

**The Islamic University of Gaza
Faculty of Engineering
Civil Engineering Department**

**Environmental Engineering
(ECIV 4324)**

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Water Treatment Part C

Sedimentation

Sedimentation is the process in which the majority of the particles will settle by gravity within a reasonable time and be removed. Particles with densities greater than $1,000 \text{ kg/m}^3$ will eventually settle, and particles with densities less than $1,000 \text{ kg/m}^3$ will float to the water surface. In water treatment, there are common types of settling: discrete particle settling and flocculant settling

Type I Settling

1. Size, shape and specific gravity of the particles do not change with time.
2. Settling velocity remains constant.

If a particle is suspended in water, it initially has two forces acting upon it:

force of gravity: $F_g = \rho_p g V_p$

buoyant force: $F_b = \rho_w g V_p$

$$F_{\text{net}} = (\rho_p - \rho_w) g V_p$$

This net force becomes the driving force. Once the motion has been initiated, a third force is created due to viscous friction. This force, called the ***drag force***, is quantified by:

$$F_d = C_D A_p \rho_w v^2 / 2$$

C_D = drag coefficient.

A_p = projected area of the particle.

the drag force acts in the opposite direction to the driving force and increases as the square of the velocity, acceleration occurs at a decreasing rate until a steady velocity is reached at a point where the drag force equals the driving force:

$$(\rho_p - \rho_w)gV_p = C_D A_p \rho_w v^2 / 2$$

C_D are:

$$C_D = \frac{24}{Re} \quad (\text{laminar})$$

For spherical particles,

$$V_p = \frac{\pi d^3}{6} \quad \text{and} \quad A_p = \frac{\pi d^2}{4}$$

$$C_D = \frac{24}{Re} + \frac{3}{Re^{1/2}} + 0.34 \quad (\text{transition})$$

$$\text{Thus, } v^2 = \frac{4g(\rho_p - \rho_w)d}{3 C_D \rho_w}$$

$$C_D = 0.4 \quad (\text{turbulent})$$

Reynolds number less than 1.0 indicate laminar flow, while values greater than 1000 indicate turbulent flow. Intermediate values indicate transitional flow.

Stokes Flow

For laminar flow, terminal settling velocity equation becomes:

$$v = \frac{(\rho_p - \rho_w)gd^2}{18\mu}$$

which is known as the ***stokes equation***.

EXAMPLE: Find the terminal setting velocity of a spherical particle with diameter 0.5 mm and specific gravity of 2.65 setting through water at 20°C.

Solution

Assume laminar flow: from Eq. (4-8) with $\rho_w = 998.2$ kg/m³ and $\mu = 1.002$ N.s-m³ at 20°C

$$v_1 = \frac{9.81 \text{ m/s} (2650 - 998.2) \text{ kg/m}^3 \times (5.0 \times 10^{-4}) \text{ m}^2}{18 \times 1.002 \times 10^{-3} \text{ N.s/m}^2}$$

$$v_1 = 0.22 \text{ m/s}$$

- Check Reynolds number:

$$Re = \frac{0.22 \text{ m/s} \times 5 \times 10^{-4} \text{ m} \times 998.2 \text{ kg/m}^3}{1.002 \times 10^{-3} \text{ N.s/m}^2}$$

= 112. Which indicates transitional flow

$$C_D = \frac{24}{112} + \frac{3}{112^{1/2}} + 0.34 = 0.84$$

$$v_1^2 = \frac{4}{3} \times 9.81 \times \frac{(2650 - 998.2)}{0.84 \times 998.2} 5 \times 10^{-4}$$

$$v_1 = 0.11 \text{ m/s}$$

with

$v_1 = 0.11$, repeat steps 2, 3, and 4.

$$\text{Re} = 55$$

$$C_D = 1.18$$

$$v_1 = 0.10 \text{ m/s} \cong 0.11 \text{ m/s} \text{ (see step 4)}$$

DISCRETE PARTICLE SETTLING

Table / 8.15

Determination of Settling Velocity of Particles Using Stokes' and Newton's Laws

Applicable Law	Settling Velocity (m/s)	Terms	Drag Coefficient	Applicability
Stokes' law	$v_s = \frac{g(\rho_p - \rho)d_p^2}{18\mu}$	g is the acceleration due to gravity (m/s^2); ρ_p is the density of the particle (kg/m^3); ρ is the density of the liquid (kg/m^3); d_p is the particle diameter (m); and μ is the dynamic viscosity of the liquid (N-s/m^2)	For laminar flow: $C_d = \frac{24}{\text{Re}}$	Applicable for spherical particles when the Reynolds number ≤ 1 (laminar flow). Has limited application in water treatment because conditions in most treatment facilities are not laminar.
Newton's law	$v_s = \sqrt{\frac{4g(\rho_p - \rho)d_p}{3C_d\rho}}$	g is the acceleration due to gravity (m/s^2); ρ_p is the density of the particle (kg/m^3); ρ is the density of the liquid (kg/m^3); C_d is the drag coefficient.	For the transition regime: $C_d = \frac{24}{\text{Re}} + \frac{3}{\sqrt{\text{Re}}} + 0.34$ C_d becomes constant in turbulent regime ($\text{Re} > 10,000$)	Applicable for particles when the Reynolds number > 1 (transition and turbulent flow).

$$\text{Re} = \frac{\rho d_p v_s}{\mu} = \frac{d_p v_s}{\nu} \approx \frac{\text{inertial forces}}{\text{viscous forces}}$$

DISCRETE PARTICLE SETTLING

example/ 8.4 Application of Stokes' Law

Calculate the terminal settling velocity for a sand particle that has a diameter of 100 μm and a density of 2,650 kg/m^3 . The water temperature is 10°C.

solution

The terminal settling velocity of the particle can be calculated using Stokes' law (Table 8.15). For water at 10°C, $\rho = 999.7 \text{ kg}/\text{m}^3$, $\mu = 1.307 \times 10^{-3} \text{ N}\cdot\text{s}/\text{m}^2$, and $\nu = 1.306 \times 10^{-6} \text{ m}^2/\text{s}$.

$$\begin{aligned} v_s &= \frac{g(\rho_p - \rho)d_p^2}{18\mu} \\ &= \frac{9.81 \text{ m/s}^2 \times (2,650 - 999.7 \text{ kg}/\text{m}^3) \times (1.0 \times 10^{-4} \text{ m})^2}{18 \times 1.307 \times 10^{-3} \text{ N}\cdot\text{s}/\text{m}^2} \times \frac{3,600 \text{ s}}{\text{h}} = 24.8 \frac{\text{m}}{\text{h}} \end{aligned}$$

We must verify the flow conditions to ensure Stokes' law is applicable. The Reynolds number is calculated to verify that the particle is settling under laminar conditions:

$$\text{Re} = \frac{d_p v_s}{\nu} = \frac{1.0 \times 10^{-4} \text{ m} \times 24.8 \text{ m/h} \times (1 \text{ h}/3,600 \text{ s})}{1.306 \times 10^{-6} \text{ m}^2/\text{s}} = 0.53$$

Because $\text{Re} < 1$, laminar flow exists, and Stokes' law is applicable.

DISCRETE PARTICLE SETTLING

example/8.5 Application of Newton's Law

Calculate the terminal settling velocity for a sand particle that has a diameter of 200 μm and a density of 2,650 kg/m^3 . The water temperature is 15°C.

solution

The terminal velocity of the particle is calculated using Stokes' law (Table 8.15). For water at 15°C, $\rho = 999.1 \text{ kg}/\text{m}^3$, $\mu = 1.139 \times 10^{-3} \text{ N}\cdot\text{s}/\text{m}^2$, and $\nu = 1.139 \times 10^{-6} \text{ m}^2/\text{s}$.

$$\begin{aligned} v_s &= \frac{g(\rho_p - \rho)d_p^2}{18\mu} \\ &= \frac{9.81 \text{ m/s}^2 \times (2,650 - 999.1 \text{ kg}/\text{m}^3) \times (2.0 \times 10^{-4} \text{ m})^2}{18 \times 1.139 \times 10^{-3} \text{ N}\cdot\text{s}/\text{m}^2} \times \frac{3,600 \text{ s}}{\text{h}} = 113.8 \frac{\text{m}}{\text{h}} \end{aligned}$$

Check the Reynolds number (Equation 8.8) to verify the particle is settling under laminar conditions:

$$\text{Re} = \frac{d_p v_s}{\nu} = \frac{(2.0 \times 10^{-4} \text{ m}) \times (113.8 \text{ m/h}) \times (1 \text{ h}/3,600 \text{ s})}{(1.139 \times 10^{-6} \text{ m}^2/\text{s})} = 5.55$$

Because $\text{Re} > 1$, Stokes' law is not valid. The equation for the drag coefficient is provided in Table 8.15, and the settling velocity can be calculated using Newton's law (Table 8.15).

DISCRETE PARTICLE SETTLING

example/ 8.5 (continued)

Because v_s cannot be determined explicitly, a trial-and-error solution must be used. Using the value of Re just obtained (that is, 5.55), the drag coefficient can be calculated as

$$C_d = \frac{24}{5.55} + \frac{3}{\sqrt{5.55}} + 0.34 = 5.94$$

The terminal settling velocity also can be recalculated:

$$\begin{aligned} v_s &= \sqrt{\frac{4(9.81 \text{ m/s}^2) \times (2,650 - 999.1 \text{ kg/m}^3) \times 2.0 \times 10^{-4} \text{ m}}{3 \times 5.94 \times 999.1 \text{ kg/m}^3}} \times 3,600 \frac{\text{s}}{\text{h}} \\ &= 97.1 \frac{\text{m}}{\text{h}} \end{aligned}$$

The Reynolds number is calculated again, and then the drag coefficient and terminal settling velocity are recalculated. After several iterations, a convergent answer is obtained, as given in Table 8.16. The settling velocity begins to converge at the sixth or seventh trial and has a value of 86.5 m/h.

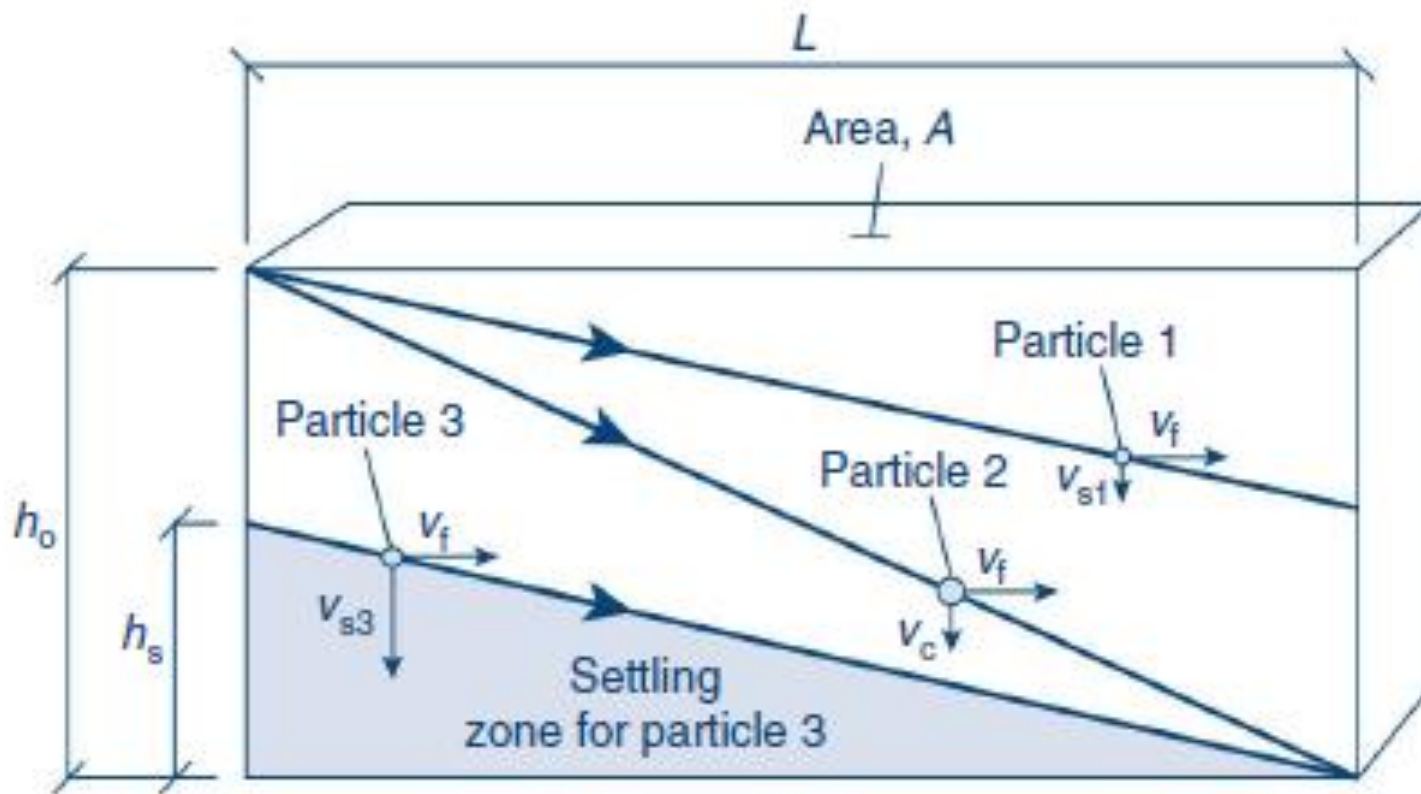
DISCRETE PARTICLE SETTLING

Table / 8.16

Iterative Process Used in Example 8.5 to Determine Settling Velocity Using Newton's Law After several iterations, a convergent answer is obtained, as shown here.

Trial	Re (dimensionless)	C_d (dimensionless)	v_s (m/h)
0	5.55	5.94	97.1
1	4.74	6.78	90.9
2	4.43	7.18	88.3
3	4.31	7.36	87.3
4	4.26	7.43	86.8
5	4.23	7.47	86.6
6	4.23	7.48	86.5
7	4.22	7.49	86.5

PARTICLE REMOVAL DURING SEDIMENTATION



$$v_c = \frac{h_o}{\theta}$$

$$v_c = \frac{h_o}{\theta} = \frac{h_o Q}{V} = \frac{h_o Q}{h_o A} = \frac{Q}{A} \text{ OR}$$

PARTICLE REMOVAL DURING SEDIMENTATION

example/ 8.6 Determining Particle Removal

A treatment plant has a horizontal flow sedimentation basin with a depth of 4 m, width of 6 m, length of 36 m, and process flow rate of 450 m³/h. What removal percentage should be expected for particles that have settling velocities of 1.0 and 2.5 m/h? What is the minimum size of particles that would be completely removed? Assume the particle density is 2,650 kg/m³. The water temperature is 10°C.

solution

First, determine the sedimentation basin overflow rate (critical settling velocity), using Equation 8.10:

$$\text{OR} = v_c = \frac{Q}{A} = \frac{450 \text{ m}^3/\text{h}}{36 \text{ m} \times 6 \text{ m}} = 2.1 \text{ m}^3/\text{m}^2\text{-h}$$

The percent removal of particles for each particle size can then be calculated as follows.

For particles with a settling velocity of 1.0 m/h, because v_s equals 1.0 m/h (which is less than the v_c of 2.1 m/h), the percentage of removal is calculated using Equation 8.11:

$$\text{fraction of particles removed} = \frac{v_s}{\text{OR}} = \frac{1.0 \text{ m/h}}{2.1 \text{ m/h}} = 0.48$$

For particles with a settling velocity of 2.5 m/h, because v_s equals 2.5 m/h (which is greater than v_c of 2.1 m/h), all particles with this settling velocity will be removed:

$$\text{fraction of particles removed} = 1.0$$

The final question is to determine the minimum particle size that would result in complete removal. This particle size can be determined using Stokes' law. The settling velocity of the minimum-size particle is equal to the critical settling velocity, $v_s = v_c = 2.1 \text{ m/h}$. Plugging v_s into Stokes' law will provide the correct particle size. For water at 10°C, $\rho = 999.7 \text{ kg/m}^3$, $\mu = 1.307 \times 10^{-3} \text{ N-s/m}^2$, and $v = 1.306 \times 10^{-6} \text{ m}^2/\text{s}$.

$$v_s = \frac{g(\rho_p - \rho)d_p^2}{18\mu} = \frac{9.81 \text{ m/s}^2 \times (2,650 - 999.7 \text{ kg/m}^3) \times (d_p)^2}{18 \times 1.307 \times 10^{-3} \text{ N-s/m}^2} \times \frac{3,600 \text{ s}}{\text{h}} = 2.1 \frac{\text{m}}{\text{h}}$$

$$d_p^2 = 8.48 \times 10^{-10} \text{ m}^2$$

$$d_p = 2.9 \times 10^{-5} \text{ m} = 2.9 \times 10^{-3} \text{ cm}$$

PARTICLE REMOVAL DURING SEDIMENTATION

$$v_s = \frac{g(\rho_p - \rho)d_p^2}{18\mu} = \frac{9.81 \text{ m/s}^2 \times (2,650 - 999.7 \text{ kg/m}^3) \times (d_p)^2}{18 \times 1.307 \times 10^{-3} \text{ N-s/m}^2} \times \frac{3,600 \text{ s}}{\text{h}} = 2.1 \frac{\text{m}}{\text{h}}$$

$$d_p^2 = 8.48 \times 10^{-10} \text{ m}^2$$

$$d_p = 2.9 \times 10^{-5} \text{ m} = 2.9 \times 10^{-3} \text{ cm}$$

To verify the flow conditions, check the Reynolds number (Equation 8.8) to see if the particle is settling under laminar conditions:

$$\begin{aligned} \text{Re} &= \frac{d_p v_s}{\nu} = \frac{8.48 \times 10^{-10} \text{ m} \times 2.1 \text{ m/h} \times 1 \text{ h}/3,600 \text{ s}}{1.306 \times 10^{-6} \text{ m}^2/\text{s}} \\ &= 3.8 \times 10^{-7} \end{aligned}$$

The Reynolds number is much less than 1. Therefore, laminar flow exists, and Stokes' law is valid.

Readers should note that horizontal settling tanks can be designed and operated without the use of mechanical rakes and other moving parts that require energy, maintenance, and funds to purchase spare parts. They may be more appropriate technology for many applications throughout the world.