

# CHAPTER 8

## CONSERVATION OF ENERGY

In this chapter we are going to discuss the following topics:

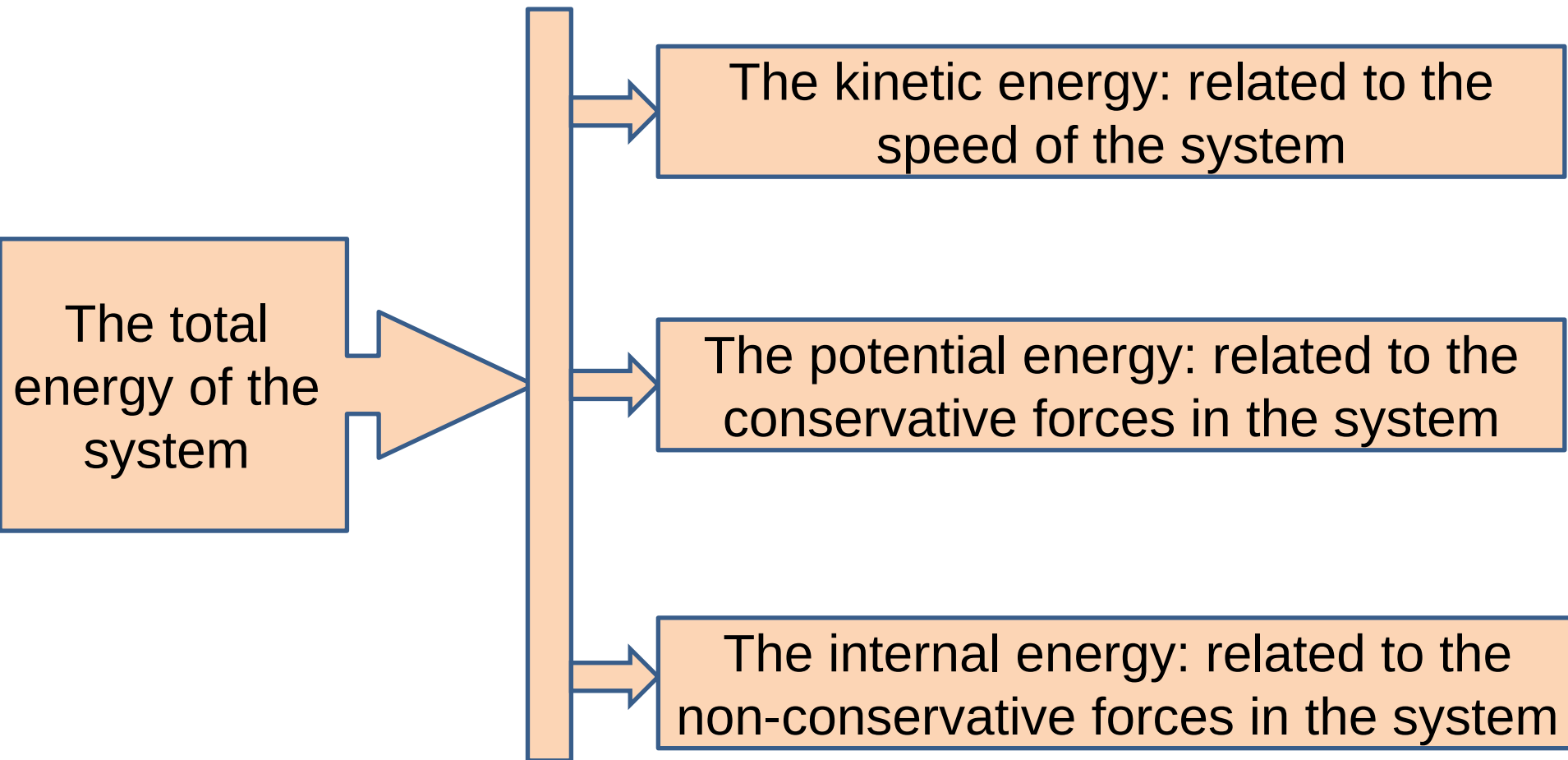
- ❑ **Energy of Non-Isolated Systems**
- ❑ **Isolated Systems & Conservation of Mechanical Energy**
- ❑ **Conservation of Energy with Non-Conservative Forces**
- ❑ **Power**

### Recommended Problems:

5, 6, 7, 8, 9, 11, 13, 14, 15, 17, 22, 23, 25, 27, 38, 43, 44, 45, 46, 59, 62, 63, 65, 68, 72, 73

## 8.1 Energy of Nonisolated System

The **nonisolated** system is the system in which energy can cross its boundary in a variety of ways so that the total energy of the system changes.



An object falling due to gravity is an example of a nonisolated system where energy is transferred to the object in the form of the work done by gravity. If the system is considered to be consist of the object and Earth, we have the isolated system. In this case the force due to gravity is an internal force.

The work-kinetic energy theorem of the previous chapter is valid only for nonisolated system where the external forces do work on the system.

### • Quick Quiz 1

Consider a block sliding over a horizontal surface with friction. Which of the following systems is isolated.

- a) The block.
- b) The surface.
- c) The Block-surface.
- d) Impossible to determine

For nonisolated system the energy can cross (into or out of) the boundary of the system by many ways:

***Work*** – transfers energy by applying a force and causing a displacement. Energy is transferred to the block by work.



***Mechanical Wave*** – transfers energy by allowing a disturbance to propagate through a medium. Energy leaves the radio from the speaker by *mechanical waves*.



***Heat*** – the mechanism of energy transfer that is driven by a temperature difference between two regions. Energy transfers to the handle of the spoon by *heat*.



***Matter Transfer*** – matter physically crosses the boundary of the system. **Energy enters the automobile gas tank by *matter transfer*.**



***Electrical Transmission*** – energy transfer into or out of a system by electric current. Energy enters the hair dryer by *electrical transmission*.



***Electromagnetic Radiation*** – energy is transferred by electromagnetic waves. Energy leaves the light-bulb by *electromagnetic radiation*.



System  
boundary

The change in the total  
amount of energy in  
the system is equal to  
the total amount of  
energy that crosses the  
boundary of the system.

Work

Heat

Mechanical  
waves

Kinetic energy  
Potential energy  
Internal energy

Matter  
transfer

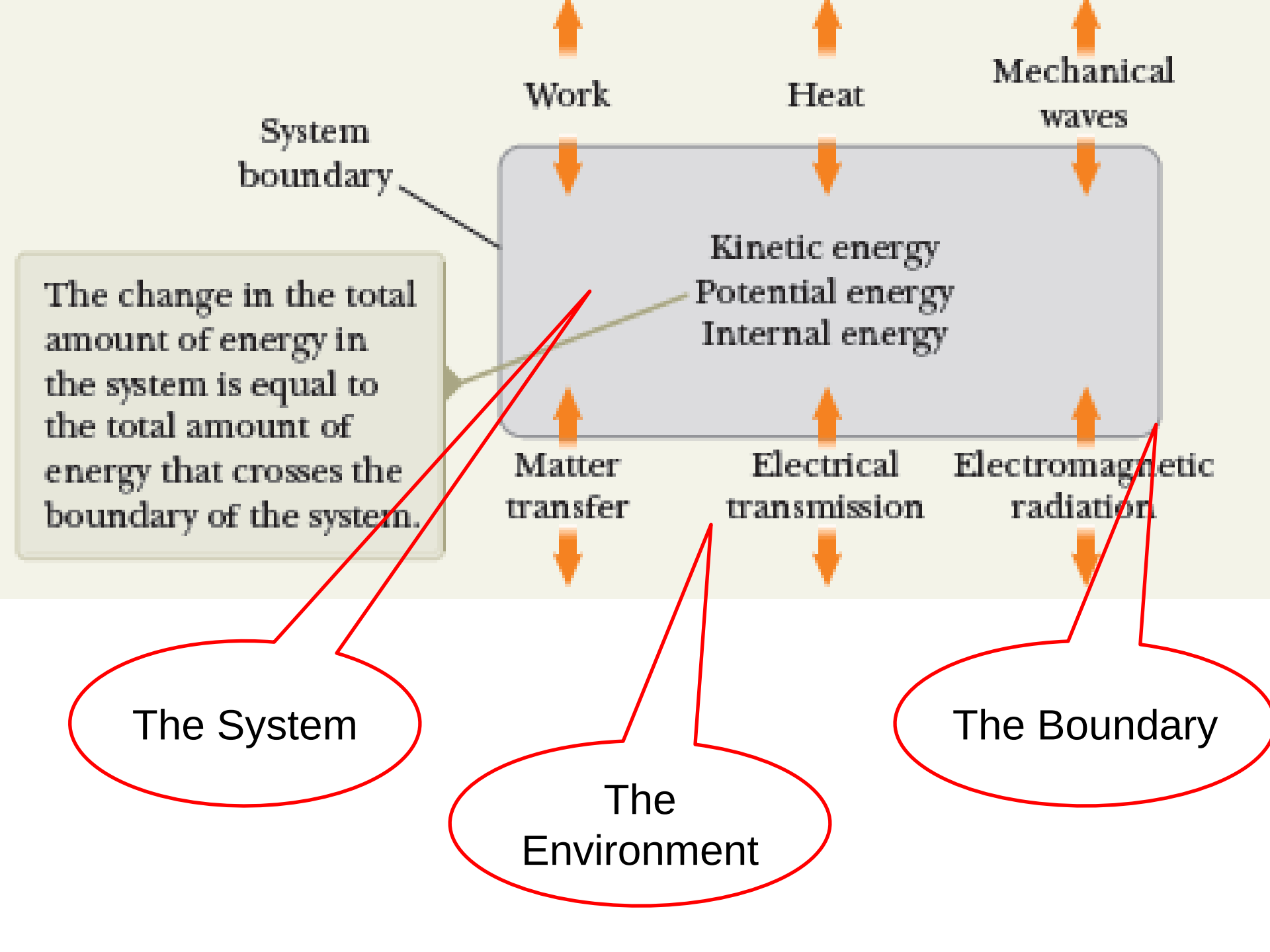
Electrical  
transmission

Electromagnetic  
radiation

The System

The  
Environment

The Boundary



In another word we can write

$$\Delta E_{system} = \sum T \quad \text{or} \quad \Delta K + \Delta U + \Delta E_{int} = \sum T$$

Where  $E_{system}$  is the total energy of the system which constitute of the kinetic energy, the potential energy, and the internal energy.

And  $T$  is the amount of energy transferred across the boundary of the system by all means mentioned before.

Suppose a force is applied to a nonisolated system such that it moves through a displacement. This means that the only transfer mechanism is work.

Then suppose the only effect on the system is to change it speed such that and the only kind of energy in the system that changes is the kinetic. In such a case the above equation reduces to

$$\Delta K = W$$

Which is the work-kinetic energy theorem discussed in the previous chapter

## 8.2 Energy of Isolated System (Mechanical Energy)

A system is called isolated if no energy or matter crosses the system boundary by any method.

This means that the net transfer of energy is zero, i.e.,  $\Sigma T=0$

In another word we say that for isolated systems, the total energy of the system is conserved.

Now if there are no work done by non-conservative forces within the system then the internal of energy of the system does not change ( $\Delta E_{\text{int}}=0$ ) so we left with the equation.

$$\Delta E_{\text{system}} = \Delta K + \Delta U = 0 \Rightarrow$$

$$(K_f + U_f) - (U_i + K_i) = 0 \Rightarrow$$

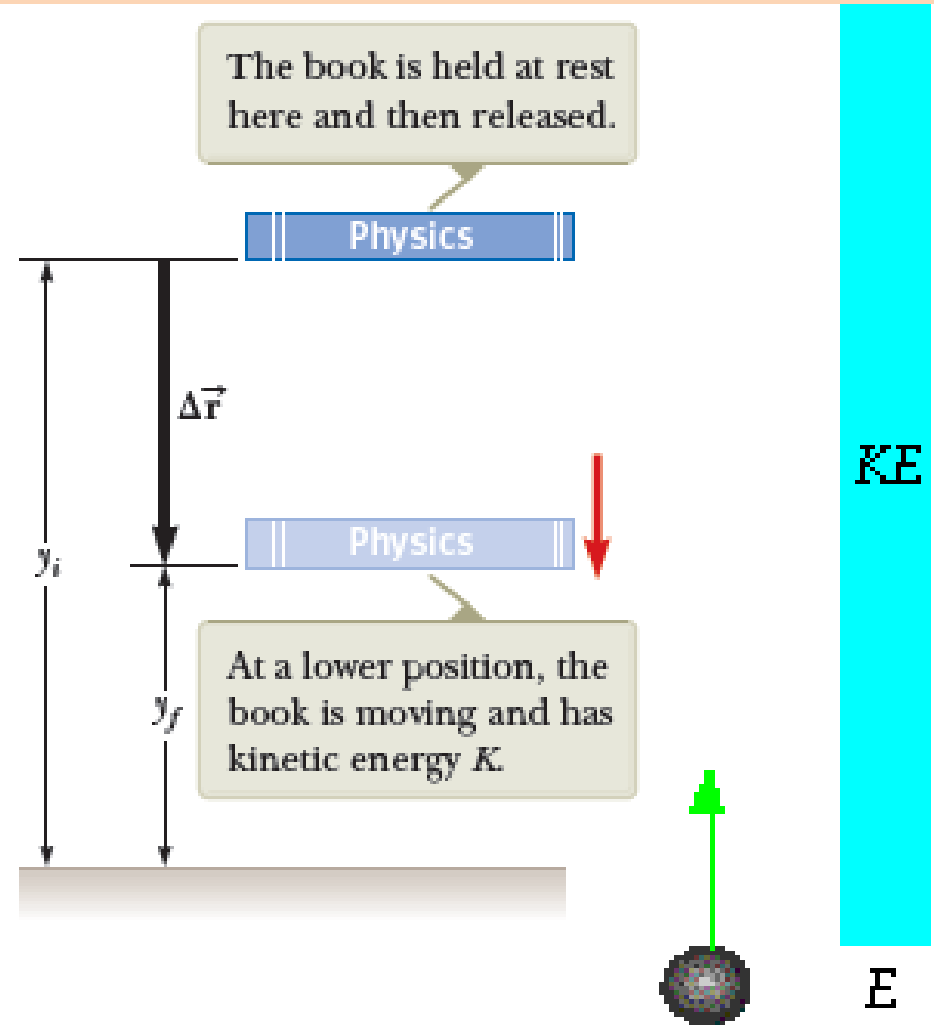
$$E_f - E_i = 0$$

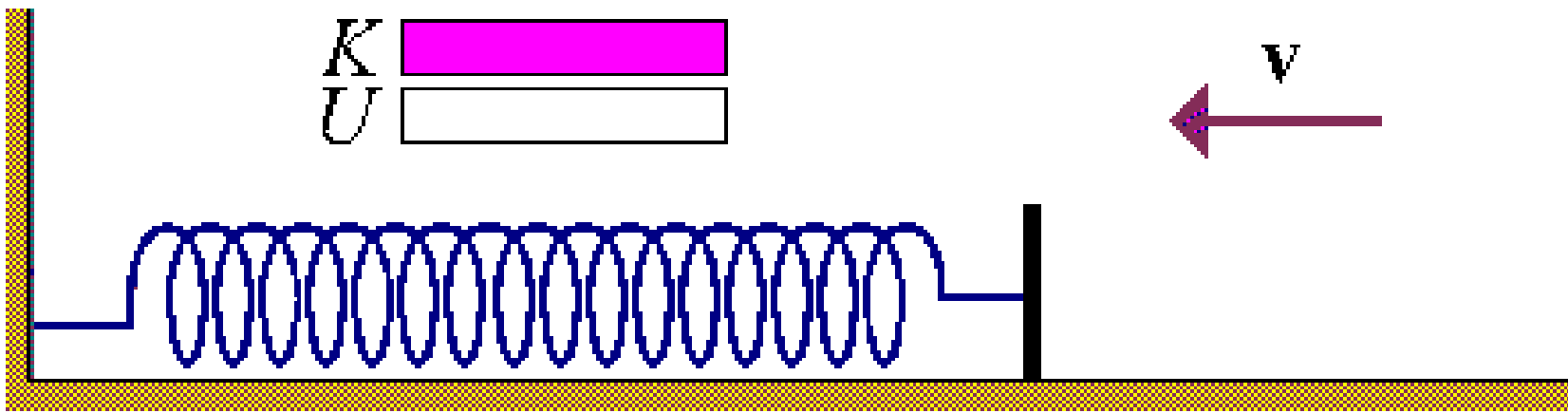
with  $E = K + U$

is called the mechanical energy.

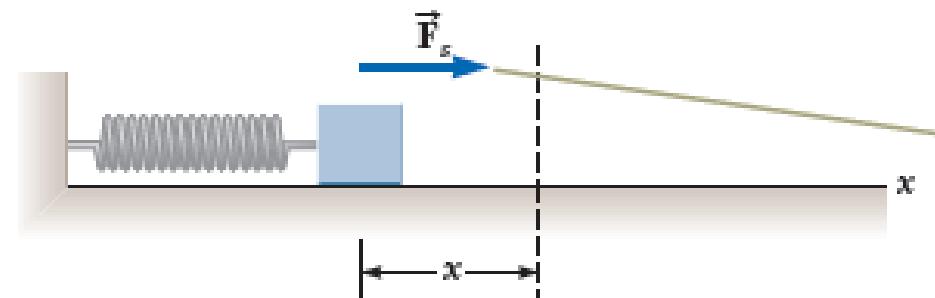
For an isolated system the mechanical energy is conserved provided that there are no work done by non-conservative forces within the system.

As an example of the isolated system, think about the book–Earth system shown. Since the Earth is not moving, the kinetic energy of the book can be considered as the kinetic energy of the system. Now as the book is released from rest, the kinetic energy increases while the potential energy decreases by the same amount such that the mechanical energy remains unchanged.





The same principle applied for a mass-spring system. If the spring is compressed (or stretched) a distance  $x$  and then released, the system gain kinetic energy by the same amount it loses potential energy.



At the equilibrium position, the potential energy is zero while the kinetic energy is maximum. At the turning points, the kinetic energy is zero while the potential energy is maximum.

## • Quick Quiz (2)

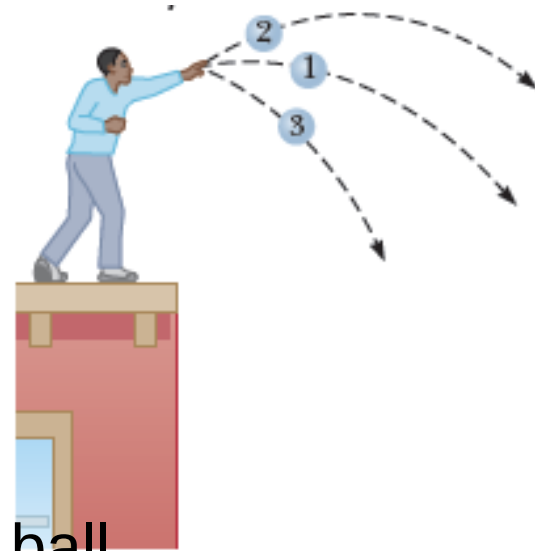
A rock of mass  $m$  is dropped to the ground from a height  $h$ . A second rock, with mass  $2m$ , is dropped from the same height. When they strike the ground, the kinetic energy of the second rock is:

- a) twice that of the first rock.
- b) half as much as that of the first.
- c) Four times that of the first.
- d) the same as that of the first.

## • Quick Quiz (3)

Three identical balls are thrown from the top of a building, all with the same initial speed but with different directions. Which of the three balls will hit the ground with the largest speed?

- a) The first ball.
- b) The second ball.
- c) The third ball.
- d) All hit with the same speed.



## • Quick Quiz (4)

A block attached to an ideal spring and oscillates along a frictionless horizontal surface. The block will have its largest speed at:

- a) its equilibrium position.
- b) its maximum compressing position.
- c) its maximum stretching position.
- d) cannot be determined.

**Example 8.1** A ball of mass  $m = 2$  kg is dropped from height of  $h$  above the ground as shown.

a) Choosing the system as the ball and the Earth, determine the speed of the block when it is at height  $y$  above the ground.

b) Find the speed of the ball at the same height  $y$  but now choosing the system as the ball only.

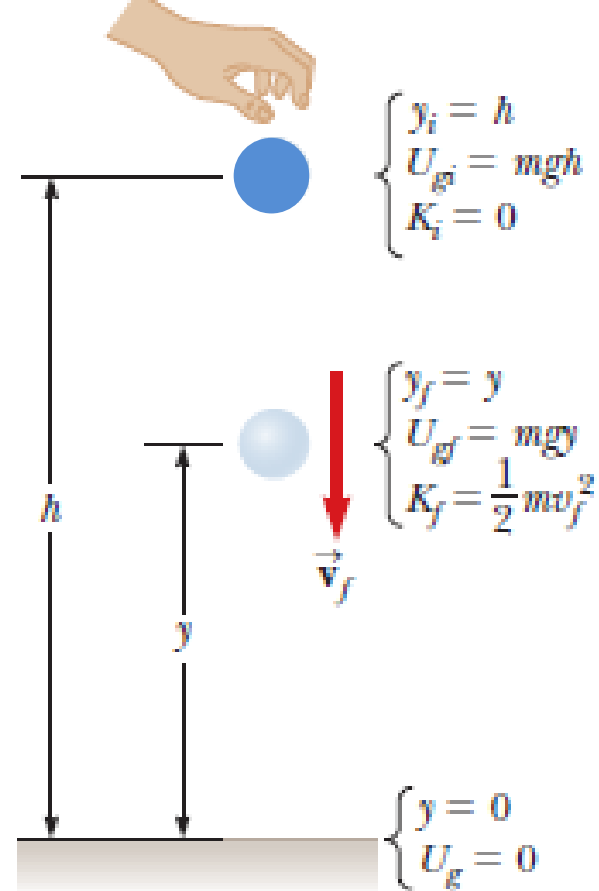
**Solution a)** Applying the conservation of mechanical energy principle  $\Rightarrow$

$$E_i = E_f \quad \Rightarrow (K_i + U_i) = (K_f + U_f)$$

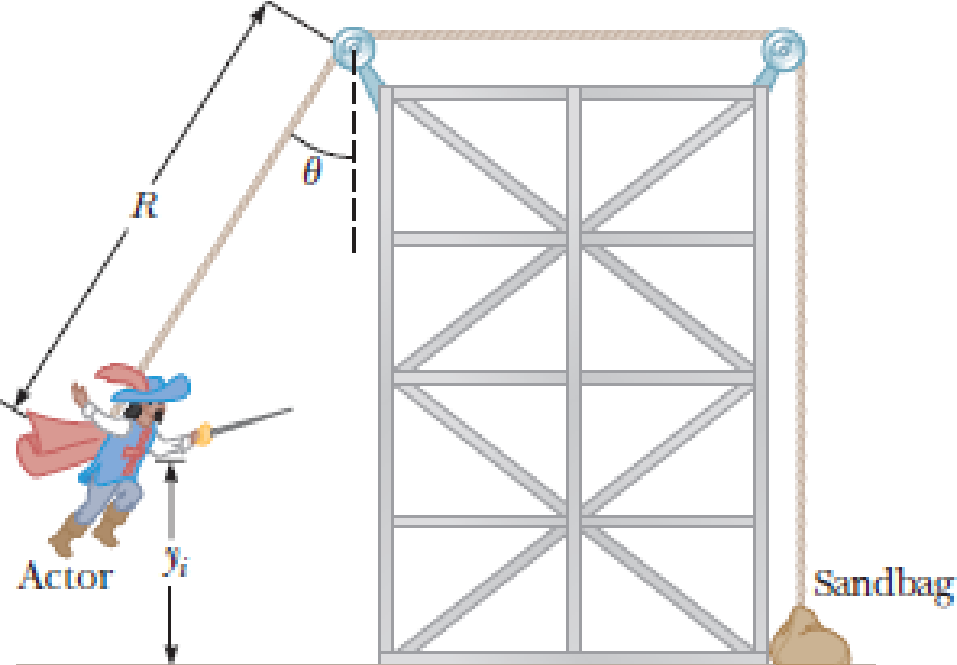
$$(0 + mgh) = \left( mgy + \frac{1}{2}mv_f^2 \right) \Rightarrow v_f = \sqrt{2g(h-y)}$$

**b)** For the ball we write  $W_{\text{ext}} = W_g = \Delta K$

$$\text{But } W_g = mg(h-y) \Rightarrow mgy = \frac{1}{2}mv_f^2 \Rightarrow v_f = \sqrt{2g(h-y)}$$

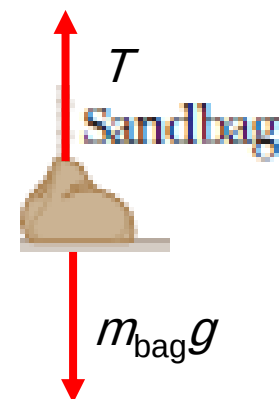


**Example 8.2** You are designing an apparatus to support an actor of mass 65.0 kg who is to fly down to the stage. You attach the actor's harness to a 130-kg sandbag by means of a light steel cable running smoothly over two frictionless pulleys as shown. You need 3.00m of cable between the harness and the nearest pulley. What is the maximum value  $\theta$  can have before the sandbag lifts off the floor?



**Solution** The sandbag lifts off the floor when the tension exceeds its weight. If the sandbag is just to be lifted off, the normal force acting on it is zero and we can write for the sandbag

$$\begin{aligned} \sum F_y = 0 & \Rightarrow T - m_{\text{bag}}g = 0 & \Rightarrow \\ T = m_{\text{bag}}g & \quad (1) \end{aligned}$$



As the actor moves in a vertical circle, the maximum value of the tension occurs at the lowest point of its swinging motion. At that point we can apply

$$\sum F_c = m \frac{v^2}{R} \Rightarrow T - m_{actor} g = m_{actor} \frac{v_b^2}{R}$$

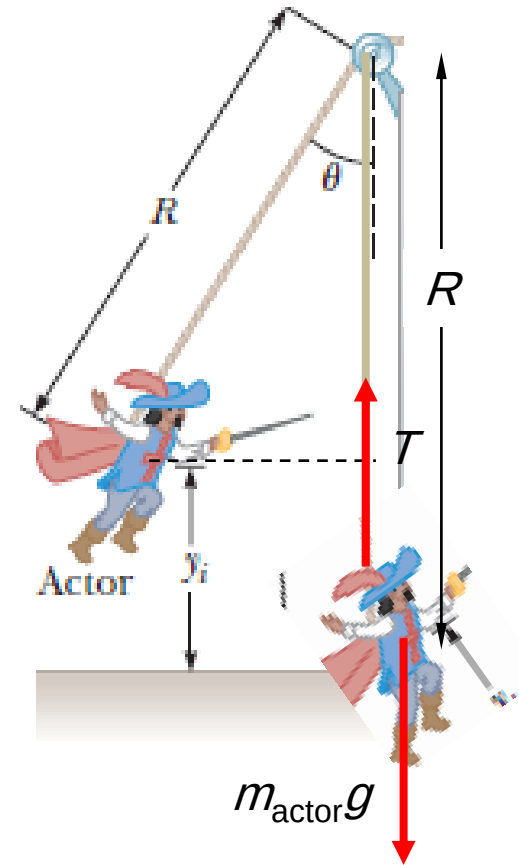
Substituting for  $T$  from Eq.(1), we get

$$(m_{bag} - m_{actor})g = m_{actor} \frac{v_b^2}{R} \quad (2)$$

To find the speed of the actor at its lowest point,  $v_b$ , we apply the conservation of energy principle taking the final point at the bottom.

$$(K_i + U_i) = (K_b + U_b) \Rightarrow$$

$$(0 + mgy_i) = (0 + \frac{1}{2}mv_b^2) \Rightarrow v_b = \sqrt{2gy_i}$$



As its clear from the figure we have

$$\text{But } y_i = R - R\cos\theta \Rightarrow$$

$$v_b = \sqrt{2gR(1-\cos\theta)} \quad (3)$$

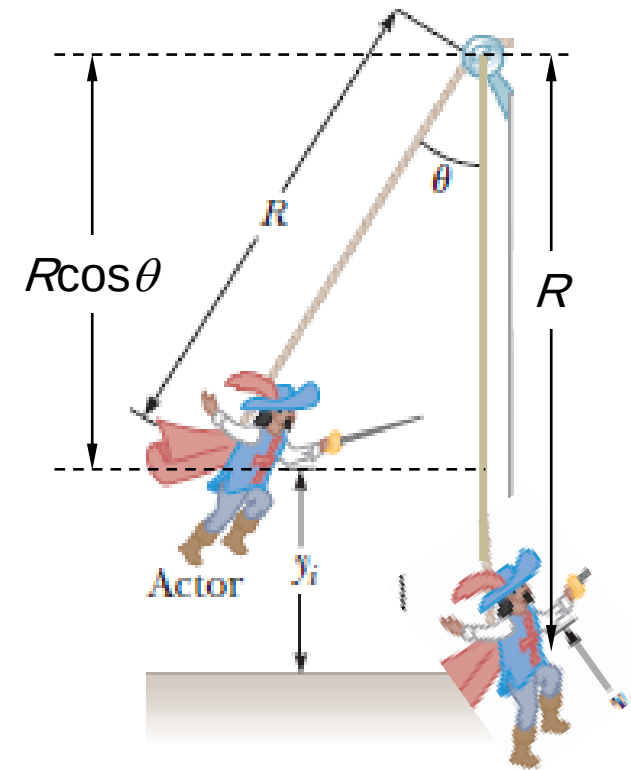
Substituting for  $v_b$  from Eq.(3) into Eq(2), we get

$$(m_{bag} - m_{actor})g = m_{actor} \frac{v_b^2}{R} \quad (2)$$

$$(m_{bag} - m_{actor})g = 2m_{actor}g(1-\cos\theta) \Rightarrow$$

$$m_{bag}g - 3m_{actor}g = -2m_{actor}g\cos\theta \Rightarrow$$

$$\cos\theta = \frac{3m_{actor} - m_{bag}}{2m_{actor}} = \frac{3(65.0) - 130}{2(65.0)} = 0.50 \Rightarrow \theta = 60.0^\circ$$

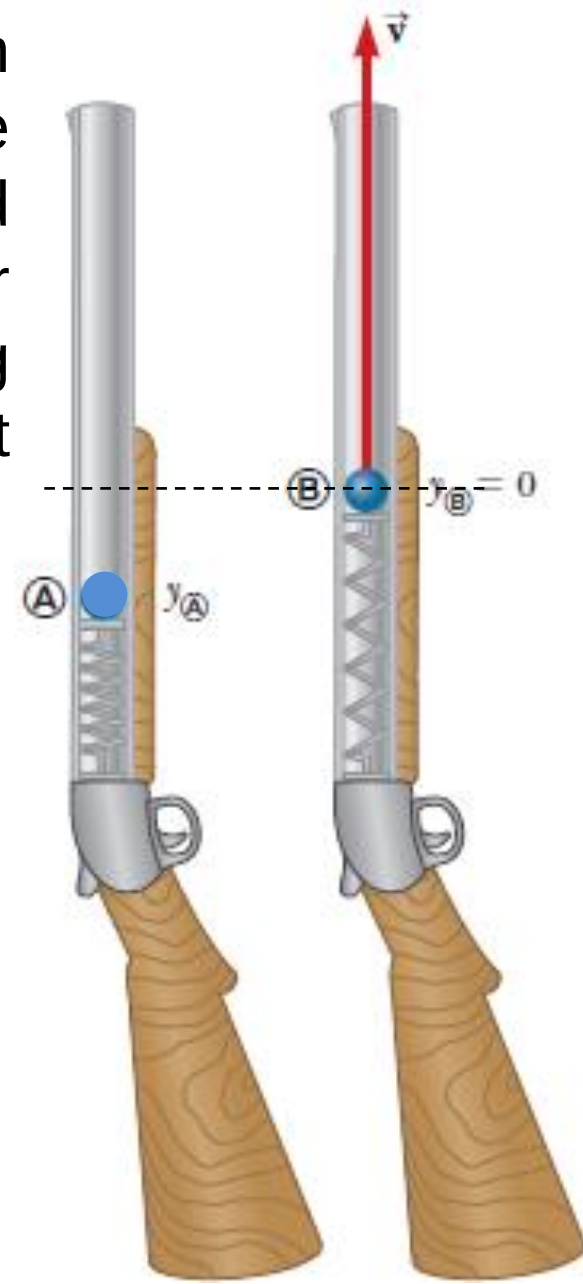


**Example 8.3** The launching mechanism of a popgun consists of a trigger-released spring. The spring is compressed to a position  $y_A = 0.120$  m, and the trigger is fired. The mass  $m$  leaves the spring at position  $B$  and rises to a maximum position  $y_C = 20.0$  m. Consider that  $m = 35.0$  g a) Determine the spring constant. b) Find the speed of the projectile at the equilibrium position,  $B$ , of the spring.

**Solution** a) Choosing the level of zero potential energy as point  $B$  and choosing points  $A$  and  $C$  as initial and final positions, respectively, and noting that we have now  $U_s$  in addition to  $U_g$  we write

$$E_i = K_A + U_{gA} + U_{sA} = -mgy_A + \frac{1}{2}kx^2$$

$$E_f = K_C + U_{gC} + U_{sC} = mgy_C$$



Using  $E_i = E_f \Rightarrow$

$$-mgy_A + \frac{1}{2}kx^2 = mgy_C \Rightarrow$$

$$mg(y_C + y_A) = \frac{1}{2}ky_A^2 \Rightarrow k = \frac{2mg(y_C + y_A)}{y_A^2}$$

$$k = \frac{2(0.035)(9.80)(20.0 + 0.120)}{(0.120)^2} = 958 \text{ N/m}$$

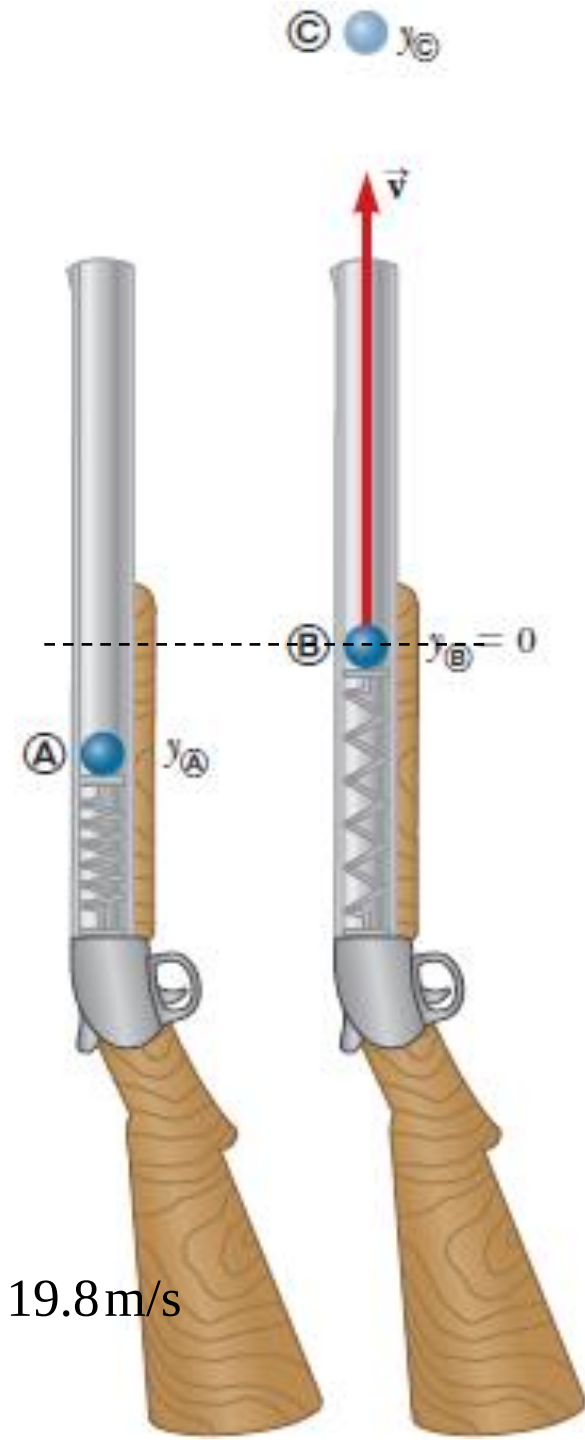
**b)** To find the speed at  $B$  we choose points  $A$  and  $B$  as initial and final positions, respectively. So we have

$$E_i = K_A + U_{gA} + U_{sA} = -mgy_A + \frac{1}{2}kx^2$$

$$E_f = K_B + U_{gB} + U_{sB} = \frac{1}{2}mv_B^2$$

$$-mgy_A + \frac{1}{2}kx^2 = \frac{1}{2}mv_B^2 \Rightarrow$$

$$v_B = \sqrt{\frac{ky_A^2 - 2mgy_A}{m}} = \sqrt{\frac{(958)(0.12)^2 - 2(0.035)(9.8)(0.12)}{0.035}} = 19.8 \text{ m/s}$$



## 8.3 Situations Involving Kinetic Friction

If there are frictional forces acting within the system and do works, the mechanical energy is no longer conserved since the force of friction dissipates some energy which appears in the form of an internal energy in the system, i.e.,

$$\Delta E_{\text{int}} = f_d d$$

The conservation of energy principle now can be written as

$$\Delta K + \Delta U + f_k d = 0$$

or  $K_i + U_i = K_f + U_f + f_k d$

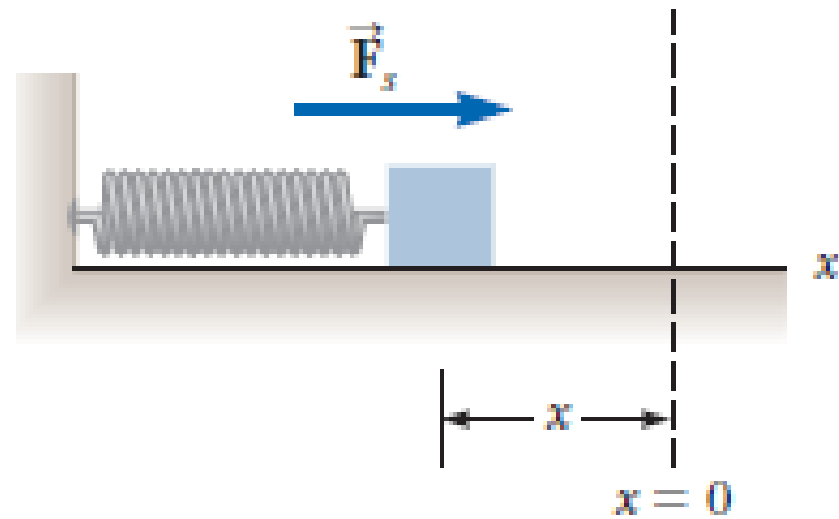
|                        |
|------------------------|
| or $E_i = E_f + f_k d$ |
|------------------------|

This equation is a general form of conservation of energy principle where  $\Delta E_{\text{int}}$  stands for the energy lost by all nonconservative forces acting within the system

# Strategy for solving problems using Conservation of Energy Principle:

- (i) Select a horizontal zero level for gravitational potential energy.
- (ii) Define two points: an initial point and a final point.
- (iii) Find the  $U$  and  $K$  at these two points.
- (iv) If there is a spring, then  $U = U_g + U_s$
- (v) If there is no friction, we use  $\Delta K + \Delta U = 0$
- (vi) If there is a friction, then we use  $E_i = E_f + f_k d$

**Example 8.6** A block of mass 1.6 kg is attached to a horizontal spring that has a force constant of 1 000 N/m. The spring is compressed 2.0 cm and is then released from rest, as shown.



- Calculate the speed of the block as it passes through the equilibrium position if the surface is frictionless.
- Repeat part (a) if a constant friction force of 4.0 N retards its motion from the moment it is released.

**Solution** a) Again we have  $U_g$  and  $U_s$  and no friction, so we use

$$E_i = K_i + U_{gi} + U_{si} = \frac{1}{2}kx^2$$

$$E_f = K_f + U_{gf} + U_{sf} = \frac{1}{2}mv^2$$

$$\text{Using } E_i = E_f \Rightarrow$$

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 \quad \Rightarrow$$

$$v = \sqrt{\frac{k}{m}}x = \sqrt{\frac{1000}{1.6}}2.0 \times 10^{-2} = 0.5 \text{ m/s}$$

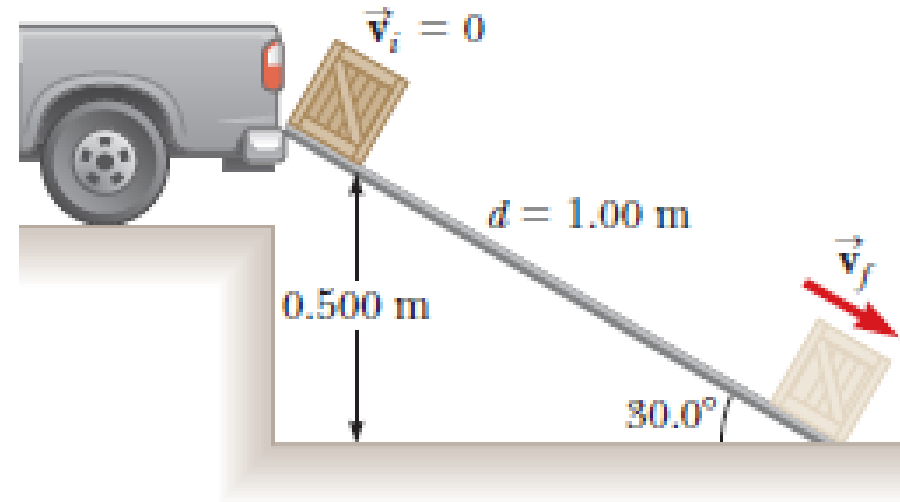
b) If we have a friction, we use

$$E_i = E_f + f_k d$$

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 + f_k x \quad \Rightarrow$$

$$v = \sqrt{\frac{kx^2 - 2f_k x}{m}} = \sqrt{\frac{1000(2.0 \times 10^{-2})^2 - 2(4.0)(2.0 \times 10^{-2})}{1.6}} = 0.39 \text{ m/s}$$

**Example 8.7** A 3.00-kg crate slides down a ramp. The ramp is 1.00 m in length and inclined at  $30.0^\circ$  as shown. The crate starts from rest at the top, experiences a constant friction force of 5.00 N, and continues to move a short distance on the horizontal floor after it leaves the ramp.



- (a) Use the energy methods to determine the speed of the crate at the bottom of the ramp.
- (b) How far does the crate slide on the horizontal floor if it continues to experience the same friction force?

**Solution** a) Considering the top and the bottom of the ramp as the initial and the final points, respectively, and choosing the bottom of the ramp as the level of zero potential energy and using

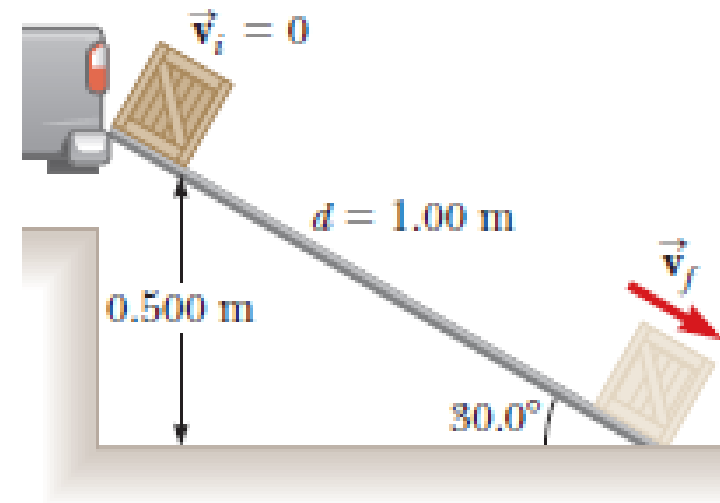
$$E_i = E_f + f_k d$$

Now  $E_i = K_i + U_i = mgh$

And  $E_f = K_f + U_f = \frac{1}{2}mv_b^2 \Rightarrow$

$$mgh = \frac{1}{2}mv_b^2 + f_k d = 0 \Rightarrow v_b = \sqrt{\frac{2(mgh - f_k d)}{m}}$$

$$v_b = \sqrt{\frac{2[(3.00)(9.80)(0.500) - (5.0)(1.00)]}{3.00}} = 2.54 \text{ m/s}$$



b) Now letting the bottom of the ramp to be the initial point while the final point is the point at the which the crate stops we have

$$E_i = K_i + U_i = \frac{1}{2}mv_b^2$$

$$\text{And } E_f = K_f + U_f = 0$$

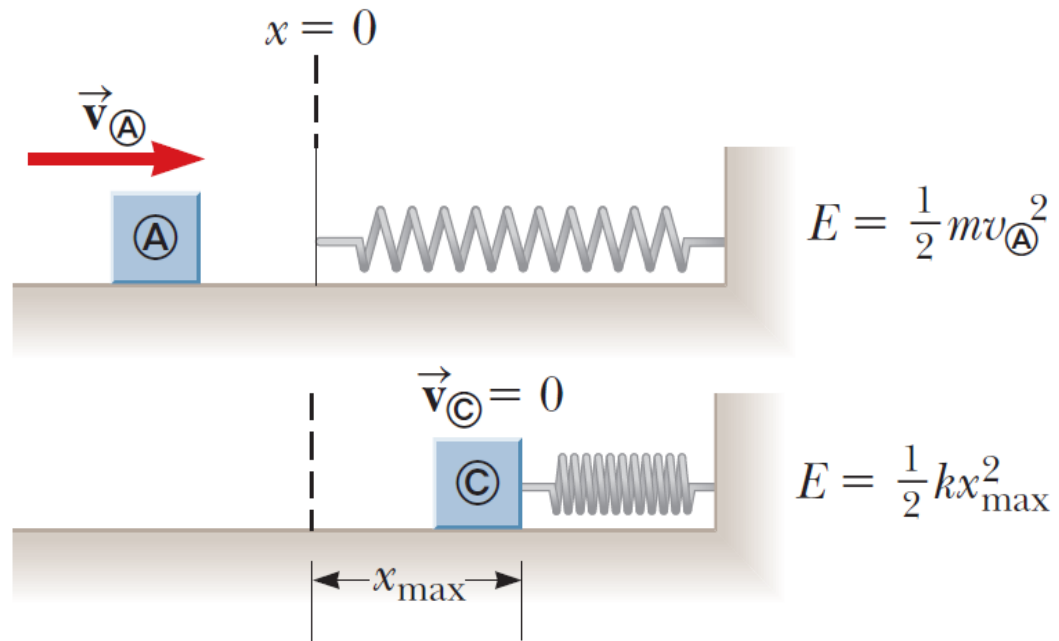
$$E_i = E_f + f_k d \Rightarrow \frac{1}{2}mv_b^2 = f_k d \Rightarrow$$

$$d = \frac{mv_b^2}{2f_k} = \frac{3.0(2.54)^2}{2(5.0)} = 1.94 \text{ m}$$

**Example 8.8** A block having a mass of 0.8-kg is given an initial velocity  $v_A = 1.2$  m/s to the right and collides with a light spring with  $k = 50$  N/m.

a) Calculate the maximum compression of the spring assuming the surface to be frictionless.

b) Repeat part (a) if the surface is rough with  $\mu_k = 0.5$ .



**Solution** a) Taking the maximum compression as the final point and noting that the speed there is zero, we have

$$\text{Now } E_i = K_i + U_i = \frac{1}{2} m v_A^2 \quad \text{And } E_f = K_f + U_f = \frac{1}{2} k x^2$$

$$\text{Using } E_i = E_f \quad \Rightarrow \quad \frac{1}{2} m v_A^2 = \frac{1}{2} k x^2 \quad \Rightarrow$$

$$x = \sqrt{\frac{m}{k}}v = \sqrt{\frac{0.80}{50.0}}1.2 = 0.15\text{m}$$

b) If we have a friction, we use  $E_i = E_f + f_k d \Rightarrow$

$$\frac{1}{2}mv_A^2 = \frac{1}{2}kx^2 + f_k d$$

$$\text{But } f_k d = \mu_k nx = \mu_k mgx \Rightarrow$$

$$\frac{1}{2}mv_A^2 = \frac{1}{2}kx^2 + \mu_k mgx \Rightarrow$$

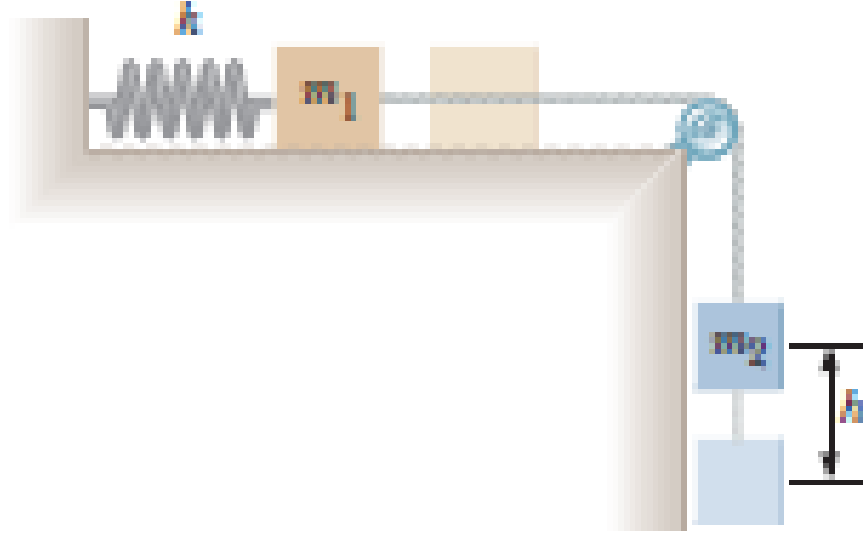
$$50x^2 + 7.84x - 1.15 = 0 \Rightarrow$$

$$x = 0.092\text{m and } x = -0.25\text{m}$$

The negative answer means that the maximum compression is at the left which is impossible, so the maximum compression is

$$x = 0.092\text{m}$$

**Example 8.9** Two blocks are connected by a light string that passes over a frictionless pulley as shown. The block of mass  $m_1$  lies on a horizontal surface and is connected to a spring of force constant  $k$ .



The system is released from rest when the spring is unstretched. If the hanging block of mass  $m_2$  falls a distance  $h$  before coming to rest, calculate the coefficient of kinetic friction between the block  $m_1$  and the surface.

**Solution** a) Here the system starts from rest and ends at rest. Taking the level of zero potential energy as the top of the surface for  $m_1$  and for  $m_2$  its final position we have

$$E_i = K_i + (U_g + U_s)_i = m_2gh$$

$$\text{And } E_f = K_f + (U_g + U_s)_f = \frac{1}{2}kh^2$$

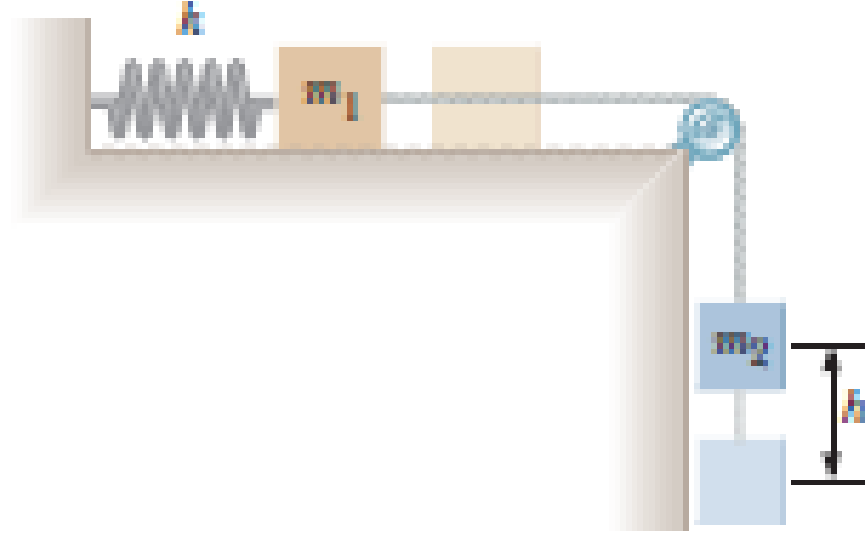
Now using

$$E_i = E_f + f_k d \quad \Rightarrow$$

$$\text{But } f_k d = \mu_k n h = \mu_k m_1 g h \quad \Rightarrow$$

$$m_2 g h = \frac{1}{2} k h^2 + \mu_k m_1 g h \quad \Rightarrow$$

$$\mu_k = \frac{m_2 g - \frac{1}{2} k h}{m_1 g}$$



## 8.5 POWER

The power is defined as time rate of energy transfer.

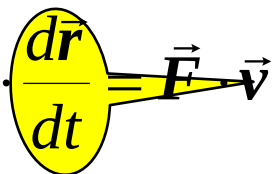
The average power of a force during a time interval is defined as

$$\bar{P} = \frac{\Delta W}{\Delta t} \quad \text{With } W \text{ is the work done by this force}$$

The instantaneous power is defined as

$$P = \lim_{\Delta t \rightarrow 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$

If  $\vec{F}$  is constant, then  $dW = \vec{F} \cdot d\vec{r} \Rightarrow$

$$P = \frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$$


The unit of power is J/s or watt, (W), with  $1W=1 \text{ J/s}=1 \text{ kg.m}^2/\text{s}^3$ .

1 hp (horsepower)= 746 W

**Example 8.11** An elevator car has a mass of 1600 kg and is carrying passengers having a combined mass of 200 kg. A constant friction force of 4000 N retards its motion.

a) How much power must a motor deliver to lift the elevator car and its passengers at a constant speed of 3.00 m/s?

b) What power must the motor deliver at the instant the speed of the elevator is  $v$  if the motor is designed to provide the elevator car with an upward acceleration of  $1.00 \text{ m/s}^2$ ?

**Solution** a) The free-body diagram of the elevator is shown, with  $T$  is the force delivered by the motor. Since the speed is constant we have

$$\begin{aligned}\sum F_y &= ma_y = 0 \quad \Rightarrow T - Mg - f_k = 0 \quad \Rightarrow \\ T &= Mg + f_k = (1600 + 200)9.80 + 4000 = 2.16 \times 10^4 \text{ N} \\ P &= Tv = (2.16 \times 10^4)(3.00) = 6.49 \times 10^4 \text{ W}\end{aligned}$$



b) If the elevator's car is accelerated we have

$$\sum F_y = ma_y \Rightarrow T - Mg - f_k = ma \Rightarrow$$

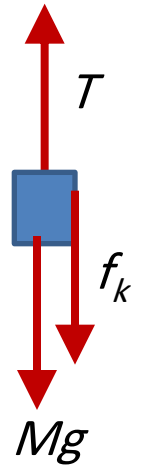
$$T = Mg + f_k + ma$$

$$= (1800)9.80 + 4000 + (1800)1.00 = 2.34 \times 10^4 \text{ N}$$

$$P = Tv = 2.34 \times 10^4 v$$

If one substitute for  $v=3.00$  m/s we get

$$P = Tv = 7.02 \times 10^4 \text{ W}$$



## • Quick Quiz (5)

Two cars, the first accelerates from rest to speed  $v$  in 10 s, while the second accelerates from rest to  $2v$  in the same time period. If the power of the first car is  $P$ , the power of the second one is:

- a)  $P$
- b)  $2P$
- c)  $4P$
- d)  $0.5P$