

Chapter 1

PHYSICS AND MEASUREMENTS

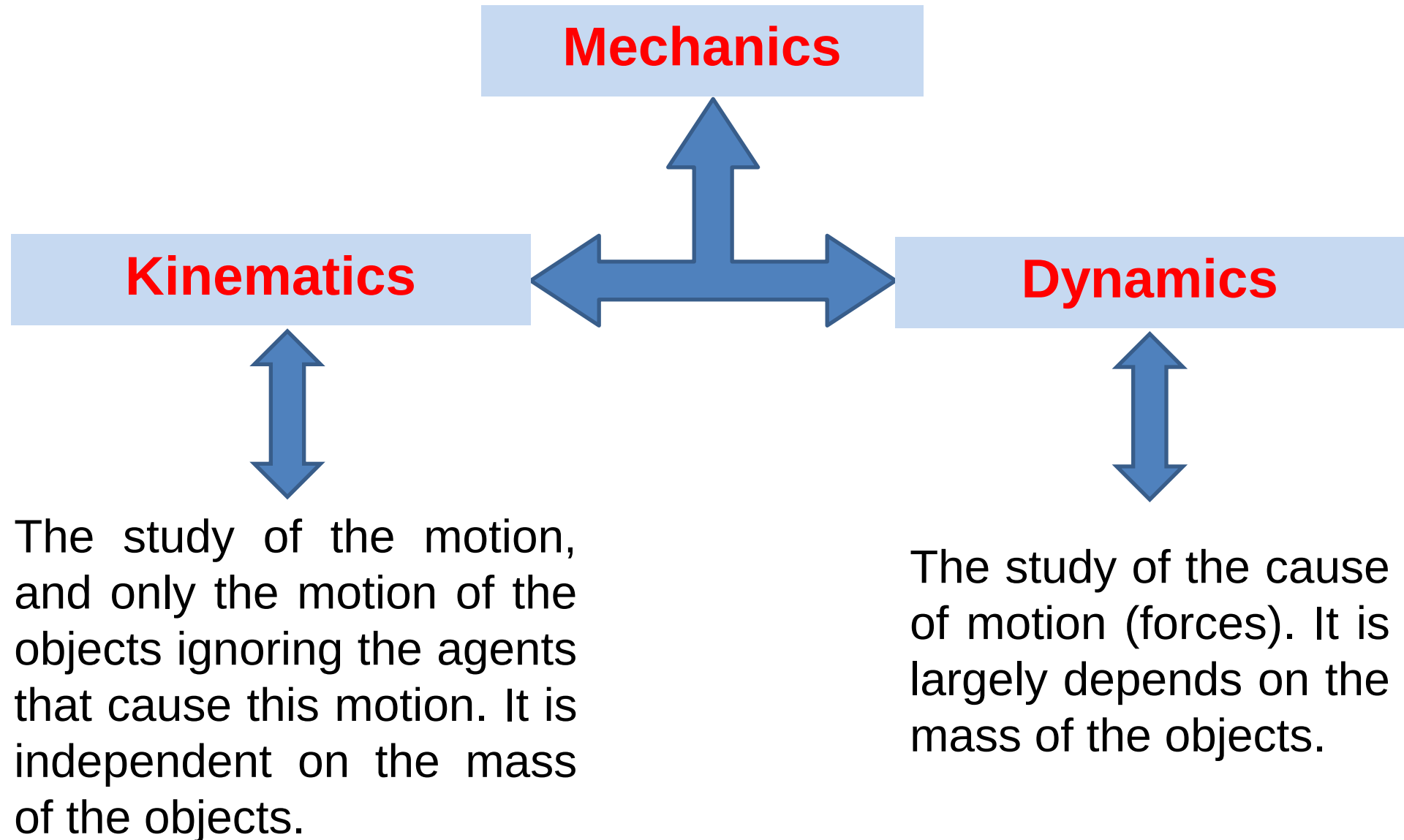
In this chapter we are going to discuss the following topics:

- ❑ Standards of Length, Mass, and Time**
- ❑ Dimensional Analysis**
- ❑ Conversion of Units**
- ❑ Significant Figures**

Recommended Problems:

1, 3, 7, 13, 21, 35, 37, 38, 41, 49, 51, 53, 65

The main theme of this course is Mechanics. What is mechanics?



1.1 Standards of Length, Mass, and Time

Any physical quantity must have, besides its numerical value, a standard unit. By standard unit, we mean a unit well defined among every one.

In 1960, an international committee established a set of standards for the fundamental quantities of science. It is called the **SI** (Système International). Its standard units are:

Meter(m) for length

Ampere(A) for current

Kilogram(kg) for mass

Kelvin(K) for temperature

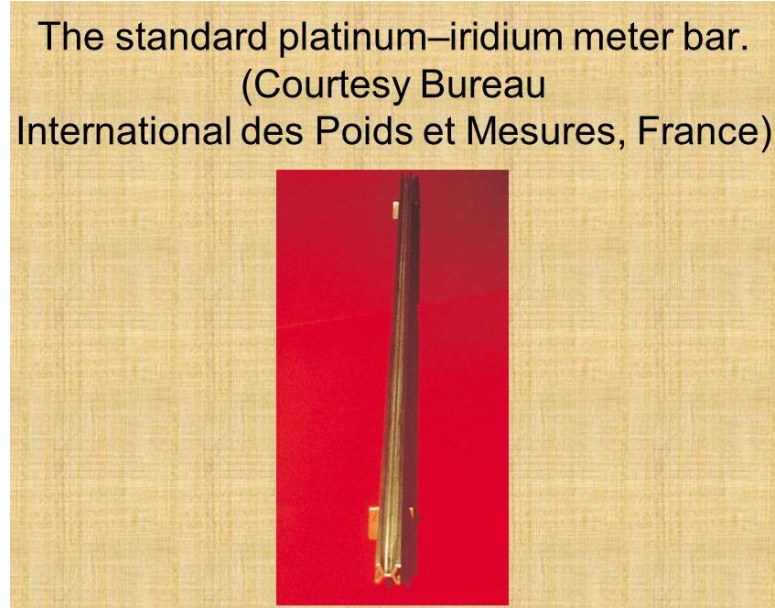
Second(s) for time

Mole(M) for substance

In mechanics the three fundamental quantities are: **length**, **mass**, and **time**. All other quantities are derived from these three quantities. Because of that, the SI system sometimes called the ***mks*** system.

Meter (m)

In 1960, the length of the meter was defined as the distance between two lines on a specific platinum–iridium bar stored under controlled conditions in France.



In 1983, however, the meter was redefined as the distance traveled by light in vacuum during a time of $1/299\,792\,458$ second. $299\,792\,458$ m/s is the speed of light in vacuum.

Kilogram (kg)

In 1887 the mass of kilogram is defined as the mass of a specific platinum–iridium alloy cylinder kept at the International Bureau of Weights and Measures at Sèvres, France.



Second (s)

Before 1967, the fundamental unit of a **second** was defined as $\left(\frac{1}{24} \frac{1}{60} \frac{1}{60}\right)$ of a mean solar day. A solar day is the time interval between successive appearances of the Sun at the highest point it reaches in the sky each day.

In 1967, One second is now defined as 9 192 631 770 times the period of vibration of radiation from the cesium-133 atom measured by the atomic clock.

There are two other unit systems used in physics:

The Gaussian (cgs) system with the standards centimeter (cm) for length, gram (g) for mass, and second (s) for time.

The British Engineering system with the standards foot (ft) for length, slug for mass, and second (s) for time.

1.2 DIMENSIONAL ANALYSIS

Dimension in physics denotes the physical nature of the quantity, whether it is a length (L), mass (M), or time (T).

We shall often use brackets [] to denote the dimensions of a physical quantity.

The dimension and units of some physical quantities are shown below.

Quantity	Dimension	Unit (mks, cgs, british)
Area [A]	L^2	m^2 , cm^2 , ft^2
Volume [V]	L^3	m^3 , cm^3 , ft^3
Speed [v]	L/T	m/s , cm/s , ft/s
Acceleration [a]	L/T^2	m/s^2 , cm/s^2 , ft/s^2
Force [F]	ML/T^2	Newton, dyne, lb
Energy [E]	ML^2/T^2	Joule, erg, lb.ft

- ❖ Quantities can be added or subtracted only if they have the same dimensions.
- ❖ The terms on both sides of an equation must have the same dimensions.

Example 1.1 Show that the equation $v = at$ is dimensionally correct.

Solution Since $[v] = L/T$ $[a] = L/T^2$ $[t] = T$


$$\frac{L}{T} = \frac{L}{T^2} T = \frac{L}{T}$$

Since the dimensions is the same on both sides then the equation is correct.

Example 1.2 If the acceleration of a particle moving with constant speed around a circle is given by $a = kr^n v^m$ with k is a dimensionless constant. Determine the values of n and m for the formula to be correct.

Solution Since $[r] = L$ $[v] = L/T$ $[a] = L/T^2$

Substituting for the dimensions in the equation we get

$$\frac{L}{T^2} = L^n \frac{L^m}{T^m} = \frac{L^{n+m}}{T^m}$$

For the dimensional equation to be balanced the exponent of L and T must be equal on both sides of the equation, i.e.,

$$n + m = 1, \quad \text{and} \quad m = 2 \Rightarrow n = -1$$

The equation then becomes $v = k \frac{v^2}{r}$

• Quick Quiz (1)

The position of a particle is given by $x=at+bt^2$, with x in (m) and t in (s). The dimension of the constant a is:

a) L

c) L/T²

b) L/T

d) has no dimension

• Quick Quiz (2)

Referring to the previous question, the dimension of the constant b is:

a) L

c) L/T²

b) L/T

d) has no dimension

1.3 Conversion of Units

If different unit systems are used in a physical equation, one system should be chosen and all the quantities must be converted to the chosen system

For example suppose that we wish to convert 15.0 in into centimeters. Since 1 in = 2.54 cm we can write

$$1\text{ in} = 2.54\text{ cm} \Rightarrow 1 = 2.54 \frac{\text{cm}}{\text{in}}$$

Since multiplying any quantity by one leaves it unchanged we write

$$15\text{ in} \times 1 = (15\text{ in}) \times \left(2.54 \frac{\text{cm}}{\text{in}} \right) = 38.1\text{ cm}$$

Example 1.3 On an interstate highway, a car is traveling at a speed of 38.0 m/s. Is the driver exceeding the speed limit of 75.0 mi/h?

Solution Since $1m = \frac{1}{1609}mi \Rightarrow 1 = \frac{1}{1609} \frac{mi}{m}$

and $1s = \frac{1}{3600}hr \Rightarrow 1 = \frac{1}{3600} \frac{hr}{s} \Rightarrow$

$$38.0 \frac{m}{s} = \left(38.0 \frac{m}{s} \right) \times \left(\frac{1}{1609} \frac{mi}{m} \right) \div \left(\frac{1}{3600} \frac{hr}{s} \right) \quad \text{or}$$

$$38.0 \frac{m}{s} = \left(38.0 \frac{m}{s} \right) \times \left(\frac{1}{1609} \frac{mi}{m} \right) \times \left(3600 \frac{s}{hr} \right) = 85.0 \text{ mi/hr}$$

The driver is indeed exceeding the speed limit and he should slow down.

1.4 SIGNIFICANT FIGURES

In science, any measurement should not be more accurate than what is allowed by the equipment used to measure that quantity.

For example when you measure a distance by a meter stick, your measurement should be written as 12.3 cm. This means that the measurement is accurate with an uncertainty of ± 0.1 cm which is the smallest division in the meter stick.

The number of significant figures in a measurement can be defined as the number of digits that are measured with certainty plus the last digit which is an estimate.

The measurement 12.3 cm has 3-significant figures: 1, and 2, are certain, and 3 is uncertain.

To determine the number of significant figures for a measurement we use the following rules:

1- Non zero digits are always significant. Thus 14 has two significant figures while 136 has three significant figures.

2- Zeros between nonzero digits are significant, for example 4036 has four significant figures and 1.06 has three significant figures.

3- Zeros placed to the left of nonzero digit are not significant; 0.02 has only one significant figure.

4- Zeros at the end of a number and to the right of a decimal are significant, for example 16.0 has three significant figures and 20.00 has four significant figures.

5- Zero at the end of a number without a decimal point is not significant. Thus 4200. has four significant figures while 4200 has only two significant figures.

6- All digits in a number in scientific notation are significant, for example 4.2×10^3 has 2-significant figures, while 4.200×10^3 has 4-significant figures

Scientific notation: a number between 1 and 10 multiplied by 10 raised to some exponent. The quantity 4200 is written in scientific notation as 4.2×10^3 .

The uncertainty in the measurement 4.2×10^3 is ± 100 , while in the measurement 4.200×10^3 is ± 1 .

When multiplying or dividing several quantities, the number of significant figures in the final answer is the same as the number of significant figures in the quantity having the smallest number of significant figures.

For example, the area of a disc with radius measured to be 6.0 cm is

$$A = \pi r^2 = \pi (6.0)^2 = 1.1 \times 10^2 \text{ cm}^2$$

The calculator gives you 113.097----, so you shouldn't report it as 113 or 113.1 but as reported above.

When numbers are added or subtracted, the number of decimal places in the result should equal the smallest number of decimal places of any term in the added or subtracted numbers.

For example $145.5 + 6.84$ equal to 152.3 and not 152.34 nor 152.

Another examples are: $1.0001 + 0.0003 = 1.0004$
and $1.002 - 0.988 = 0.004$

In the first the answer has five significant figures even though one of the numbers has only one significant figure. In the second, the answer has only one significant figure.

Example 1.5 Find the area of a room whose length is measured to be 12.71 m and whose width is measured to be 3.46 m.

Solution $A = 12.71 \times 3.46 = 43.9766 \text{ m}^2$

But since the lowest significant figures in the multiplied quantity is three we must report the result as 44.0 m^2

• Quick Quiz (3)

How many significant figures are in the number 0.003?

- a) 1 b) 2 c) 3 d) 4

• Quick Quiz (4)

The result of the arithmetic operation 2.34 multiplied by 2.0 is:

- a) 4 b) 4.7 c) 4.68 d) 4.6

• Quick Quiz (5)

The result of the arithmetic operation $6.467 + 9.4$ is:

- a) 15.867 b) 15.8 c) 16 d) 15.9