

CHAPTER 6

CIRCULAR MOTION and OTHER APPLICATIONS OF NEWTON'S LAWS

In this chapter we are going to discuss the following topics:

- Uniform Circular Motion**
- Non-Uniform Circular Motion**
- Motion in Accelerated Frames**

Recommended Problems:

1, 8, 9, 10, 11, 15, 16, 17, 18, 21, 23, 37, 39, 40, 42, 45, 54, 57,
59, 61, 68

6.1 The Particle in Uniform Circular Motion

It is known that if a particle moves with constant speed v in a circular path of radius r , it acquires a centripetal acceleration toward the center of the circle due to the change in the direction of the particle's velocity. The magnitude of such acceleration is

$$a_c = \frac{v^2}{r}$$

From Newton's 2nd Law this acceleration should be due to a force called the centripetal Force.

$$\sum \vec{F}_c = ma_c = m \frac{v^2}{r}$$

It should be noted that any force in nature can be considered a centripetal force if it acts on a particle in a direction toward the center of the circular path followed by that particle.

Remark The word centripetal refers to the direction of the forces and not to a new kind of forces. It is like horizontal or vertical.

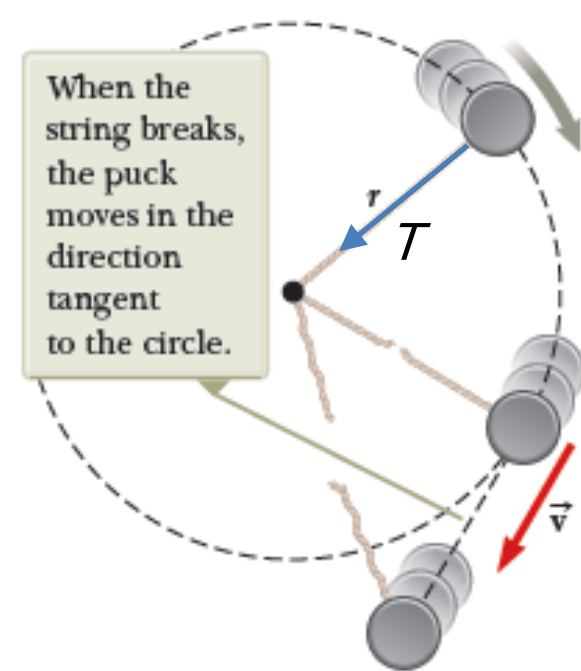
Example 6.2 Consider a puck, of mass 0.50 kg, is tied to a string of length $r=1.50\text{m}$, and moves at constant speed in a horizontal, circular path as shown. What is the maximum speed at which the puck can move before the string breaks? The string can withstand a maximum tension of 50.0 N.

Solution Since the tension is the only centripetal force acting on the puck we have

$$\sum \vec{F}_c = m \frac{v^2}{r} \Rightarrow$$

$$T = m \frac{v^2}{r} \Rightarrow v = \sqrt{\frac{Tr}{m}} \Rightarrow$$

$$v_{\text{max}} = \sqrt{\frac{T_{\text{max}} r}{m}} = \sqrt{\frac{(50.0)(1.50)}{0.50}} = 12.2 \text{ m/s}$$



If that force vanish, by breaking the string, the puck would move along the straight path that is tangent to the circle at the instant of breaking. This is due to Newton's 1st law.

• Quick Quiz (1)

To keep a particle moving on a circular path:

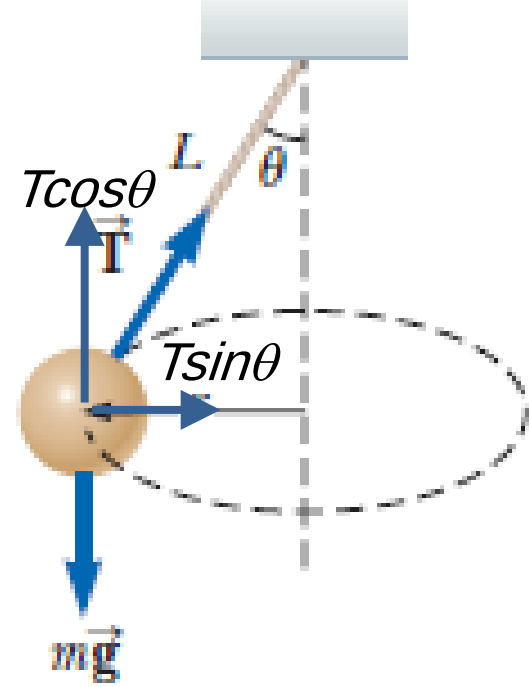
- a) The net force along the radius must be zero.
- b) A net force must act toward the center.
- c) A net force must act outward from the center.
- d) A centrifugal force must balance the centripetal force.

The wave swinger



Example 6.1 A small ball of mass m is suspended from a string of length L . The ball revolves with constant speed v in a horizontal circle of radius r as shown. (*conical pendulum*.) Find an expression for v .

Solution The tension exerted by the string on the ball is resolved into a vertical component $T\cos\theta$ and a horizontal component $T\sin\theta$ acting toward the center of the circular path.



$T\sin\theta$ is the centripetal force acting on the ball, so we write

$$\sum \vec{F}_c = m \frac{v^2}{r} \Rightarrow$$
$$T \sin \theta = m \frac{v^2}{r} \quad (1)$$

Since there is no vertical motion for the ball we have

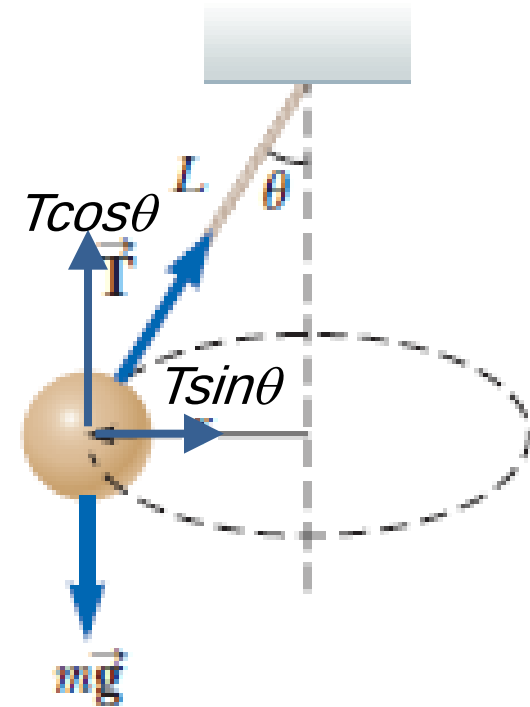
$$\sum \vec{F}_y = 0 \Rightarrow T \cos \theta - mg = 0 \quad (2)$$

Dividing Eq.(1) by Eq.(2)

$$\tan \theta = \frac{v^2}{gr}$$

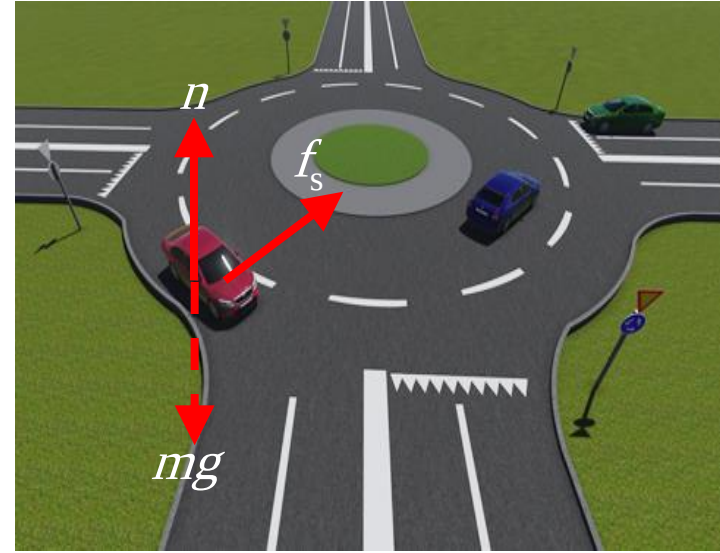
But as it is clear from the geometry of the figure that $r = L \sin \theta \Rightarrow$

$$\tan \theta = \frac{v^2}{gL \sin \theta} \Rightarrow v = \sqrt{gL \tan \theta \sin \theta}$$



Consider what happens when θ goes to 90° so that the string is horizontal. Because the tangent of 90° is ∞ , the speed v is infinite, which tells us the string cannot possibly be horizontal.

Example 6.3 A 1500-kg car moving on a flat, horizontal road negotiates a curve as shown. If the radius of the curve is 35.0 m and the coefficient of static friction between the tires and dry pavement is 0.523, find the maximum speed the car can have and still make the turn successfully.



Solution Here there are three forces acting on the car: The weight and the normal force act perpendicular to the plane of motion, and the static frictional force parallel to the road. The static frictional force is the only force that acts along the radius of the curve $\Rightarrow$

$$\sum \vec{F}_c = m \frac{v^2}{r} \Rightarrow f_s = m \frac{v^2}{r}$$

$$\text{But } f_s \leq \mu_s n \Rightarrow \mu_s n \geq m \frac{v^2}{r}$$

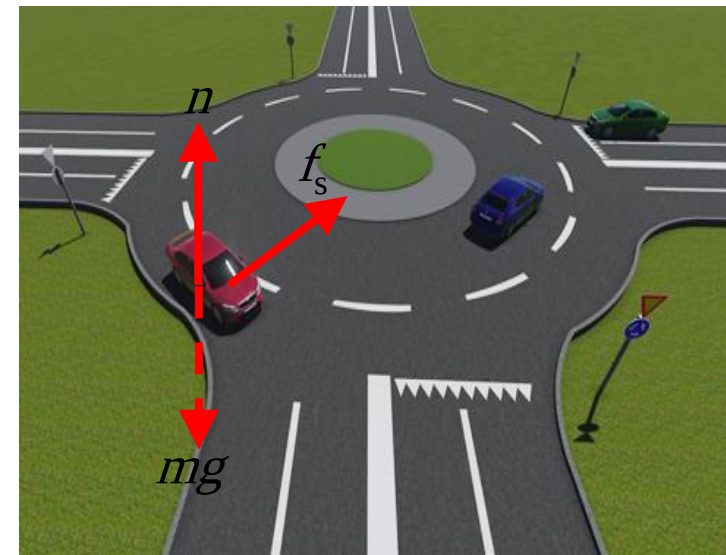
since there is no motion in the vertical direction $\Rightarrow$

$$n = mg \Rightarrow$$

$$\mu_s(\cancel{mg}) \geq \cancel{m} \frac{v^2}{r} \Rightarrow$$

$$v \leq \sqrt{\mu_s gr} \Rightarrow$$

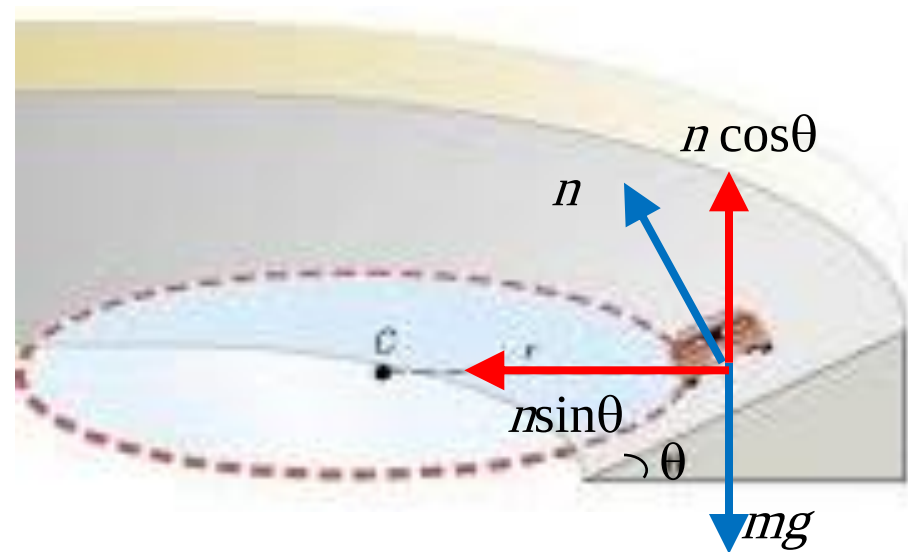
$$v_{\max} = \sqrt{\mu_s gr} = 13.4 \text{ m/s} = 48.2 \text{ km/h}$$



Notice that the maximum speed does not depend on the mass of the car.

If we try to take the curve going *faster* than $v_{\max}$, we will skid due to the lack of sufficient centripetal force. We could still go in a circle without skidding at this higher speed, but the radius would have to be larger.

Example 6.4 A circular curve on a road is designed for traffic moving at 48.2 km/hr (13.4 m/s) without relying on the friction. If the radius of the curve is 35.0 m, what is the correct angle of road's banking?

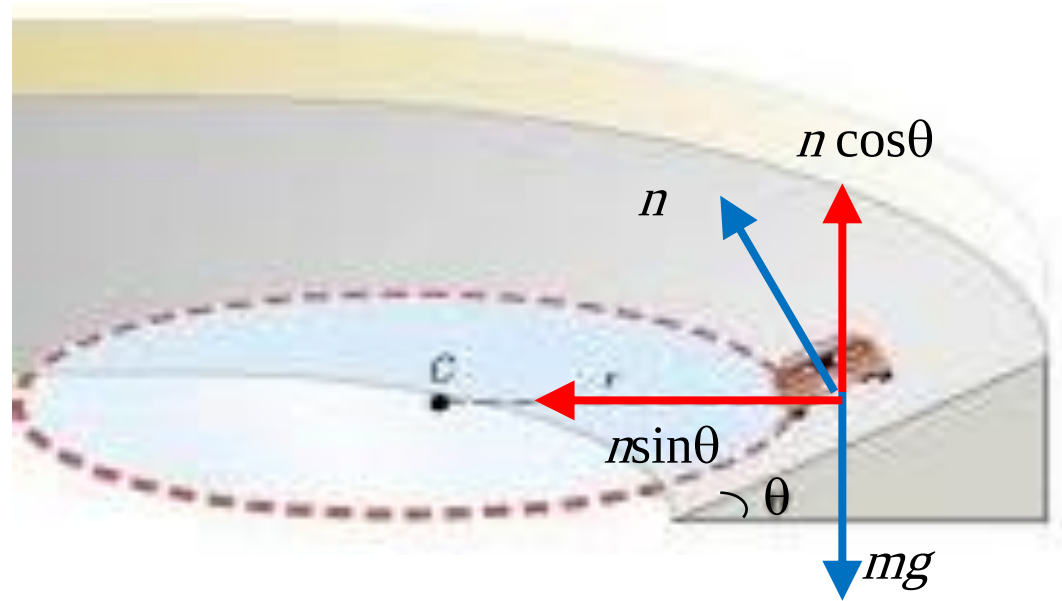


Solution Note that even though the car is on an inclined surface, it will revolve on a horizontal circle. This means that the weight is perpendicular to the plane of motion.

The normal force n should be resolved into two components: one toward the center of the curve (horizontal), and the other vertical. The centripetal force is then the horizontal component of this normal force. Now applying the law

$$\sum \vec{F}_c = m \frac{v^2}{r} \quad \Rightarrow \quad n \sin \theta = m \frac{v^2}{r} \quad (1)$$

$$n \sin \theta = m \frac{v^2}{r} \quad (1)$$



since there is no motion in the vertical direction $\Rightarrow$

$$n \cos \theta = mg \quad \Rightarrow \quad n = \frac{mg}{\cos \theta} \quad (2)$$

Substituting for n from eq.(2) into eq.(1) we get

$$mg \tan \theta = m \frac{v^2}{r} \quad \Rightarrow \quad \theta = \tan^{-1} \frac{v^2}{gR} = 27.6^\circ$$

The Ferris wheel



Example 6.5 A child of mass m rides on a Ferris wheel as shown. The child moves in a vertical circle of radius 10.0 m at a constant speed of 3.00 m/s.

(a) Determine the force exerted by the seat on the child at the bottom of the ride. Express your answer in terms of the weight of the child, mg .

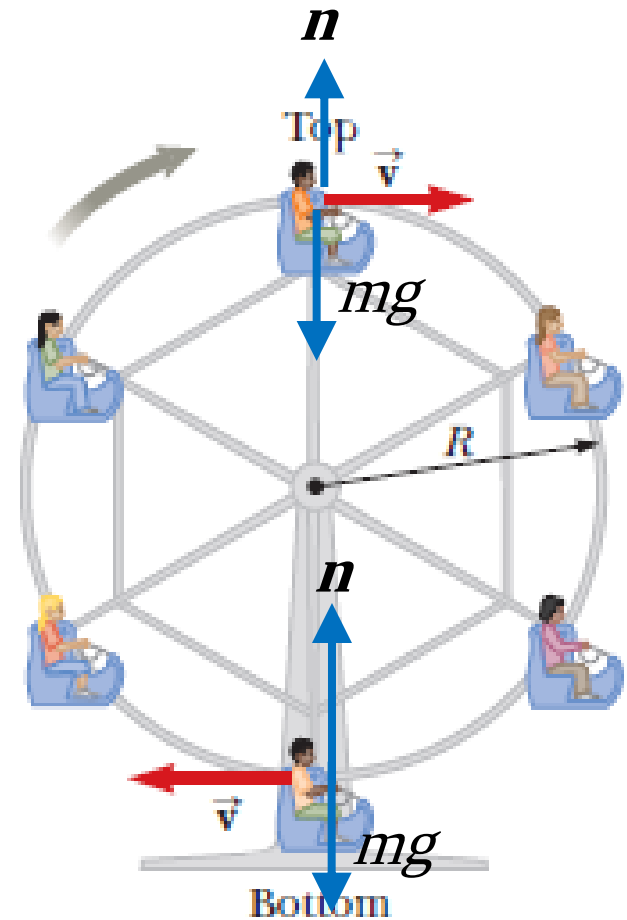
(b) Determine the force exerted by the seat on the child at the top of the ride.

Solution The free-body diagrams of the child at the two positions are shown. Using

$$\sum \vec{F}_c = m \frac{v^2}{r}$$

(a) At the bottom of the ride we write

$$n - mg = m \frac{v^2}{r} \Rightarrow n = m \left(g + \frac{v^2}{r} \right) = 1.09 mg$$



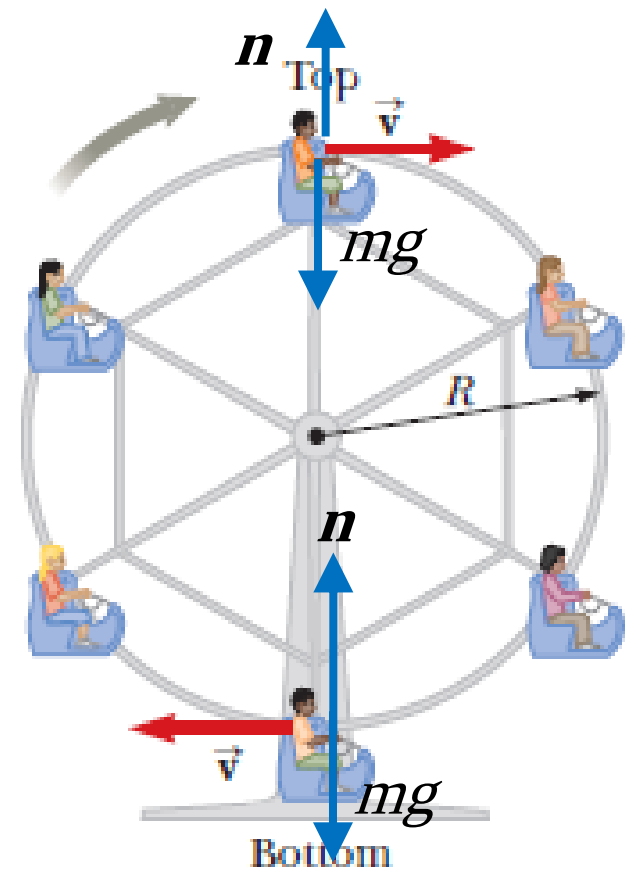
(b) At the top of the ride we write

$$mg - n = m \frac{v^2}{r} \Rightarrow n = m \left(g - \frac{v^2}{r} \right) = 0.908 mg$$

Note that at the bottom, the normal force is larger than its weight, while at the top the normal force is less than its weight.

Suppose a defect in the Ferris wheel causes the speed of the child to increase. What does the child experience at the top of the ride in this case?

If the speed is increased such that the normal force at the top of the ride is negative, which is impossible, it means that the child will lose contact with the seat and will only stay in his circular path if there is a safety bar or a seat belt that provides a downward force on him to keep him in his seat.

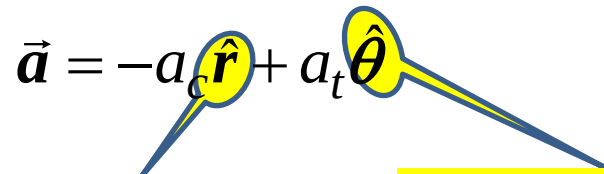


6.2 Nonuniform Circular Motion

When the magnitude of the velocity is not constant but change with time we have the nonuniform circular motion.

The change in the speed will result a tangential acceleration.

Resolving the acceleration vector into two perpendicular components: radial component and tangential component $\Rightarrow$

$$\vec{a} = -a_c \hat{r} + a_t \hat{\theta}$$
The diagram shows the equation $\vec{a} = -a_c \hat{r} + a_t \hat{\theta}$. The unit vector $\hat{r}$ is circled in blue, and the unit vector $\hat{\theta}$ is also circled in blue. Two yellow boxes with blue arrows point from the text below to these circled unit vectors.

Unit vector along the radius

Unit vector along the tangent

The radial component is the centripetal acceleration and the tangential component is due to the change in the particle's speed, i.e.,

$$a_c = \frac{v^2}{r}$$

$$a_t = \frac{d|\vec{v}|}{dt}$$

Remark In applying Newton's second law for the circular motion, the coordinate axes will be the radius-axis and the tangent-axis, so all the applied forces have to be resolved accordingly. The law now reads

$$\sum F_r = ma_r \qquad \sum F_t = ma_t$$

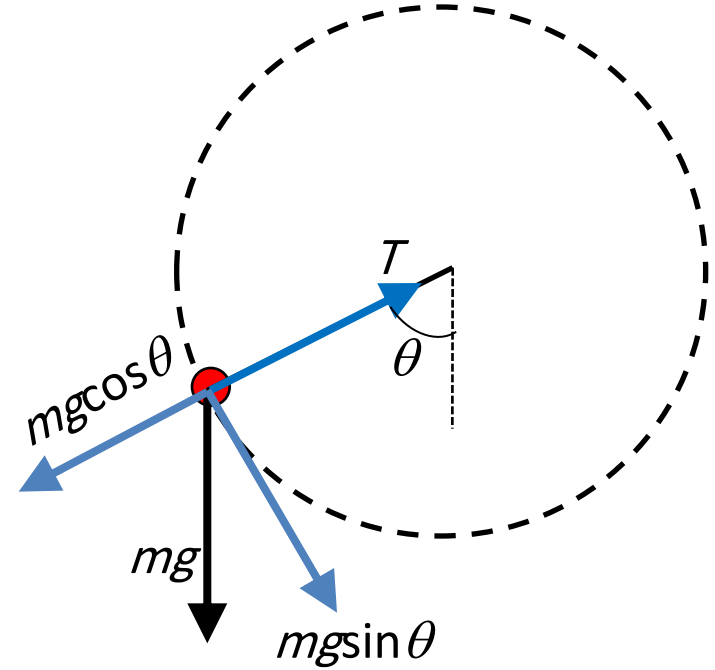
The positive senses of r and θ will be chosen away from the center, and counterclockwise respectively.

• Quick Quiz (2)

Which of the following is impossible concerning a car moving in a circular path?

- a) the car may has centripetal acceleration only.
- b) the car may has a tangential acceleration only.
- c) the car may has both centripetal and tangential accelerations.
- d) the car may move with constant speed.

Example 6.6 A small sphere of mass m is attached to the end of a cord of length R and set into motion in a *vertical* circle about a fixed point O shown. Determine the tangential acceleration of the sphere and the tension in the cord at any instant when the speed of the sphere is v and the cord makes an angle θ with the vertical.



Solution The free body diagram of the block is shown.

mg has to be resolved into a radial and a tangential component. For the tangential direction, we have

$$\begin{aligned} \sum F_t = ma_t &\Rightarrow mg \sin \theta = ma_t \Rightarrow \\ a_t &= g \sin \theta \end{aligned}$$

To find the tension we use

$$\sum F_r = ma_r \Rightarrow mg \cos \theta - T = -m \frac{v^2}{R} \Rightarrow$$

$$T = m \left(\frac{v^2}{R} + g \cos \theta \right)$$

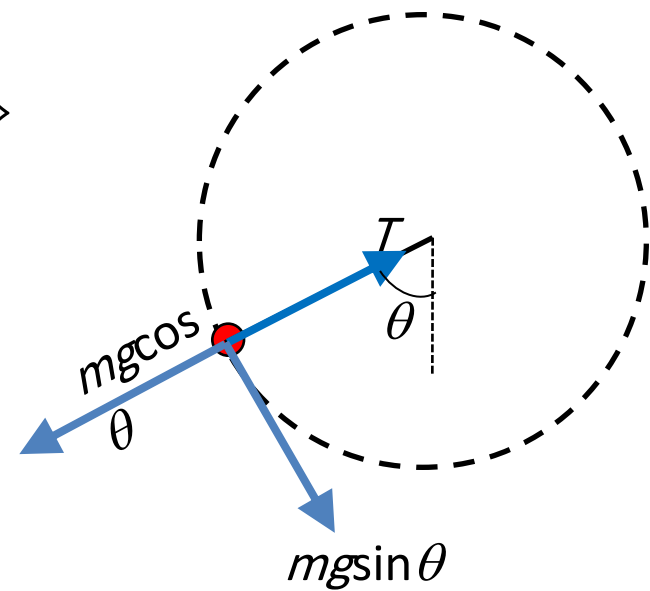
Note that at the bottom point $\theta=0$ the tension is maximum with

$$T_{top} = m \left(\frac{v^2}{R} + g \right)$$

While at the top point $\theta=\pi$ the tension is minimum with

$$T_{bottom} = m \left(\frac{v^2}{R} - g \right) \quad \text{What happen if at the top point } v^2 \leq gR$$

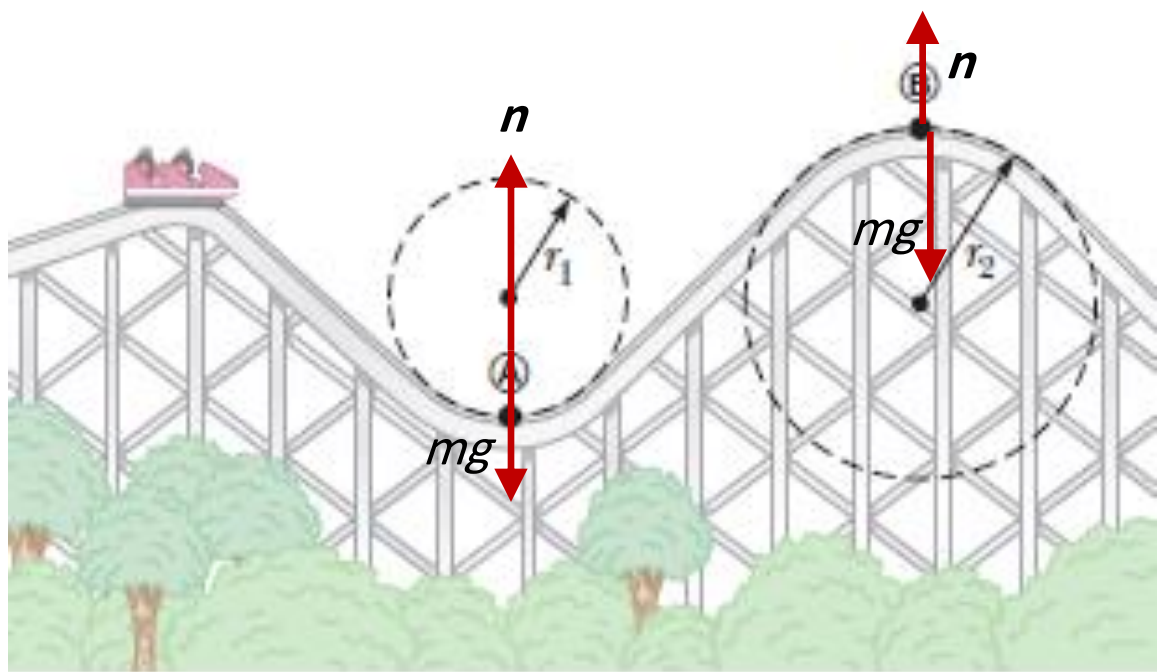
In this case, T is negative, which is impossible, and this means that the ball never reaches the top of the circle. At some point on the way up, the tension in the string goes to zero and the ball becomes a projectile.





Example (Problem 6.16)

A roller-coaster car has a mass of 500 kg when fully loaded with passengers. The tracks at points A and B are parts of vertical circles of radius $r_1=10.0$ m and $r_2=15.0$ m, respectively.



- (a) If the vehicle has a speed of 20.0 m/s at point A , what is the force exerted by the track on the car at this point?
- (b) What is the maximum speed the vehicle can have at point B and still remain on the track?

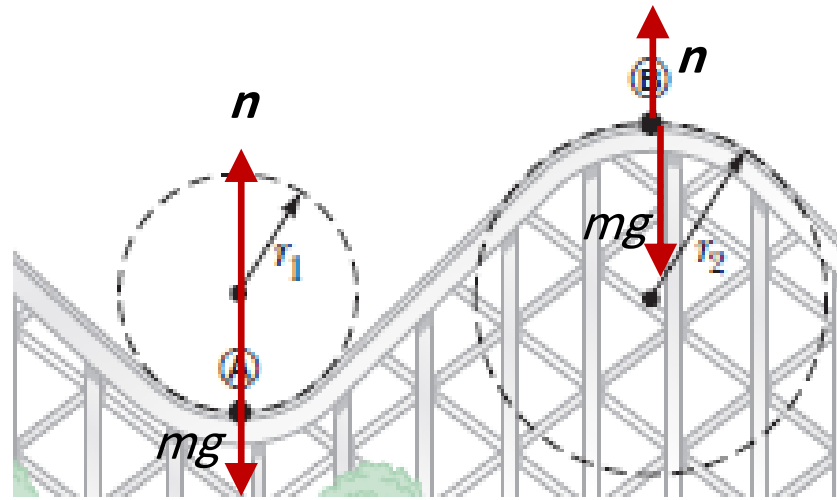
Solution The free body diagram of the vehicle is shown at the two positions. At point A , n is toward the center while mg is outward from the center, and vice versa at point B . Now we use

$$\sum \vec{F}_c = m \frac{v^2}{r}$$

a) At point A: $n - mg = m \frac{v^2}{r} \Rightarrow$

$$n = m \left(\frac{v^2}{r} + g \right)$$

$$= 500 \left(\frac{(20.0)^2}{10.0} + 9.8 \right) = 2.49 \times 10^4 \text{ N}$$



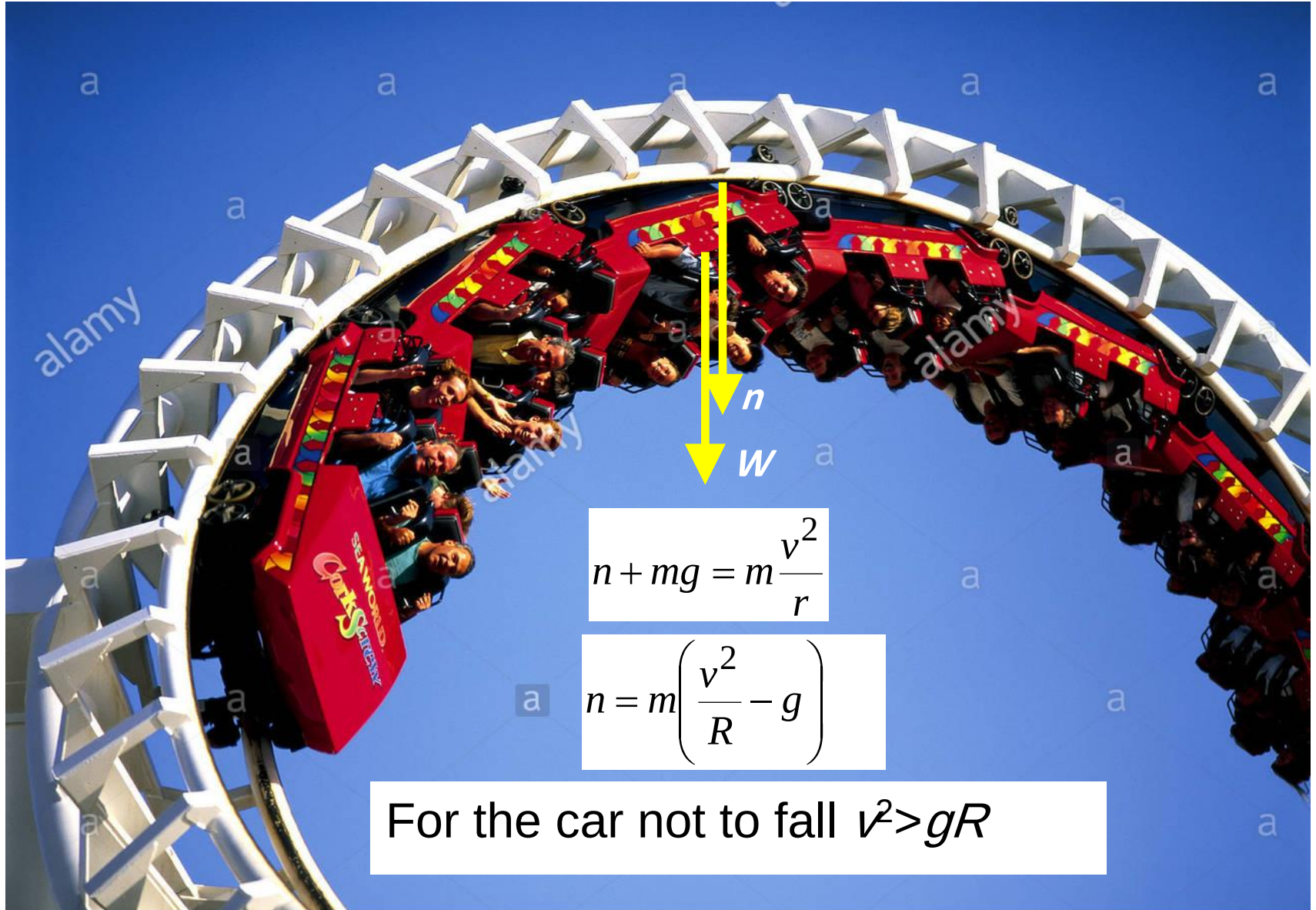
b) At point B: $mg - n = m \frac{v^2}{r} \Rightarrow n = m \left(g - \frac{v^2}{r} \right)$

For the vehicle to be on track, the normal force must have a positive value, that is, $n > 0$.

$$m \left(g - \frac{v^2}{r} \right) > 0 \Rightarrow v < \sqrt{gr} = 12.1 \text{ m/s}$$

This means that if the vehicle remains move with the speed of 20.0 m/s it will never reaches point B.

The roller coaster



$$n + mg = m \frac{v^2}{r}$$

$$n = m \left(\frac{v^2}{R} - g \right)$$

For the car not to fall $v^2 > gR$

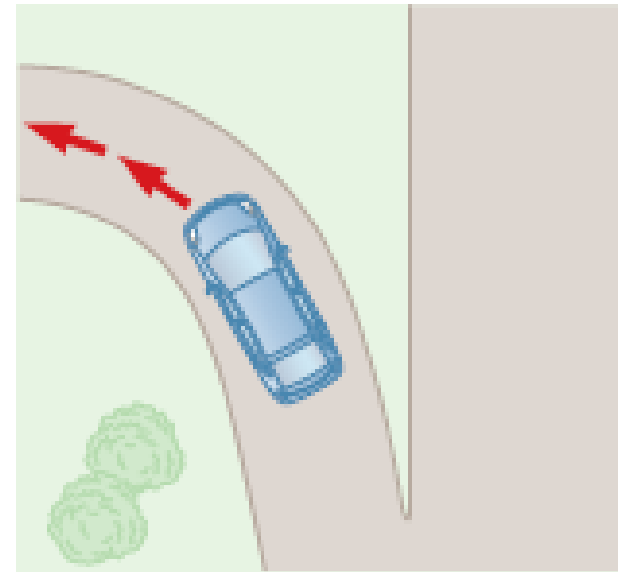
6.3 Motion in Accelerated Motion

We have mentioned that Newton's laws are applied only in inertial frames. Let us show how an observer in an accelerated frame, which is not inertial, can analyze the motion of an object.

In the example of the accelerated train and the roller skater in the last chapter, an observer inside the train sees you moving backward even though there are no forces responsible for your horizontal motion. A fictitious force appears to act on you in a direction opposite to the direction of the train's acceleration.

As another situation, consider a car traveling along a highway at a high speed and then makes a turn to the left, as shown.

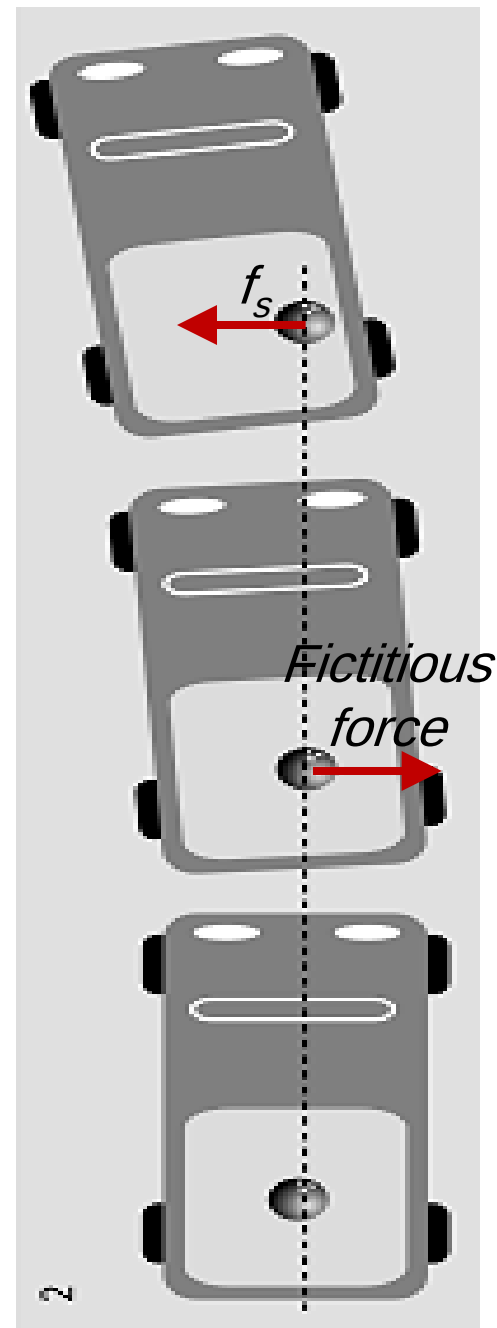
According to an observer inside the car (non-inertial frame), a passenger inside the car would slide to the right and would hit the door of the car.



What causing the passenger to move outward?
According to the observer inside the car there is no forces responsible to the horizontal motion of the passenger, and so Newton's laws of motion are not valid.

A popular but *incorrect* explanation is that a force acting toward the right pushes the passenger outward from the center of the circular path. Although often called the "centrifugal force," it is a fictitious force.

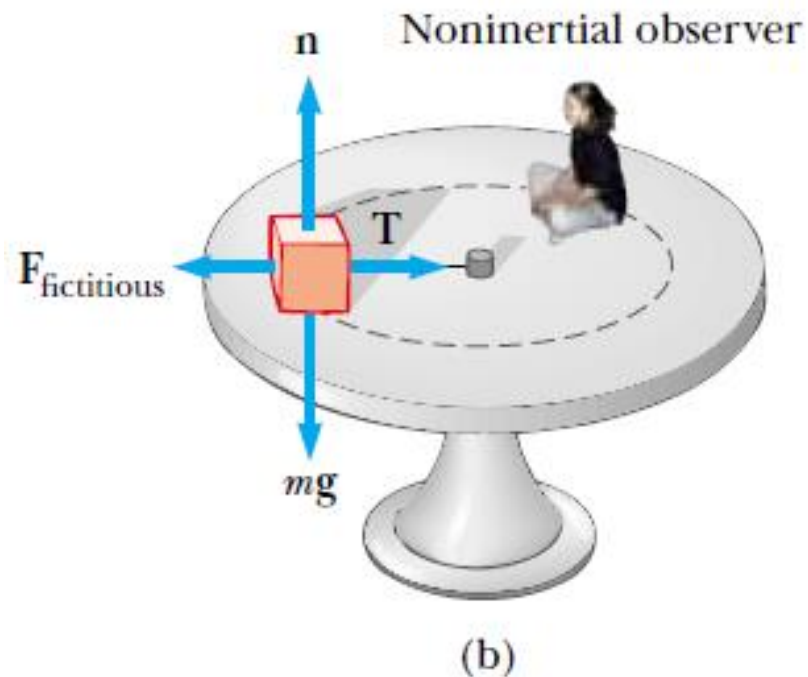
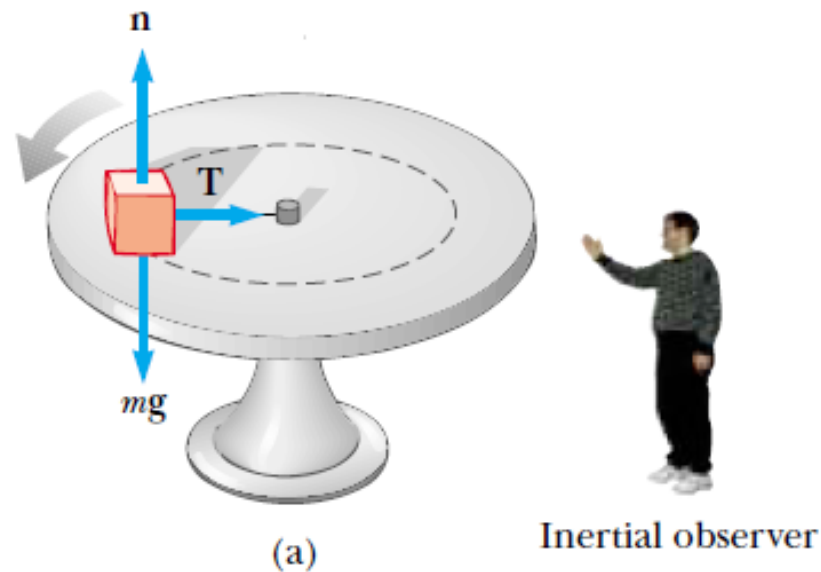
For an observer in the ground (inertial frame), the passenger tends to move along the original straight-line path, which is in accordance with Newton's first law: the natural tendency of an object is to continue moving in a straight line.



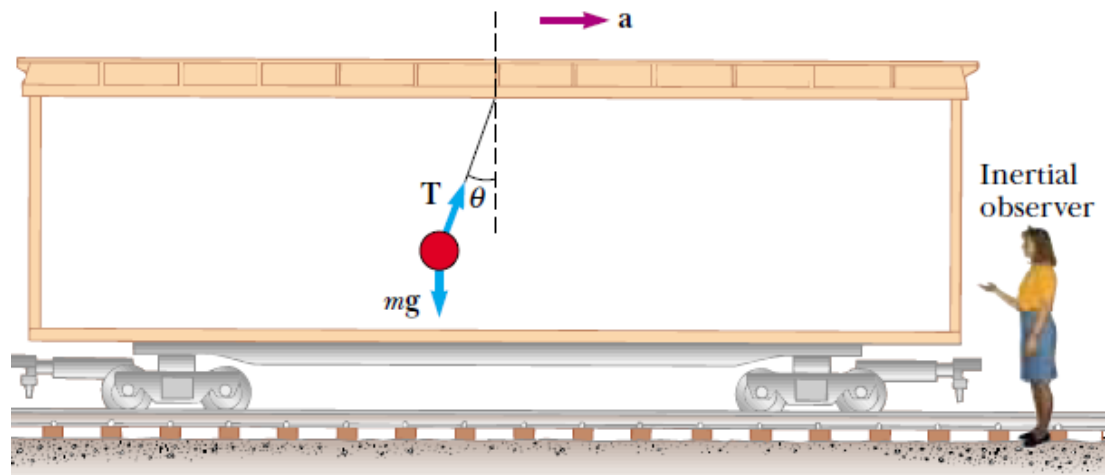
As another situation, consider a block of mass m lying on a horizontal, frictionless turntable is connected to a string attached to the center of the turntable, as shown. How would each of the observers analyze the motion of the block?

According to an inertial observer, the block undergoes a circular motion with the tension serves as the centripetal force.

According to a non-inertial observer attached to the turntable, the block is at rest. Therefore, she must introduce a fictitious outward force (centrifugal force) to balance the inward force exerted by the string.

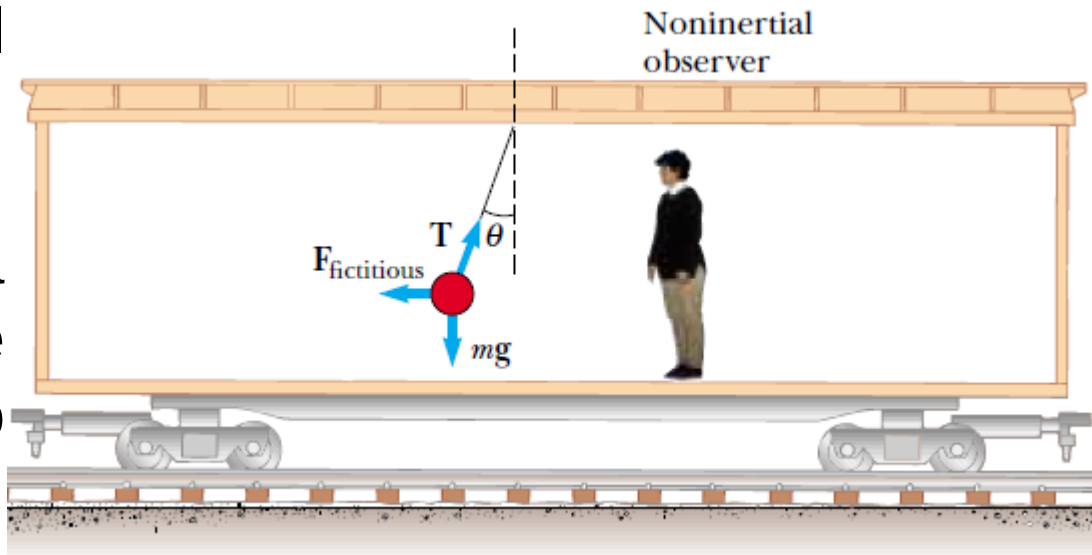


As a last example, consider a small sphere of mass m is hung by a cord from the ceiling of a boxcar that is accelerating to the right, as shown.



The inertial observer concludes that the acceleration of the sphere is the same as that of the boxcar and that this acceleration is provided by the horizontal component of T .

According to the non-inertial observer riding in the car, the sphere is at rest. Therefore, he introduces a fictitious force in the horizontal direction to balance the horizontal component of T .



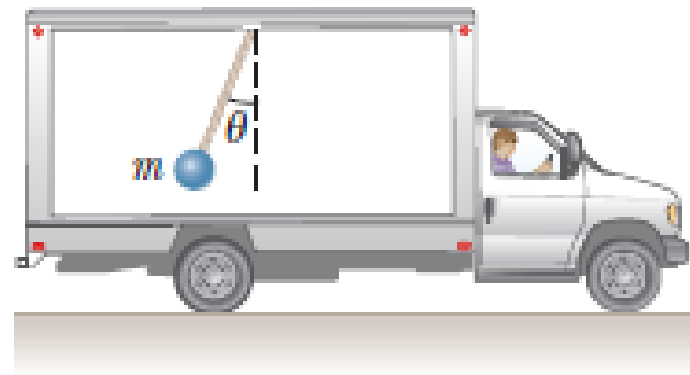
• Quick Quiz (3)

Consider the passenger in the car making a left turn mentioned previously. Which of the following is correct about forces in the horizontal direction if he is making contact with the right-hand door?

- a) The passenger is in equilibrium between real forces acting to the right and real forces acting to the left.
- b) The passenger is subject only to real forces acting to the right.
- c) The passenger is subject only to real forces acting to the left.
- d) None of those statements is true.

• Quick Quiz (4)

An object of mass m is suspended from the ceiling of a moving truck. The ball fluctuates forthward and backward during its journey. This means that:



- a) The truck is accelerating.
- b) The truck is decelerating.
- c) The truck is moving with constant velocity.
- d) The truck accelerates and decelerates frequently during its journey.