

Answer the following questions:

Q.1 (a) Find $\lim_{x \rightarrow 0} \frac{1 - \cos 4x}{x^2}$.

(b) Show that the function $f(x) = 2x^3 + x^2 + 2$ has a root in the interval $[-2, -1]$.

Q.2 (a) If $y = f(\frac{1}{x})$ and $f'(x) = \frac{1}{x}$, find $\frac{dy}{dx}$.

(b) Find the linearization of the curve $f(x) = 2x \sin x$ at $x = \frac{\pi}{2}$, and use it to approximate $f(\frac{3}{2})$.

(c) Find $\frac{dy}{dx}$ if $x^2y + y^2x = 3x$.

Q.3 Evaluate the following integrals:

(a) $\int \sec^4 x \, dx.$

(b) $\int \frac{3x \cos(\sqrt{x^2 + 1})}{\sqrt{x^2 + 1}} \, dx.$

Q.4 Let $f(x) = \frac{x^2}{x+1}$.

(a) Find the critical points and intervals on which f is increasing and decreasing.

(b) Find the points where f has local maximum and local minimum values.

(c) Find the inflection points of f and the intervals on which the graph of f is concave up and concave down.

(d) Find all asymptotes of the graph of f .

(e) Graph the function.

Q.5 (a) For what values of a and b that the function $f(x) = ax^2 + 2x + b$ has the tangent $y = 4x + 14$ at $x = 1$.

(b) Find the length of the curve $x = \frac{y^3}{6} + \frac{1}{2y}$ from $y = 2$ to $y = 3$.

Q.6 Choose the correct answer for each of the following:

- (1) If $f(x) = x - \sqrt{x+1}$ and $g(x) = \sqrt{x-1}$, then the domain of $\left(\frac{f}{g}\right)$ is
(a) $[1, \infty)$ (b) $(1, \infty)$
(c) $(-1, \infty)$ (d) $[-1, 1]$
- (2) The graph of $f(x) = (x-1)^2 + 3$ is the graph of $g(x) = x^3$ after shifting
(a) one unit to the right and 3 unit down (b) one unit to the left and 3 unit up
(c) one unit to the right and 3 unit up (d) one unit to the left and 3 unit down
- (3) If f and g are differentiable functions, then $(f \cdot g)''$ equals
(a) $f g'' + f'' g$ (b) $f g'' - f'' g$
(c) $f' g'' + f'' g'$ (d) $f g'' + 2 f' g' + f'' g$
- (4) If f is a differentiable function on $[a, b]$, then
(a) f is integrable on $[a, b]$ (b) f has a cusp at $c \in [a, b]$
(c) f must be an even function (d) f must be increasing and decreasing on $[a, b]$
- (5) If f is defined on $[a, b]$, then
(a) f is continuous on $[a, b]$ (b) f is differentiable on $[a, b]$
(c) if $c \in [a, b]$, then $\lim_{x \rightarrow c} f(x)$ exists (d) none of the above
- (6) If $f(x) = \sec^2 x$ and $g(x) = \tan^2 x$, $x \in (-\frac{\pi}{2}, \frac{\pi}{2})$, then
(a) $f'(x) = g'(x)$ (b) $f(x) - g(x)$ is a constant function
(c) $f^2(x) - g^2(x) = f(x) + g(x)$ (d) all of the above
- (7) If $f(x) = \frac{\sin x}{1+x^2}$, then
(a) $\int_{-1}^1 \frac{\sin x}{1+x^2} dx = 2 \int_0^1 \frac{\sin x}{1+x^2} dx$ (b) $\int_{-1}^1 \frac{\sin x}{1+x^2} dx = \int_1^1 \frac{\sin x}{1+x^2} dx$
(c) $\int_{-1}^1 \frac{\sin x}{1+x^2} dx = 2 \int_{-1}^0 \frac{\sin x}{1+x^2} dx$ (d) all of the above are true
- (8) Let $F(x) = f(g(x))$ $f'(-2) = 4$ $g'(5) = 6$, then $F'(5) =$
(a) 24 (b) 8 (c) 12 (d) 20
- (9) If $\lim_{x \rightarrow \infty} \frac{x+2}{9x^2+1} =$, then there is a number $c \in (0, 1)$ such that
(a) 0 (b) $\frac{1}{9}$ (c) $\frac{2}{9}$ (d) ∞
- (10) For an interior point c , if $f'(c)$ does not exist, then at $x = c$, f has
(a) a critical point (b) a local extreme value
(c) an inflection point (d) none of the above

Q.7 (a) Find the area between the curves $y = x$ and $y = x^2$ from $x = 0$ to $x = 2$.

(b) Find the volume of the solid generated by revolving the region bounded by the curves $x = 2\sqrt{y}$, y -axis, and $y = 9$ about $x = -1$.