

Student Name :	Q.1	Q.2	Q.3	Q.4	Q.5	Q.6	Total
Student Number :							
Instructor :	14	16	16	20	14	20	100

Answer the following questions:

Q.1 (a) Find $\frac{dy}{dx}$ if $y = \csc(x^2 + 1)^3 + 2\sqrt{x} \tan^2 x$.

(b) If $f'(x) = \frac{x}{x^2 + 1}$ and $g(x) = \sqrt{3x - 1}$, then show that $(f \circ g)'(x) = \frac{1}{2x}$.

Q.2 (a) Evaluate $\lim_{x \rightarrow 0} \frac{x^2}{\sec x - 1}$

(b) Use the mean value theorem for $f(x) = \cot x$ on $[\frac{\pi}{4}, \theta]$ to show that $|\cot \theta - 1| \geq |\theta - \frac{\pi}{4}|$.

Q.3 Evaluate the following integrals:

(a) $\int \frac{\sin(2x)}{(3 + 2 \cos^2 x)^3} dx$

(b) $\int_1^2 \sqrt[3]{\frac{2-x}{x^7}} dx$

Q.4 Let $f(x) = \frac{x^2 - x + 1}{x - 1}$.

a) Find the domain of the function f .

b) Find the intervals on which the function f is increasing and decreasing.

c) Identify the function's local extreme values, if any, saying where they are taken on.

d) Find where the graph of f is concave up and where it is concave down and find the inflection points on the graph of f , if any.

e) Find all asymptotes of the graph of f .

f) Find the x-intercept(s) and the y-intercept(s), if any, and sketch the graph of f .

Q.5 (a) Find the area of the surface generated by revolving the curve $x = \sqrt{9 - y^2}$, $-2 \leq y \leq 2$ about the y -axis.

(b) Find the volume of the solid generated by revolving the region bounded by $y = (x - 1)^2$, $y = 0$, and $x = 3$ about the y -axis.

Q.6 Choose the correct answer for each of the following:

- (i) The range of the function $f(x) = \lceil x \rceil$ is the set of all
(a) real numbers $\mathbb{R}$ (b) integers $\mathbb{Z}$
(c) natural numbers $\mathbb{N}$ (d) rational numbers $\mathbb{Q}$
- (ii) The domain of $f(x) = \sqrt{3 - \sqrt{x}}$ is
(a) $[0, 9]$ (b) $(-\infty, 9]$ (c) $[9, \infty)$ (d) $(-\infty, 3]$
- (iii) Let $f(x)$ be a continuous function on $\mathbb{R}$ and assume that f is increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$. Then
(a) f has an absolute maximum value at $x = 0$.
(b) f a local maximum value at $x = 0$.
(c) f does not have an absolute minimum value.
(d) all of the above.
- (vi) If $f(x) = x^2$, then $f(\cos x) - f(\sin x)$ equals
(a) $\cos x - \sin x$ (b) $\cos(2x)$
(c) $\sin(2x)$ (d) 1
- (v) If $\lim_{x \rightarrow c} f(x)$ exists, then $\lim_{x \rightarrow c} \frac{1}{f(x)}$
(a) exists (b) does not exist
(c) may or may not exists (d) equals $\frac{1}{f(c)}$
- (vi) Assume that f is a continuous and decreasing function on $[a, b]$. If $f(a) = 1$ and $f(b) = -1$, then $f(x)$ has
(a) no zero in (a, b) (b) exactly one zero in (a, b)
(c) exactly two zeros in (a, b) (d) more than two zeros in (a, b)
- (vii) If $f(x) = \sec^2 x$ and $g(x) = \tan^2 x$, then
(a) $f(x) - g(x)$ is constant (b) $f'(x) = g'(x)$
(c) $f(x) - g(x)$ is continuous on its domain (d) all of the above
- (viii) $\int [f(x)g'(x) + f'(x)g(x)] dx$ equals
(a) $f(x)g(x) + C$ (b) $f(x) + g(x) + C$
(c) $2f(x)g(x) + C$ (d) none of the above
- (ix) If f is differentiable at $x = 0$, then
(a) $f(0) = 0$ (b) $\lim_{x \rightarrow 0} f(x) = f(0)$
(c) $\lim_{x \rightarrow 0} f(x) = f'(0)$ (d) none of the above
- (x) If for every $\epsilon > 0$ there is a corresponding $\delta > 0$ such that for all x , $0 < |x + 2| < \delta \Rightarrow |f(x) - 5| < \epsilon$, then
(a) $\lim_{x \rightarrow 2} f(x) = 5$ (b) $\lim_{x \rightarrow 2} f(x) = -5$
(c) $\lim_{x \rightarrow -2} f(x) = 5$ (d) $\lim_{x \rightarrow -2} f(x) = -5$