

CHAPTER 7

ENERGY OF A SYSTEM

In this chapter we are going to discuss the following topics:

- ❑ **Work of Constant Force**
- ❑ **Work of Varying Force**
- ❑ **Work-Kinetic Energy Theorem**
- ❑ **Conservative & Non-Conservative Forces**
- ❑ **Potential Energy (Gravitational & Elastic)**

Recommended Problems:

5, 14, 15, 25, 33, 39, 41, 42, 59, 63, 64

7.1 Systems and Environments

A *system* is a small portion of the Universe.

A valid system may be:

A single object or particle.

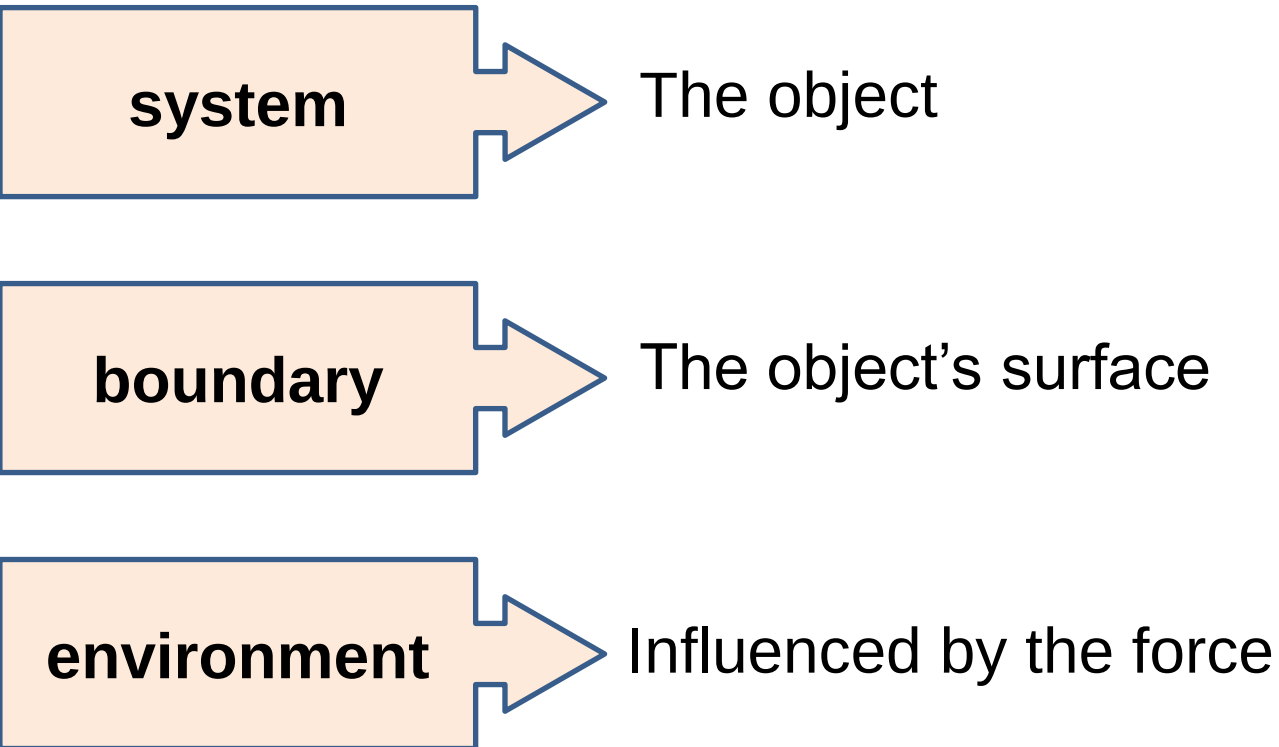
A collection of objects or particles.

A region of space.

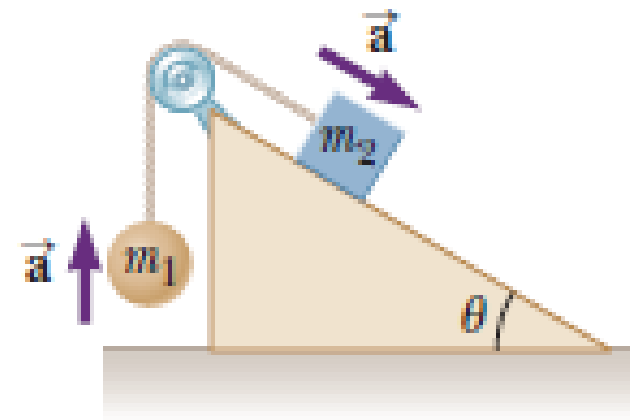
Vary with times, in size, and shape.

A *boundary* is an imaginary surface (not necessarily coinciding with a physical surface) that divides the Universe into the system and the *environment* surrounding the system.

Example: A force applied to an object in empty space.



Example: A block and a ball connected by a cord that passes over a pulley, as shown.



system

The block, the ball and the cord

boundary

imaginary

environment

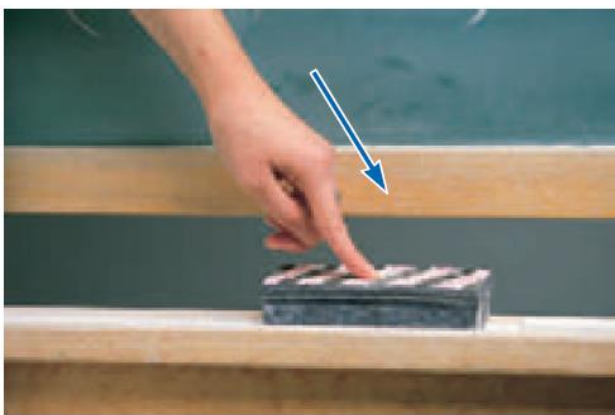
Influenced by the gravitational forces on the ball and the block, the normal and friction forces on the block, and the force exerted by the pulley on the cord.

7.2 Work Done by a Constant Force

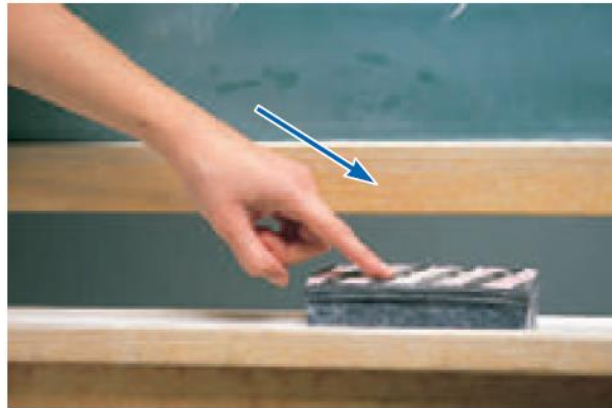
Work is one of the mechanisms by which a system can be influenced by its environment.

Work in physics has a meaning different from its everyday meaning.

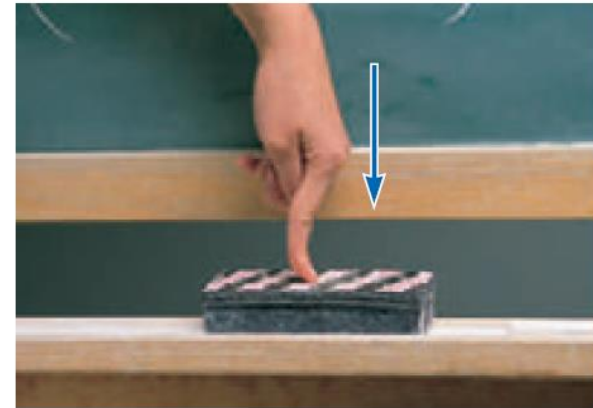
To understand the concept of work, consider a force applied to a chalkboard eraser that can slide along a tray, as shown.



a



b

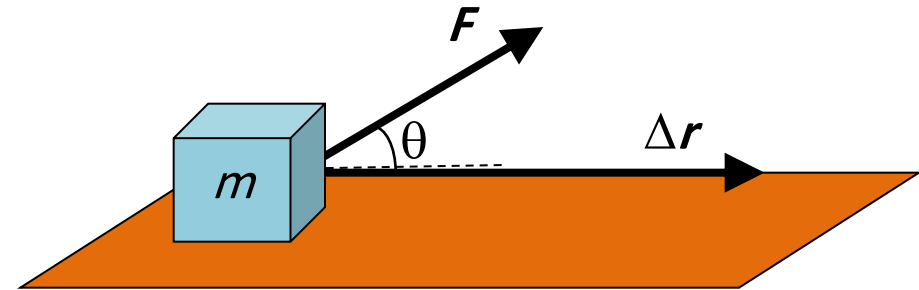


c

The push applied in Figure (b) does more to move the eraser than the push in Figure (a), while in Figure (c) the applied force does not move the eraser at all, regardless of how hard it is pushed.

If we want to know how effective the force is in moving the eraser, we must consider not only the magnitude of the force but also its direction.

Consider an object displaced horizontally under the action of the constant force as shown



The work done by this force is defined as

$$W = \vec{F} \cdot \Delta \vec{r} = F \Delta r \cos \theta$$

⇒ The work is a scalar quantity with its SI unit of N.m or Joule (J).

The work may be positive, negative, or zero depending on the direction of $\mathbf{F}$ relative to $\Delta \mathbf{r}$. It is positive if $\mathbf{F}$ is in the direction of $\Delta \mathbf{r}$, and is negative if $\mathbf{F}$ is in the opposite direction of $\Delta \mathbf{r}$. An example of the negative work is the work done by a frictional force. It is zero when F and $\Delta \mathbf{r}$ are perpendicular.

Work can be considered as an energy transfer.

- ❖ If the work is done on a system and it is positive, energy is transferred to the system.
- ❖ If the work is done on a system and it is negative, energy is transferred from the system.

This will result in a change in the amount of energy (different types) inside the system.

• Quick Quiz 1

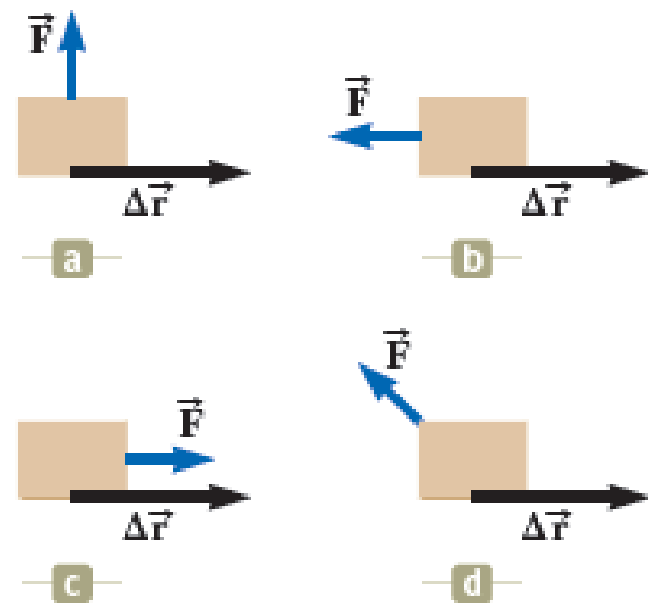
The gravitational force exerted by the Sun on the Earth holds the Earth in an orbit around the Sun. Assuming it is perfectly circular. The work done by this gravitational force as the Earth moves through a displacement in its orbital path is

- a) positive.
- b) negative.
- c) zero.
- d) Impossible to determine

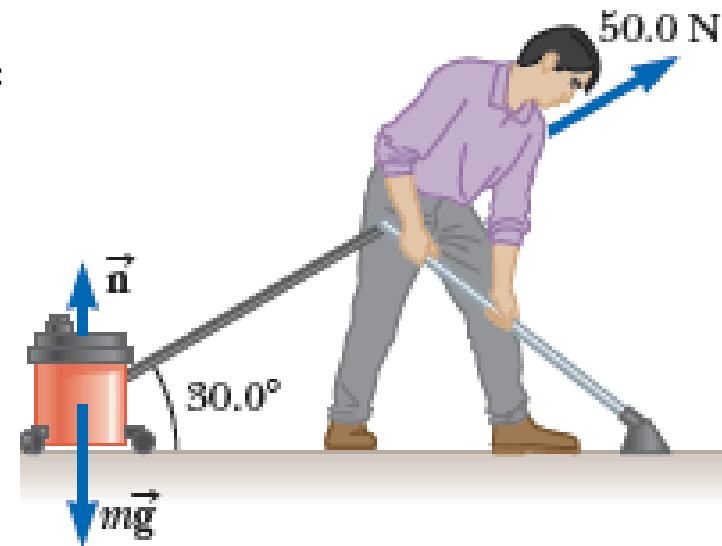
• Quick Quiz 2

The figure shows four situations in which a force is applied to an object. Rank the situations in order of the work done by the force on the object, from most positive to most negative.

(c) then (a) then (d) and then (b)



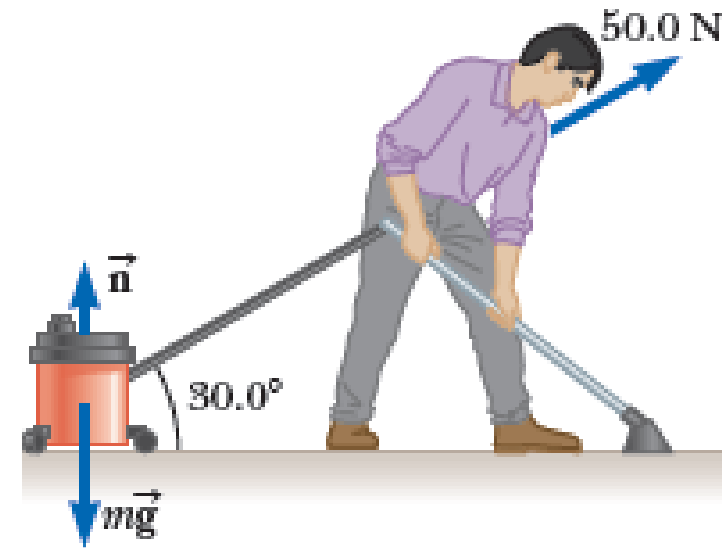
Example 7.1 A man cleaning a floor pulls a vacuum cleaner with a force of magnitude $F=50.0$ N at an angle of 30.0° with the horizontal as shown. Calculate the work done by the force on the vacuum cleaner as the vacuum cleaner is displaced 3.00 m to the right.



Solution As the displacement is horizontal so we write

$$W = F\Delta r \cos \theta$$
$$= 50.0(3.00)\cos 30.0^\circ = 130 \text{ J}$$

Note that both n and mg do no work on the vacuum.



Example 7.3 A particle moving in the x - y plane undergoes a displacement given by $\Delta\vec{r} = (2.0\hat{i} + 3.0\hat{j})\text{m}$ as a constant force acts on the particle given by $\vec{F} = (5.0\hat{i} + 2.0\hat{j})\text{N}$. Calculate the work done by $\vec{F}$ on the particle.

Solution Using the relation

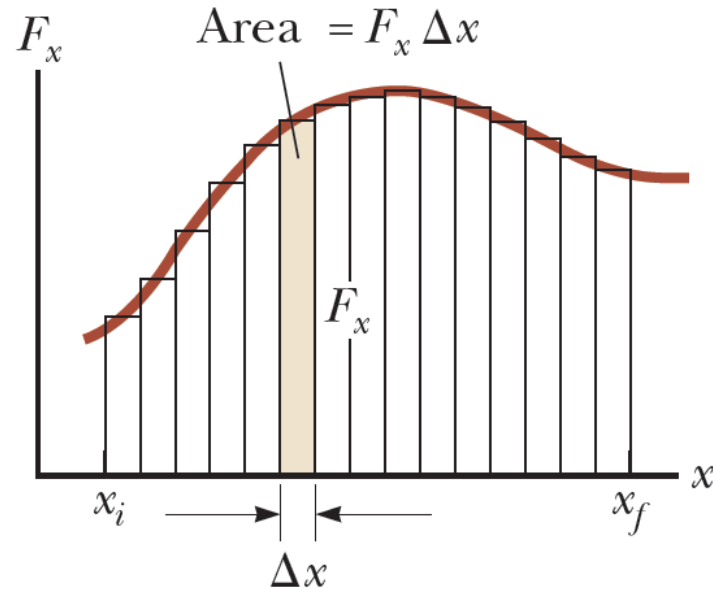
$$W = \vec{F} \cdot \Delta\vec{r} \quad \Rightarrow$$

$$W = (5.0)(2.0) + (2.0)(3.0) = 16 \text{ J}$$

7.4 Work Done by a Varying Force

Consider a particle displaced along the x -axis under a force that is not constant over the displacement. In this case we have to divide the displacement into small elements each of Δx . The work during one of these elements is

$$W \cong F_x \Delta x$$



Now the total work done during the total displacement is

$$W \cong \sum_{x_i}^{x_f} F_x \Delta x$$

If $\Delta x \rightarrow 0$, the total work during the total displacement is

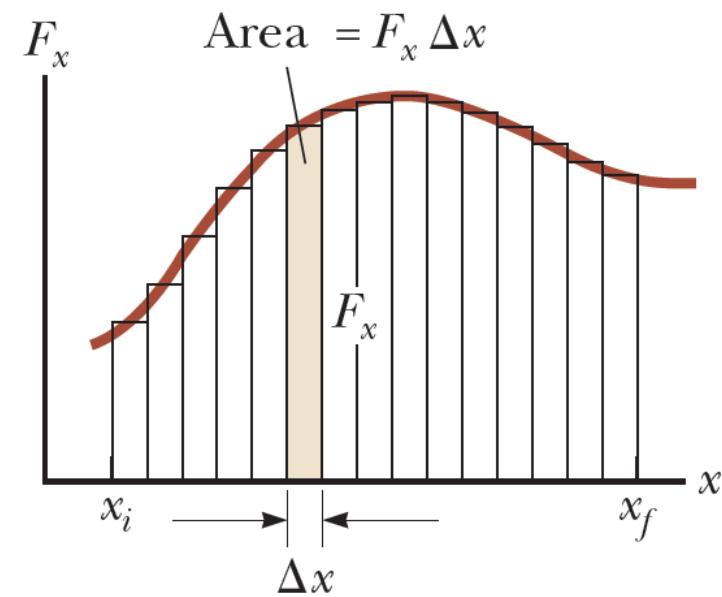
$$W = \int_i^f F_x dx$$

In general we write

$$W = \int_i^f \vec{F} \cdot d\vec{r}$$

Note that if $\vec{F}$ is constant we have

$$W = \vec{F} \cdot \int_i^f d\vec{r} = \vec{F} \cdot (\vec{r}_f - \vec{r}_i) = \vec{F} \cdot \Delta\vec{r}$$



Which is the equation of the work done by a constant force defined previously.

It is clear from the figure that the work done by the component F_x is equal to the area under the force vs. x curve.

If more than one force acts on a system, the total work done on the system is just the work done by the net force.

Example 7.4 force acting on a particle varies with x as shown. Calculate the work done by the force on the particle as it moves from $x=0$ to $x=6.0$ m.

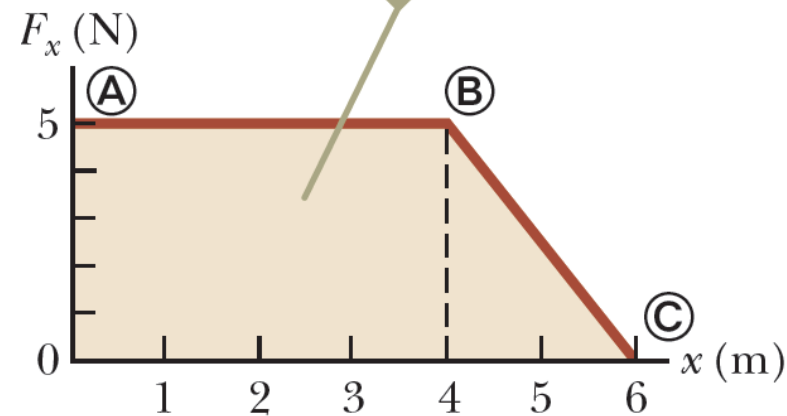
Solution The work done by the force is equal to the area under the curve from $x=0$ to $x=6.0$ m. This is the same as the area from A to B plus the area from B to C .

$$W_{A \rightarrow B} = 5 \times 4 = 20 \text{ J}$$

$$W_{B \rightarrow C} = \frac{1}{2}(5 \times 2) = 5.0 \text{ J}$$

$$W_{A \rightarrow C} = W_{A \rightarrow B} + W_{B \rightarrow C} = 25 \text{ J}$$

The net work done by this force is the area under the curve.



Work Done by a Spring

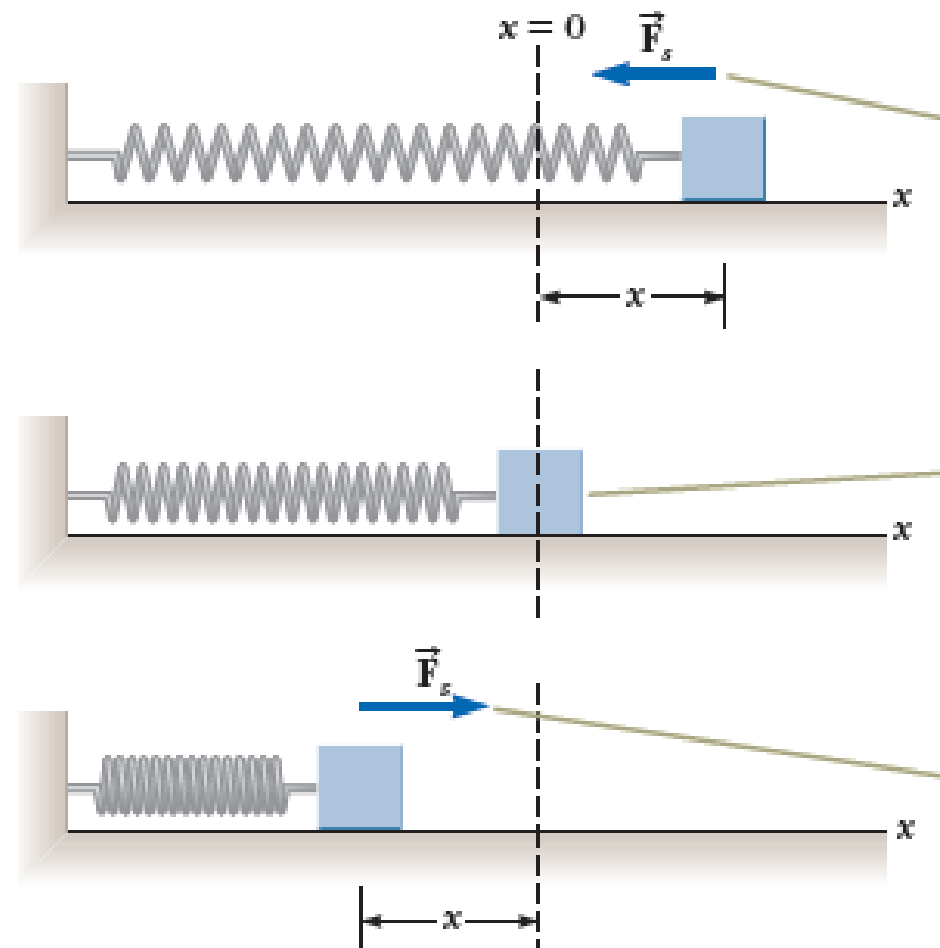
When a block attached to a spring the spring affect the block with a force given by Hook's law:

$$F_s = -kx$$

where x is the position of the block from its equilibrium position ($x=0$) and the force constant k is a measure of the stiffness of the spring.

The minus sign in Hook's law tells that the force of the spring is always opposes the displacement.

Let us calculate the work done by the force $\mathbf{F}_s$, as the body moves from an initial position x_i to a final position x_f



$$W_s = \int_i^f F_s dx = -k \int_{x_i}^{x_f} x dx \quad \Rightarrow \quad W_s = \frac{1}{2} k (x_i^2 - x_f^2)$$

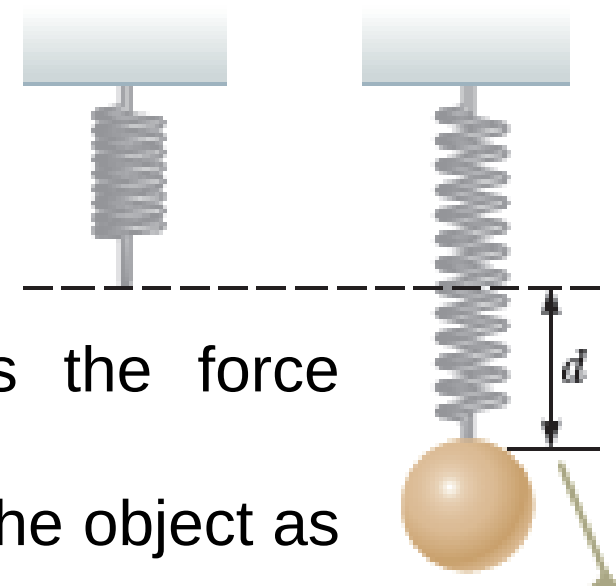
From the last eq. we can conclude that

(i) If the mass moves toward the equilibrium position then $x_f = 0$ and the work is positive. This is due to the fact that the force and the displacement, in this case, are in the same direction.

(ii) If the mass moves away from the equilibrium position then $x_i = 0$, and the work is negative. Here the force and the displacement are in opposite directions.

Remark: The work done by an external force in compressing or stretching a spring is equal to the negative of the work done by the spring's force during the corresponding displacement.

Example 7.5 A spring is hung vertically, and an object of mass m is attached to its lower end. The spring stretches a distance d from its equilibrium position as shown.

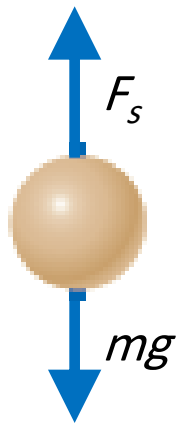


- (a) If $m=0.55$ kg, and $d=2.0$ cm, what is the force constant of the spring?
- b) How much work is done by the spring on the object as it stretches through this distance?

Solution a) The free-body diagram of the mass is shown. Since the object is in equilibrium, the net force is zero, i.e.,

$$\sum \vec{F}_y = 0 \Rightarrow F_s = mg = kd \Rightarrow$$

$$k = \frac{mg}{d} = \frac{0.55(9.80)}{2.0 \times 10^{-2}} = 2.7 \times 10^2 \text{ N/m}$$



b) To find the work we use

$$W_s = \frac{1}{2} k (x_i^2 - x_f^2) = \frac{1}{2} (2.7 \times 10^2) \left(0 - (2.0 \times 10^{-2})^2 \right) = -5.4 \times 10^{-2} \text{ J}$$

7.5 Work-Kinetic Energy Theorem

Consider a system consisting of an object moves through a displacement along the x-axis under the action of a net external force F_x . We write

$$W_{ext} = \int_i^f F_x dx$$

Using Newton's second law we have $F_x = ma_x = m \frac{dv}{dt}$

$$\text{but } \frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt} = \frac{dv}{dx} v \Rightarrow F_x = mv \frac{dv}{dx} \Rightarrow$$

$$W_{ext} = m \int_i^f v \frac{dv}{dx} dx = m \int_i^f v dv = \frac{1}{2} mv^2 \Big|_{v_i}^{v_f} = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2 \Rightarrow$$

$W_{ext} = K_f - K_i = \Delta K$

 The work-kinetic energy theorem.

$$\text{with } K = \frac{1}{2}mv^2$$

It is called the kinetic energy of the object. It represents the energy associated with the motion of the particle.

The kinetic energy is a scalar quantity with the unit of J, the same unit of work.

Another way to write the theorem is $K_f = K_i + W_{ext}$

Which tells that the final kinetic energy of an object is equal to its initial kinetic energy plus the work done on it.

It is clear that if W_{ext} is positive then $K_f > K_i$.

The theorem tells that One possible result of doing work on a system is changing the kinetic energy of the system.

• Quick Quiz (3)

For a particle moving with constant speed the net work on the particle:

- a) Is always positive.
- b) Is always negative.
- c) Is always zero.
- d) it depends on the path followed.

• Quick Quiz (4)

A dart is inserted into a spring-loaded dart gun by pushing the spring in by a distance x . For the next loading, the spring is compressed a distance $2x$. How much faster does the second dart leave the gun compared with the first?

- a) Two times as fast.
- b) Four times as fast.
- c) He same..
- d) Half as fast.

Example 7.6 A 6.0-kg block initially at rest is pulled along a frictionless, horizontal surface by a constant force of $F = 12$ N. Find the block's speed after it moved a horizontal distance of 3.0 m.

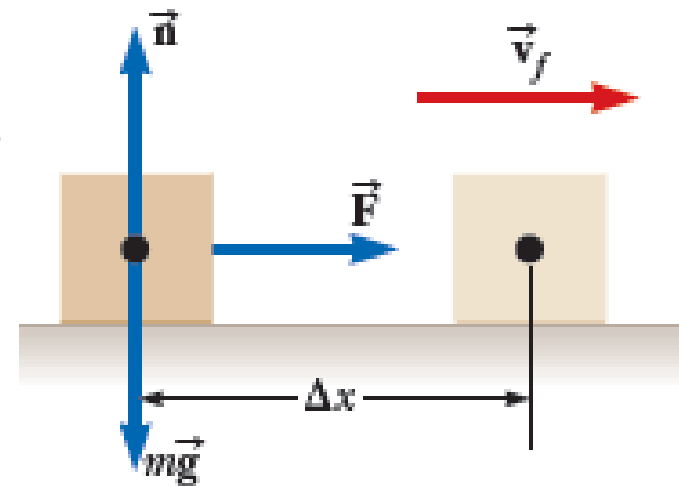
Solution Three external forces acting on the block. The normal force and its weight do no work since they are perpendicular to the displacement. The work done by $\mathbf{F}$ is

$$W_{ext} = F\Delta x = 12(3.0) = 36 \text{ J}$$

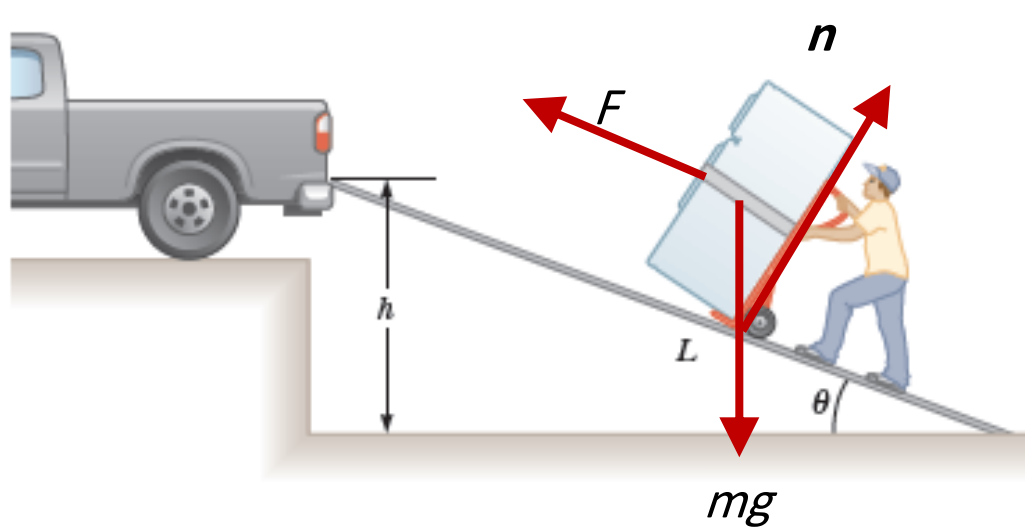
Applying the work-energy theorem, we have

$$W_{ext} = K_f - K_i = \frac{1}{2} m(v_f^2 - v_i^2) \Rightarrow$$

$$36 = \frac{1}{2} (6.0) v_f^2 \Rightarrow v_f = \sqrt{12} = 3.5 \text{ m/s}$$



Example 7.7 A man wishes to load a refrigerator onto a truck using a ramp at angle θ as shown. He claims that less work would be required to load the truck if the length L of the ramp were increased. Is his claim correct?



Solution Suppose that the refrigerator is moved up the ramp at constant speed. In this case $\Delta K=0$, so we write

$$W_{ext} = K_f - K_i = \Delta K = 0$$

As it is clear from the figure, the forces that do work are the force of the man F , and the weight of the refrigerator. So we have

$$W_{ext} = W_{man} + W_{gravity} = 0 \Rightarrow W_{man} = -W_{gravity}$$

$$\text{But } W_{gravity} = -mg(L \sin \theta) = -mgh \Rightarrow W_{man} = mgh$$

That is the work required depends only on the height of the ramp and not on L or θ .

7.7 Conservative and Non-conservative Forces

Conservative Force: The force is conservative if the work done by it on a particle that moves between two points depends only on these points and not on the path followed. Or the work done by a conservative force through any closed path is zero.

As an example of a conservative force is the force of the spring.

The work done by such a force is

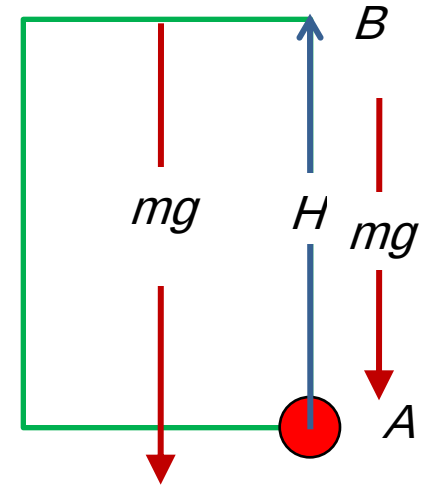
$$W_s = \frac{1}{2}k(x_i^2 - x_f^2)$$

As it is clear from the Equation, the work done by a spring depends only on x_i and x_f .

As another example of a conservative force is the force of gravity. Consider a particle goes from point A to a point B .

The work done by gravity if the particle followed path 1, (the blue path) is simply

$$W_1 = -mgH$$



If the particle followed another path, the green path, we see again that the work done by gravity is also

$$W_2 = -mgH$$

This means that the work done by gravity is independent on the path followed but depends only on the initial and final point of the motion.

Non-conservative Force: The force is non-conservative if the work done by that force on a particle that moves between two points depends on the path taken between those points.

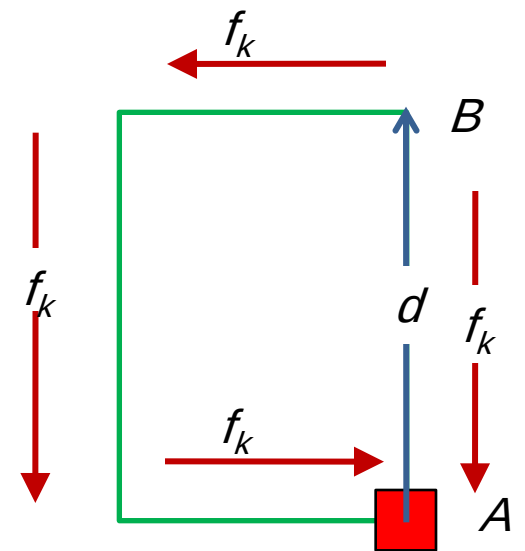
The Frictional force is an example of a non-conservative force.

If a block moves a distance d from A to B along a rough horizontal surface following two paths then for the blue path,

$$W_1 = -f_k H$$

while for the green path,

$$\text{while } W_2 = -f_k H - 2 f_k d$$



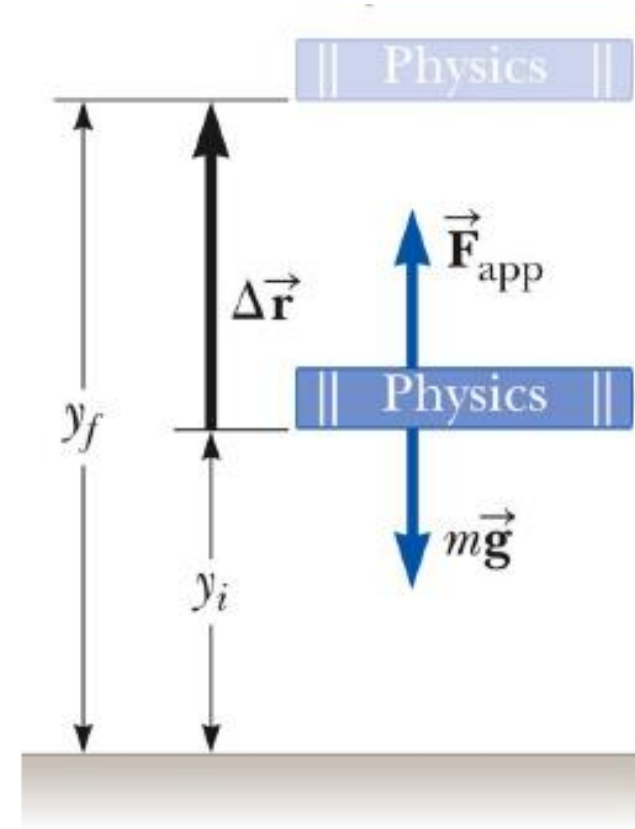
Where d is the horizontal distance along the surface. It is clear now that the work is different depending on the path followed.

7.6 Potential Energy of a System

Consider the system of a book and the Earth. Let us do work, by applying a force F_{app} to lift the book through a vertical displacement $\Delta\vec{r}$

This work must change the energy of the system. Since the initial and the final kinetic energy of the system is zero, the kinetic energy of the system is not changed. So what type of energy is changed? This type is called the potential energy of the system.

So we can say that, the work done by a conservative force, W_C is related to the change in potential energy as



$$W_C = -\Delta U = -(U_f - U_i)$$

With U is called the potential energy.

The potential energy, like work and kinetic energy, is a scalar quantity with the SI unit of J.

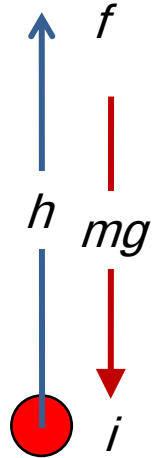
- ❖ The potential energy is associated only with conservative forces.
- ❖ The potential energy is associated with internal forces within the system.
- ❖ The potential energy of an object is due to its position.

Gravitational Potential Energy

Since the force of gravity is conservative, we can associate a gravitational potential energy function U_g to this force. To calculate such potential energy, we consider a particle that makes a displacement from an initial point i to a final point f ,

The force of gravity does a negative work given by $-mgh$ (why negative?).

$$\begin{aligned} -mgh &= -(U_f - U_i) \Rightarrow \\ mgh &= U_f - U_i \end{aligned}$$



Since we are concerned only by the change in the potential energy, U_i can be chosen to be zero.

The horizontal level at which U_i is zero is called the level of zero potential energy. $\Rightarrow$

$$U_g = mgh$$

where h is the vertical displacement above an arbitrary horizontal level (level of zero potential energy).

Remark: The gravitational potential energy is positive if the body is above the level of zero potential energy, and negative if the body is below the level.

Example 7.8 A trophy being shown off by a careless athlete slips from the athlete's hands and drops, from a height of 1.4 m, on his foot. Choosing floor as the level of zero potential energy, estimate the change in gravitational potential energy of the trophy as it falls. Repeat the calculation, using the top of the athlete's hand as the level of zero potential energy. If the trophy has a mass of 2.0 kg, and the top of a person's foot is about 0.05 m above the floor.

Solution (a) If the floor is the level of zero P.E. $\Rightarrow$

$$U_i = mgh_i = 2.0(9.80)(1.40) = 27 \text{ J}$$

$$U_f = mgh_f = 2.0(9.80)(0.05) = 1.0 \text{ J}$$

$$\Rightarrow \Delta U = U_f - U_i = 0.98 - 27.4 = -26 \text{ J}$$

(b) If the top of the athlete's hand is the level of zero P.E. $\Rightarrow$

$$U_i = mgh_i = 0$$

$$U_f = -mgh_f = 2.00(9.80)(1.35) = -26 \text{ J}$$

$$\Rightarrow \Delta U = -26 - 0 = -26 \text{ J}$$

This means ΔU is independent of the choice of the level representing the zero of potential energy.

Elastic Potential Energy

Consider the system of a particle attached to a spring and goes from the equilibrium position ($x_i = 0$) to a position where the spring is compressed or stretched a distance x ($x_f = x$), the force of spring does a work given by

$$W_s = \frac{1}{2}k(x_i^2 - x_f^2) = -\frac{1}{2}kx^2$$

$$U_s = -W_s = \frac{1}{2}kx^2$$

With U_i is taken to be zero at the equilibrium position.

Remarks: (i) The potential energy of the spring is zero at the equilibrium point ($x = 0$).

(ii) U_s is always positive, since x^2 is always positive.