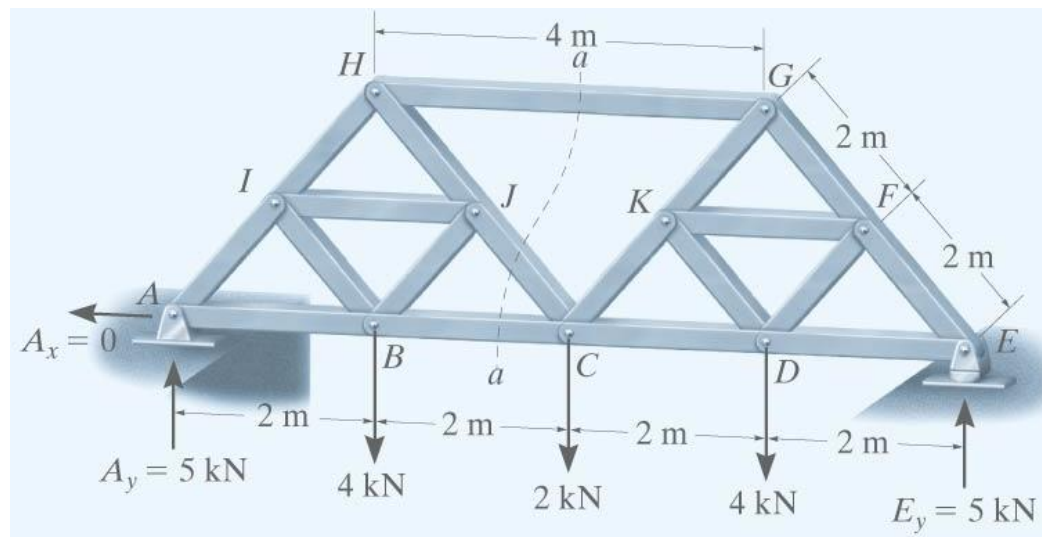
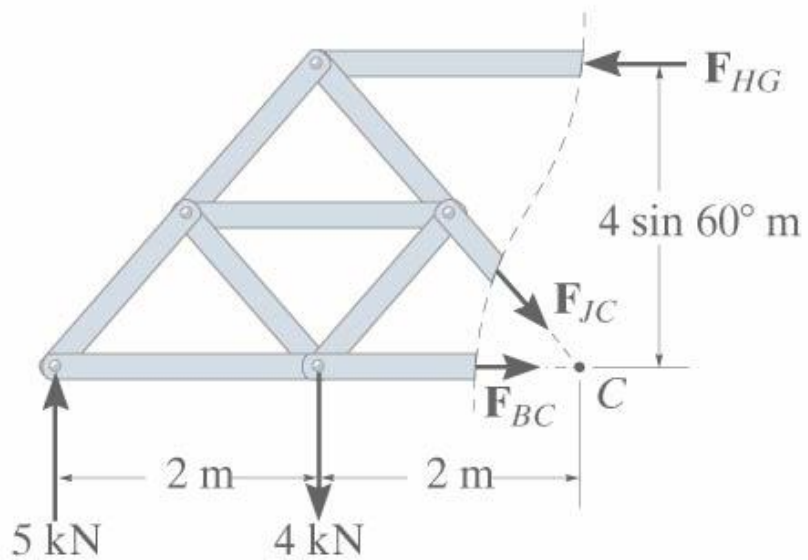
Solve members CH, CI



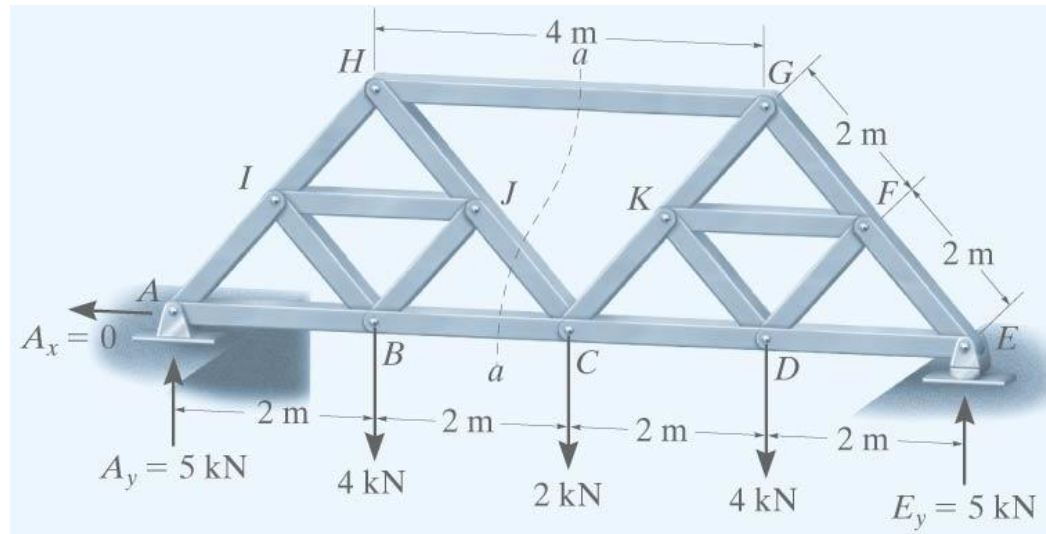
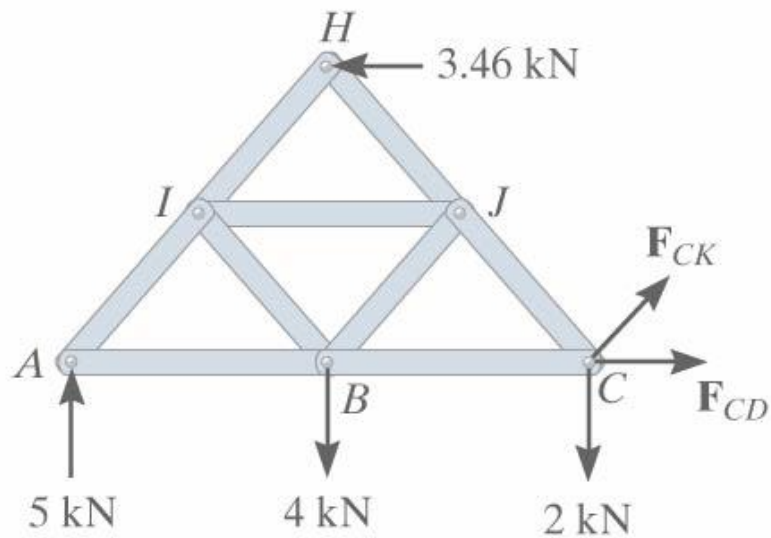
Example 1 Compound Truss

Solve the truss





$$\sum M_C = 0 \rightarrow -5(4) + 4(2) + F_{HG}(4 \sin 60) = 0 \Rightarrow F_{HG} = 3.46 \text{ kN} \quad C$$



Joint A $\Rightarrow F_{AB}$ & F_{AI}

Joint H $\Rightarrow F_{HI}$ & F_{HJ}

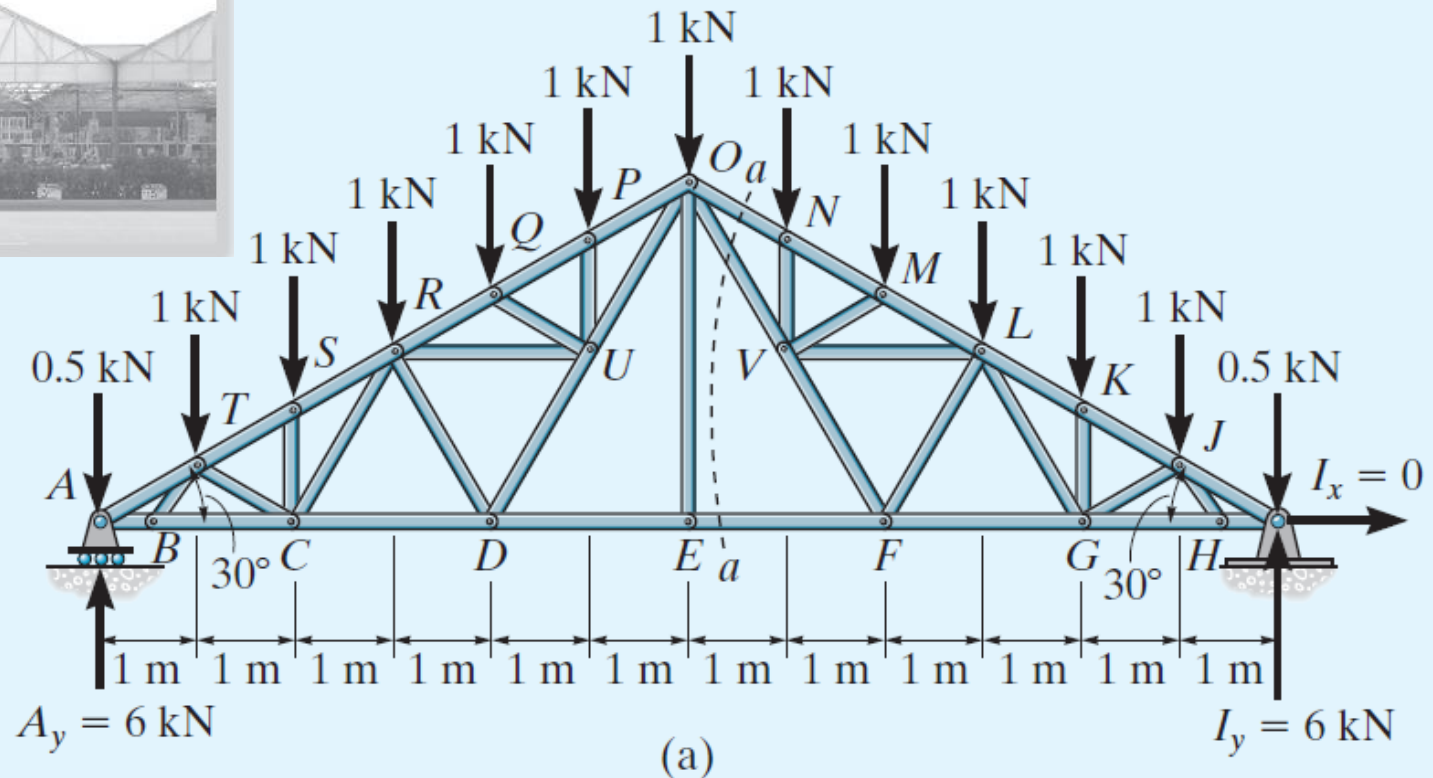
Joint I $\Rightarrow F_{IJ}$ & F_{IB}

Joint B $\Rightarrow F_{BC}$ & F_{BJ}

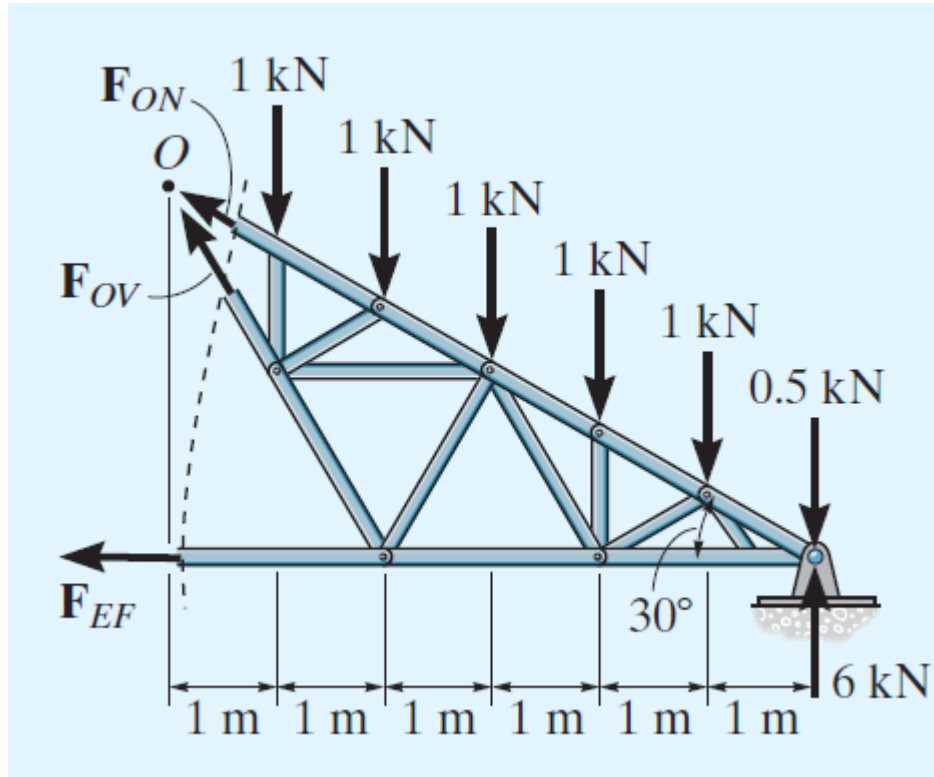
Joint J $\Rightarrow F_{JC}$

Example 2 Compound Truss

Compound roof trusses are used in a garden center, as shown in the photo. They have the dimensions and loading shown in Figure. Indicate how to analyze this truss. The reactions at the supports are given.

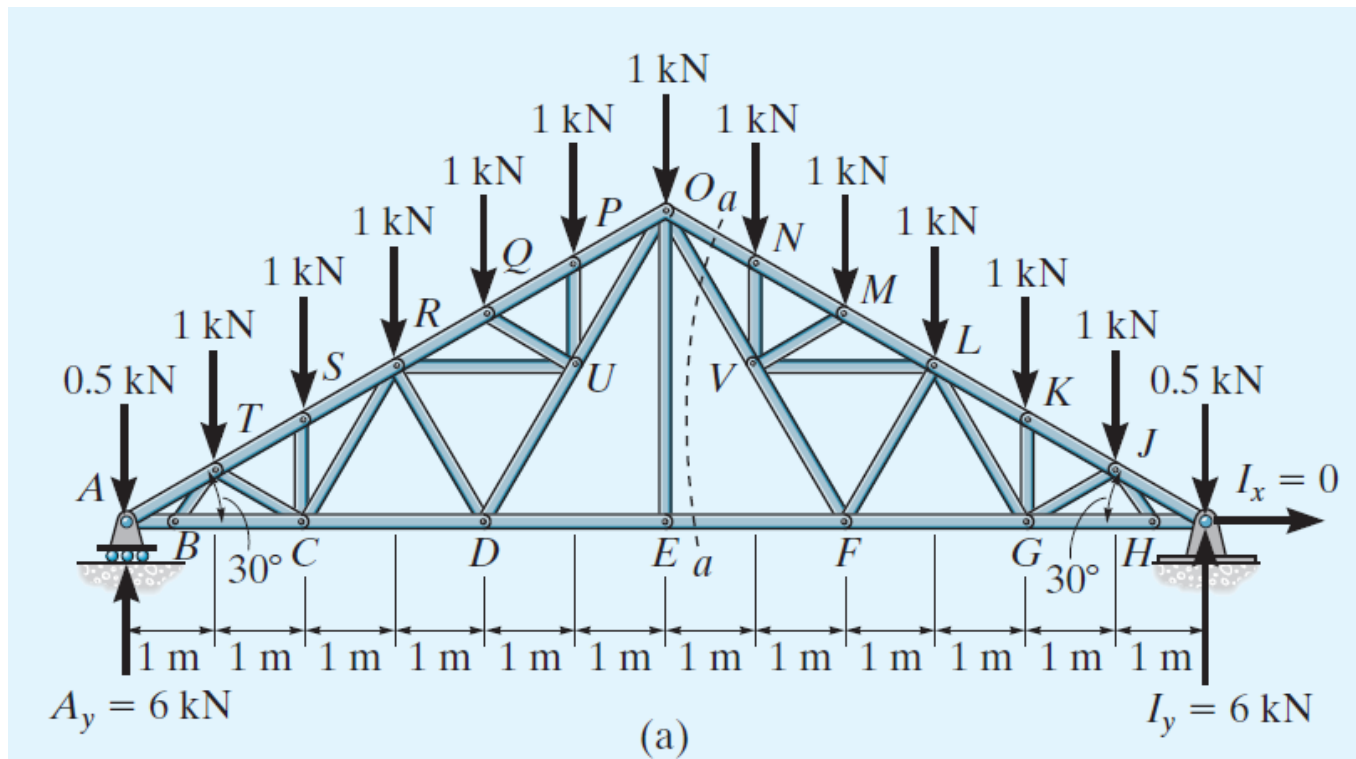


Example 2 Compound Truss



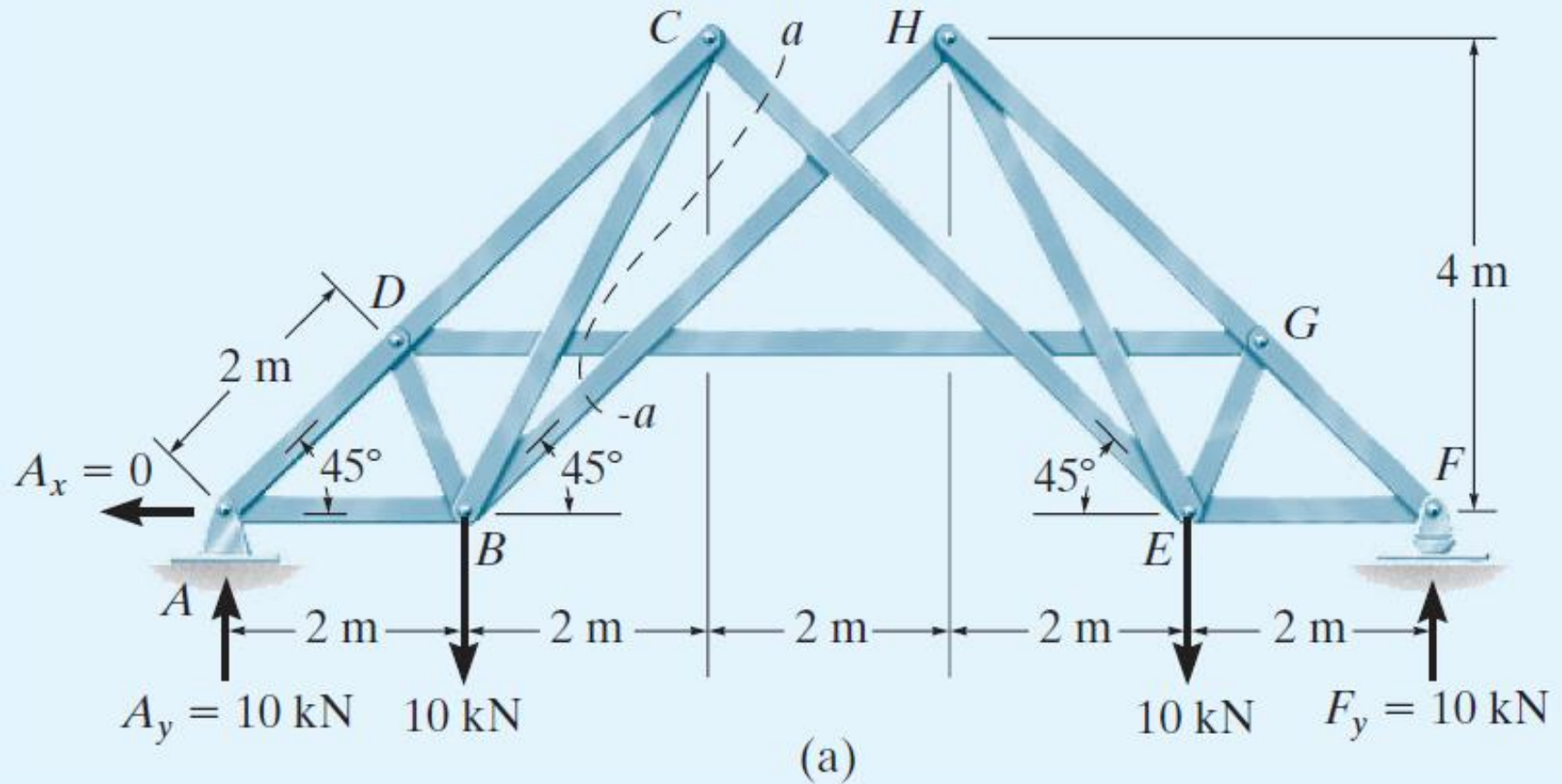
$$\begin{aligned} \downarrow + \Sigma M_O &= 0; & -1(1) - 1(2) - 1(3) - 1(4) - 1(5) - 0.5(6) \\ & & + 6(6) - F_{EF}(6 \tan 30^\circ) = 0 \\ & & F_{EF} = 5.20 \text{ kN (T)} \end{aligned}$$

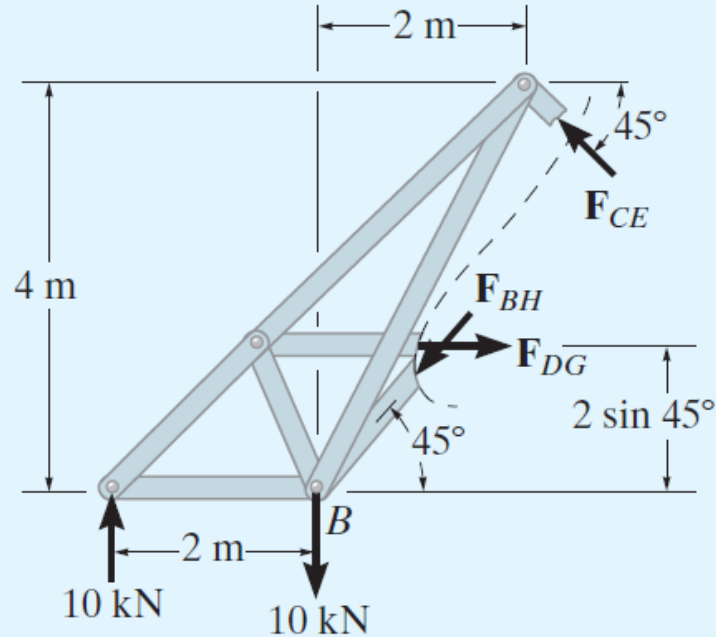
Example 2 Compound Truss



By inspection notice that BT , EO , and HJ are zero-force members since we require $+\uparrow \Sigma F_y = 0$ at joints B , E , and H , respectively. Also, by applying $+\nwarrow \Sigma F_y = 0$ (perpendicular to AO) at joints P , Q , S , and T , we can directly determine the force in members PU , QU , SC , and TC , respectively.

Example 3 Compound Truss





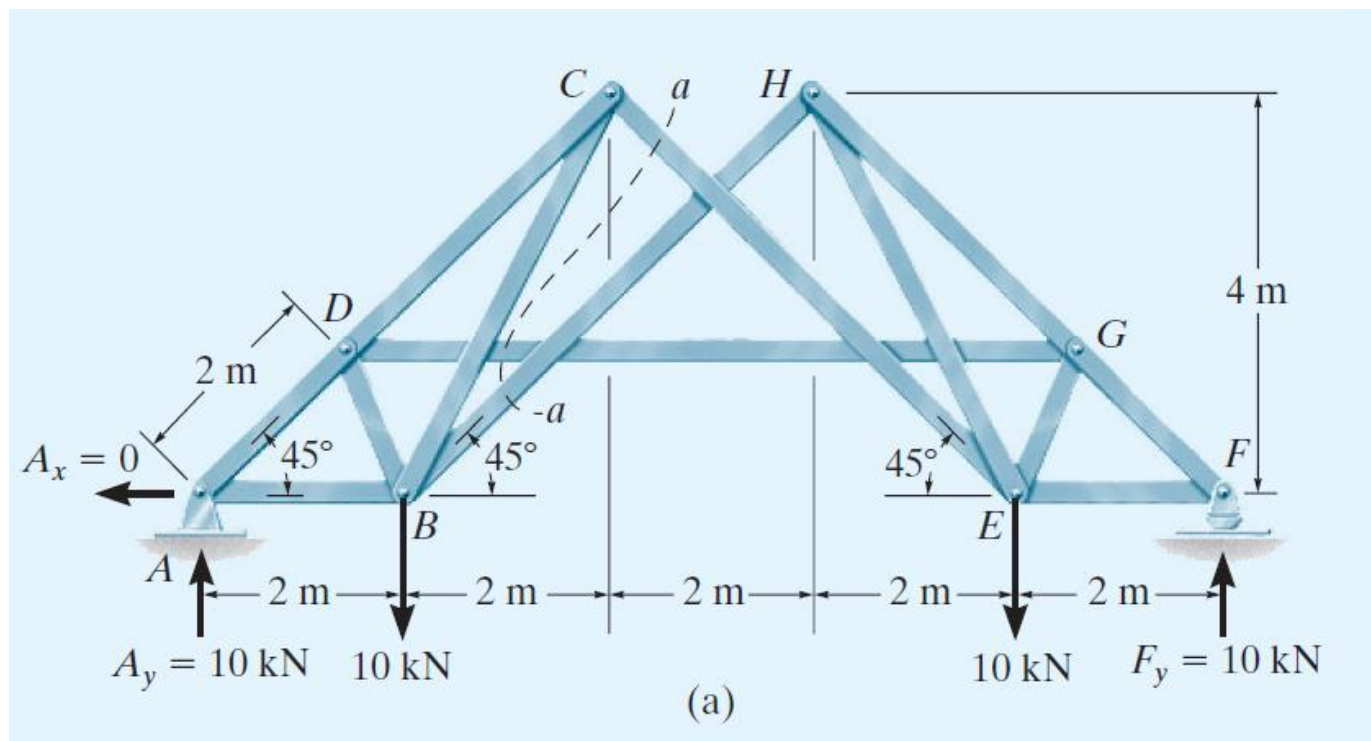
$$\downarrow + \Sigma M_B = 0; \quad -10(2) - F_{DG}(2 \sin 45^\circ) + F_{CE} \cos 45^\circ(4) + F_{CE} \sin 45^\circ(2) = 0 \quad (1)$$

$$+ \uparrow \Sigma F_y = 0; \quad 10 - 10 - F_{BH} \sin 45^\circ + F_{CE} \sin 45^\circ = 0 \quad (2)$$

$$\rightarrow \Sigma F_x = 0; \quad -F_{BH} \cos 45^\circ + F_{DG} - F_{CE} \cos 45^\circ = 0 \quad (3)$$

From Eq. (2), $F_{BH} = F_{CE}$; then solving Eqs. (1) and (3) simultaneously yields

$$F_{BH} = F_{CE} = 8.92 \text{ kN (C)} \quad F_{DG} = 12.6 \text{ kN (T)}$$



Analysis of each connected simple truss can now be performed using the method of joints. For example, from Fig. 3–30c, this can be done in the following sequence.

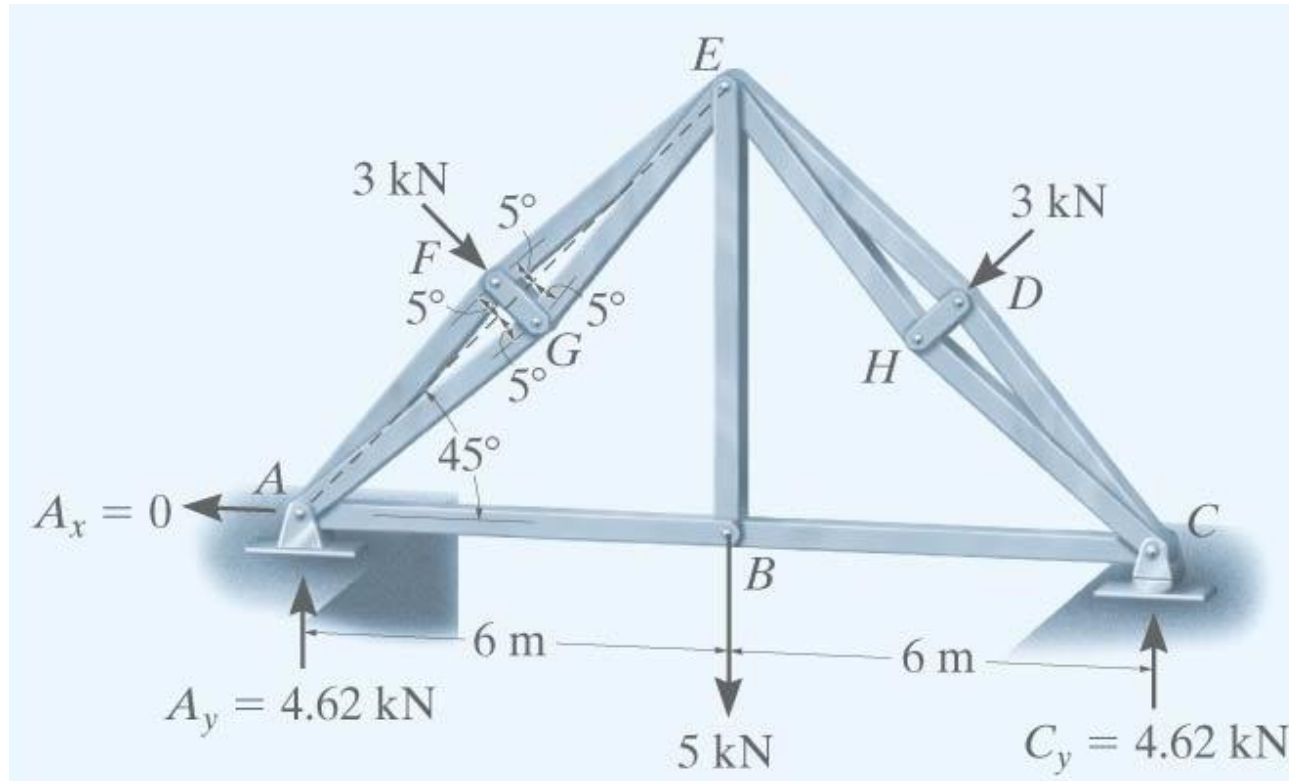
Joint A: Determine the force in AB and AD .

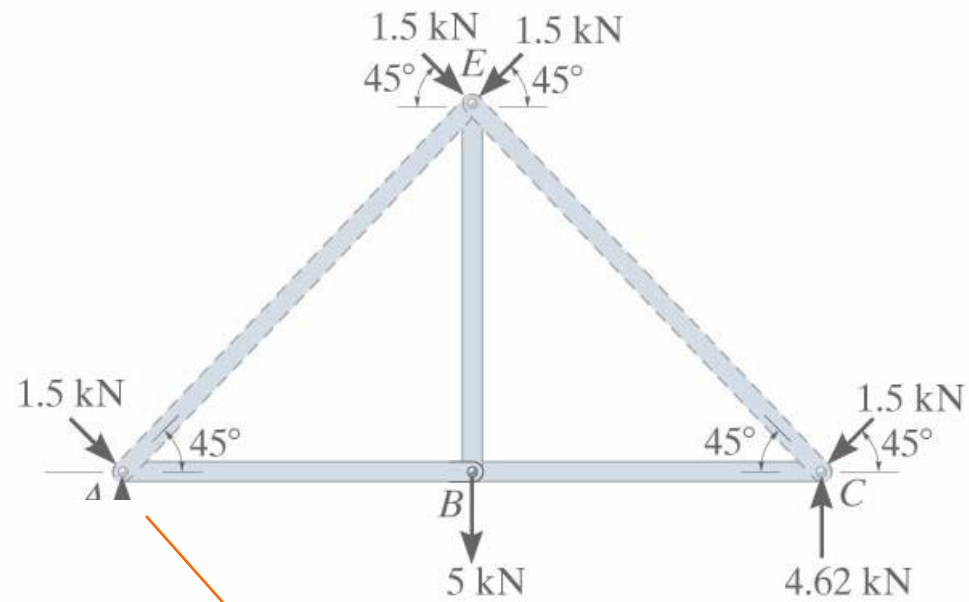
Joint D: Determine the force in DC and DB .

Joint C: Determine the force in CB .

Example 4 Compound Truss

Solve the truss





Joint A $\Rightarrow F_{AE}$ & F_{AB}

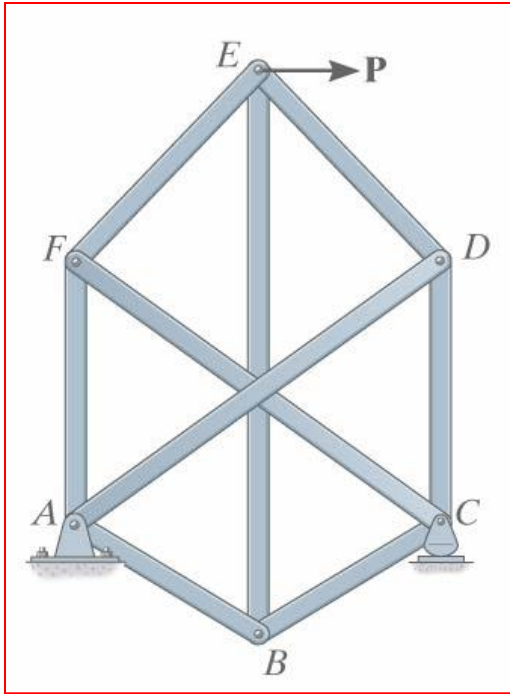
Joint B $\Rightarrow F_{EB}$

(b)

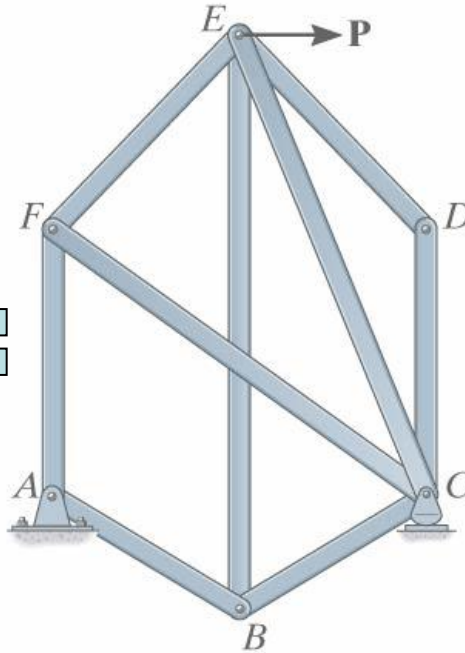
After Solve F_{AE}

Joint A $\Rightarrow F_{AF}$ & F_{AG}

Complex Truss

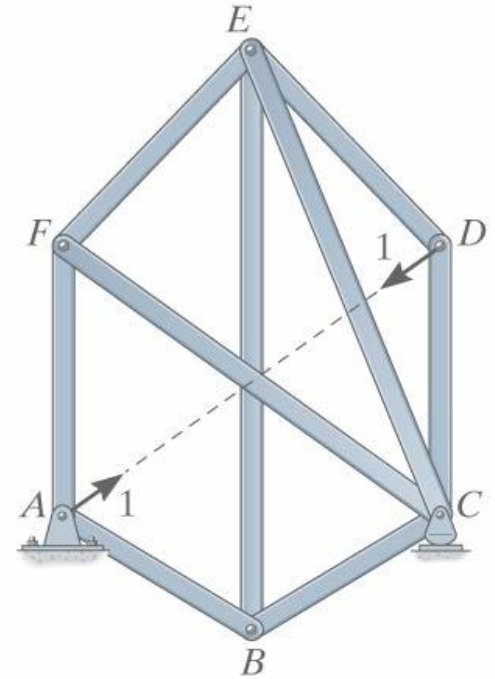


=



S'_i forces

+



s_i forces

$$S_i = S'_i + xS_i$$

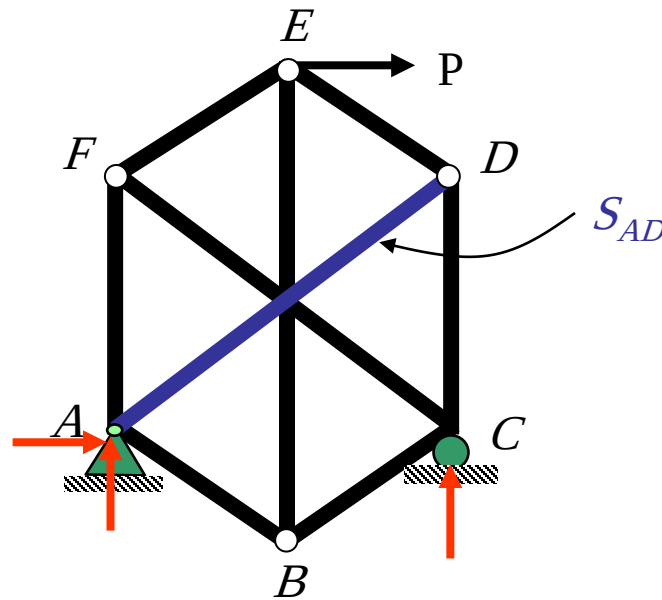


Force in members

$$S_{EC} = S'_{EC} + xS_{EC} = 0$$

$$x = ?$$

Complex Trusses

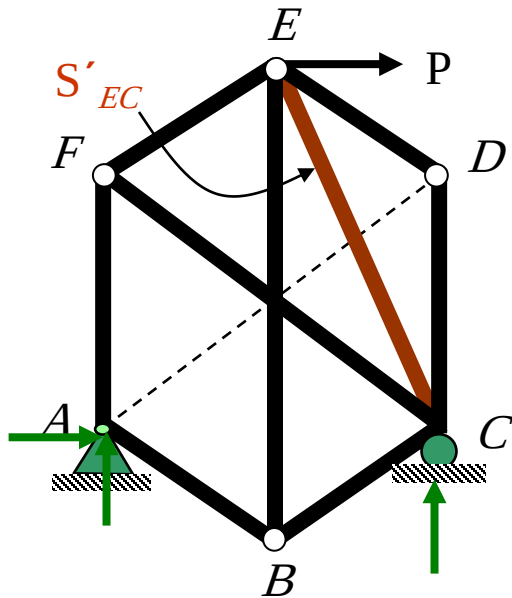


$$r + b = 2j,$$

$$\begin{matrix} \nearrow & \nearrow & \nearrow \\ 3 & 9 & 2(6) \end{matrix}$$

↓

Determinate •
Stable •



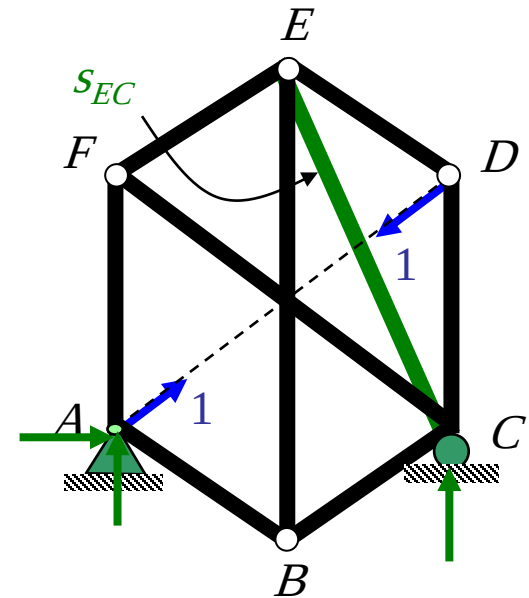
+

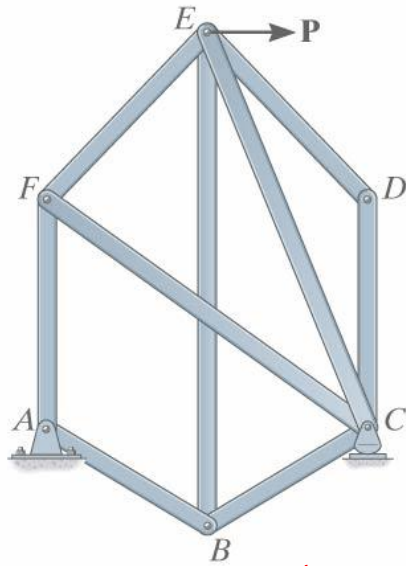
X x

$$S'_{EC} + X s_{EC} = 0$$

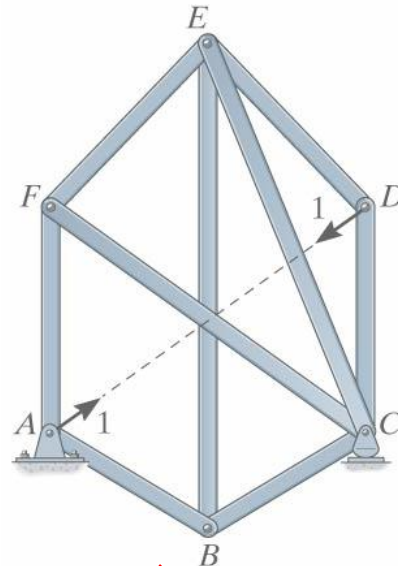
$$X = -\frac{S'_{EC}}{s_{EC}} = F_{AD}$$

$$S_i = S'_i + X s_i$$





S'_i forces



s_i forces

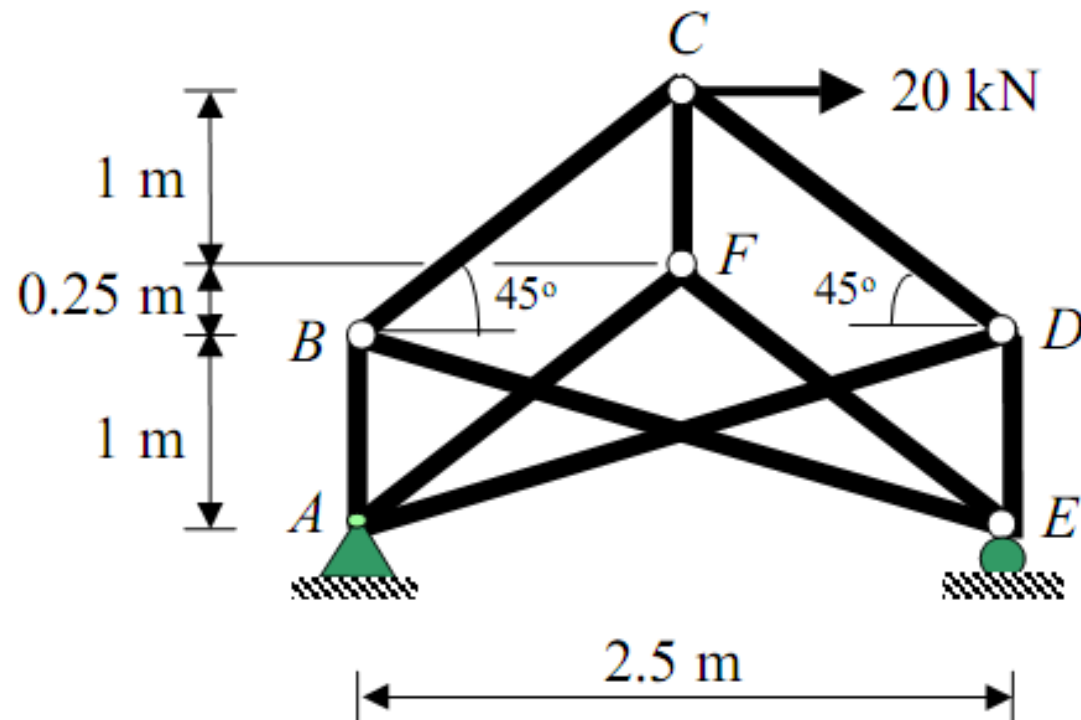
$$S_{EC} = S'_{EC} + xS_{EC} = 0$$

$$x = ?$$

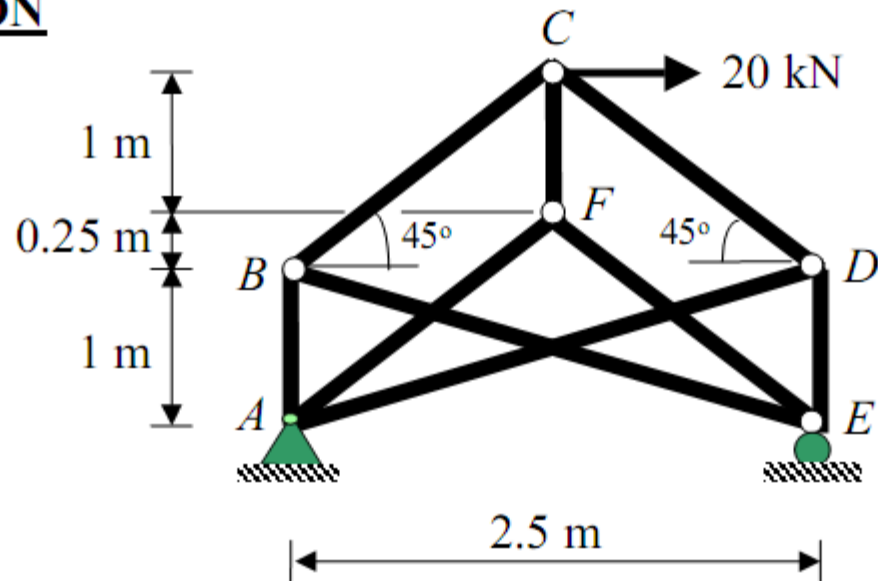
$$S_i = S'_i + xS_i$$

Member	S'_i	S_i	xS_i	S_i
AB				
AC				
AF				
FE				
BE				
ED				
FC				
EC				

Example 1 Complex Truss



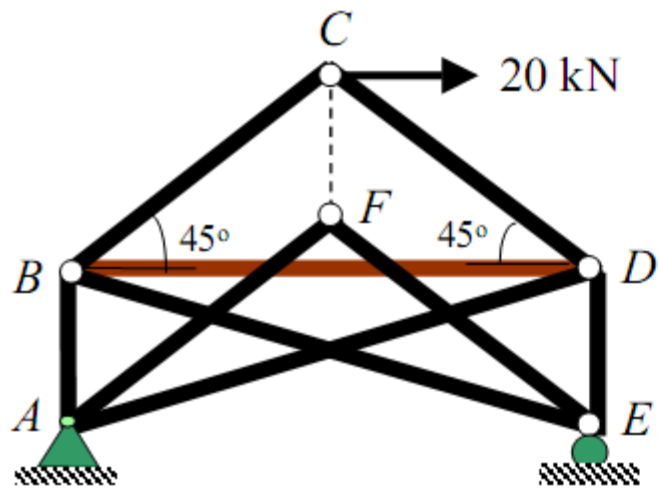
SOLUTION



$$F_{BD} + x f_{BD} = 0$$

$$x = \frac{F_{BD}}{f_{BD}}$$

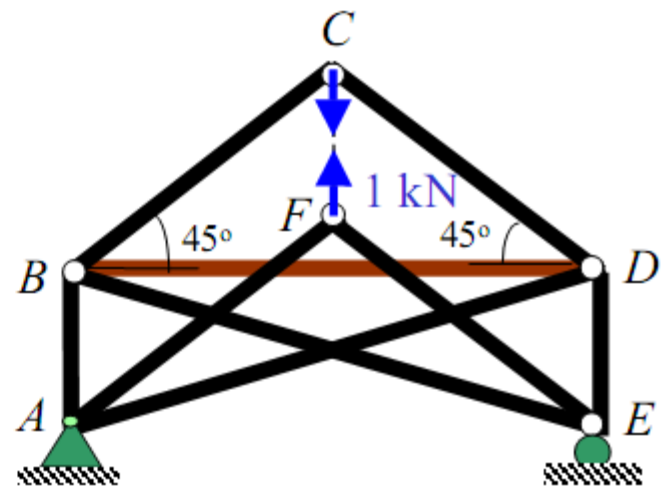
$$F_i = F_i + x f_i$$

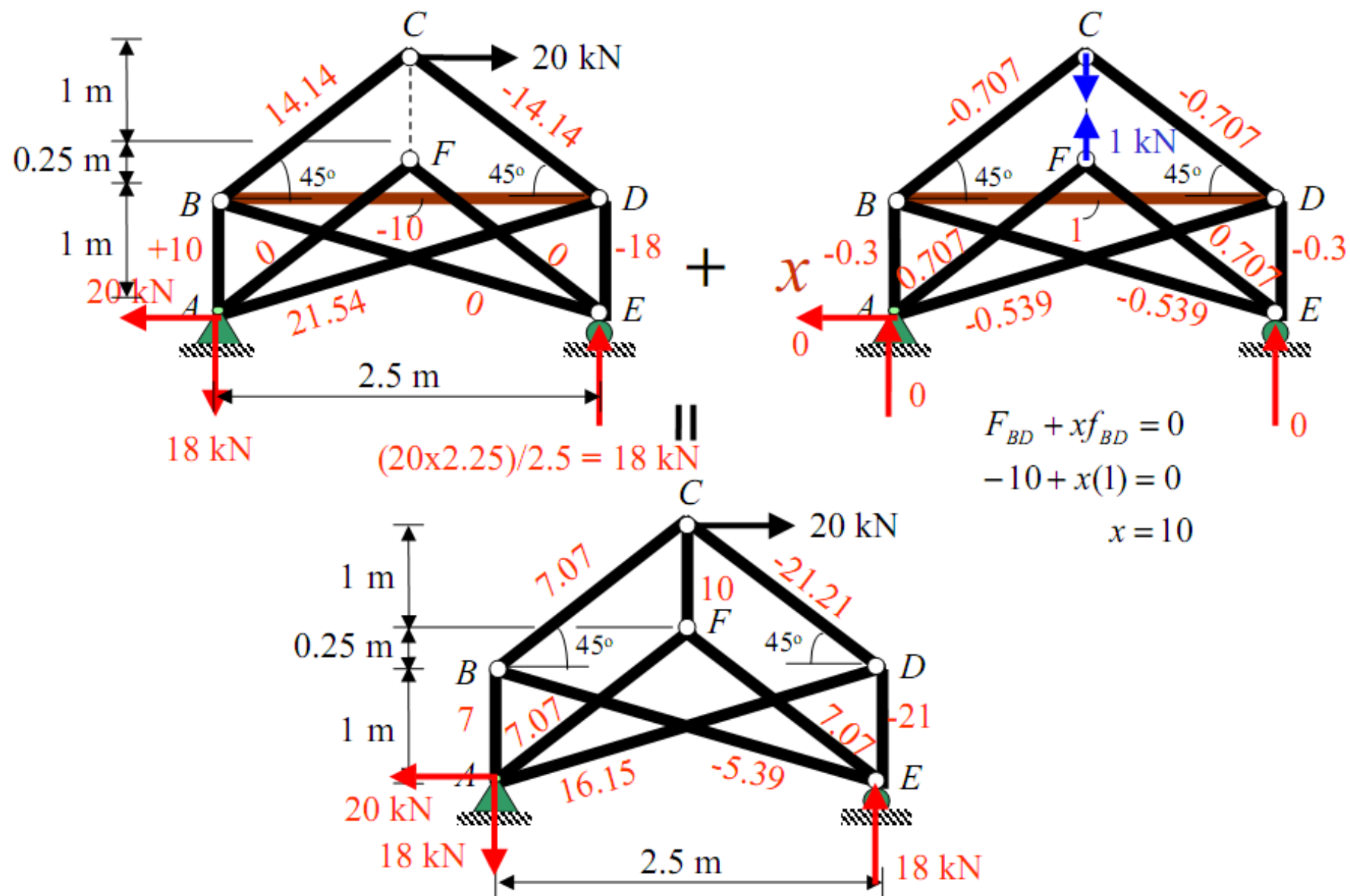


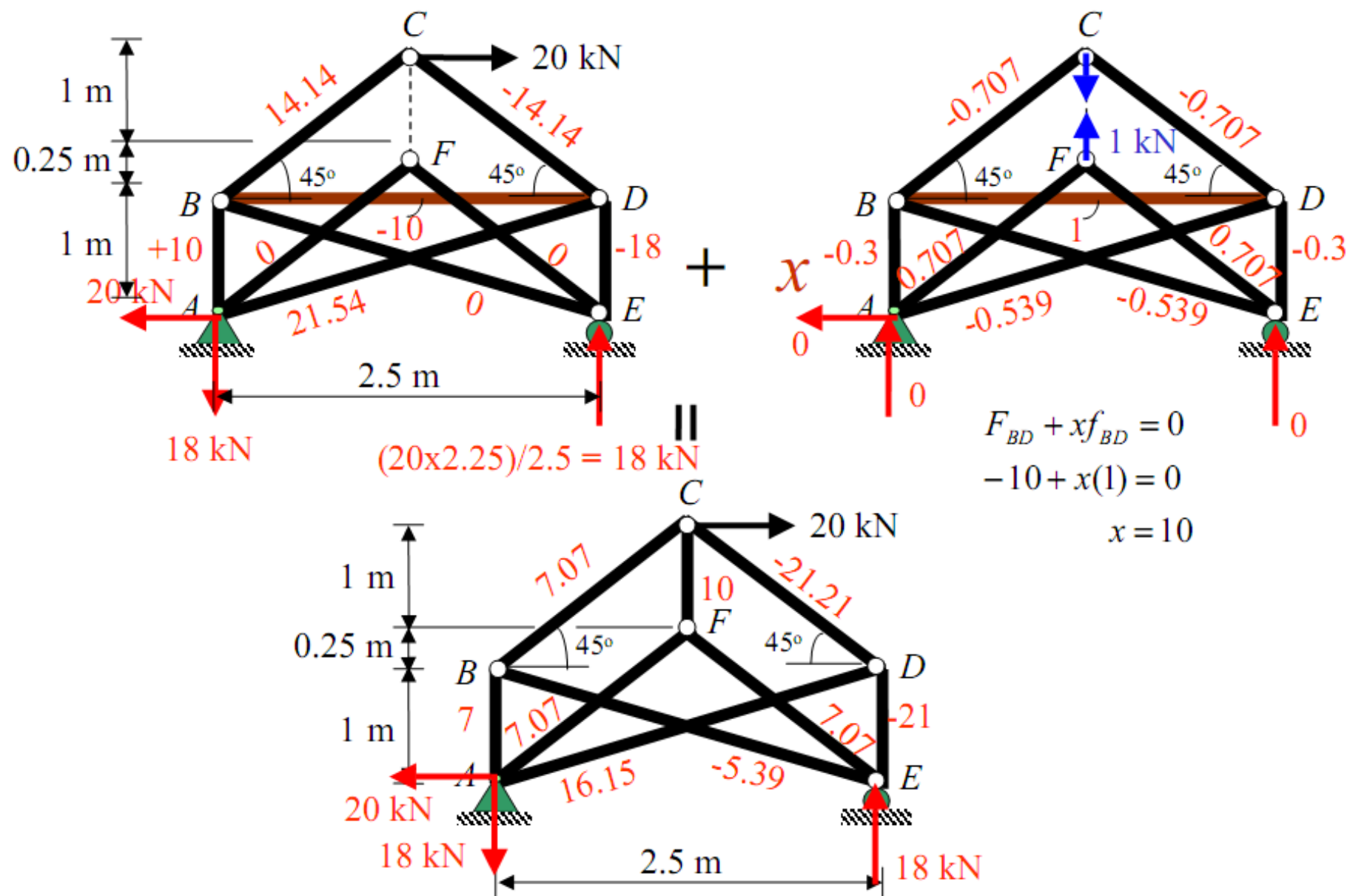
||

+

x







Member	S'_i	s^i	$x s^i$	S_i
AB	10	-0.3	-3.0	7.0
AD	21.54	-0.539	-5.39	16.15
AF	0	0.707	7.07	7.07
BC	14.14	-0.707	-7.07	7.07
BE	0	-0.539	-5.39	-5.39
CD	-14.14	-0.707	-7.07	-21.21
CF	0	1	10.0	10.0
ED	-18	-0.3	-3.0	-21.0
EF	0	0.707	7.07	7.07
BD	-10	1.0	10.0	0

$$S'_{BD} + x S_{BD} = 0$$

$$-10 + x \cdot (1) = 0$$

$$\Rightarrow x = 10$$

Space Truss

- Determinacy and Stability



$b + r < 3j$ Unstable truss

$b + r = 3j$ Statically determinate -Check stability

$b + r > 3j$ Statically indeterminate -Check stability

Space Truss

- x, y, z, Force Components

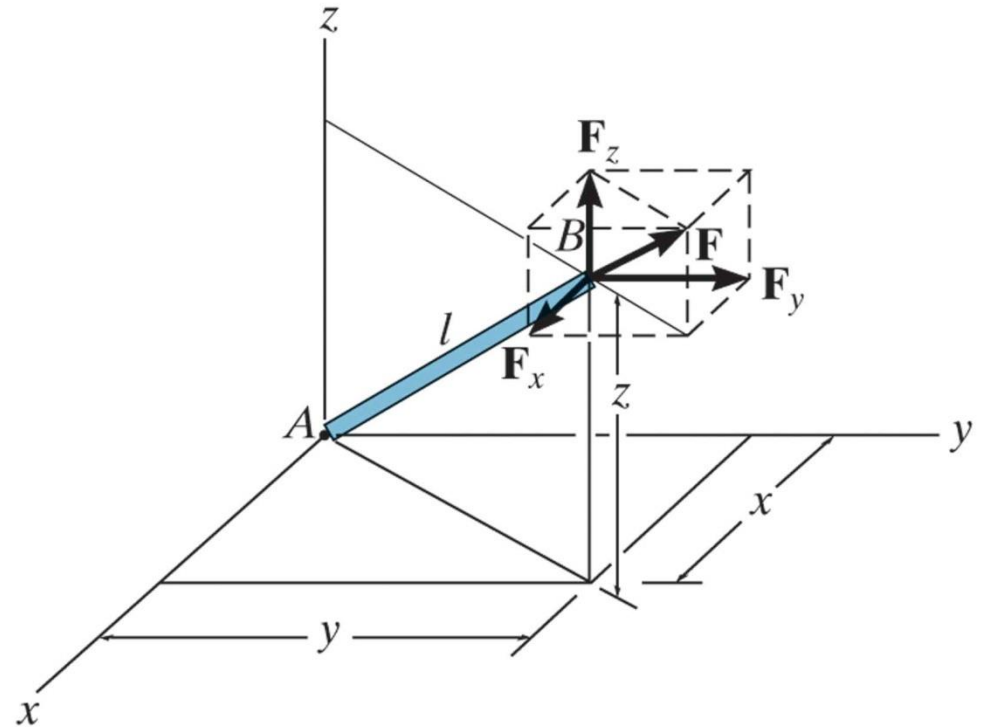
$$l = \sqrt{x^2 + y^2 + z^2}$$

$$F_x = F\left(\frac{x}{l}\right)$$

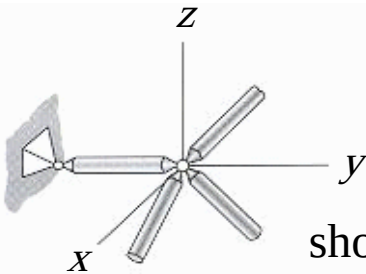
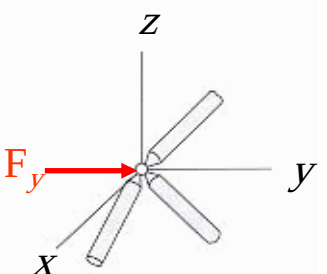
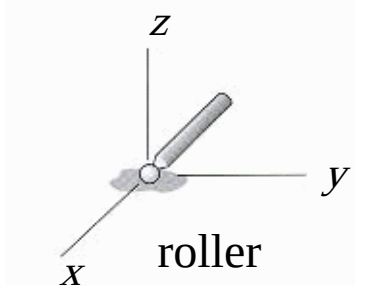
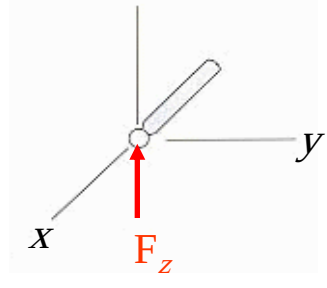
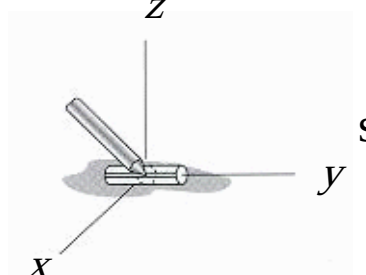
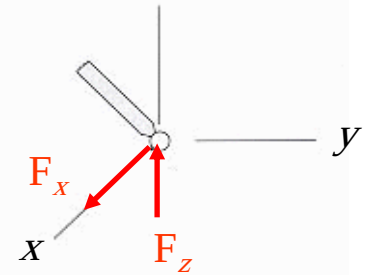
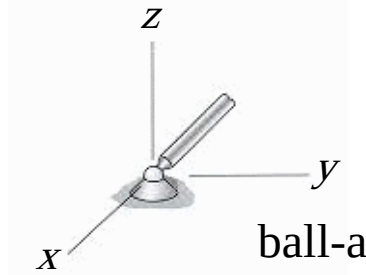
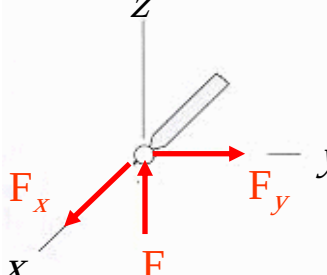
$$F_y = F\left(\frac{y}{l}\right)$$

$$F_z = F\left(\frac{z}{l}\right)$$

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$



Support Reactions

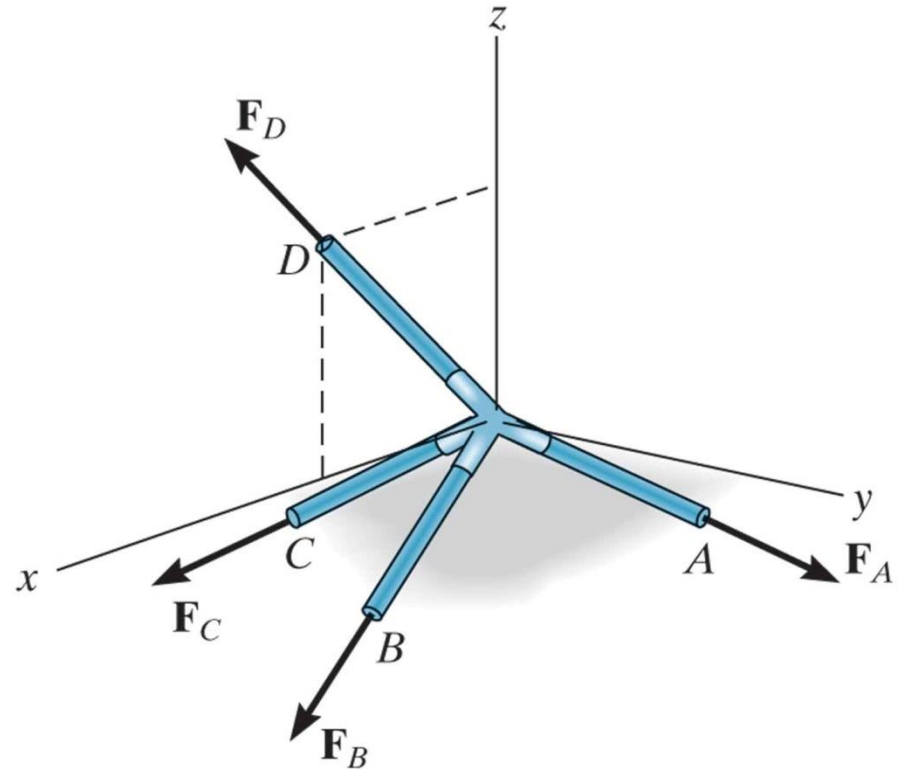
 short link	
 roller	
 slotted roller constrained in a cylinder	
 ball-and -socket	

Zero Force Member

1- If all but one of the members connected to a joint lie on the same plane, and provided no external load act on the joint, then the member not lying in the plane of the other members must be subjected to zero force.

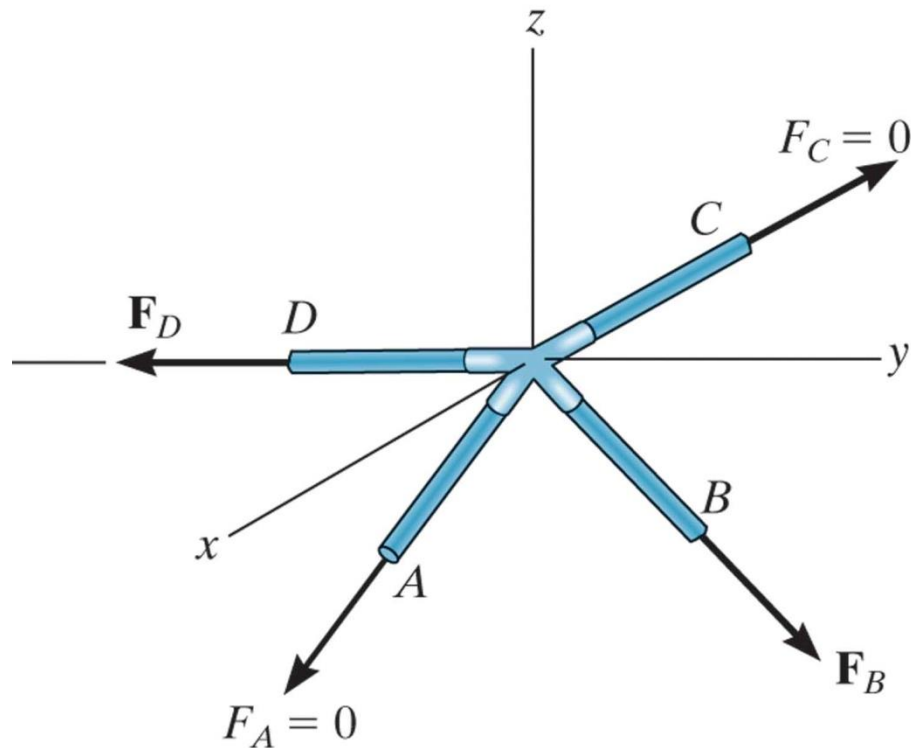
$$\Sigma F_z = 0 ,$$

$$F_D = 0$$



Zero Force Member

2- If it has been determined that all but two of several members connected at a joint support zero force, then the two remaining members must also support zero force, provided they don't lie along the same line and no external load act on the joint.



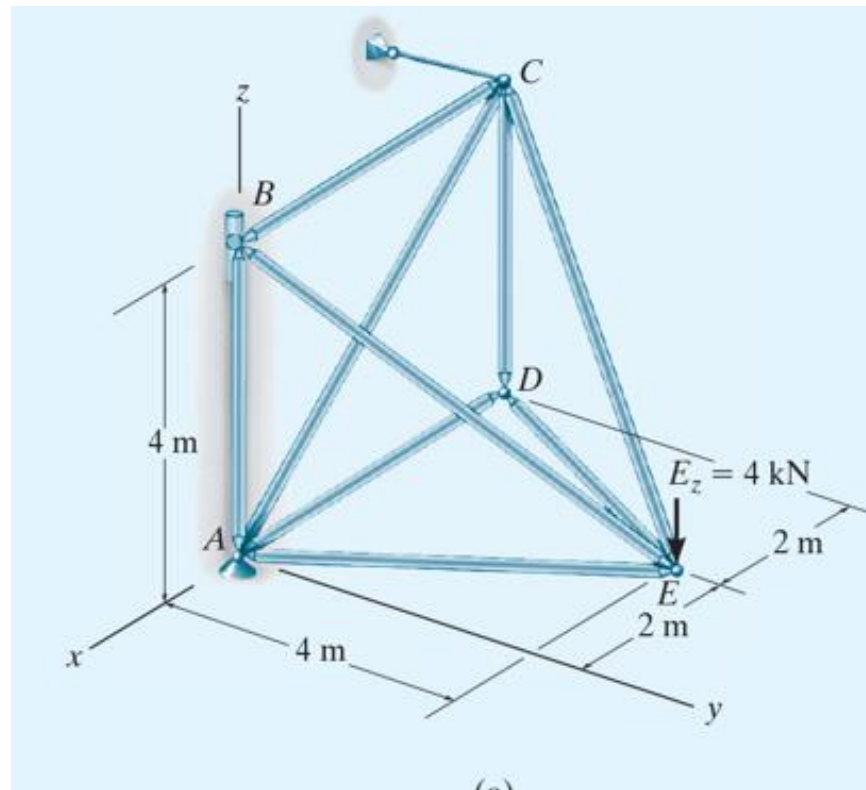
$$\Sigma F_z = 0 ,$$

$$F_B = 0$$

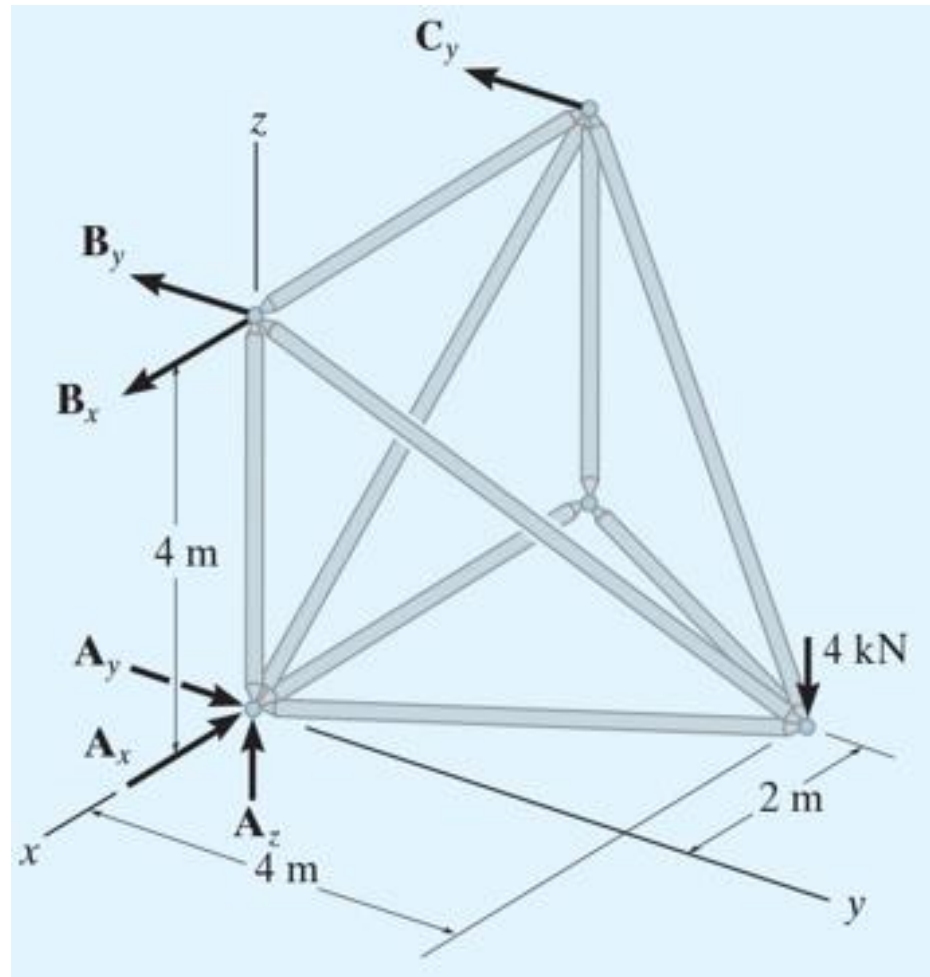
$$\Sigma F_y = 0 ,$$

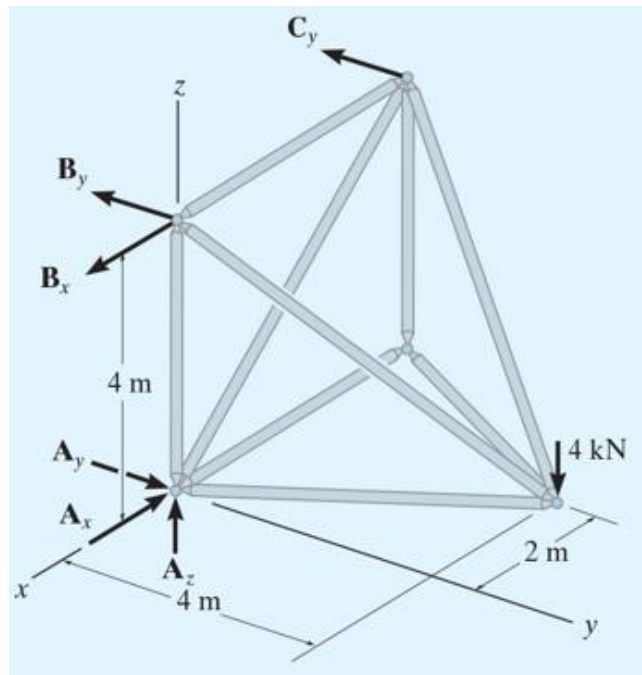
$$F_D = 0$$

Example 1 Space Truss

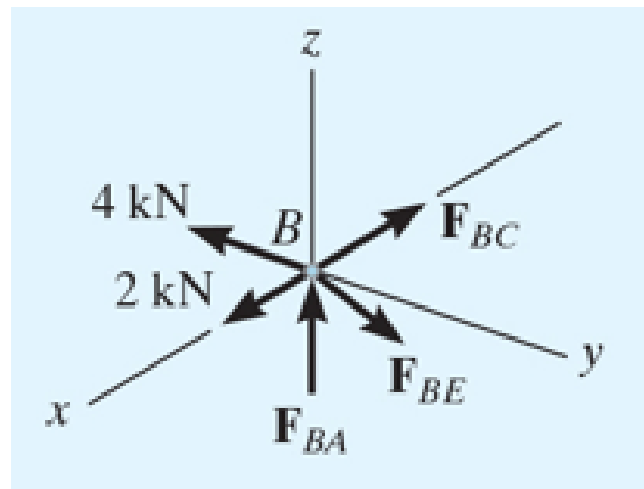


Example 1 Space Truss

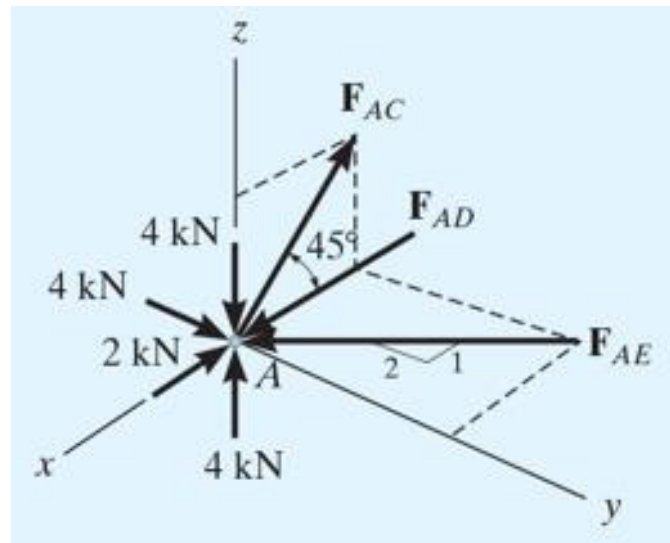




$$\begin{array}{lll}
 \Sigma M_y = 0; & -4(2) + B_x(4) = 0 & B_x = 2 \text{ kN} \\
 \Sigma M_z = 0; & C_y = 0 & \\
 \Sigma M_x = 0; & B_y(4) - 4(4) = 0 & B_y = 4 \text{ kN} \\
 \Sigma F_x = 0; & 2 - A_x = 0 & A_x = 2 \text{ kN} \\
 \Sigma F_y = 0; & A_y - 4 = 0 & A_y = 4 \text{ kN} \\
 \Sigma F_z = 0; & A_z - 4 = 0 & A_z = 4 \text{ kN}
 \end{array}$$



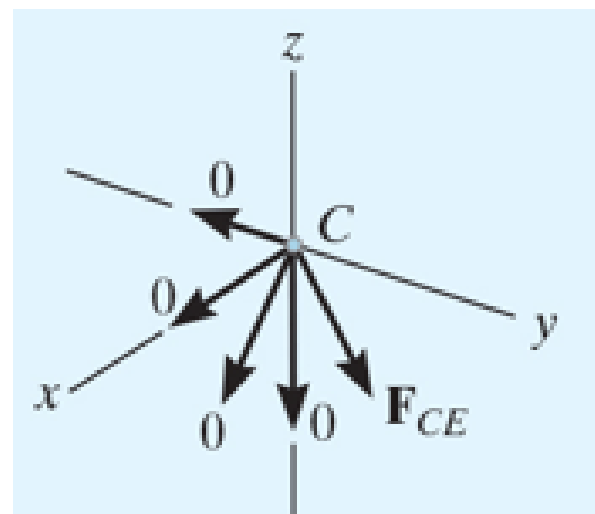
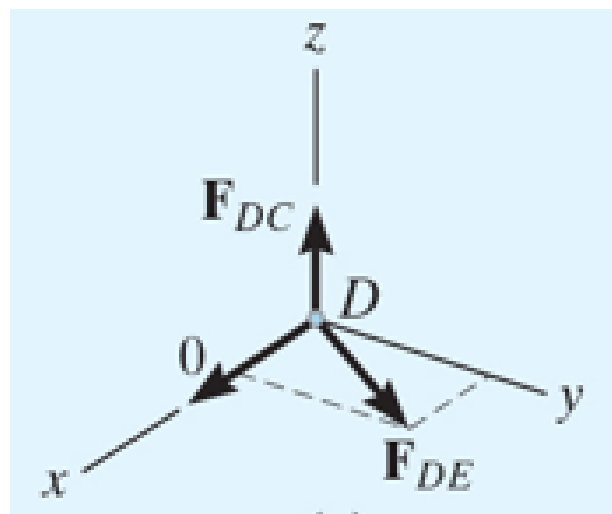
$$\begin{array}{lll}
 \Sigma F_y = 0; & -4 + F_{BE}\left(\frac{4}{6}\right) = 0 & F_{BE} = 6 \text{ kN (T)} \\
 \Sigma F_x = 0; & 2 - F_{BC} - 6\left(\frac{2}{6}\right) = 0 & F_{BC} = 0 \\
 \Sigma F_z = 0; & F_{BA} - 6\left(\frac{4}{6}\right) = 0 & F_{BA} = 4 \text{ kN (C)}
 \end{array}$$



$$\begin{aligned}\Sigma F_z &= 0; & 4 - 4 + F_{AC} \sin 45^\circ &= 0 \\ & & F_{AC} &= 0\end{aligned}$$

$$\begin{aligned}\Sigma F_y &= 0; & -F_{AE} \left(\frac{2}{\sqrt{5}} \right) + 4 &= 0 \\ & & F_{AE} &= 4.47 \text{ kN (C)}\end{aligned}$$

$$\begin{aligned}\Sigma F_x &= 0; & -2 + F_{AD} + 4.47 \left(\frac{1}{\sqrt{5}} \right) &= 0 \\ & & F_{AD} &= 0\end{aligned}$$



$$\Sigma F_x = 0;$$

$$F_{DE} = 0$$

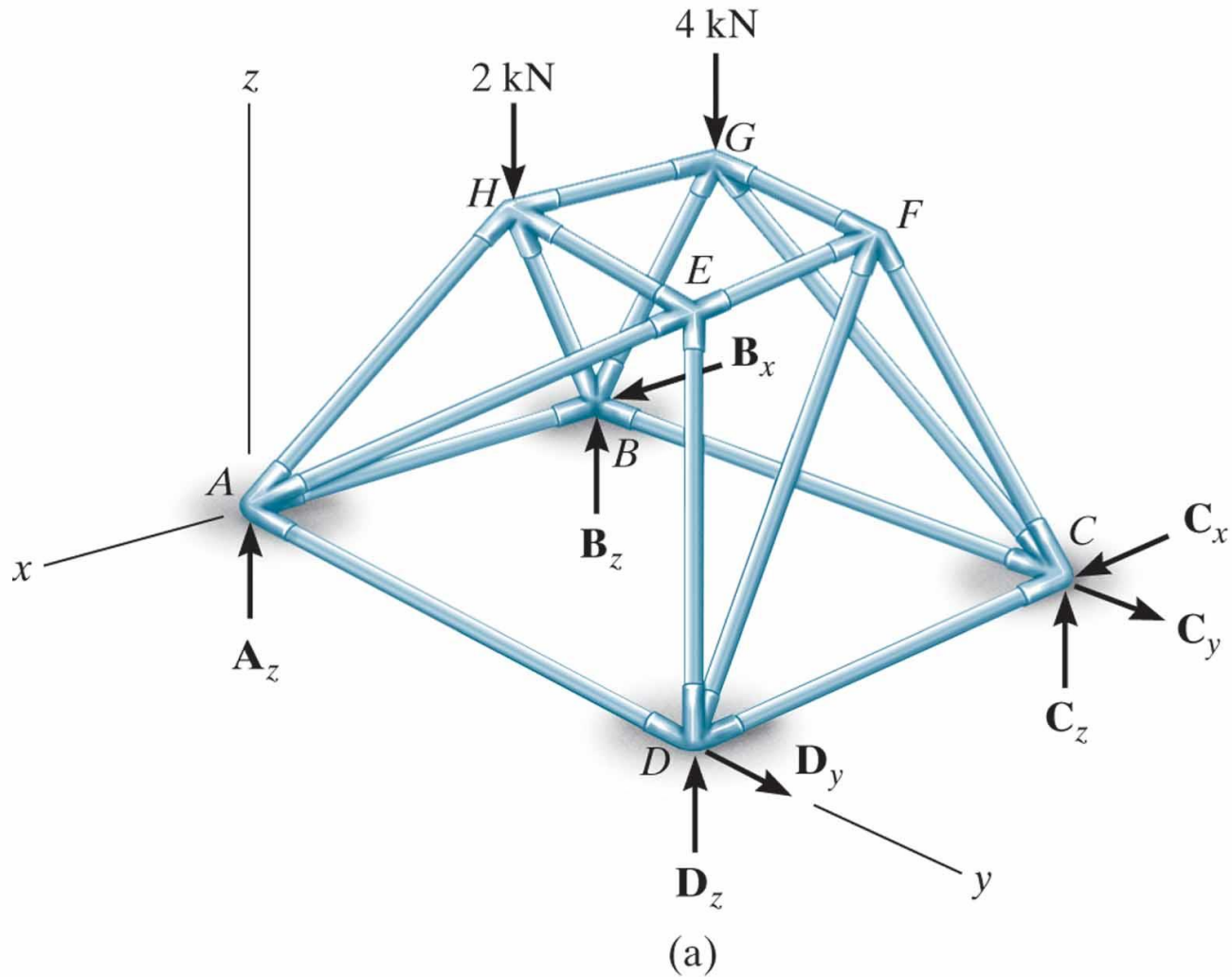
$$\Sigma F_z = 0;$$

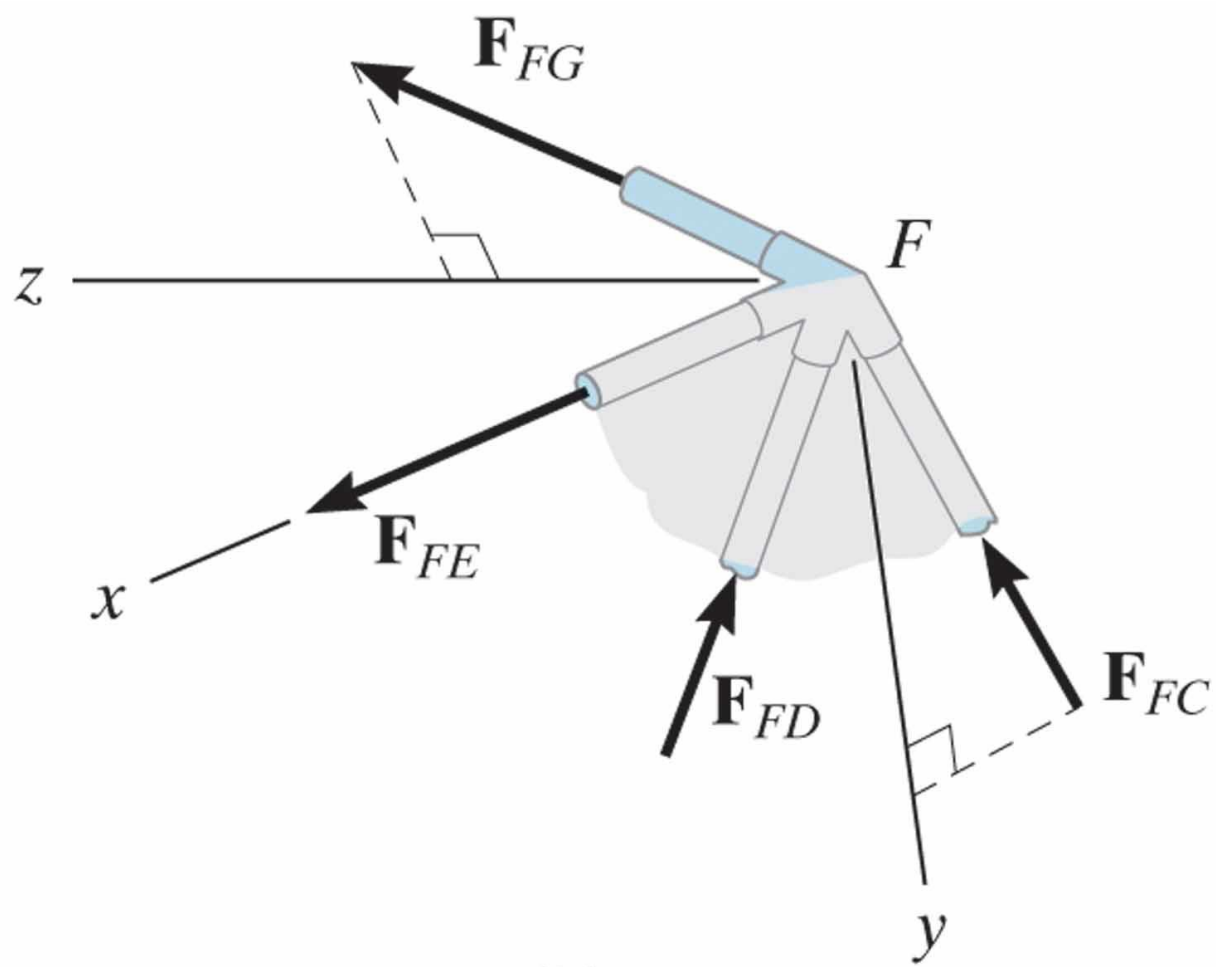
$$F_{DC} = 0$$

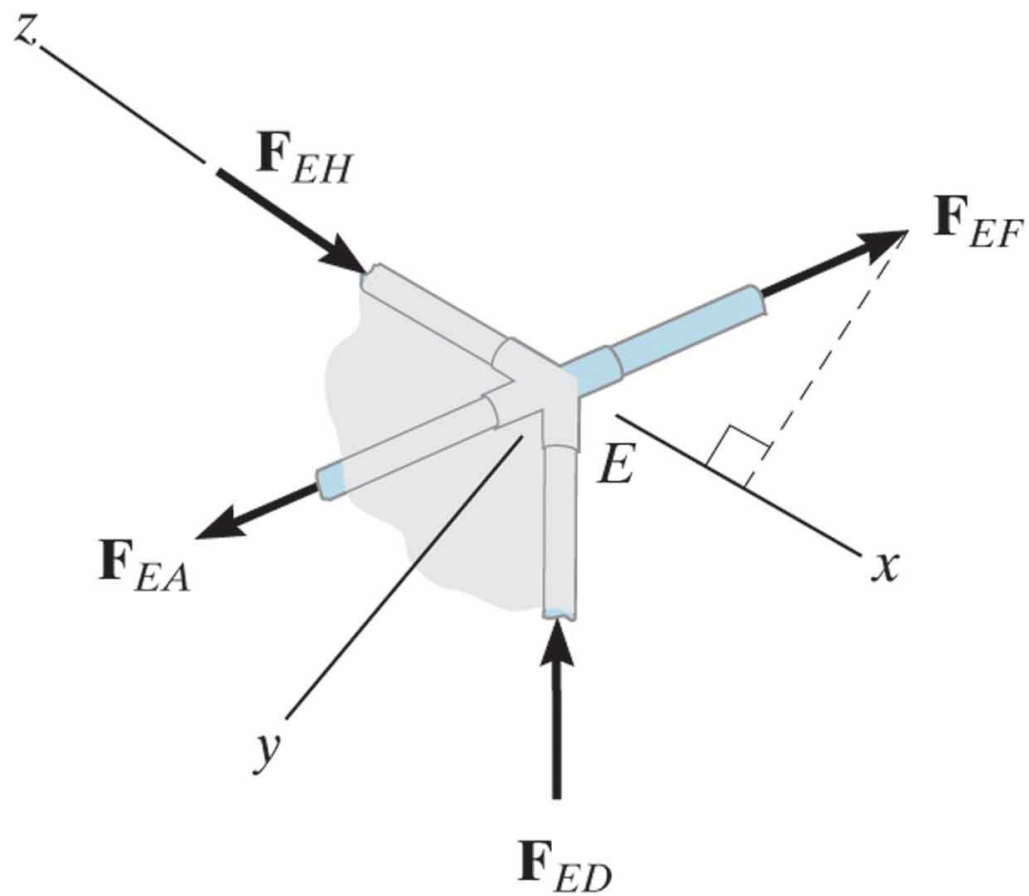
Joint C. By observation of the free-body diagram, Fig. 3–38f,

$$F_{CE} = 0$$

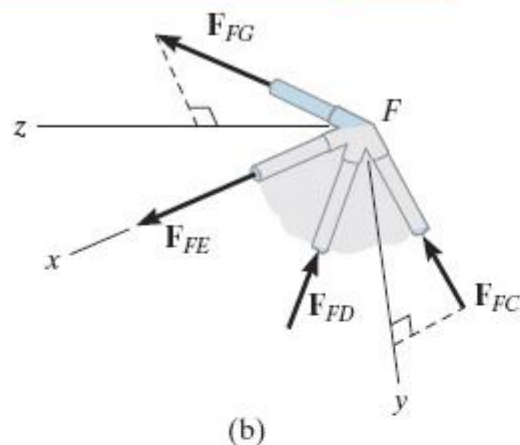
Example 2 Space Truss







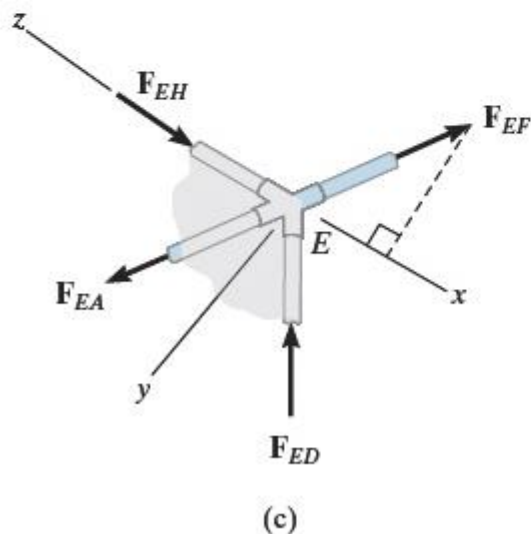
EXAMPLE 3-13 (Continued)



Solution

The free-body diagram, Fig. 3-39a, indicates there are eight unknown reactions for which only six equations of equilibrium are available for solution. Although this is the case, the reactions can be determined, since $b + r = 3j$ or $16 + 8 = 3(8)$.

To spot the zero-force members, we must compare the conditions of joint geometry and loading to those of Figs. 3-36 and 3-37. Consider joint F, Fig. 3-39b. Since members FC, FD, FE lie in the x-y plane and FG is not in this plane, FG is a zero-force member. ($\sum F_z = 0$ must be satisfied.) In the same manner, from joint E, Fig. 3-39c, FE is a zero-force member, since it does not lie in the y-z plane. ($\sum F_x = 0$ must be satisfied.) Returning to joint F, Fig. 3-39b, it can be seen that $F_{FD} = F_{FC} = 0$, since $F_{FE} = F_{FG} = 0$, and there are no external forces acting on the joint.



The numerical force analysis of the joints can now proceed by analyzing joint G ($F_{GF} = 0$) to determine the forces in GH, GB, GC. Then analyze joint H to determine the forces in HE, HB, HA; joint E to determine the forces in EA, ED; joint A to determine the forces in AB, AD, and A_z ; joint B to determine the force in BC and B_x , B_z ; joint D to determine the force in DC and D_y , D_z ; and finally, joint C to determine C_x , C_y , C_z .