

CHAPTER 5

THE LAWS OF MOTION

In this chapter we are going to discuss the following topics:

- ❑ **The Concept of Force**
- ❑ **Mass, Inertia & weight**
- ❑ **Newton's Laws of Motion**
- ❑ **Force of Friction**

Recommended Problems:

15, 21, 23, 28, 29, 31, 33, 40, 42, 43, 44, 45, 46, 47, 49, 55, 60, 63, 64, 65, 66, 70, 83, 85, 88, 89, 93, 101, 103

5.1 The Concept of Force

When a body affects a second body, we say that the first body exerts a force on the second.

This means that only force can cause the body to change its state of uniform motion or rest.

The force may be contact force or field force.

Contact force is the force that result from the physical contact between the objects, e.g., frictional force

Field force is the force that doesn't require physical contact between the objects, e.g., gravitational force

The force is a vector quantity with the SI unit of Newton (N). The cgs unit is dyne and its British unit is pound (lb).

5.2 Newton's First Law and Inertial Frames

Newton's 1st law states that *an object continues in its state of rest or uniform motion until it is forced to change that state by an external force.*

The frame in which Newton's first law is valid is called **inertial frame**. All accelerated frames are not inertial frames.

Newton's first law sometimes is called the **law of inertia**

To explain how Newton's laws are not valid in a non inertial frame consider the following situations:

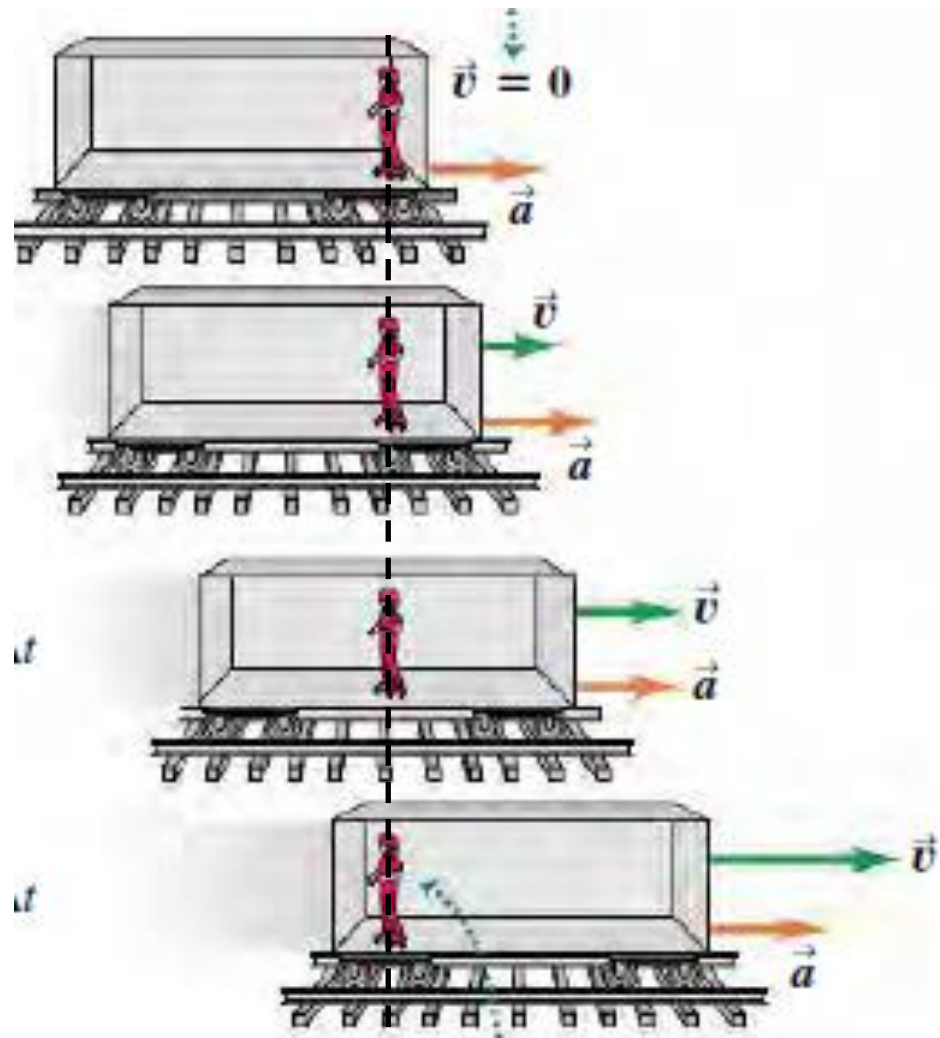
Suppose that you are riding roller skates (the friction is neglected) inside a train that is traveling on a straight road with some acceleration. There are no horizontal forces acted on you. According to Newton's law you should stay as you are placed without changing your initial velocity.

When the train starts to accelerate, you would start moving backward, relative to an observer inside the train (non-inertial frame). If instead the train was slowing to a stop, you would start moving forward.

According to the observer inside the train there are no forces responsible to your horizontal motion, and so Newton's laws of motion are not valid.

Note that to an observer in the ground (inertial frame), Newton's laws hold since you are still fixed relative to that observer.

The ground can be considered as an approximate inertial frame regardless of its rotation about the sun and about its own axis.



5.3 Mass and Inertia

Inertia is the property of matter that resists the change of its state and mass is the measure of this inertia.

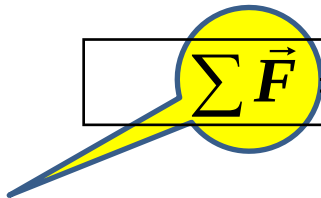
Mass is an inherent property of an object and is independent of the object's surroundings and of the method used to measure it.

The mass is a scalar quantity with a unit of kilogram (kg), in the SI unit system and gram (g) in the *cgs* unit system.

Remark There is a difference between mass and weight. They are completely different quantities. While the mass of a body is the measure of inertia of that body (the same everywhere), its weight is the force exerted by gravity on it (depends on the body's position).

5.4 Newton's Second Law

The acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass, i.e.,


$$\sum \vec{F} = m\vec{a}$$

The net force
acting on the body.

The last eq. can be written as.

$$\sum F_x = ma_x$$

$$\sum F_y = ma_y$$

$$\sum F_z = ma_z$$

• Quick Quiz (1)

Which of the following statements is correct?

- a) It is possible for an object to have motion in the absence of forces on the object.
- b) It is possible to have forces on an object in the absence of motion of the object.
- c) Both of them
- d) Non of them.

• Quick Quiz (2)

An object experiences no acceleration. Which of the following *cannot* be true for the object?

- a) A single force acts on the object.
- b) No forces act on the object.
- c) Forces act on the object, but the forces cancel.
- d) Non of them.

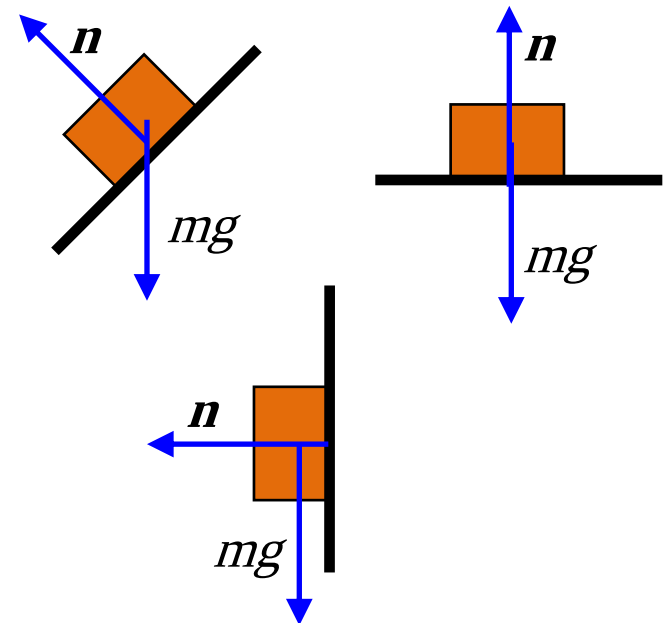
5.5 The Gravitational Force and Weight

The attractive force exerted by the Earth on an object is called the **gravitational force** F_g . This force is directed toward the center of the Earth, and its magnitude is called the **weight** of the object.

$$\vec{F}_g = m\vec{g}$$

The direction of the weight is always vertically downward regardless of the location of the body.

The Normal Force Whenever an object is in contact with a surface, the surface exerts a normal force on the object (*Normal means perpendicular to the surface*). The normal force is non-zero, positive value (a negative value for n means an opposite direction which is unacceptable physically).



5.6 Newton's Third Law

for every action there is an equal, but opposite reaction.

if two bodies interact, the force exerted by body number 1 on body number 2 ($\mathbf{F}_{21}$) is equal and opposite to the force exerted on body number 1 by body number 2 $\mathbf{F}_{12} \Rightarrow$

$$\vec{F}_{21} = -\vec{F}_{12}$$



Remark Action and reaction act on different bodies.

Free Body Diagram: It is a diagram showing all the forces acting on the body and do not include the forces the body exert on other bodies.

• Quick Quiz (3)

If a fly collides with the windshield of a fast-moving bus, which experiences an impact force with a larger magnitude?

- a) The fly.
- b) The bus.
- c) Both experience the same force.
- d) can't be known.

• Quick Quiz (4)

If a fly collides with the windshield of a fast-moving bus, which experiences the greater acceleration?

- a) The fly.
- b) The bus.
- c) Both have the same acceleration.
- d) can't be known.

Strategy for solving problems using Newton's laws:

(i) Chose a suitable coordinate system with the positive direction is the direction of the acceleration, if it is known.

(ii) Draw a free-body diagram of each body of the system separately.

(iii) Resolve each force into its components according to the chosen coordinates.

(iv) Identify the known and the unknown quantities.

(v) Now you can apply Newton's second law for one body or more of the system according to the unknown quantities.

Example 5.4 A traffic light weighing 122 N hangs from a cable tied to two other cables fastened to a support as shown. The upper cables make angles of $\theta_1=37.0^\circ$ and $\theta_2=53.0^\circ$ with the horizontal. These upper cables will break if the tension in them exceeds 100 N. Does the traffic light remain hanging in this situation, or will one of the cables break?

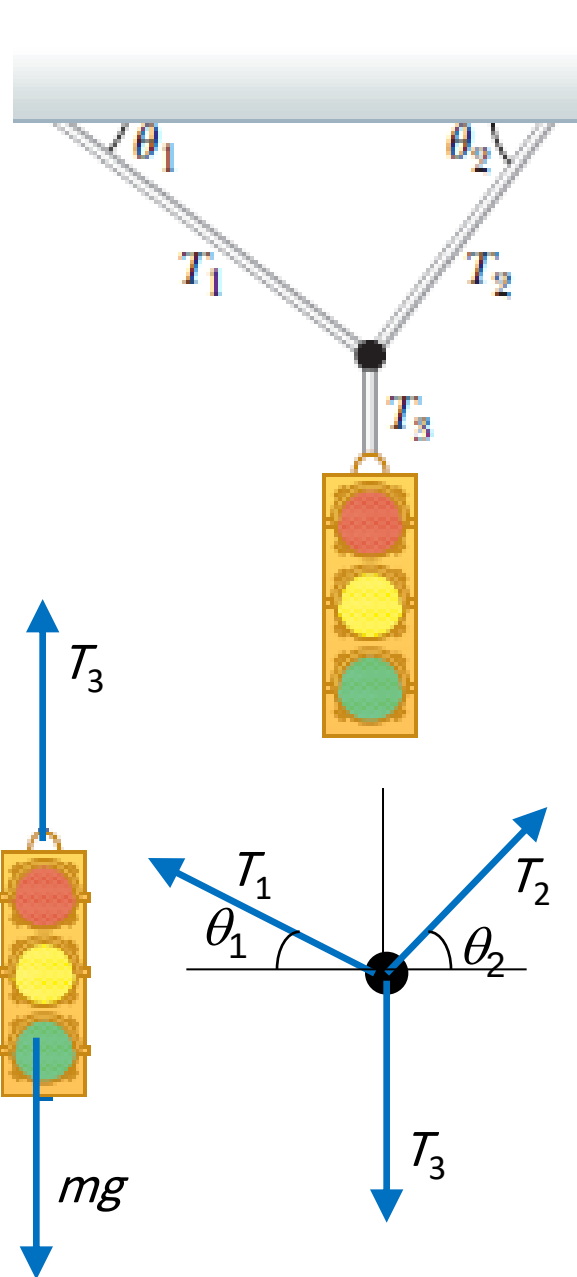
Solution Let us first draw the free-body diagram of the traffic light and the knot that holds the three cables together.

Assuming the traffic to be in equilibrium, Newton's 2nd law for the traffic gives

$$\sum F_y = ma_y = 0 \quad \Rightarrow$$

$$T_3 - mg = 0 \quad \Rightarrow$$

$$T_3 = mg = 122\text{ N}$$



And for the knot we have

$$\sum F_x = ma_x = 0 \Rightarrow$$

$$T_2 \cos \theta_2 - T_1 \cos \theta_1 = 0 \quad (1)$$

$$\sum F_y = ma_y = 0 \Rightarrow$$

$$T_2 \sin \theta_2 + T_1 \sin \theta_1 - mg = 0 \quad (2)$$

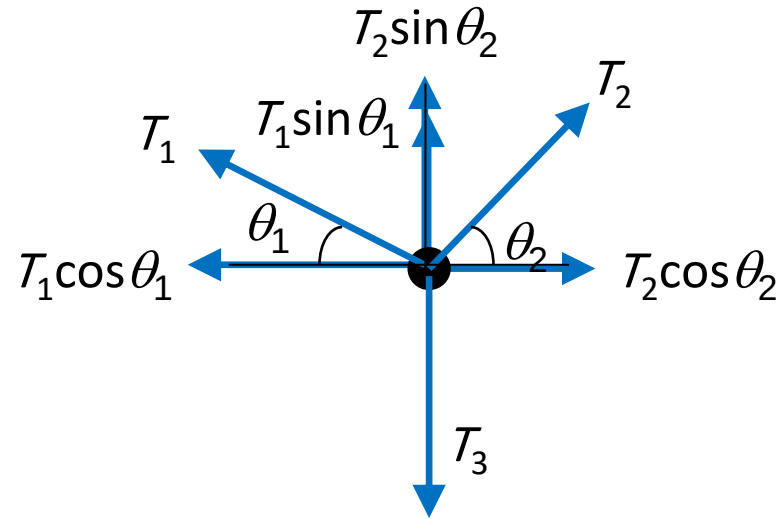
From Eq.(1) we get $T_2 = T_1 \frac{\cos \theta_1}{\cos \theta_2} \quad (3)$

It is clear that if $\theta_1 = \theta_2$ then $T_1 = T_2$

Now substituting Eq.(3) into Eq.(2) we get

$$T_1 \left(\frac{\cos \theta_1}{\cos \theta_2} \right) (\sin \theta_2) + T_1 \sin \theta_1 - mg = 0$$

$$\text{or } T_1 = \frac{mg}{\sin \theta_1 + \cos \theta_1 \tan \theta_2} = 73.4 \text{ N}$$



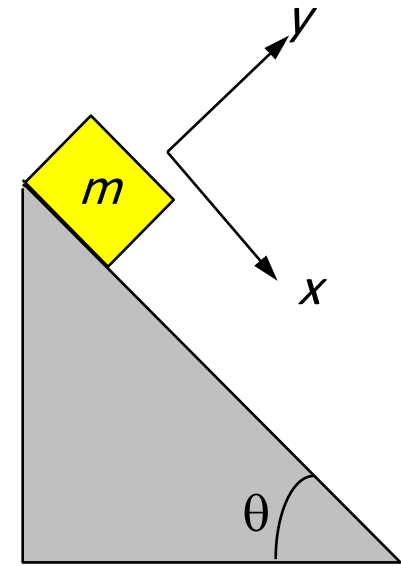
From Eq.(3) we get $T_2 = 73.4 \frac{\cos 37^\circ}{\cos 53^\circ} = 97.4 \text{ N}$

Both values are less than 100 N, so neither of the cables will break.

Example 5.6 A car of mass m is on an icy driveway inclined at an angle θ as shown.

(a) Find the acceleration of the car, assuming the driveway is frictionless.

b) Suppose the car is released from rest at the top of the incline and the distance from the car's front bumper to the bottom of the incline is d . How long does it take the front bumper to reach the bottom of the hill, and what is the car's speed as it arrives there?



Solution a) Let us first draw the free-body diagram of the car. It is convenient to choose the coordinate axes with x along the incline and y perpendicular to it.

$$\sum F_x = ma_x \Rightarrow$$

$$mg \sin \theta = ma \Rightarrow a = g \sin \theta$$

b) Since the acceleration is constant, we use

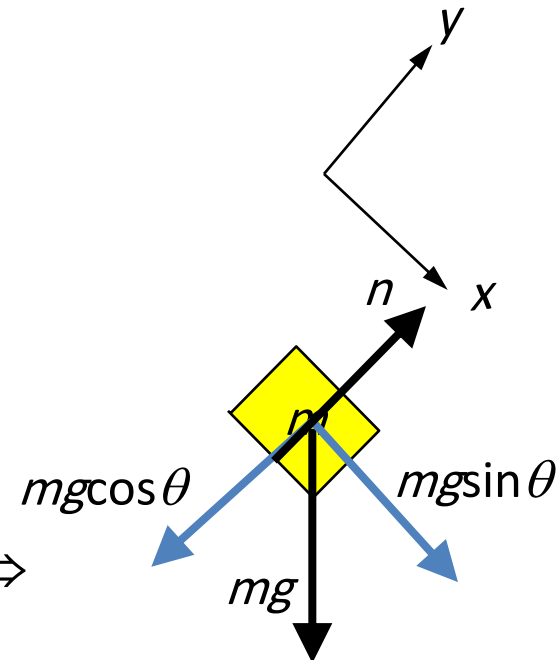
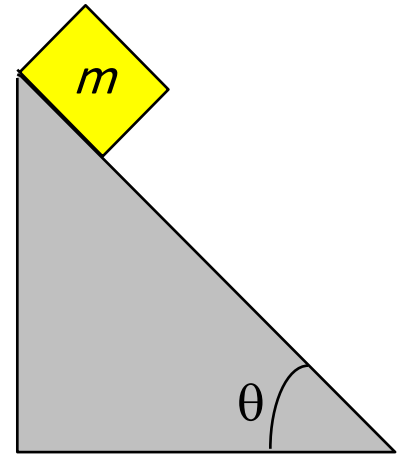
$$x_f = v_{xi}t + \frac{1}{2}a_x t^2 \Rightarrow$$

$$d = \frac{1}{2}g \sin \theta t^2 \Rightarrow t = \sqrt{\frac{2d}{g \sin \theta}}$$

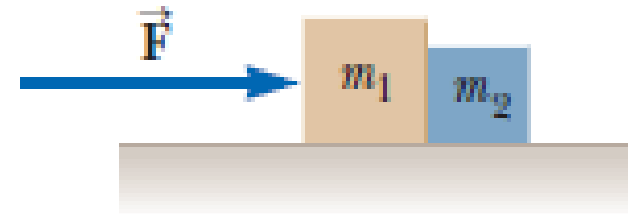
To find the final speed of the car, we use

$$v_f^2 = v_i^2 + 2ax_f \Rightarrow v^2 = 0 + 2gd \sin \theta \Rightarrow$$

$$v = \sqrt{2gd \sin \theta}$$



Example 5.7 Two blocks of masses m_1 and m_2 , with $m_1 > m_2$ are placed in contact on a smooth horizontal surface, as shown. A constant horizontal force $\vec{F}$ is applied to m_1 as shown.



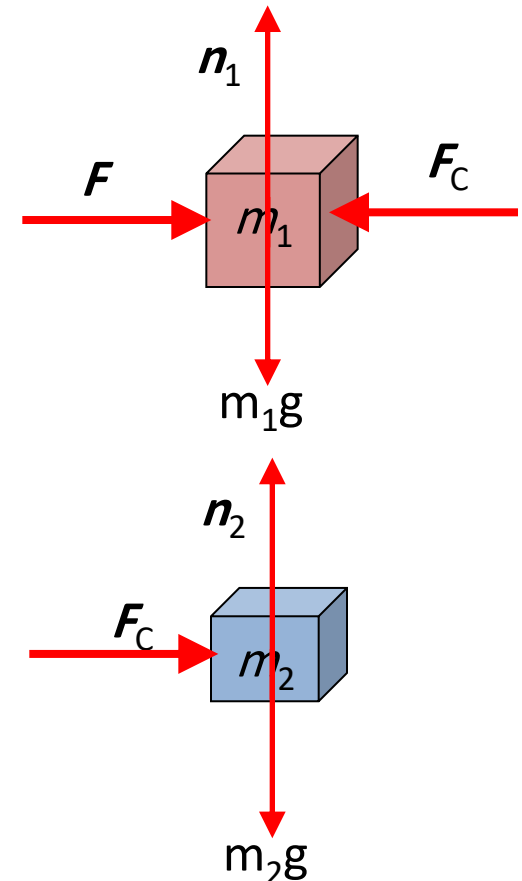
- Find the acceleration of the system.
- Find the contact force between the two blocks

Solution Let us first draw the free-body diagram of the system.

Note that $\vec{F}_C$ represents the contact force due to the contact between the two blocks. They are action and reaction forces.

Now using Newton's 2nd law we get for m_1 .

$$\begin{aligned}\sum F_x &= ma_x \quad \Rightarrow \\ F - F_C &= m_1 a \quad (1)\end{aligned}$$



$$F - F_C = m_1 a \quad (1)$$

and for m_2 we get.

$$\sum F_x = m a_x \Rightarrow$$

$$F_C = m_2 a \quad (2)$$

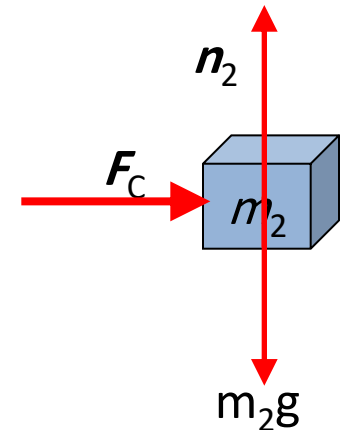
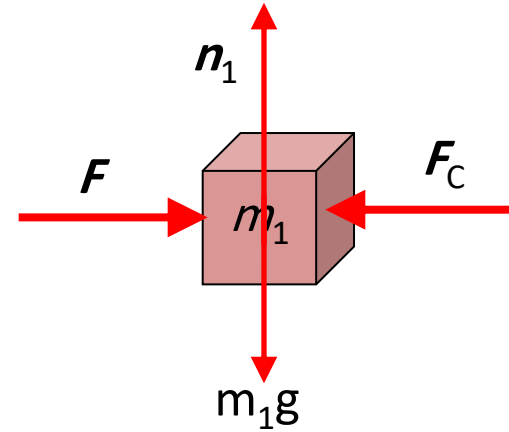
Adding the 2-eqs. $\Rightarrow$

$$F = (m_1 + m_2) a \Rightarrow$$

$$a = \frac{F}{(m_1 + m_2)}$$

b) Substituting for a in eq. (2) $\Rightarrow$

$$F_C = \frac{m_2 F}{(m_1 + m_2)}$$



Example 5.8 A person weighs a fish of mass m on a spring scale attached to the ceiling of an elevator as shown.

a) Show that if the elevator accelerates either upward or downward, the scale gives a reading that is different from the weight of the fish.

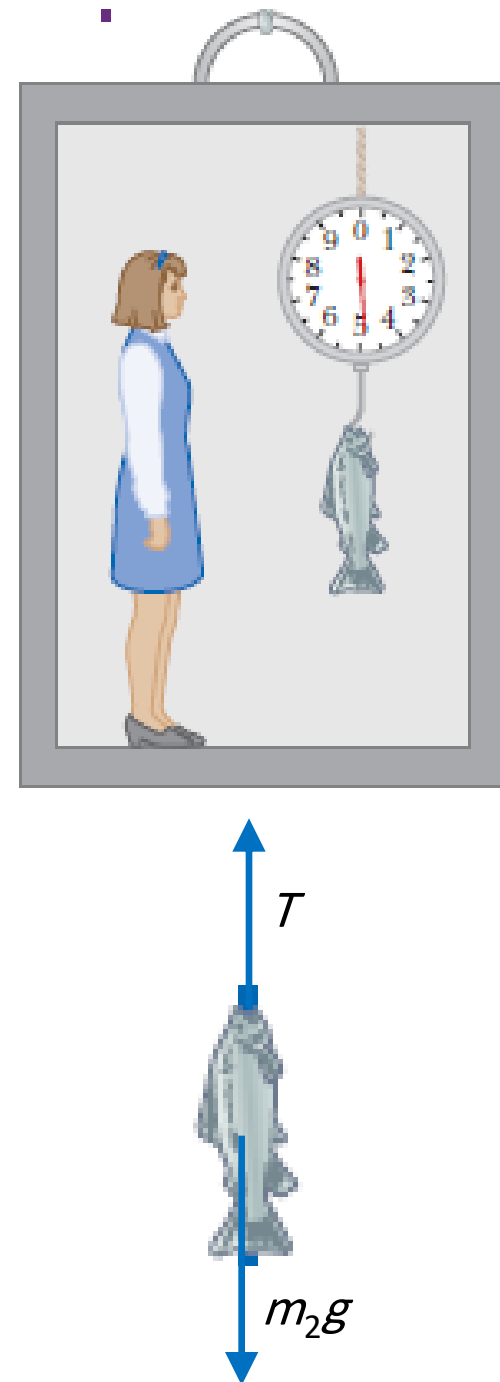
Solution Let us first draw the free-body diagram of fish. Noting that the scale reads the tension, we get for upward acceleration

$$\sum F_y = ma \quad \Rightarrow \quad T - mg = ma \quad \Rightarrow \quad T = m(g + a)$$

This means that the scale reading T is greater than the fish's weight mg if a is upward. If a is downward we get

$$T - mg = m(-a) \quad \Rightarrow \quad T = m(g - a)$$

This means that the scale reading T is less than the fish's weight mg if a is downward.



b) Evaluate the scale readings for a 40.0-N fish if the elevator moves with an acceleration $a=\pm 2.00 \text{ m/s}^2$.

For upward acceleration $a=+ 2.00 \text{ m/s}^2$ we have.

$$T = m(g + a) = mg\left(1 + \frac{a}{g}\right) = 40.0\left(1 + \frac{2.00}{9.80}\right) = 48.2 \text{ N}$$

For downward acceleration $a=- 2.00 \text{ m/s}^2$ we have.

$$T = m(g - a) = mg\left(1 - \frac{a}{g}\right) = 40.0\left(1 - \frac{2.00}{9.80}\right) = 31.8 \text{ N}$$

Suppose the elevator cable breaks and the elevator and its contents are in free fall. What happens to the reading of the scale?

If the elevator falls freely, the fish's acceleration is $a=-g$. We see that the scale reading T is zero in this case; that is, the fish *appears* to be weightless.

Example 5.9 Two masses m_1 and m_2 ($m_2 > m_1$) are suspended vertically by a light string that passes over a light, frictionless pulley (Atwood's machine). Find the acceleration of the masses and the tension in the string.

Solution Again let us first draw the free-body diagram of the system. Using

$$\sum F_y = ma_y \Rightarrow$$

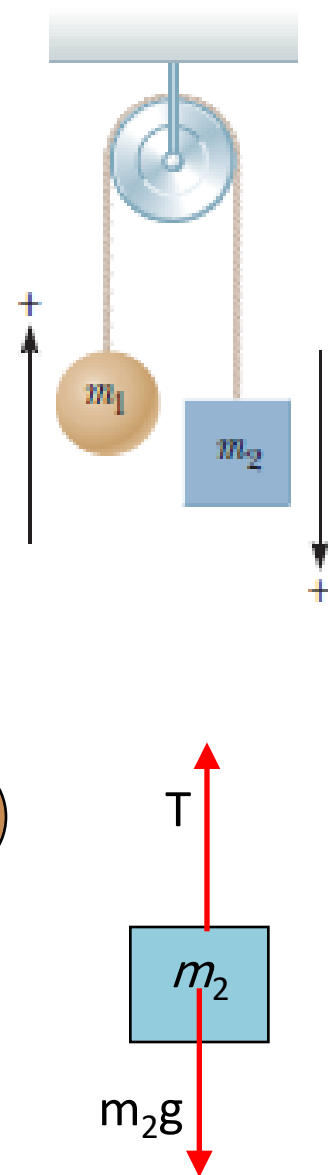
For m_1 we take the +ve sense to be upward

$$T - m_1g = m_1a \quad (1)$$

For m_2 we take the +ve sense to be downward

$$m_2g - T = m_2a \quad (2)$$

Adding the 2-eqs. $\Rightarrow$



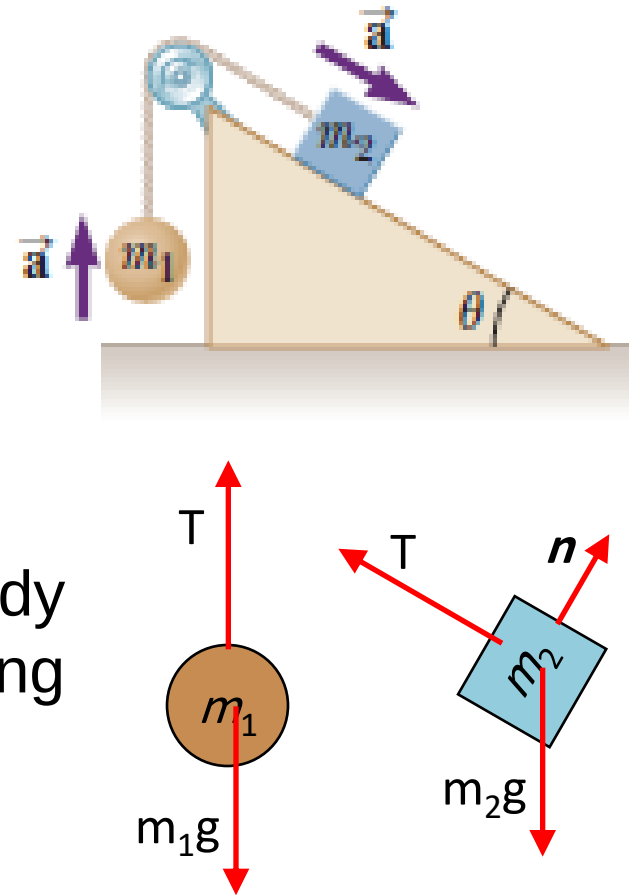
$$(m_2 - m_1)g = (m_1 + m_2)a \quad \Rightarrow \quad a = \frac{(m_2 - m_1)g}{(m_1 + m_2)}$$

And for T we have $T = \frac{(2m_1m_2)g}{(m_1 + m_2)}$

Example 5.9 A ball of mass m_1 and a block of mass m_2 are attached by a light cord that passes over a frictionless pulley of negligible mass as shown. The block lies on a frictionless incline of angle θ . Find the magnitude of the acceleration of the two objects and the tension in the cord.

Solution Let us first draw the free-body diagram of the 2-masses. For m_1 and using the +ve direction upward we use

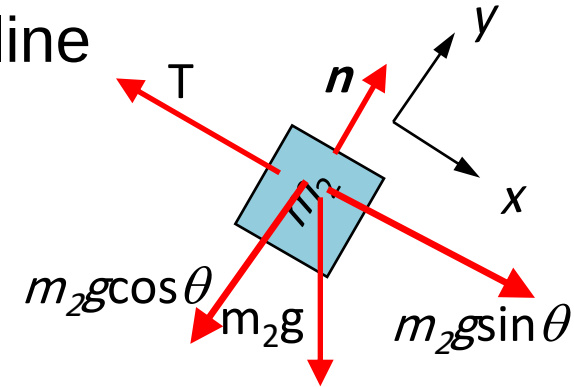
$$\sum F_y = ma_y \quad \Rightarrow \quad T - m_1g = m_1a \quad (1)$$



$$T - m_1g = m_1a \quad (1)$$

For m_2 we choose the x-axis along the incline with the +ve direction down the incline. Using

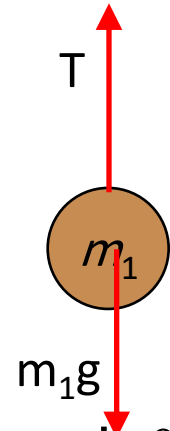
$$\begin{aligned} \sum F_x &= ma_x \Rightarrow \\ m_2g \sin \theta - T &= m_2a \end{aligned} \quad (2)$$



Adding the 2-eqs. $\Rightarrow$

$$(m_2 \sin \theta - m_1)g = (m_1 + m_2)a \quad \Rightarrow$$

$$a = \frac{(m_2 \sin \theta - m_1)g}{(m_1 + m_2)}$$



Note that the block accelerate down the incline only if $m_2 \sin \theta > m_1$ otherwise it will accelerate up the incline.

And for T we have

$$T = \left(\frac{m_1 m_2 (\sin \theta + 1)}{m_1 + m_2} \right) g$$

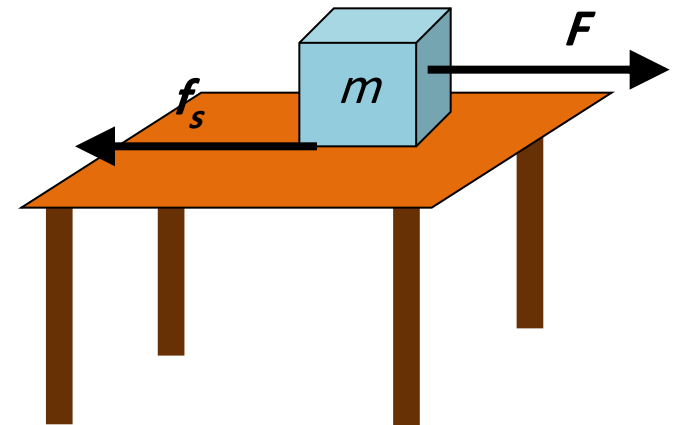
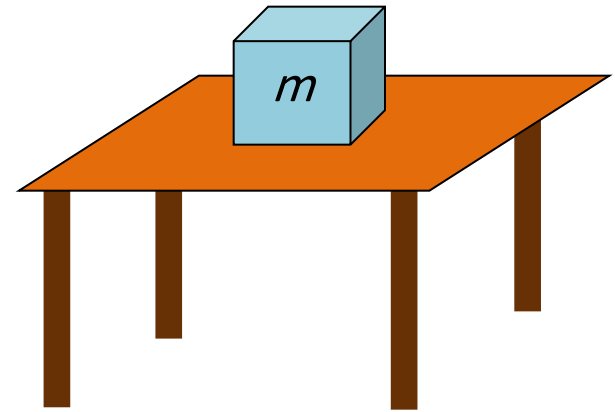
5.8 Forces of Friction

The frictional forces is the forces that two surfaces in contact exert on each other to oppose the sliding of one surface over the other.

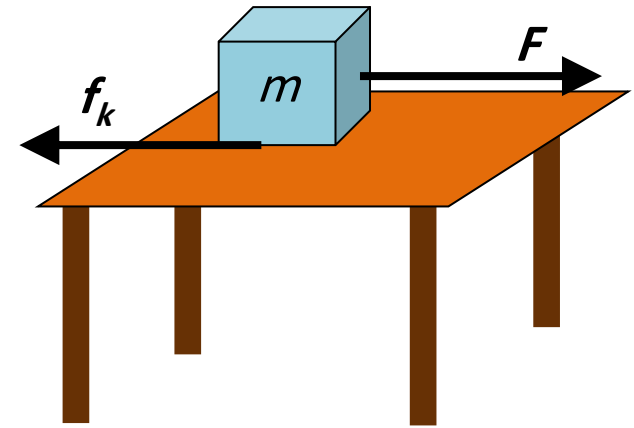
If you want to push a block along a horizontal table three cases have to be considered:

(i) No external force is applied. Since $a=0 \Rightarrow$ The frictional force is zero.

(ii) An external force is applied but no motion. Since $a=0 \Rightarrow f_s = F$. This frictional force is called the static frictional force.



(iii) A larger external force is applied $\{F > f_s(max)\}$ so that the block is moving. Since $a \neq 0 \Rightarrow F - f_k = ma$. This frictional force is called the kinetic frictional force.



It is found, experimentally, that the frictional forces f_s and f_k , between two surfaces, are independent on the area of contact between the two surfaces but proportional to the normal force N pressing the two surfaces together. i.e.,

$$f_s \leq \mu_s n$$

$$f_k = \mu_k n$$

Where the dimensionless constants μ_s and μ_k are, respectively, the coefficient of static friction and the coefficient of kinetic friction. It is found that $\mu_s > \mu_k$

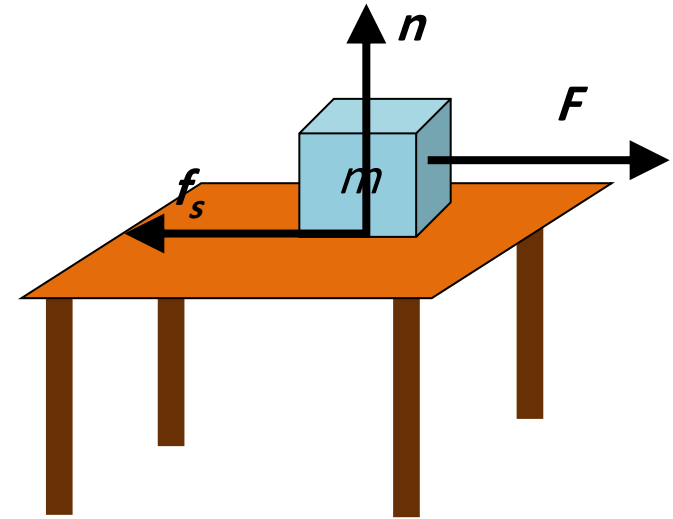
Ramarks:

(i) The frictional force is always parallel to the surfaces in contact.

(ii) The force of static-friction is always directed to oppose the supposed relative motion between the contact surfaces if there were no friction.

(iii) The force of kinetic friction is always opposite to the direction of motion.

(iv) The frictional force, together with the normal force constitute the two perpendicular components of the reaction force exerted by one of the contact bodies on the other.



• Quick Quiz (5)

You press your book against a vertical wall with your hand. What is the direction of the friction force by the wall on the book?

- a) Upward.
- b) downward.
- c) Out from the wall.
- d) Into the wall.

• Quick Quiz (6)

A crate is located in the center of a flatbed truck. The truck accelerates to the east, and the crate moves with it without sliding. Which of the following is correct concerning the friction force exerted by the truck on the crate?

- a) It is static and directed to the west.
- b) It is static and directed to the east.
- c) It is kinetic and directed to the west.
- d) It is kinetic and directed to the east.

Example 5.11 Suppose a block is placed on a rough surface inclined relative to the horizontal as shown. The incline angle is increased until the block starts to move. Show that you can obtain μ_s by measuring the critical angle θ_c at which this slipping just occurs.

Solution The free body diagram of the block is shown. Taking the x -axis along the plane and using Newton's second law, we get

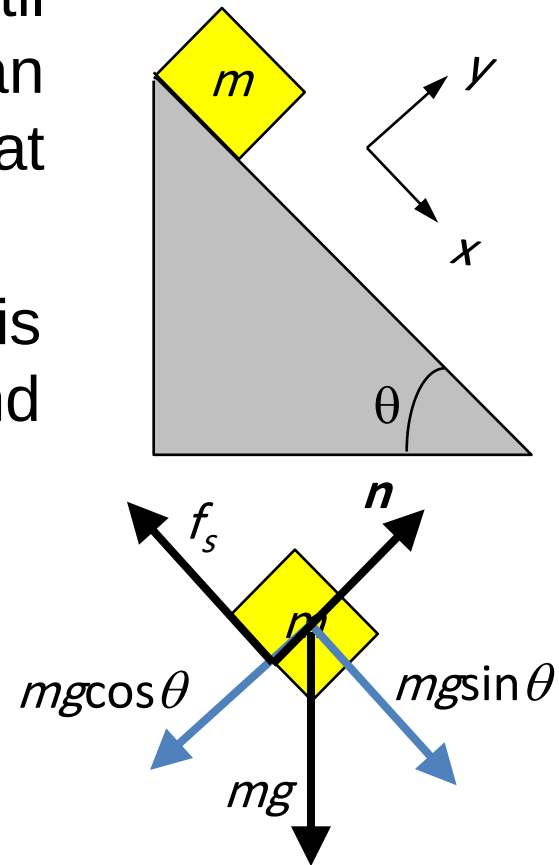
$$\sum F_x = ma_x \quad \Rightarrow \quad mg \sin \theta - f_s = 0 \quad (1)$$

$$\sum F_y = ma_y \quad \Rightarrow \quad n - mg \cos \theta = 0 \quad (2)$$

From Eq.(2) we have $mg = \frac{n}{\cos \theta}$

Now substituting for mg into Eq.(1) we get

$$f_s = \frac{n}{\cos \theta} \sin \theta = n \tan \theta \quad (3)$$



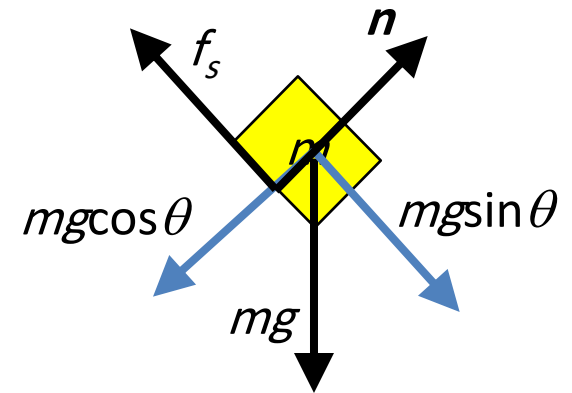
When $\theta = \theta_c$ the block is on the verge of slipping and the force of static friction reaches its maximum value $\mu_s n$. Make these substitutions in Equation (3):

$$\mu_s n = n \tan \theta_c \Rightarrow \mu_s = \tan \theta_c$$

When $\theta > \theta_c$ the block accelerates down the incline and the force of friction is $f_k = \mu_k n$. To determine μ_k we reduce θ until the block moves with constant speed ($a=0$). This new critical angle is θ'_c . Using Eqs. (1) and (2) with f_s replaced by f_k we find

$$\mu_k = \tan \theta'_c$$

It should be noted that $\theta'_c < \theta_c$.

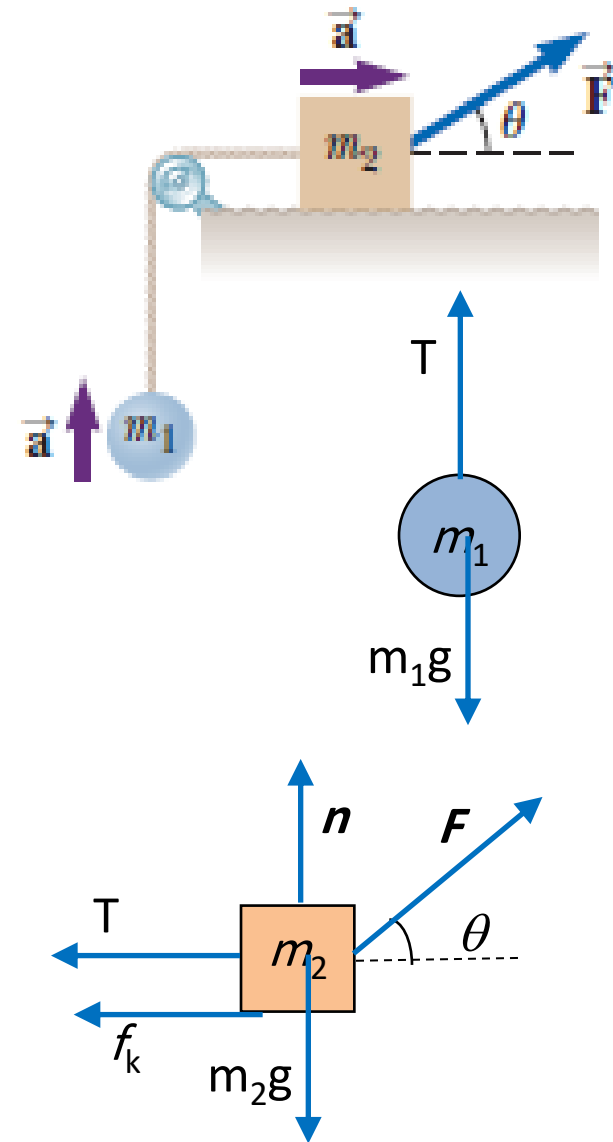


Example 5.13 A block of mass m_2 on a rough, horizontal surface is connected to a ball of mass m_1 by a lightweight cord over a light, frictionless pulley as shown. A force of magnitude F at an angle θ with the horizontal is applied to the block as shown, and the block slides to the right. The coefficient of kinetic friction between the block and surface is μ_k . Determine the magnitude of the acceleration of the two objects.

Solution The free body diagram of the system is shown. Applying Newton's 2nd law for the ball we get

$$\sum F_y = ma_y \Rightarrow T - m_1g = m_1a \quad (1)$$

The block moves horizontally such that $a_x = a$, and $a_y = 0$. Applying Newton's 2nd in the two axes we get



$$\sum F_x = ma_x \Rightarrow F \cos \theta - T - f_k = m_2 a \quad (2)$$

$$\sum F_y = ma_y \Rightarrow F \sin \theta + n - m_2 g = 0 \quad (3)$$

From Eq.(3) we have $n = m_2 g - F \sin \theta \Rightarrow$

$$f_k = \mu_k n = \mu_k (m_2 g - F \sin \theta)$$

Substituting for f_k into Eq.(2) we have

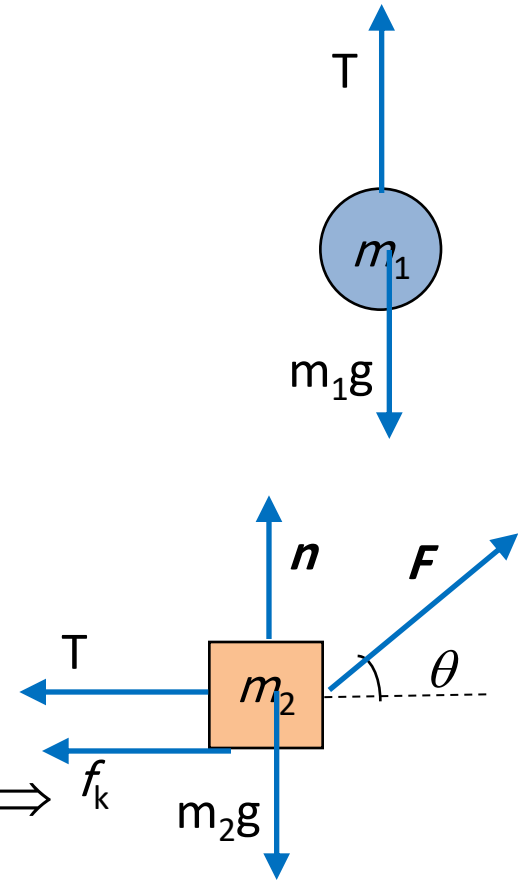
$$F \cos \theta - T - \mu_k (m_2 g - F \sin \theta) = m_2 a \quad (4)$$

$$T - m_1 g = m_1 a \quad (1)$$

Adding Eq.(1) and Eq.(4) we get

$$F \cos \theta - \mu_k (m_2 g - F \sin \theta) - m_1 g = (m_1 + m_2) a \Rightarrow$$

$$a = \frac{F(\cos \theta + \mu_k \sin \theta) - (m_1 + \mu_k m_2)g}{m_1 + m_2}$$



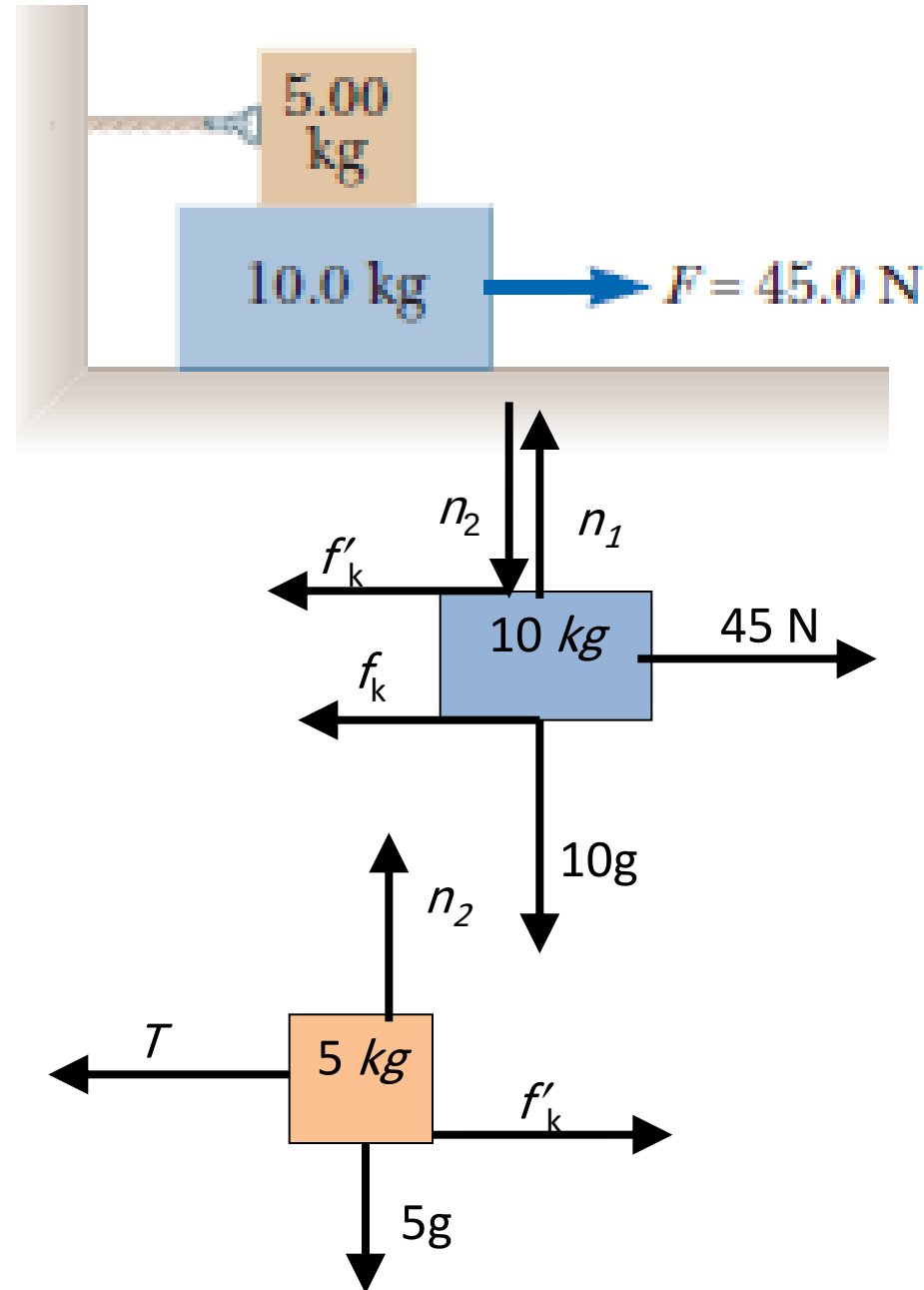
Example (problem 70) A 5.00-kg block is placed on top of a 10.0-kg block. A horizontal force of 45.0 N is applied to the 10-kg block, and the 5.00-kg block is tied to the wall. The coefficient of kinetic friction between all moving surfaces is 0.200.

(a) Draw a free-body diagram for each block and identify the action–reaction forces between the blocks.

(b) Determine the tension in the string and the magnitude of the acceleration of the 10.0-kg block.

Solution The free-body diagram of the two blocks are shown.

Note that f'_k and n_2 form action-reaction forces.



b) Applying Newton's 2nd law to the 5-kg block which is held with no motion, we get

$$\sum F_x = ma_x \Rightarrow f'_k - T = 0 \quad (1)$$

$$\sum F_y = ma_y \Rightarrow n_2 - 5.00g = 0 \quad (2)$$

For the 10-kg block we have

$$\sum F_y = ma_y \Rightarrow n_1 - n_2 - 10.0g = 0 \quad (3)$$

$$\sum F_x = ma_x \Rightarrow 45 - f_k - f'_k = 10.0a \quad (4)$$

From Eq.(2) we have $n_2 = 5.00g = 49.0\text{N}$

$$\Rightarrow f'_k = \mu_k n_2 = 0.200(490) = 9.80\text{N}$$

From Eq.(3) we have $n_1 = n_2 + 10.0g = 147\text{N}$

$$\Rightarrow f_k = \mu_k n_1 = 0.200(147) = 29.4\text{N}$$

From Eq.(4) we obtain

$$45 - 29.4 - 9.8 = 10.0a \Rightarrow a = \frac{5.80}{10.0} = 0.58\text{m/s}^2$$

