

Chapter 2

Motion in One Dimension

In this chapter we are going to discuss the following topics:

- ☐ **Position, Velocity, and Speed**
- ☐ **Acceleration**
- ☐ **Motion with Constant Acceleration**
- ☐ **Free Falling Objects**

Recommended Problems:

1, 3, 7, 13, 21, 35, 37, 38, 41, 49, 51, 53, 65

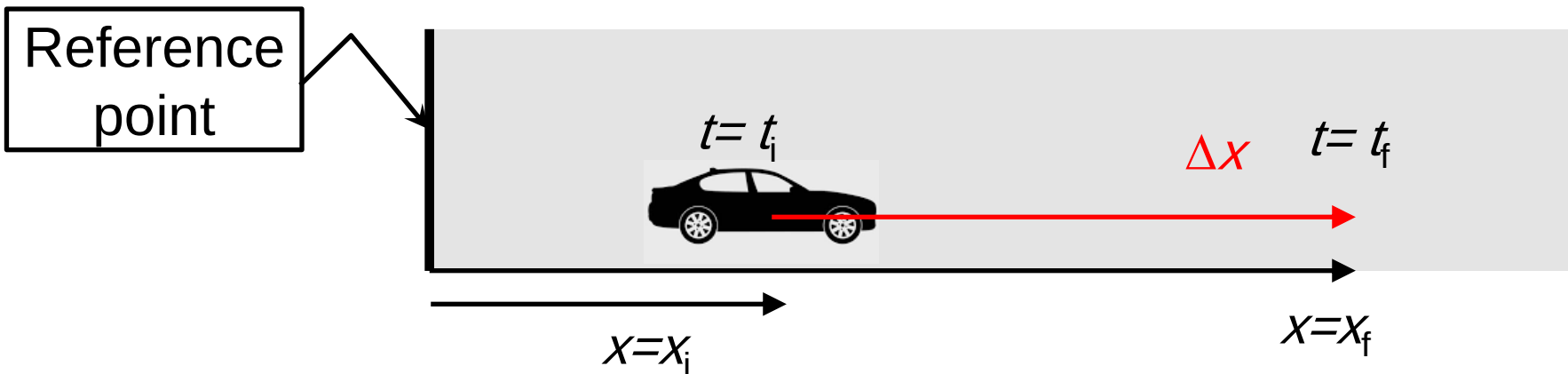
2.1 Position, Velocity, and Speed

The position of a particle x is its location with respect to a reference point

As a particle moves from an initial position x_i to a final position x_f , its displacement is defined as its change in position in some time interval.

So the **displacement** Δx of a particle is given by

$$\Delta x = x_f - x_i$$

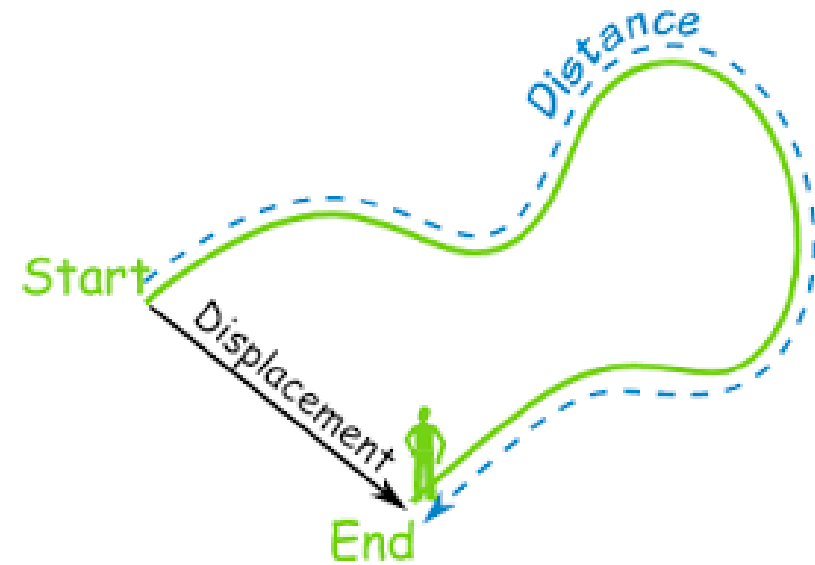


Remark: There is a difference between distance and displacement.

Distance, a scalar quantity, is the actual length of the path followed by a particle, but **displacement**, a vector quantity, is the shortest distance between the initial and the final positions of the particle. We shall treat vector quantities in detail in chapter 3.

In this chapter, we use positive (+) and negative (-) signs to indicate vector direction.

For horizontal motion we specify to the right as being the positive direction, while for vertical motion we specify the upward as being the positive direction.



Average Velocity

The average velocity of a particle is defined as

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

The average velocity depends only on the initial and the final positions of the particle.

This means that if a particle starts from a point and returns back to the same point, its displacement, and so its average velocity is zero.

The average speed differs from average velocity in that it covers the total distance rather than the total displacement, that is

$$\text{average speed} = \frac{\text{total distance}}{\Delta t}$$

The SI unit of velocity and speed is m/s.

❑ **Remark** Even though the speed is the magnitude of the velocity, the average speed is not the magnitude of the average velocity.

• Quick Quiz (1)

Under which of the following conditions is the magnitude of the average velocity of a particle moving in one dimension smaller than the average speed over some time interval?

- a) A particle moves in the $+x$ direction without reversing.
- b) A particle moves in the $-x$ direction without reversing.
- c) A particle moves in the $+x$ direction and then reverses the direction of its motion.
- d) Under no condition.

Example 2.1 The figure shown represents the variation of position with time for a car. Find the displacement, the average velocity, and the average speed of the car between positions A and F .

Solution It is clear from the figure that

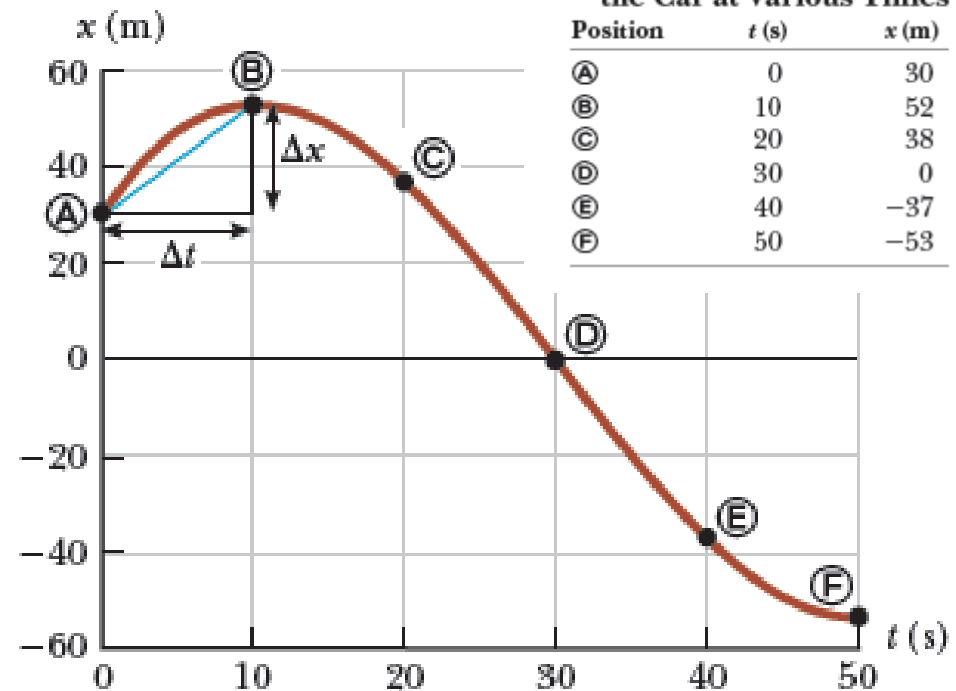
$$t_A = 0, \text{ and } x_A = 30\text{m}$$

$$t_F = 50\text{s}, \text{ and } x_F = -53\text{m}$$

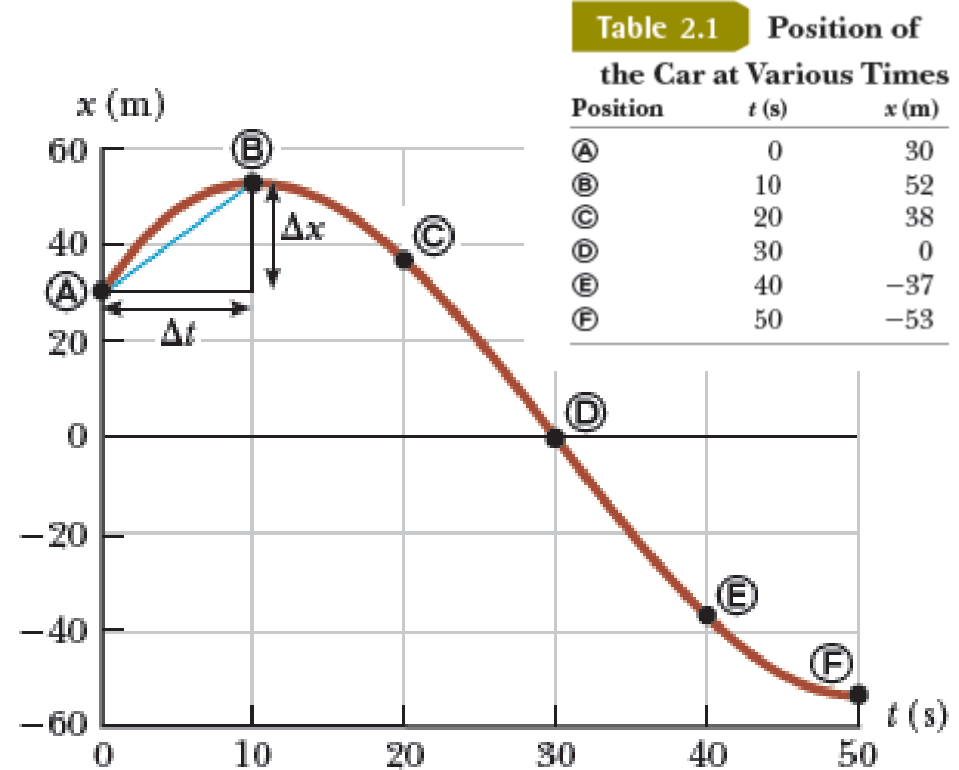
For the displacement we have $\Delta x = x_F - x_A = -53 - 30 = -83\text{m}$

For the average velocity we have

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_F - x_A}{t_F - t_A} = \frac{-83}{50 - 0} = -1.7\text{m/s}$$



For the average speed we want the total distance covered by the car which is the sum of the distance from A to B (22m) and the distance from B to F (105m), i.e., the total distance $d=127\text{m}$. $\Rightarrow$



$$\text{average speed} = \frac{\text{total distance}}{\Delta t} = \frac{127}{50} = 2.5 \text{ m/s}$$

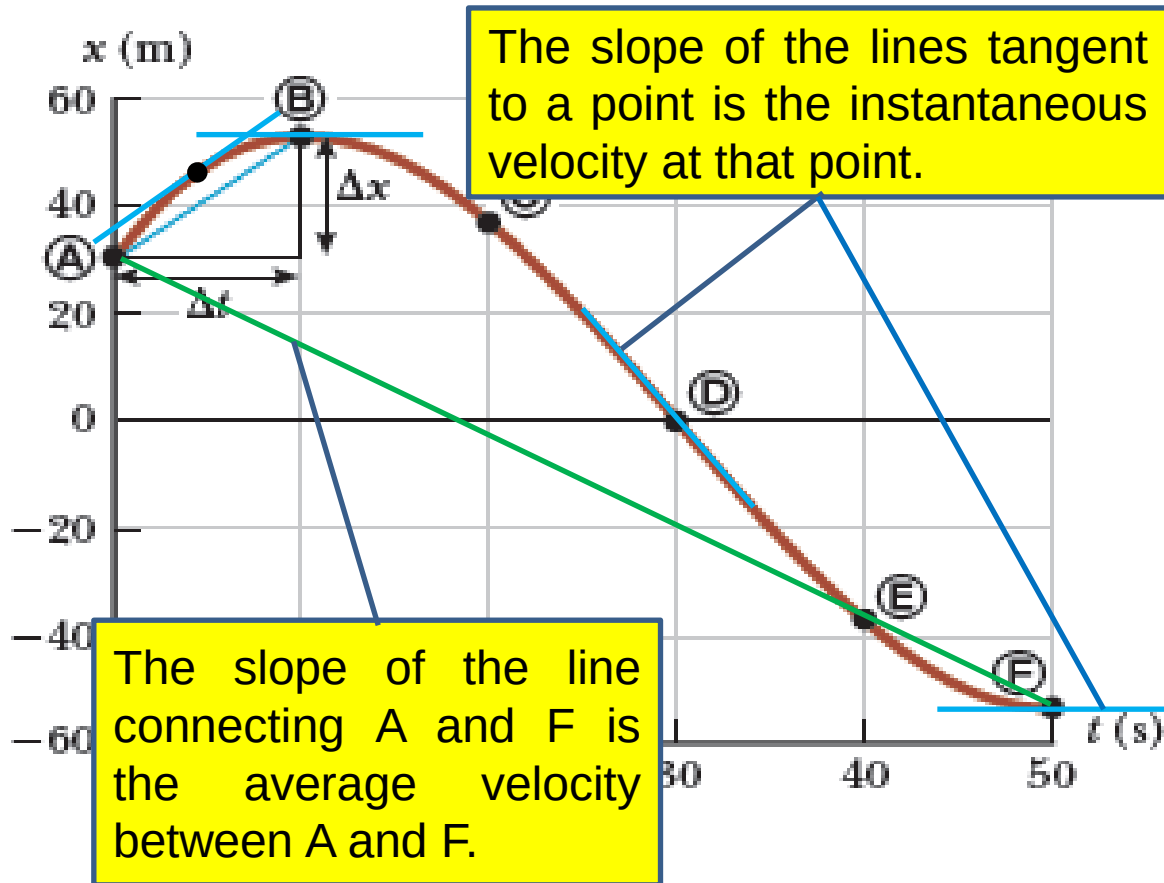
2.1 Instantaneous Velocity and Speed

The instantaneous velocity of a particle is defined as

become instant $v = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t}$ $\rightarrow$ $v = \frac{dx}{dt}$

In the position-time graph the **average velocity** is the slope of the line between the initial and the final points while the **instantaneous velocity** at some instant is the slope of the tangent at that instant.

The instantaneous **speed** is the magnitude of the instantaneous velocity.



Example 2.3 A particle moves along the x -axis. Its position varies with time according to $x = -4t + 2t^2$ with x in m and t in s.

- Find the displacement between $t=0$ to $t=1$ s and between $t=1$ s to $t=3$ s.
- Calculate the average velocity during these two time intervals.
- Calculate the instantaneous velocity at $t=2.5$ s

Solution a) to find x_i and x_f , we have to substitute for t_i and t_f in the x - t relation, that is, for the interval $t_i = 0$, to $t_f = 1$ s $\Rightarrow$

$$x_i = -4(0) + 2(0)^2 = 0 \quad \text{and} \quad x_f = -4(1) + 2(1)^2 = -2\text{m}$$

$$\Rightarrow \quad \Delta x = x_f - x_i = -2 - 0 = -2\text{m}$$

for the interval $t_i = 1$ s, to $t_f = 3$ s $\Rightarrow$

$$x_i = -4(1) + 2(1)^2 = -2\text{m} \quad \text{and} \quad x_f = -4(3) + 2(3)^2 = 6\text{m}$$

$$\Rightarrow \quad \Delta x = x_f - x_i = 6 + 2 = 8\text{m}$$

b) Now for the average velocity in the first interval

$$\bar{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{-2}{1-0} = -2 \text{ m/s}$$

And the in the second interval

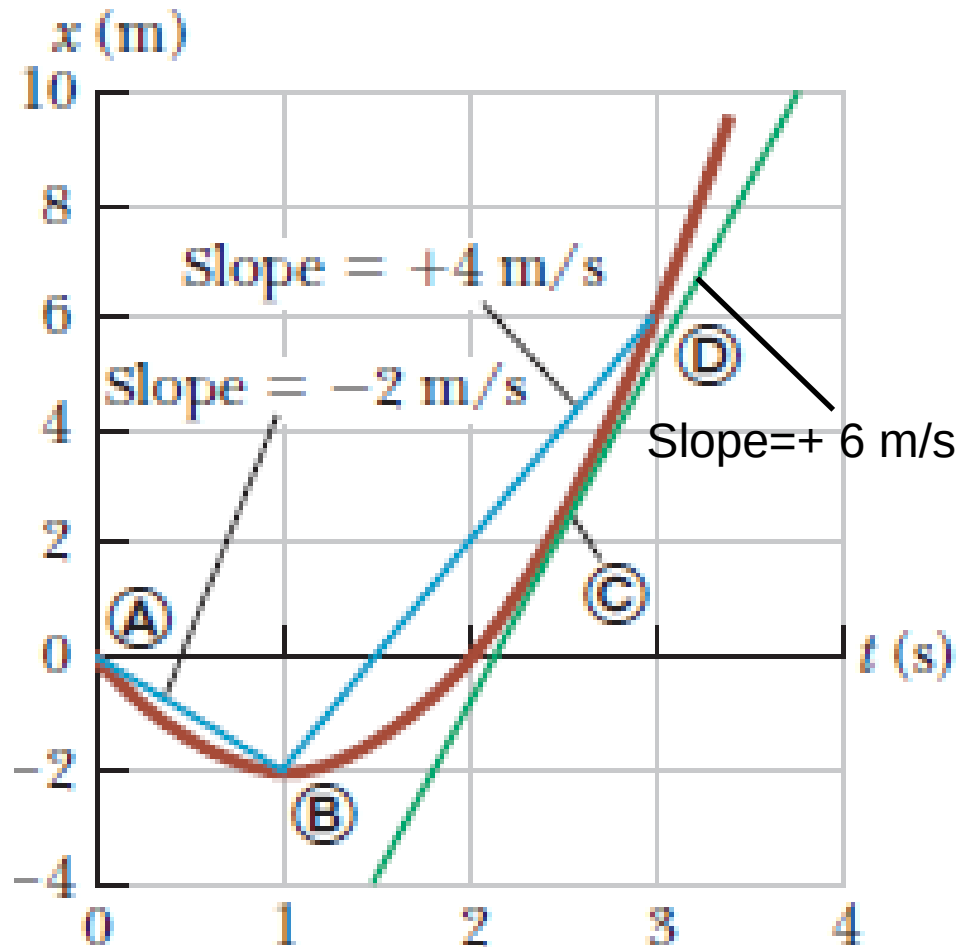
$$\bar{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{8}{3-1} = 4 \text{ m/s}$$

c) To find the instantaneous speed at $t=2.5$ s we have

$$v = \frac{dx}{dt} = \frac{d(-4t + 2t^2)}{dt} = -4 + 4t$$

Substituting for $t=2.5$ s we get

$$v = -4 + 4(2.5) = 6 \text{ m/s}$$



• Quick Quiz (2)

Consider the following one dimensional motion:

- (i) a ball thrown directly upward rises to a highest point and falls back into the thrower's hand;
- (ii) a race car starts from rest and speeds up to 100 m/s; and
- (iii) a spacecraft drifts through space at constant velocity.

In which of these three motions there are any points at which the instantaneous velocity has the same value as the average velocity over the entire motion?

- a) In (i) b) In (ii) c) In (iii) d) In all of them

2.3 Acceleration

When the velocity of a particle changes with time, the particle is said to be *accelerating*, (has an acceleration).

The average acceleration of a particle is defined as

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

The SI unit of velocity is m/s².

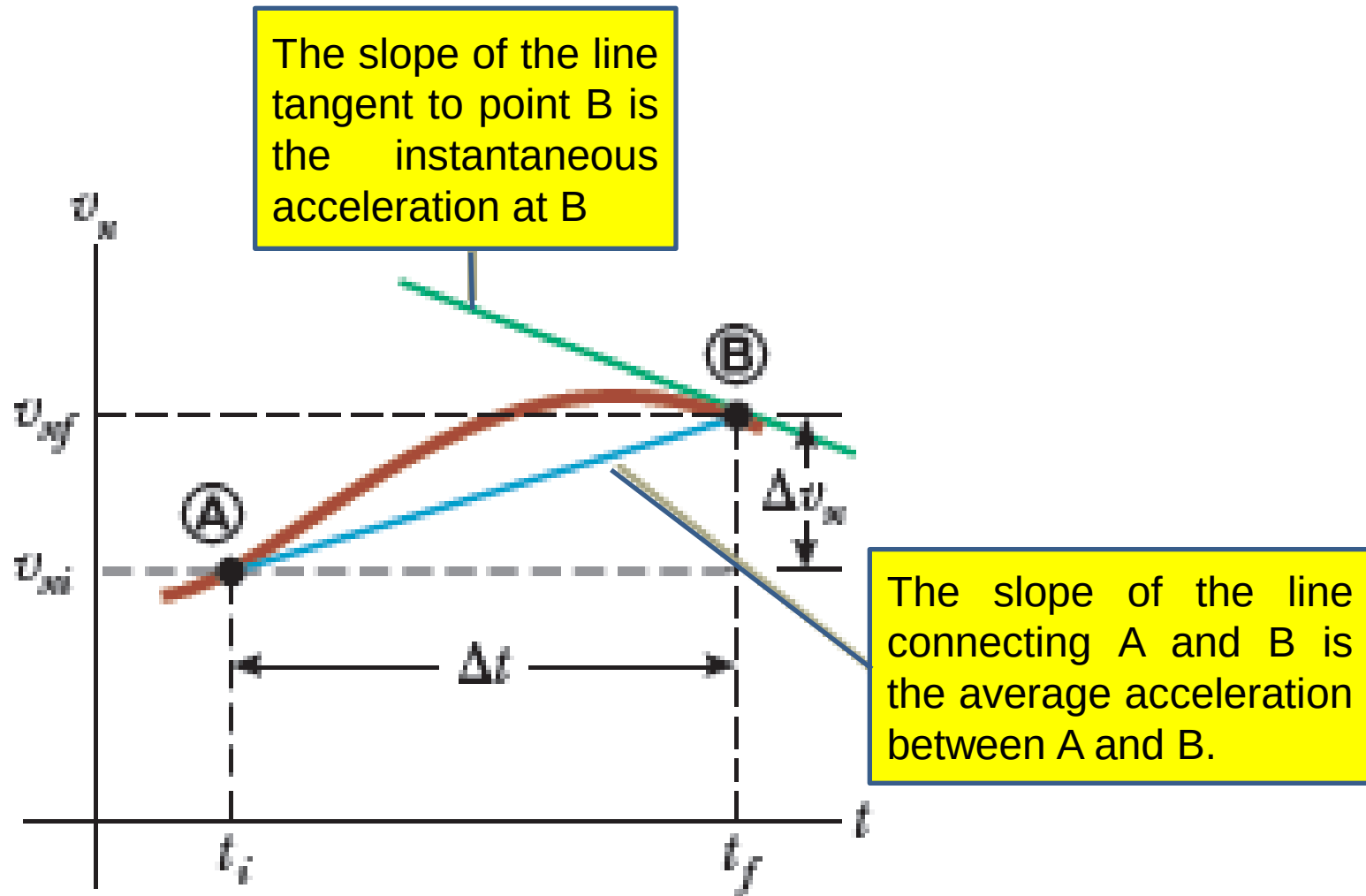
The instantaneous acceleration of a particle is defined as



become instant

$$a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} \longrightarrow a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

In the velocity–time graph, the **average acceleration** is the slope of the line connecting the initial and the final points while the **instantaneous acceleration** at any time is the slope of the tangent at that time.



- ❖ The acceleration is positive when the velocity is increasing in the positive x direction (speeding up).
- ❖ The acceleration is negative when the velocity is decreasing in the positive x direction (slowing down).

• Quick Quiz (3)

If a car is traveling eastward and slowing down, what is the direction of the acceleration of the car?

- a) eastward
- b) westward
- c) neither eastward nor westward
- d) eastward then westward

• Quick Quiz (4)

Which of the following statements is true?

- a) If a car is traveling eastward, its acceleration must be eastward.
- b) If a car is slowing down, its acceleration must be negative.
- c) A particle with constant acceleration can never stop.
- d) A particle with constant acceleration can never stop and stay stopped.

Example 2.6 The velocity of a particle moving along the x axis varies with time according to the expression $v=40-5t^2$ with v in m/s and t in s.

a) Find the average acceleration for the interval $t=0$ to $t=2.0$ s

b) Determine the acceleration at $t=2.0$ s

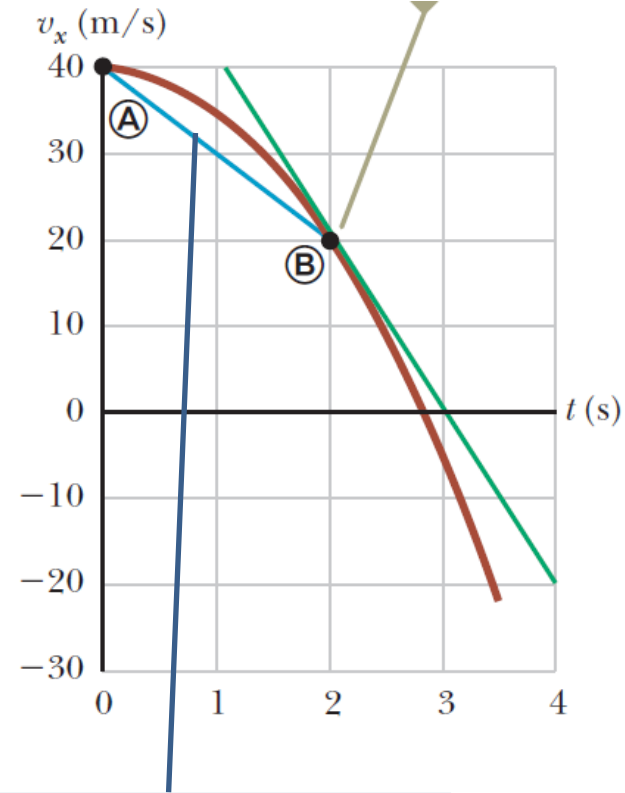
Solution a) to find v_i and v_f , we have to substitute for t_i and t_f in the v - t relation, that is,

$$v_i = 40 - 5(0)^2 = 40 \text{ m/s}$$

$$\text{and } v_f = 40 - 5(2)^2 = 20 \text{ m/s}$$

Now we have for the average acceleration

$$\bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{20 - 40}{2 - 0} = -10 \text{ m/s}^2$$



The slope of this line is the average acceleration between A and B.

b) And for the instantaneous acceleration we have

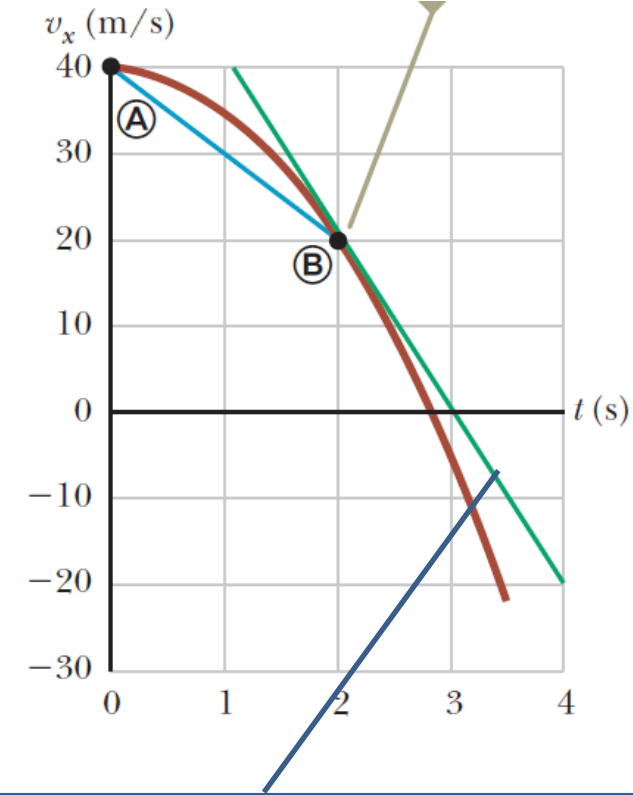
$$a = \frac{dv}{dt} = \frac{d(40 - 5t^2)}{dt}$$

$$\Rightarrow a = \frac{dv}{dt} = -10t$$

Which gives the acceleration at any time.

To find the acceleration at $t=2.0$ s we substitute for $t=2.0$ s in the last expression to get

$$a = -20 \text{ m/s}^2$$



The slope of this line is the instantaneous acceleration at $t=2.0$ s.

2.6 Particle Under Constant Acceleration

The simplest type of one-dimensional motion is that in which the acceleration is constant. In such case $\bar{a} = a \Rightarrow$

$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{v_f - v_i}{t}$$

Where we set $t_i = 0$. The above equation now is

$$v_f = v_i + at$$

Also, because a is constant, we can write

$$\bar{v} = \frac{v_f + v_i}{2} \Rightarrow \bar{v} = \frac{x_f - x_i}{t} = \frac{v_f + v_i}{2} = \frac{v_i + at + v_i}{2} \Rightarrow$$

$$x_f - x_i = v_i t + \frac{1}{2} at^2$$

$$\text{Also } \bar{v} = \frac{x_f - x_i}{t} = \frac{v_f + v_i}{2} \Rightarrow$$

$$x_f - x_i = \frac{1}{2}(v_f + v_i)t$$

Substituting for t from the 1st equation into the 2nd one we get

$$v_f = v_i + at \quad \Rightarrow \quad t = \frac{v_f - v_i}{a} \Rightarrow$$

$$x_f - x_i = v_i \left(\frac{v_f - v_i}{a} \right) + \frac{1}{2} a \left(\frac{v_f - v_i}{a} \right)^2$$

$$x_f - x_i = \frac{v_f^2 - v_i^2}{2a} \Rightarrow$$

$$v_f^2 = v_i^2 + 2a(x_f - x_i)$$

To summarize, for motion with constant acceleration we have

$$v_f = v_i + at$$

$$x_f - x_i = v_i t + \frac{1}{2} at^2$$

$$x_f - x_i = \frac{1}{2} (v_f + v_i) t$$

$$v_f^2 = v_i^2 + 2a(x_f - x_i)$$

Example 2.7 A jet lands on an aircraft carrier at a speed of 140 mi/h (≈ 63 m/s).

- a) What is its acceleration (assumed constant) if it stops in 2.0 s?
- b) If the jet touches down at position $x_i=0$, what is its final position?

Solution a) The initial point is the point at which the jet hits the ground. At this point $v_i=63$ m/s while $v_f=0$. Now using

$$v_f = v_i + at \quad \Rightarrow \quad a = \frac{v_f - v_i}{t} = \frac{0 - 63}{2.0} = -32 \text{ m/s}^2$$

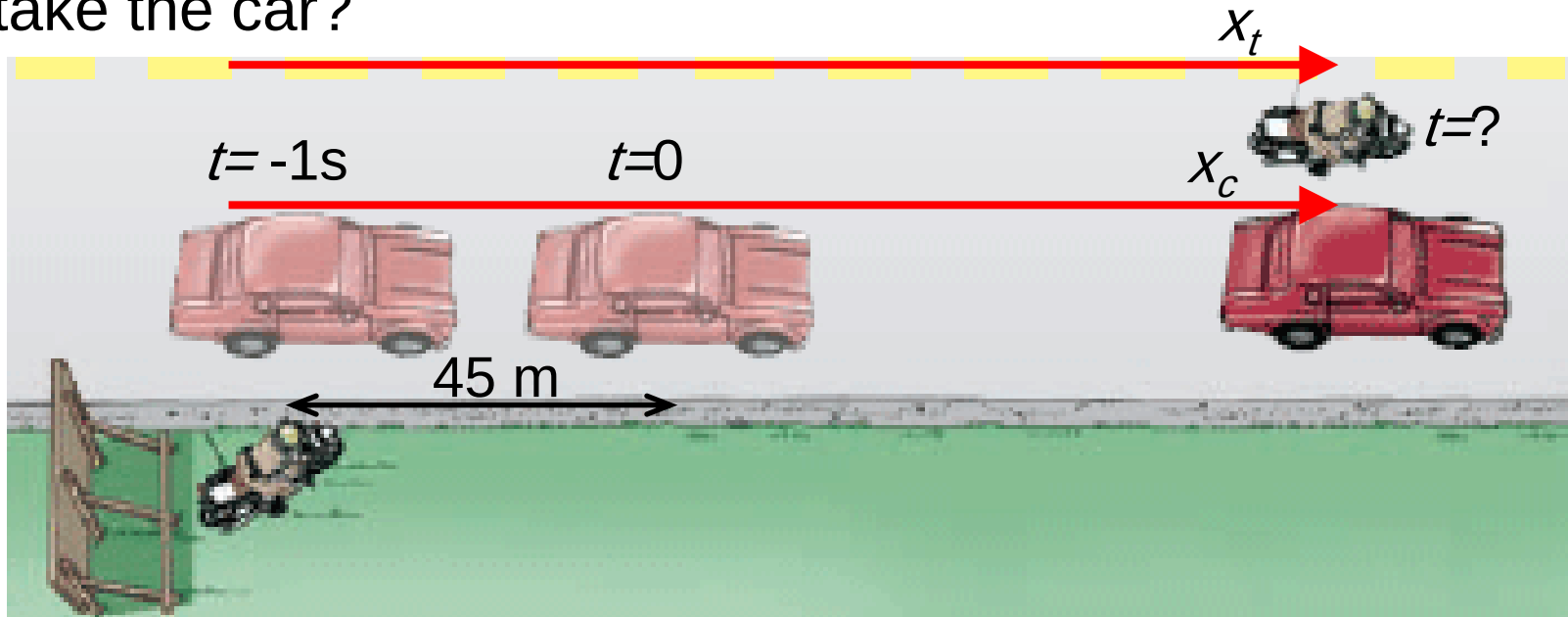
b) Using the formula $x_f - x_i = +\frac{1}{2}(v_i + v_f)t \Rightarrow$

$$x_f - 0 = +\frac{1}{2}(63 + 0)2 \Rightarrow x_f = 63 \text{ m}$$

Note that we can also use the formula $x_f - x_i = v_i t + \frac{1}{2}at^2 \Rightarrow$

$$x_f = 63(2.0) + \frac{1}{2}\left(-\frac{63}{2}\right)(2.0)^2 = 63 \text{ m}$$

Example 2.8 A car traveling at a constant speed of 45.0 m/s passes a trooper on a motorcycle hidden behind a billboard. One second after the speeding car passes the billboard, the trooper sets out from the billboard to catch the car, accelerating at a constant rate of 3.00 m/s². How long does it take the trooper to overtake the car?



Solution Let us choose the position of the billboard as the origin and to set $t=0$ as the time the trooper begins moving. For the trooper to catch the car his position should be equal to the position of the car, i.e., $x_f^c = x_f^t$

At this instant the car is travelled for 1s, so its initial position is

$$x_i^c = v_c t = 45.0\text{m}$$

The position of the car as the trooper overtake it is now can be obtained from

$$x_f^c = x_i^c + v_c t = 45.0 + 45.0t$$

The position of the trooper can be obtained from

$$x_f^t = x_i^t + v_i^t t + \frac{1}{2} a t^2 = 1.50t^2$$

Equating the 2-equations we get

$$x_f^c = x_f^t \quad \Rightarrow \quad 45.0 + 45.0t = 1.50t^2$$

Solving for t we get $t=31.0\text{s}$

Substituting for $t=31.0\text{s}$ to find the positions of the car and the trooper we get

$$x_f^c = 45 + 45(31.0) = 1.44 \times 10^3 \text{m}$$

$$x_f^t = 1.5(31.5)^2 = 1.44 \times 10^3 \text{m}$$

2.7 Free Falling Objects

Any object moving freely under the influence of gravity is said to be in free falling motion.

Assuming that the gravitational acceleration, denoted by g , is constant, we can consider the motion of a free falling body as a linear motion with constant acceleration.

$$g = 9.8 \text{ m/s}^2 = 32 \text{ ft/s}^2$$

Since the motion is vertical we take our axis to be the y -axis with the +ve sense is upward.

So we replace x by y and the acceleration a by $-g \Rightarrow$

The $-ve$ sign is because the gravitational acceleration is always downward. $\Rightarrow$

$$v_f = v_i - gt$$

$$y_f - y_i = v_i t - \frac{1}{2} gt^2$$

$$y_f - y_i = \frac{1}{2} (v_f + v_i) t$$

$$v_f^2 = v_i^2 - 2g(y_f - y_i)$$

Remark: Remember that the gravitational acceleration, g , is constant in magnitude and in direction and this means that the negative sign of g in the last equations will not be changed unless you change your positive sense, regardless of the direction of motion.

• Test Your Understanding (5)

A ball is thrown upward in free falling motion. Its acceleration:

- a) increases.
- b) decreases
- c) decreases and then increases
- d) Stays constant.

• Test Your Understanding (6)

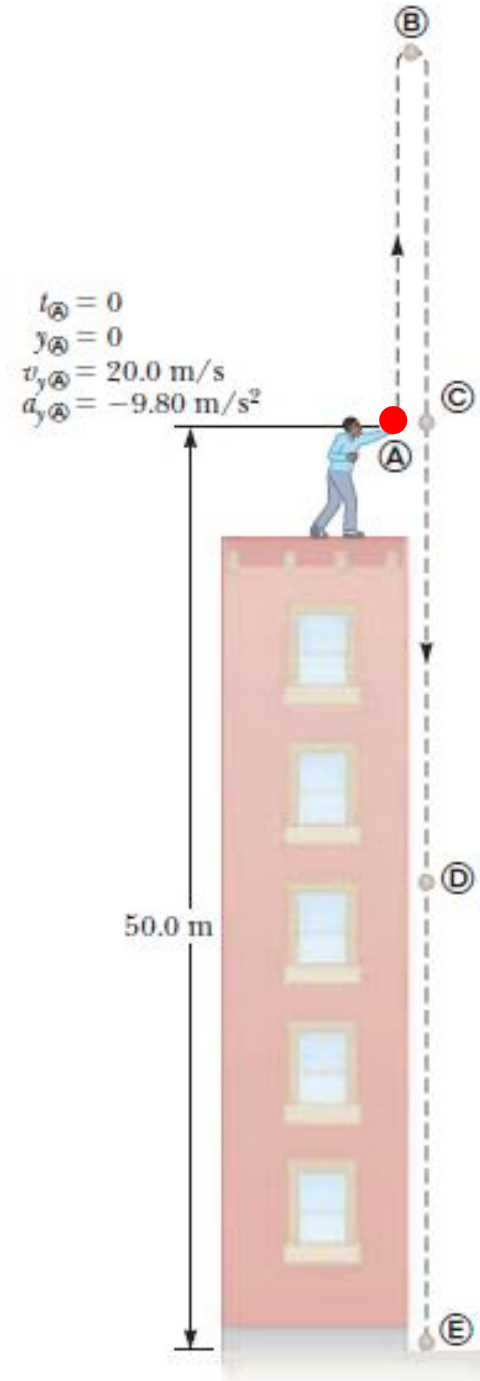
A ball is thrown upward in free falling motion. Its velocity:

- a) increases.
- b) decreases
- c) decreases and then increases
- d) Stays constant.

Example 2.10 A stone thrown from the top of a building with an initial velocity of 20.0 m/s straight upward. The stone is launched 50.0 m above the ground, and the stone just misses the edge of the roof on its way down as shown in the Figure.

- Determine the time at which the stone reaches its maximum height.
- Find the maximum height?
- Determine the velocity of the stone when it returns to the height from which it was thrown.
- Find the velocity and the position of the stone at $t = 5.00\text{s}$.
- Find the velocity of the stone just before hitting the ground.

Solution a) Choose point A as initial point and point B as final point (at maximum height) with $v_f = 0$. Using the equation



$$v_f = v_i - gt \quad \Rightarrow \quad t = \frac{v_i - v_f}{g} = \frac{20.0 - 0}{9.8} = 2.04 \text{ s}$$

b) The maximum high is the vertical distance y between points A and B. Using

$$v_f^2 = v_i^2 - 2g(y_f - y_i) \Rightarrow$$

$$y_f = \frac{v_i^2 - v_f^2}{2g} \Rightarrow h = \frac{(20.0)^2 - 0}{2(9.80)} = 20.4 \text{ m}$$

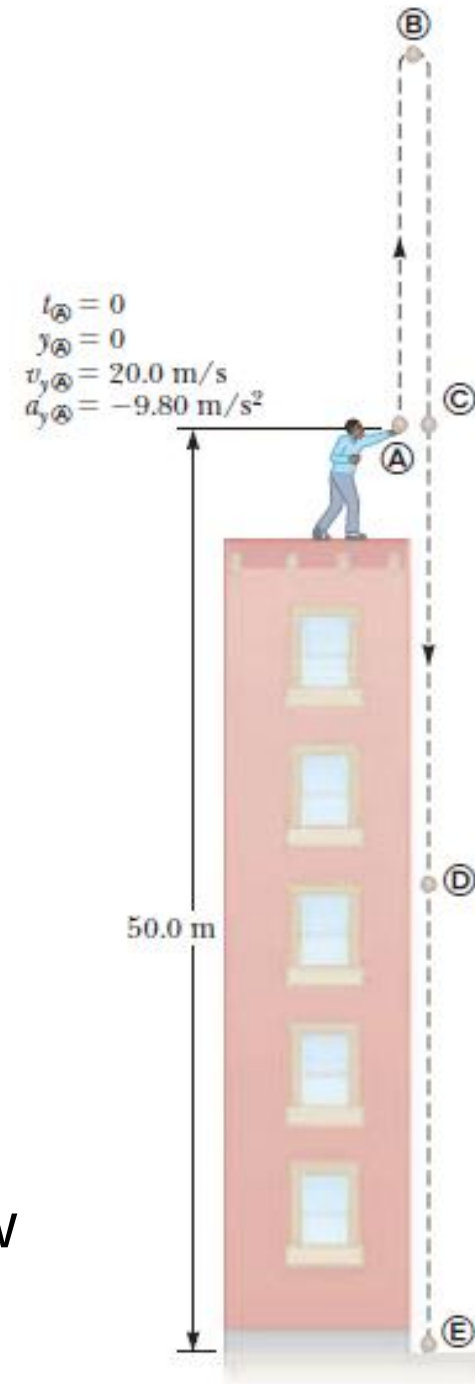
c) Now choose the final point at point C with

$$y_f = y_i = 0 \quad \Rightarrow$$

$$v_f^2 = v_i^2 - 2g(y_f - y_i) = v_i^2 - 2g(0) \Rightarrow$$

$$v_f = -v_i = -20.0 \text{ m/s}$$

We choose the negative root because we know that the stone is moving downward at point C.



d) Choose the final point after 5.00 s (point D) we get

$$y_f - y_i = v_i t - \frac{1}{2} g t^2 \Rightarrow$$

$$y_f = (20.0)(5.00) - \frac{1}{2} (9.80)(5.00)^2 = -22.5 \text{ m}$$

Below the
launching
point

Now using the equation

downward

$$v_f = v_i - g t = 20.0 - (9.80)(5.00) = -29.0 \text{ m/s}$$

e) Now the final point is point E , just before fitting the ground. So we have

$$y_f = -50.0 \text{ m} \Rightarrow$$

$$v_f^2 = v_i^2 - 2g(y_f - y_i) \Rightarrow$$

downward

$$v_f^2 = (20.0)^2 - 2(9.80)(-50.0) \Rightarrow v = -37.1 \text{ m/s}$$

The positive solution is rejected because the ball hits the ground while falling.

