



ECIV 3302
Engineering Geology

Chapter 3

WEIGHT-VOLUME RELATIONSHIPS

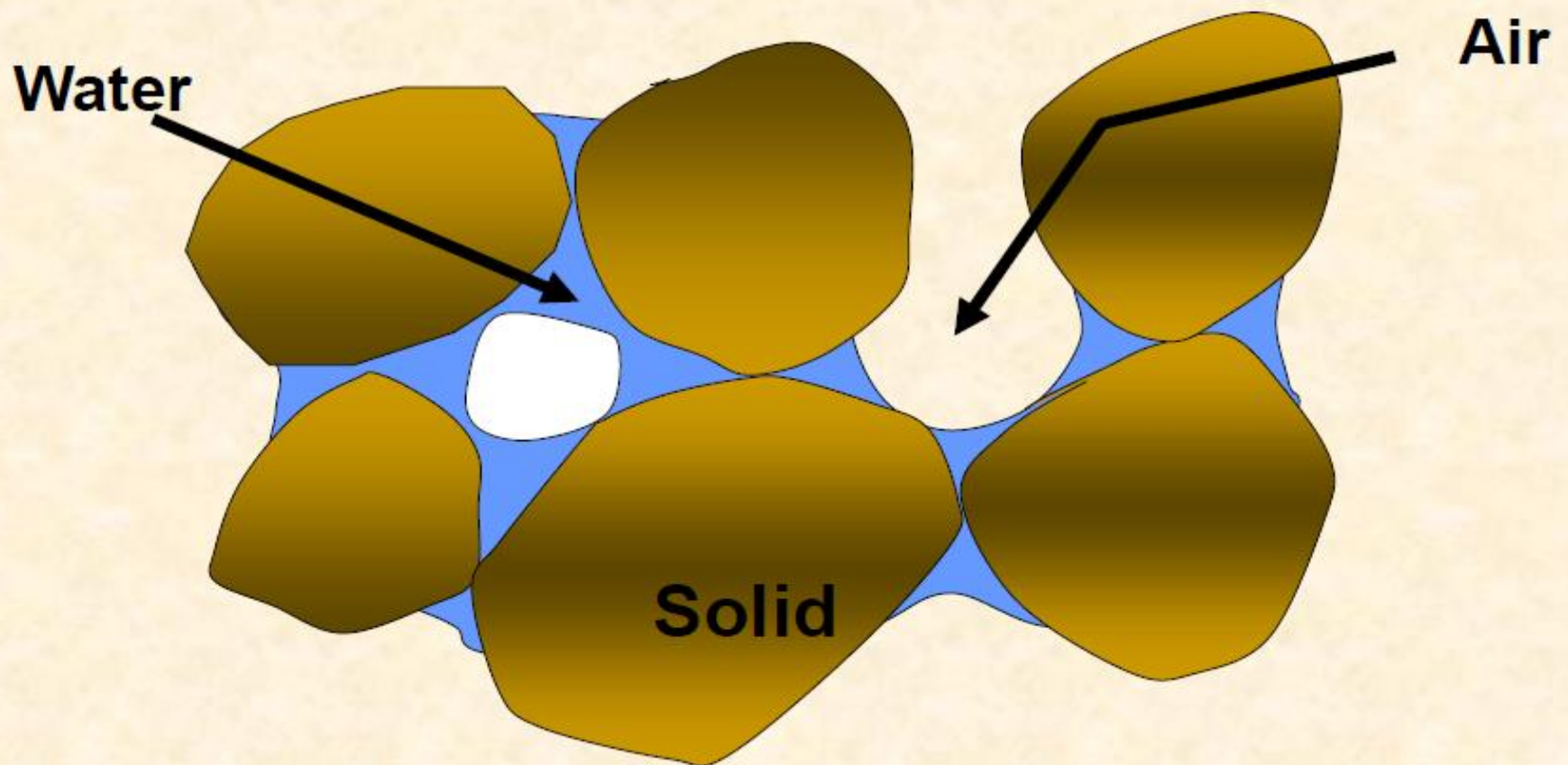
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The materials are mostly adapted from the presentation of Ahmed Abu Foul

Weight-Volume Relationship

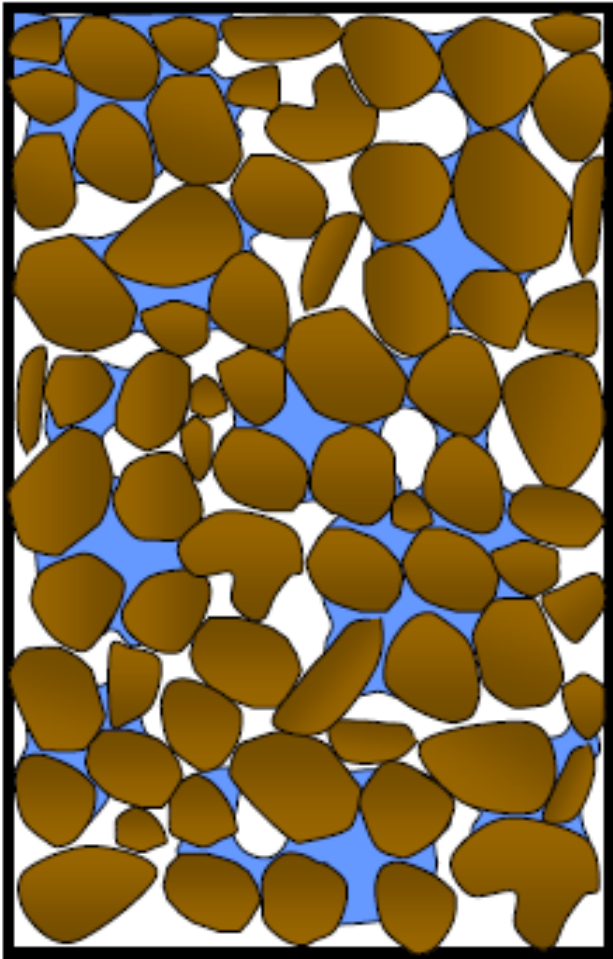


Soil Phase Relationships

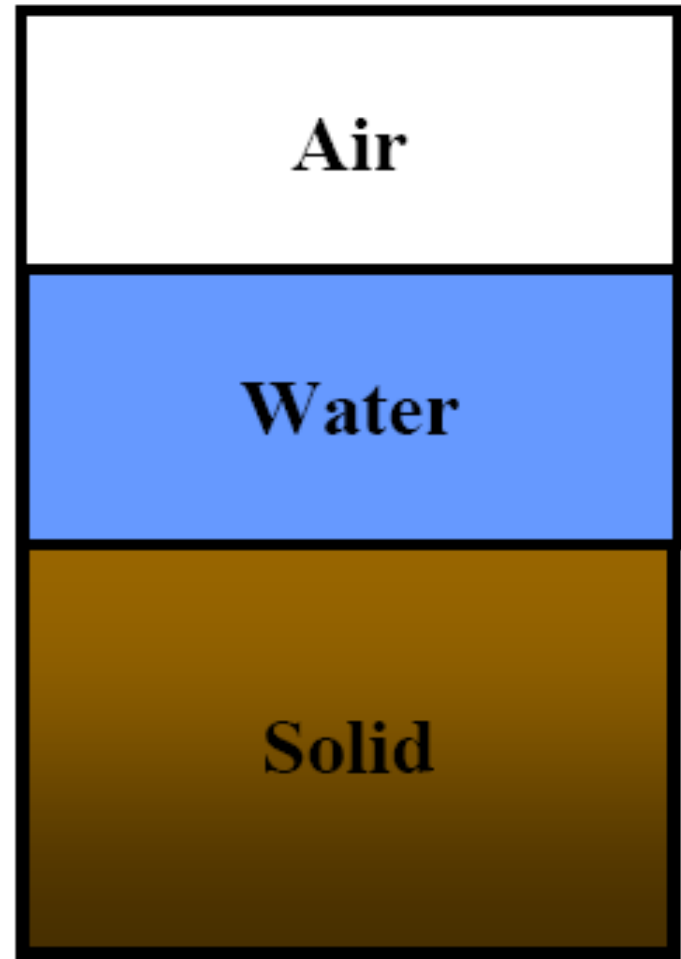
Soil mass is generally a three phase system. It consists of **solid** particles, **liquid** and **gas**. For all practical purposes, the liquid may be considered to be water (although in some cases, the water may contain some dissolved salts) and the gas as air.

The inter relationships of the different phases are important since they help to define the condition or the physical make-up of the soil.

Three Phase Diagram

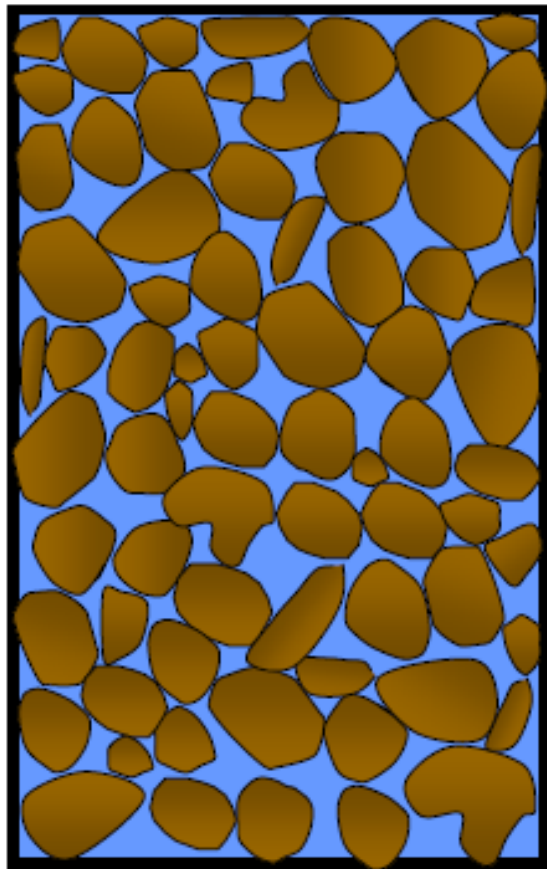


Mineral Skeleton

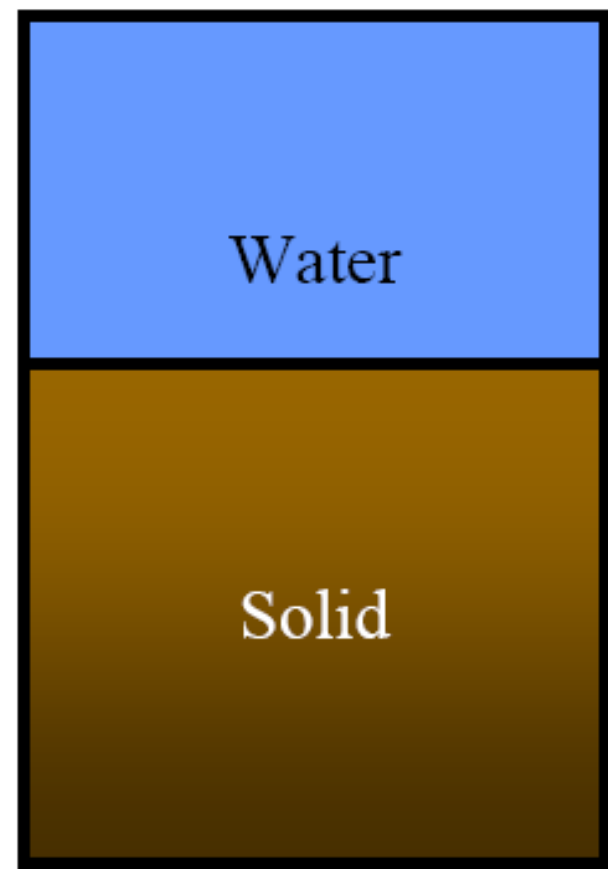


Idealization:
Three Phase Diagram

Fully Saturated Soils (Two Phase)

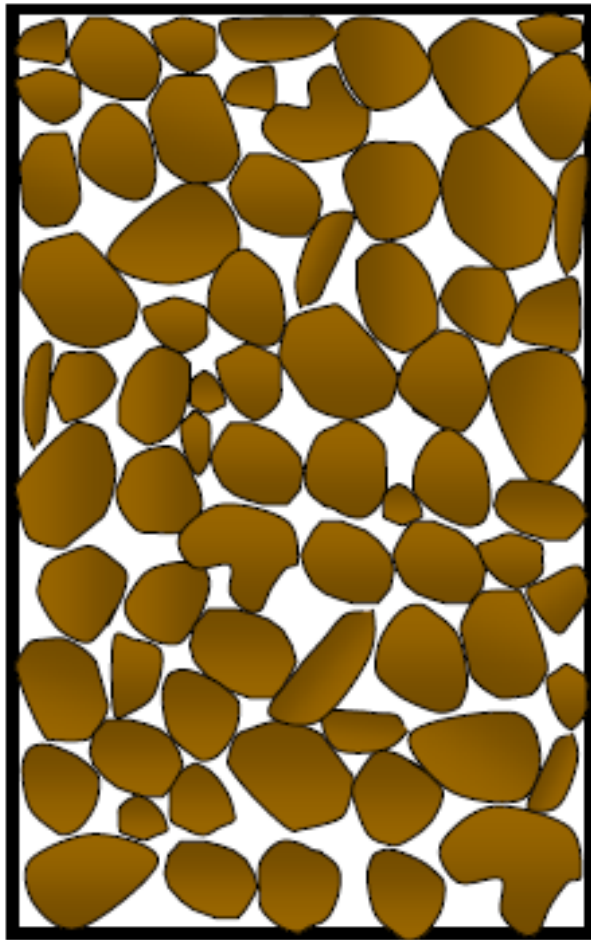


Mineral Skeleton

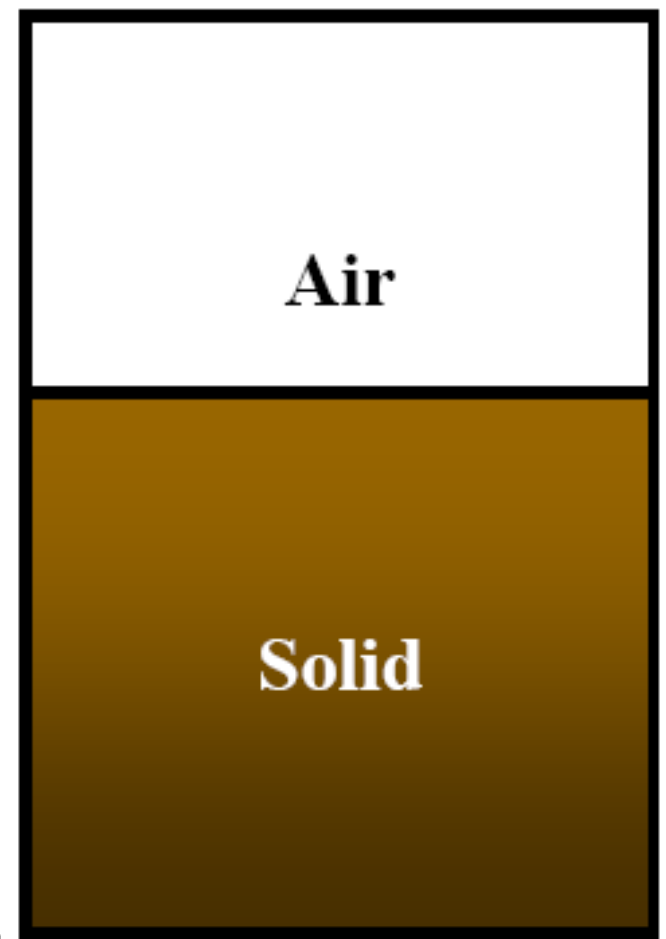


Fully Saturated

Dry Soils (Two phase) [Oven Dried]

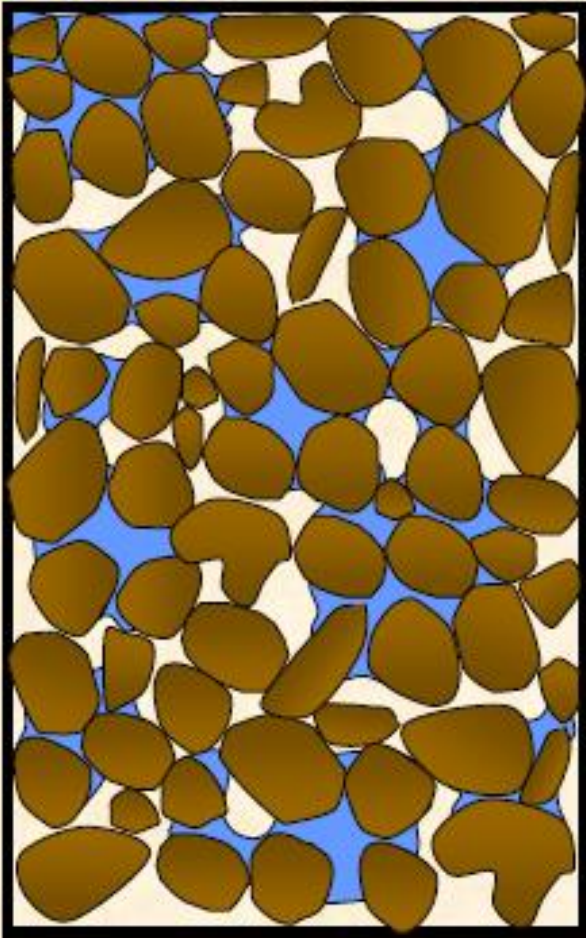


Mineral Skeleton

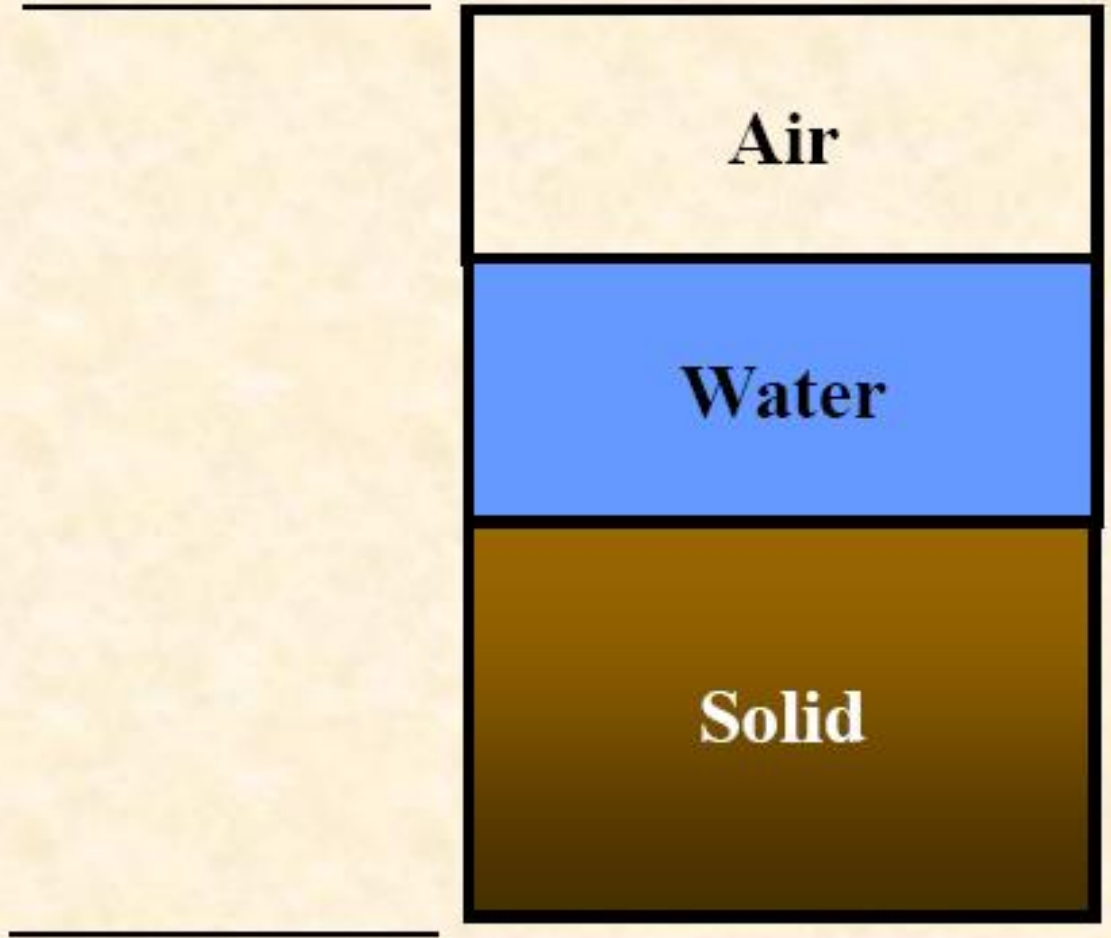


Dry Soil

Partly Saturated Soils



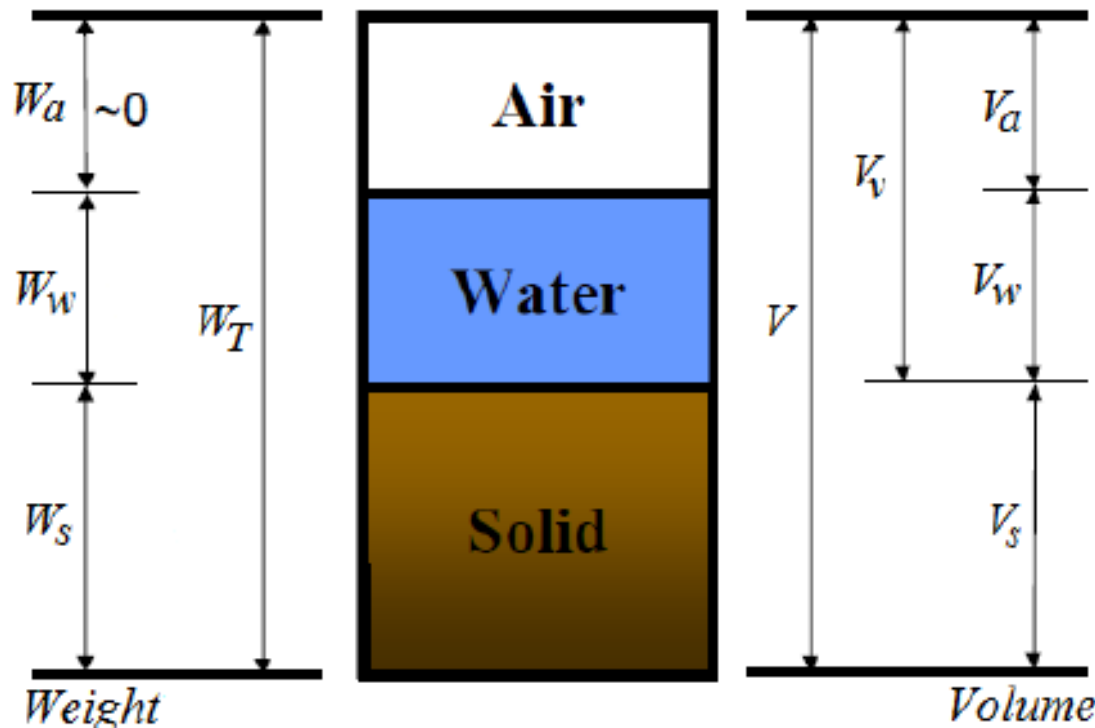
Mineral Skeleton



Partly Saturated Soils

Weight-Volume Relationships

To develop the weight–volume relationships, we must separate the three phases (that is, solid, water, and air) as shown:



W_s = weight of soil solid

W_w = weight of water

W_a = weight of air ≈ 0

V_s = volume of soil solid

V_v = volume of voids

V_w = volume of water

V_a = volume of air

V = total volume

$$W = W_s + W_w$$

$$V = V_s + V_v = V_s + V_w + V_a$$

Weight-Volume Relationships

- There are three volumetric ratios that are very useful in geotechnical engineering and these can be determined directly from the phase diagram:

- Void ratio** (e), defined as the ratio of the volume of voids to the volume of solids:

$$e = \frac{V_v}{V_s}$$

- Porosity** (n), defined as the ratio of the volume of voids to the total volume:

$$n = \frac{V_v}{V_T}$$

- Degree of saturation** (S), defined as the ratio of the volume of water to the volume of voids:

$$S = \frac{V_w}{V_v} \times 100\%$$

- (S) is always expressed as a percentage. When $S = 0\%$, the soil is completely dry, and when $S = 100\%$, the soil is fully saturated.

Weight-Volume Relationships

- The relationship between void ratio and porosity can be derived:

$$e = \frac{V_v}{V_s} = \frac{V_v}{V - V_v} = \frac{\left(\frac{V_v}{V}\right)}{1 - \left(\frac{V_v}{V}\right)} = \frac{n}{1 - n}$$
$$n = \frac{V_v}{V_T} = \frac{V_v}{V_s + V_v} = \frac{\left(\frac{V_v}{V_s}\right)}{1 + \left(\frac{V_v}{V_s}\right)} = \frac{e}{1 + e}$$

- The common terms used for weight relationships are moisture content and unit weight.

- Moisture content** (w), also referred to as water content, is defined as the ratio of the weight of water to the weight of solids in a given volume of soil:

$$w = \frac{W_w}{W_s}$$

Weight-Volume Relationships

2. **Unit weight** (γ) is the weight of soil per unit volume. There are several commonly used unit weights:

a) **Total unit weight** (moisture/wet/bulk unit weight): $\gamma = \frac{W}{V} = \frac{W_s + W_w}{V}$

b) **Dry unit weight** (when saturation $S = 0$): $\gamma_d = \frac{W_s}{V}$

c) **Saturated unit weight** (when saturation $S = 100\%$): $\gamma_{sat} = \frac{W}{V}$

d) **Solid unit weight**: $\gamma_s = \frac{W_s}{V_s}$

• The moisture unit weight and the dry unit weight can also be expressed as:

$$\gamma = \frac{W}{V} = \frac{W_s + W_w}{V} = \frac{W_s \left[1 + \left(\frac{W_w}{W_s} \right) \right]}{V} = \frac{W_s (1 + w)}{V} \quad \text{and} \quad \gamma_d = \frac{\gamma}{1 + w}$$

Weight-Volume Relationships

- Another very useful concept in geotechnical engineering is the **density** (equivalent to unit weight) which is expressed as mass per unit volume. There are several commonly used densities:

a) **Total density**: $\rho = \frac{M}{V}$

b) **Dry density** (when saturation $S = 0$): $\rho_d = \frac{M_s}{V}$

c) **Saturated density** (when saturation $S = 100\%$): $\rho_{sat} = \frac{M}{V}$

d) **Density of solids**: $\rho_s = \frac{M_s}{V_s}$

- The unit weight can be obtained from densities as:

$$\gamma = \rho g$$

$$\gamma_d = \rho_d g$$

M = total mass of the soil sample (kg)

M_s = mass of soil solids in the sample (kg)

V = total volume of the soil (m^3)

Specific Gravity

- The **Specific Gravity** (G) which is defined as the ratio of unit weight (or density) of a given material to the unit weight (or density) of water.
- a) The specific gravity of a mass of soil (including air, water and solids) is termed as **bulk specific gravity** (G_m). It is expressed as:

$$G_m = \frac{\gamma}{\gamma_w} = \frac{\rho g}{\rho_w g} = \frac{\rho}{\rho_w}$$

- b) The **specific gravity of solids** (G_s), excluding air and water, is expressed by:

$$G_s = \frac{\gamma_s}{\gamma_w} = \frac{\rho_s g}{\rho_w g} = \frac{\rho_s}{\rho_w}$$

- Note that:

$$\gamma_w = \rho_w g = \left(1000 \frac{\text{kg}}{\text{m}^3}\right) \left(9.81 \frac{\text{m}}{\text{s}^2}\right) = 9810 \frac{\text{kg} \cdot \text{m}}{\text{m}^3 \cdot \text{s}^2} = 9810 \frac{\text{N}}{\text{m}^3} = 9.81 \frac{\text{kN}}{\text{m}^3}$$

Specific Gravity

- We can use G_s to calculate the density or unit weight of the solid particles:

$$\rho_s = G_s \rho_w \quad \text{and} \quad \gamma_s = G_s \gamma_w$$

- and hence the volume of the solid particles if the mass or weight is known:

$$V_s = \frac{W_s}{\gamma_s} = \frac{M_s}{\rho_s} = \frac{W_s}{G_s \gamma_w} = \frac{M_s}{G_s \rho_w}$$

- Expected values for G_s :

Type of Soil	G_s
Sand	2.65 – 2.67
Silty Sand	2.67 – 2.70
Inorganic Clay	2.70 – 2.80
Soils with Mica or Iron	2.75 – 3.00
Organic Soils	< 2.00

γ , e , w , G_s Relationships

Solids Weight in Terms of Specific Gravity G_s

$$G_s = \frac{\gamma_s}{\gamma_w} = \frac{W_s / V_s}{\gamma_w} = \frac{W_s}{V_s \gamma_w}$$

$$W_s = G_s \gamma_w V_s$$

Water Weight in Terms of Specific Gravity G_s

$$W_w = w W_s = w G_s \gamma_w V_s$$

γ , e , w , G_s Relationships

Soil Unit Weight in terms of G_s

$$\begin{aligned}\gamma &= \frac{W}{V} = \frac{W_s + W_w}{V} \\ &= \frac{G_s \gamma_w V_s + w G_s \gamma_w V_s}{V_s + V_v} \quad \text{Divide by } V_s \\ &= \frac{G_s \gamma_w + w G_s \gamma_w}{1 + e} = \frac{(1 + w) G_s \gamma_w}{1 + e}\end{aligned}$$

γ , e , w , G_s Relationships

Dry Unit Weight in terms of G_s

$$\begin{aligned}\gamma_d &= \frac{W_s}{V} \\ &= \frac{G_s \gamma_w V_s}{V_s + V_v} \quad \text{Divide by } V_s \\ &= \frac{G_s \gamma_w}{1 + e}\end{aligned}$$

γ , e , w , G_s Relationships

Void Ratio(e)

$$e = \frac{V_v}{V_s} = \frac{V - V_s}{V_s} \quad \text{Divivde by } V_s$$

$$= \frac{V}{V_s} - 1 \quad \text{by subsituting } V = \frac{W_s}{\gamma_d}$$

$$= \frac{\left(\frac{W_s}{\gamma_d} \right)}{V_s} - 1 = \frac{\left(\frac{G_s \gamma_w V_s}{\gamma_d} \right)}{V_s} - 1 = \frac{G_s \gamma_w}{\gamma_d} - 1$$

γ , e , w , G_s Relationships

Saturation (S)

$$S = \frac{V_w}{V_v}$$

$$e = \frac{V_v}{V_s}$$

$$Se = \frac{V_w}{V_v} * \frac{V_v}{V_s} = \frac{V_w}{V_s} = \frac{\left(\frac{W_w}{\gamma_w} \right)}{\left(\frac{W_s}{\gamma_s} \right)} = \frac{W_w}{W_s} * \frac{\gamma_s}{\gamma_w} = w * \frac{\gamma_s}{\gamma_w}$$

$$= w * \frac{G_s * \gamma_w}{\gamma_w} = wG_s$$

$$Se = wG_s$$

Note

IF $S=100\%$, $e=wG_s$

γ, e, w, G_s Relationships

Saturation (S)

How you can proof this relationship ($S, e = G_s, w$)???

$$S = \frac{V_w}{V_v} \rightarrow \text{Eq. (1)}$$

$$V_{\text{water}} = \frac{W_w}{\gamma_w} \rightarrow \text{Eq. (2)}$$

$$w = \frac{W_w}{W_s} \rightarrow W_w = w \times W_s \rightarrow \text{Eq. (3)}$$

$$W_s = ???$$

$$\gamma_s = \frac{W_s}{V_s} \rightarrow W_s = \gamma_s \times V_s, \text{ but we know that } G_s = \frac{\gamma_s}{\gamma_w} \rightarrow \gamma_s = G_s \times \gamma_w$$

$$\rightarrow W_s = G_s \times \gamma_w \times V_s \text{ (substitute in Eq. (3)) } \rightarrow \rightarrow$$

$$W_w = w \times G_s \times \gamma_w \times V_s \text{ (substitute in Eq. (2)) } \rightarrow \rightarrow$$

$$V_w = \frac{w \times G_s \times \gamma_w \times V_s}{\gamma_w} = w \times G_s \times V_s \text{ (substitute in Eq. (1)) } \rightarrow \rightarrow$$

$$S = \frac{w \times G_s \times V_s}{V_v} \rightarrow S \times \frac{V_v}{V_s} = w \times G_s \text{ (but we know that } \frac{V_v}{V_s} = e) \rightarrow$$

$$S \times e = w \times G_s \text{ (proofed)} \odot \odot \odot$$

γ , e , w , G s Relationships

Group Work



Prove that

$$\gamma_{sat} = \frac{(G_s + e)\gamma_w}{1 + e}$$

$$\gamma = G_s \gamma_w (1 + w)(1 - n)$$

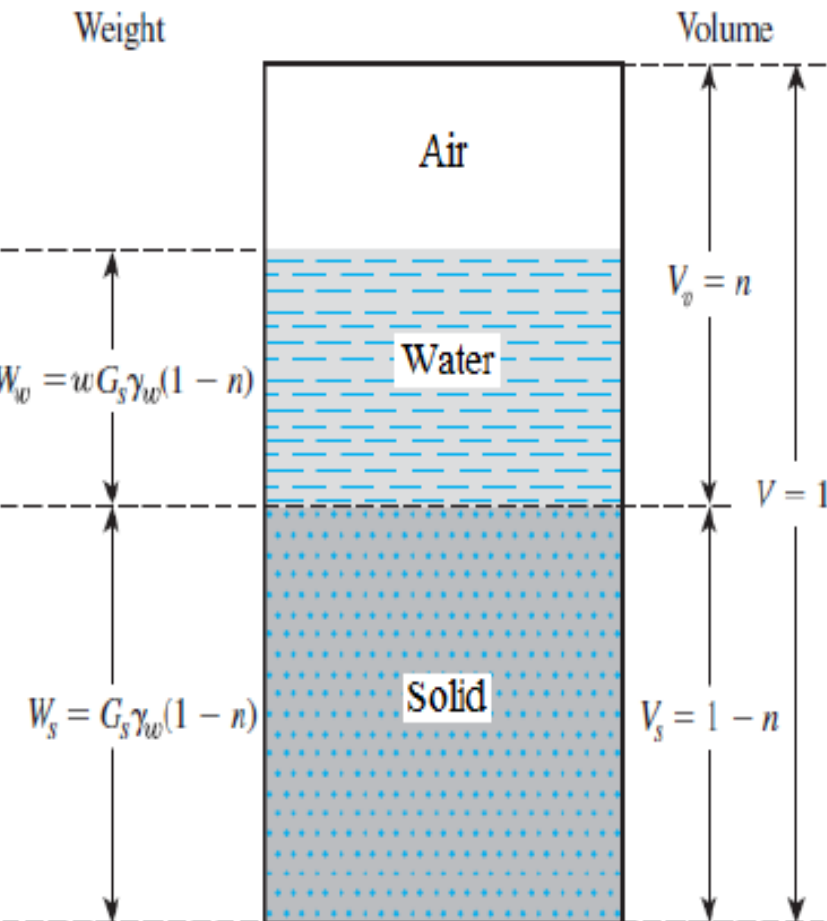
- By assuming $V = 1$, we have: $n = \frac{V_v}{V} = V_v$ $V_s = 1 - n$

$$W_s = \gamma_s V_s = G_s \gamma_w (1 - n)$$

$$W_w = w W_s = w G_s \gamma_w (1 - n)$$

$$\gamma_d = \frac{W_s}{V} = G_s \gamma_w (1 - n)$$

$$\gamma = \frac{W_s + W_w}{V} = G_s \gamma_w (1 - n)(1 + w)$$



$$\gamma_{sat} = \frac{W_s + W_w}{V} = \frac{(1 - n)G_s \gamma_w + n \gamma_w}{1} = [(1 - n)G_s + n] \gamma_w$$

- When the soil is saturated ($S = 100\%$):

$$w = \frac{W_w}{W_s} = \frac{n \gamma_w}{(1 - n) \gamma_w G_s} = \frac{n}{(1 - n) G_s} = \frac{e}{G_s}$$

Weight-Volume Relationships (Table 3.1)

Table 3.1 Various Forms of Relationships for γ , γ_d , and γ_{sat}

Moist unit weight (γ)		Dry unit weight (γ_d)		Saturated unit weight (γ_{sat})	
Given	Relationship	Given	Relationship	Given	Relationship
w, G_s, e	$\frac{(1 + w)G_s\gamma_w}{1 + e}$	γ, w	$\frac{\gamma}{1 + w}$	G_s, e	$\frac{(G_s + e)\gamma_w}{1 + e}$
S, G_s, e	$\frac{(G_s + Se)\gamma_w}{1 + e}$	G_s, e	$\frac{G_s\gamma_w}{1 + e}$	G_s, n	$[(1 - n)G_s + n]\gamma_w$
w, G_s, S	$\frac{(1 + w)G_s\gamma_w}{1 + \frac{wG_s}{S}}$	G_s, n	$G_s\gamma_w(1 - n)$	G_s, w_{sat}	$\left(\frac{1 + w_{\text{sat}}}{1 + w_{\text{sat}}G_s}\right)G_s\gamma_w$
w, G_s, n	$G_s\gamma_w(1 - n)(1 + w)$	G_s, w, S	$\frac{G_s\gamma_w}{1 + \left(\frac{wG_s}{S}\right)}$	e, w_{sat}	$\left(\frac{e}{w_{\text{sat}}}\right)\left(\frac{1 + w_{\text{sat}}}{1 + e}\right)\gamma_w$
S, G_s, n	$G_s\gamma_w(1 - n) + nS\gamma_w$	e, w, S	$\frac{eS\gamma_w}{(1 + e)w}$	n, w_{sat}	$n\left(\frac{1 + w_{\text{sat}}}{w_{\text{sat}}}\right)\gamma_w$
		γ_{sat}, e	$\gamma_{\text{sat}} - \frac{e\gamma_w}{1 + e}$	γ_d, e	$\gamma_d + \left(\frac{e}{1 + e}\right)\gamma_w$
		γ_{sat}, n	$\gamma_{\text{sat}} - n\gamma_w$	γ_d, n	$\gamma_d + n\gamma_w$
		γ_{sat}, G_s	$\frac{(\gamma_{\text{sat}} - \gamma_w)G_s}{(G_s - 1)}$	γ_d, G_s	$\left(1 - \frac{1}{G_s}\right)\gamma_d + \gamma_w$
				γ_d, w_{sat}	$\gamma_d(1 + w_{\text{sat}})$

The 3rd column is a special case of the 1st column when $S = 1$.

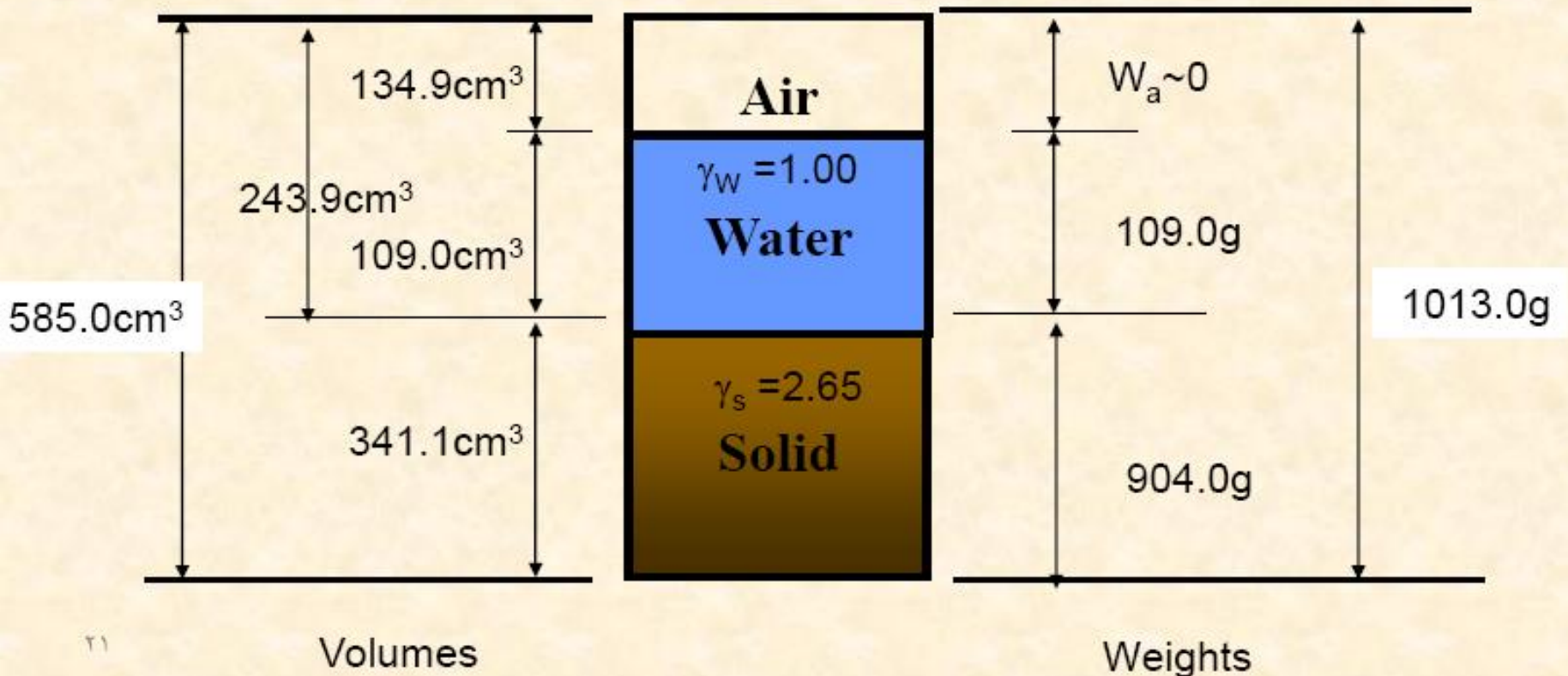
Relationships Typical Values

Table 3.2 Void Ratio, Moisture Content, and Dry Unit Weight for Some Typical Soils in a Natural State

Type of soil	Void ratio, e	Natural moisture content in a saturated state (%)	Dry unit weight, γ_d
			kN/m ³
Loose uniform sand	0.8	30	14.5
Dense uniform sand	0.45	16	18
Loose angular-grained silty sand	0.65	25	16
Dense angular-grained silty sand	0.4	15	19
Stiff clay	0.6	21	17
Soft clay	0.9–1.4	30–50	11.5–14.5
Loess	0.9	25	13.5
Soft organic clay	2.5–3.2	90–120	6–8
Glacial till	0.3	10	21

Example

Determine void ratio, porosity and degree of saturation of a soil core sample



Solution

$$e = \frac{V_v}{V_s} = \frac{243.9}{341.1} = 0.72$$

$$n(\%) = \frac{V_v}{V_T} \times 100\% = \frac{243.9}{585.0} \times 100 = 41.7\%$$

$$S(\%) = \frac{V_w}{V_v} \times 100\% = \frac{109.0}{243.9} \times 100 = 44.7\%$$

Question (3.9) (Text Book page 79)

For a given soil : $G_s = 2.67$, $\gamma_{moist} = 17.6 \frac{KN}{m^3}$, $w = 10.8\%$

- Find:** (a) Dry Unit Weight (γ_d)
(b) Void ratio (e)
(c) Porosity (n)
(d) The degree of saturation (S)
(e) Find the required weight of water to be added so that **S=80%**
($V_T = 1m^3$)

$$(a) \gamma_d = \frac{\gamma}{1+w} = \frac{17.6}{1+0.108} = 15.88 \frac{KN}{m^3}$$

$$(b) \gamma_{dry} = \frac{W_s}{V_T} = \frac{G_s \gamma_w}{1+e} \rightarrow 15.88 = \frac{2.67*9.81}{1+e} \rightarrow e = 0.648$$

$$(c) n = \frac{e}{1+e} = \frac{0.648}{1+0.648} = 0.393$$

$$(d) \text{ } S e = G_s w \rightarrow S = \frac{G_s w}{e} = \frac{2.67 * 0.108}{0.648} * 100 = 44.5\%$$

At first case : $w=10.8\%$

$$\gamma_d = 15.88 \rightarrow \frac{W_s}{V_T} = 15.88 \rightarrow W_s = 15.88 * 1 = 15.88 \text{ KN}$$

$$\frac{W_w}{W_s} = \frac{10.8}{100} \rightarrow W_w = 0.108 * 15.88 = 1.715 \text{ KN}$$

At second case : $S= 80\%$

$$S e = G_s w \rightarrow 0.8 * 0.648 = 2.67 * w \rightarrow w = 0.194$$

$$W_w = 0.194 * 15.88 = 3.08 \text{ KN}$$

$$(\Delta w)_{required} = 3.08 - 1.715 = 1.365 \text{ KN}$$

Example 3

Given:

$$V = 1.2 \text{ m}^3$$

$$M = 2350 \text{ kg}$$

$$w = 8.6 \%$$

$$G_s = 2.71$$

Required:

$$\rho$$

$$\gamma_d$$

$$e$$

$$n$$

$$s$$

$$V_w$$

Solution

$$\text{Density } \rho = \frac{M}{V} = \frac{2350}{1.2} = 1958.3 \text{ kg/m}^3$$

$$\text{Dry density } \rho_d = \frac{\rho}{1+w} = \frac{1958.3}{1+0.086} = 1803.3 \text{ kg/m}^3$$

$$M_s = \frac{M}{1+w} = \frac{2350}{1+0.086} = 2163.9 \text{ kg}$$

$$G_s = \frac{M_s}{V_s * \rho_w}$$

$$2.71 = \frac{2163.9}{V_s * 1000}$$

$$V_s = 0.798 \text{ m}^3 \quad V_v = V - V_s = 1.2 - 0.798 = 0.402 \text{ m}^3$$

$$\text{Void ratio } e = \frac{V_v}{V_s} = \frac{0.402}{0.798} = 0.503$$

$$\text{Porosity } n = \frac{V_v}{V} = \frac{0.402}{1.2} = 0.335 = 33.5\%$$

Degree of Saturation $S = \frac{V_w}{V_v}$

$$V_w = \frac{M_w}{\rho_w} \quad M_w = M - M_s = 2350 - 2163.9 = 186.1 \text{ kg}$$

$$V_w = \frac{186.1}{1000} = 0.186 \text{ m}^3$$

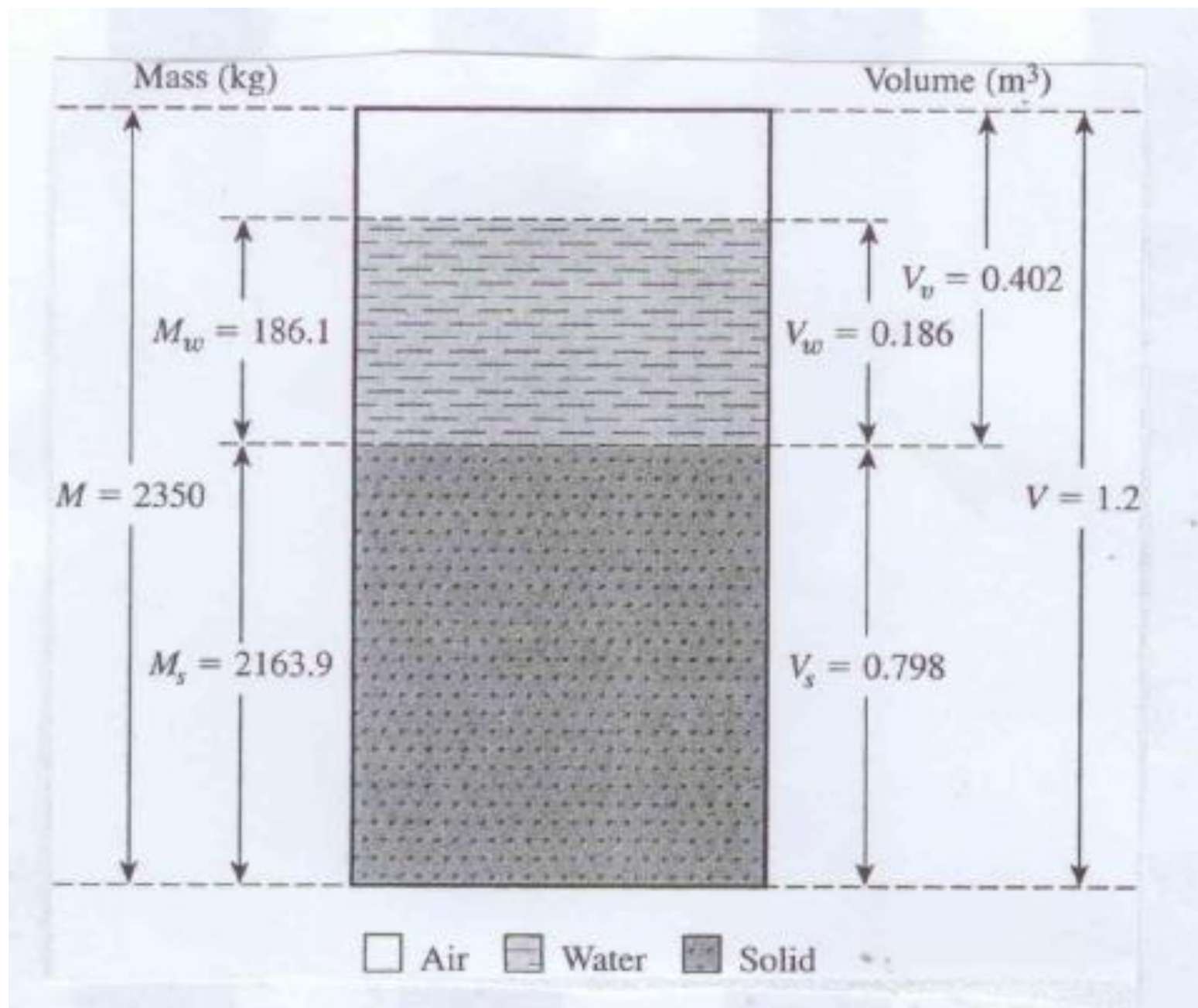
$$S = \frac{V_w}{V_v} = \frac{0.186}{0.402} = 0.463 = 46.3\%$$

Volume of water in the soil $V_w = 0.186 \text{ m}^3$

Degree of Saturation

$$S * e = w * G_s$$

$$S = \frac{w * G_s}{e} = \frac{0.086 * 2.71}{0.503} = 0.463 = 46.3\%$$

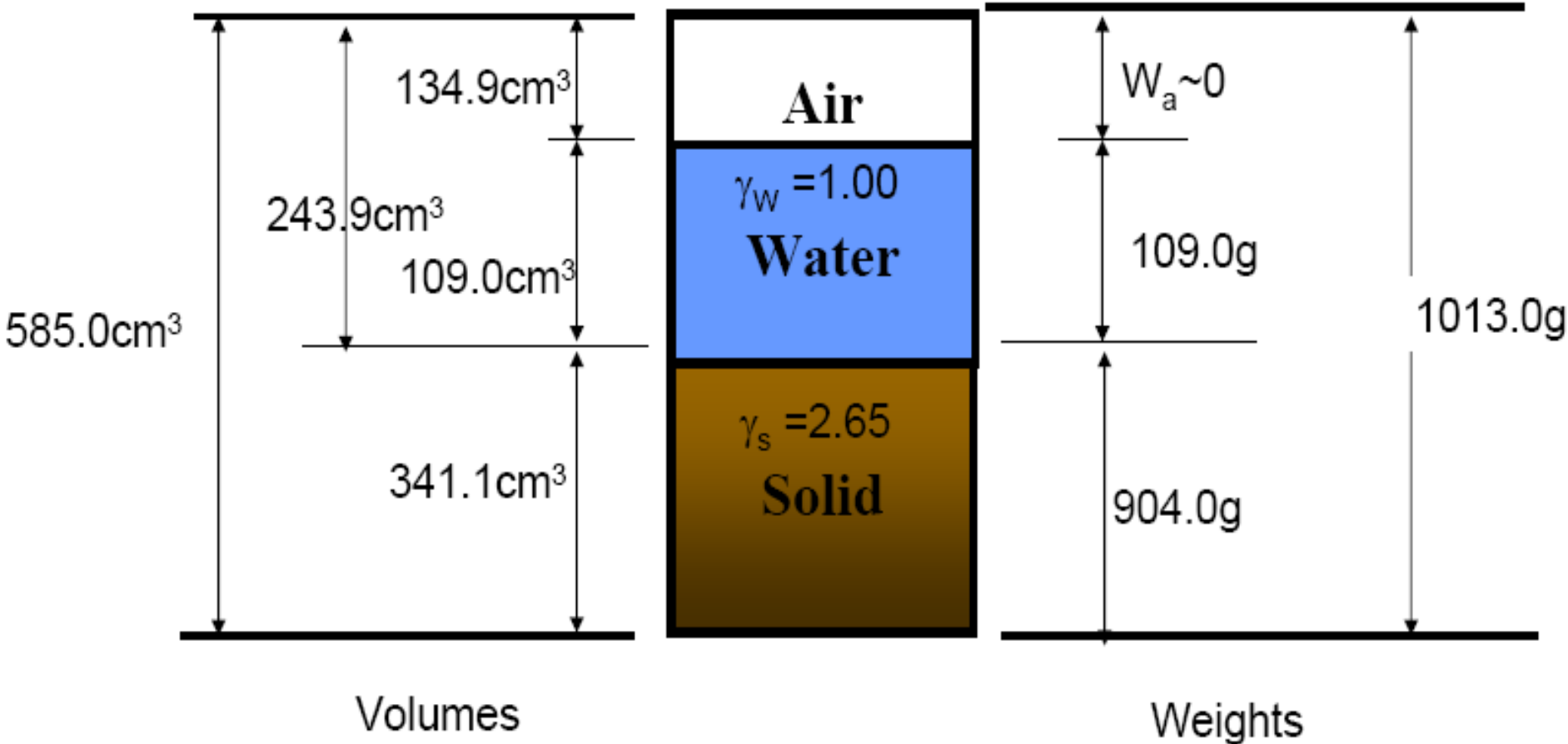


Example 4

Determine moisture content, void ratio, porosity and degree of saturation of a soil core sample. Also determine the dry unit weight, γ_d

Data:

- Weight of soil sample = 1013 g
- Volume of soil sample = 585.0 cm³
- Specific gravity, $G_s = 2.65$
- Dry weight of soil = 904.0 g



From the three phase diagram we can find:

- Moisture content, w
$$w = \frac{W_w}{W_s} = \frac{109(g)}{904(g)} \times 100 = 12.1\%$$
- Void ratio, e
$$e = \frac{V_v}{V_s} = \frac{243.9cm^3}{341.1cm^3} = 0.715$$
- Porosity, n
$$n = \frac{V_v}{V_T} = \frac{243.9(cm^3)}{585.0(cm^3)} \times 100 = 41.7\%$$
- Degree of saturation, S
$$S = \frac{V_w}{V_v} = \frac{109}{243.9} \times 100 = 44.7\%$$
- Dry unit weight, γ_d
$$\gamma_d = \frac{W_s}{V_T} = \frac{904}{585} = 1.55 \frac{g}{cm^3}$$

Relative Density

The term relative density is commonly used to indicate the in situ denseness or looseness' of granular soil.

$$D_r = \frac{e_{\max} - e}{e_{\max} - e_{\min}}$$

where

D_r = Relative density

e = in situ void ratio of the soil

$e_{\max}$ = void ratio of the soil in loosest state

$e_{\min}$ = void ratio of the soil in densest state

The values of D_r may vary from a minimum of 0% for very loose soil to a maximum of 100% for very dense soils. Soils engineers qualitatively describe the granular soil deposits according to the relative densities, as show:

Relative density (%)	Description of soil deposit
0 -15	Very loose
25 – 50	Loose
50 – 70	Medium
70 – 85	Dense
85 –100	Very dense

Relative Density Definition in other terms

Porosity Term

$$D_r = \frac{(1 - n_{\min})(n_{\max} - n)}{(n_{\max} - n_{\min})(1 - n)}$$

Dry Unit Term

$$D_r = \left[\frac{\gamma_d - \gamma_{d(\min)}}{\gamma_{d(\max)} - \gamma_{d(\min)}} \right] \left[\frac{\gamma_{d(\max)}}{\gamma_d} \right]$$

Example

For a given sandy soil, $e_{\max}=0.82$ and $e_{\min}=0.42$. Let $G_s=2.66$. In the field, the soil is compacted to a moist density of 1720 kg/m^3 at a moisture content of 9%. Determine the relative density of compaction

$$e = \frac{G_s \rho_w (1 + w)}{1 + e} - 1 = \frac{(2.66)(1000)(1 + 0.09)}{1760} - 1$$

$$e = 0.686$$

$$D_r = \frac{e_{\max} - e}{e_{\max} - e_{\min}} = \frac{0.82 - 0.686}{0.82 - 0.42} = 0.335 = 33.55\%$$

Example

For a given sandy soil, $e_{\max} = 0.75$ and $e_{\min} = 0.4$. Let $G_s = 2.68$. In the field, the soil is compacted to a moist density of 112 lb/ft^3 at a moisture content of 12%. Determine the relative density of compaction.

Solution

From Eq. (3.21),

$$\rho = \frac{(1 + w)G_s\gamma_w}{1 + e}$$

or

$$e = \frac{G_s\gamma_w(1 + w)}{\gamma} - 1 = \frac{(2.68)(62.4)(1 + 0.12)}{112} - 1 = 0.67$$

From Eq. (3.30),

$$D_r = \frac{e_{\max} - e}{e_{\max} - e_{\min}} = \frac{0.75 - 0.67}{0.75 - 0.4} = 0.229 = 22.9\%$$

H.W

All Problems