

CHAPTER 10

ROTATION OF A RIGID OBJECT ABOUT A FIXED AXIS

In this chapter we are going to discuss the following topics:

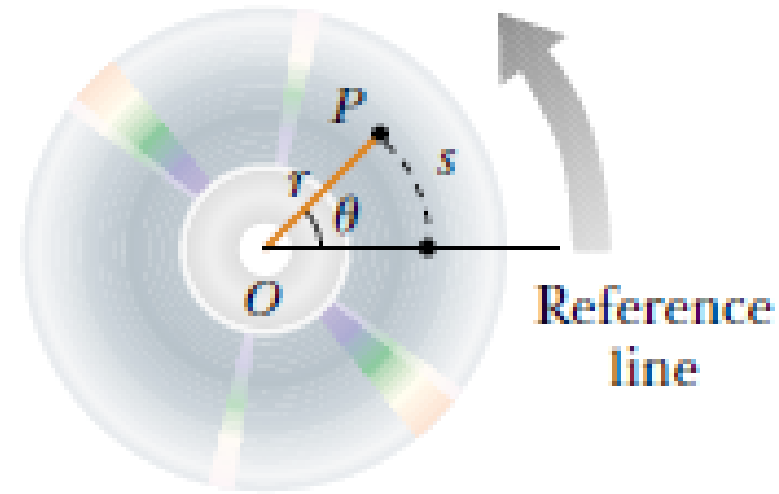
- ☐ **Angular Position, Velocity, & Acceleration**
- ☐ **Rotation with Constant Angular Acceleration**
- ☐ **Relations Between Angular & Translational Quantities**
- ☐ **Torque**
- ☐ **Dynamics of Rotational Motion (Moment of Inertia)**
- ☐ **Energy in Rotational Motion (Work & Kinetic Energy)**

Recommended Problems:

3, 5, 11, 13, 19, 25, 27, 29, 32, 35, 40, 50, 51, 53, 55, 57, 73, 77, 78, 79, 80, 92

10.1 Angular position, Velocity, and Acceleration

Consider a rigid body rotates about a fixed axis through point O in x - y plane as shown. Any particle at point P in this rigid body rotates in a circle of radius r about O . The angle between r and the positive x -axis is θ .



As the point P moves around the circle the angle θ changes while r remains constant. Thus θ serves as a coordinate to describe the rotational position of a particle. The angle is related to the displacement (arc length) s , through the relation

$$s=r\theta$$

Here the unit of θ is radian with $1rad.=\frac{360^{\circ}}{2\pi}\approx 57.3^{\circ}$

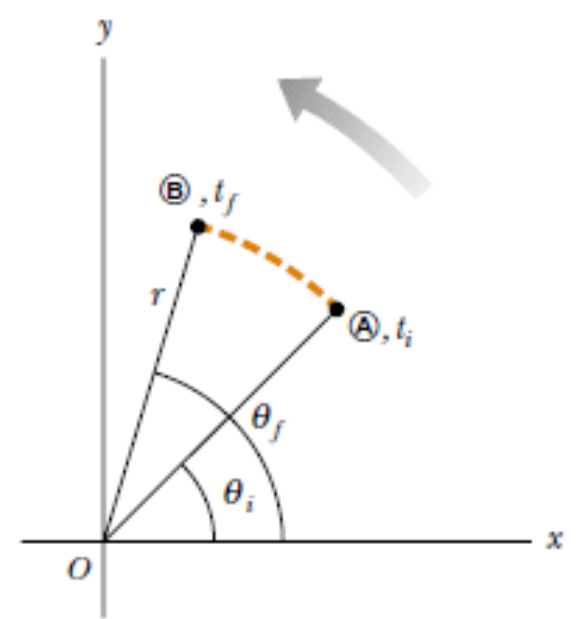
In analogy to the linear velocity, the **average angular velocity** of a particle moves from point A to point B in the figure is defined as

$$\bar{\omega} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$

The **instantaneous angular velocity** is defined as

$$\omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

As it is clear from the above equations, the unit of ω is rad./s, or simply s^{-1} . The direction of ω is along the axis of rotation with its sense can be determined from the **right hand rule**: turn the four fingers of your right hand with the direction of rotation; your thumb then gives the direction of .



The **average angular acceleration** of a particle is defined as

$$\bar{\alpha} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

And the **instantaneous angular acceleration** is defined as

$$\alpha = \lim_{\Delta t \rightarrow 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

The unit of α is rad./s^2 , or simply s^{-2} .

As it is clear from the last equation that the direction of α is also along the axis of rotation with its sense is determined whether the angular velocity increases or decreases.

If the rotation is speeding up, $(d\omega/dt)$ is positive and α is in the same direction as ω , and if the rotation is slowing down, $(d\omega/dt)$ is negative and α is opposite to ω .

10.2 Rotation with Constant Angular Acceleration

$$\alpha = \frac{d\omega}{dt} \Rightarrow d\omega = \alpha dt \Rightarrow \int_{\omega_i}^{\omega_f} d\omega = \alpha \int_0^t dt \Rightarrow$$

$$\boxed{\omega_f = \omega_i + \alpha t}$$

$$\text{Now } \omega = \frac{d\theta}{dt} = \omega_i + \alpha t \Rightarrow d\theta = (\omega_i + \alpha t) dt \Rightarrow$$

$$\int_{\theta_i}^{\theta_f} d\theta = \int_0^t (\omega_i + \alpha t) dt \Rightarrow$$

$$\boxed{\theta_f = \theta_i + \omega_i t + \frac{1}{2} \alpha t^2}$$

From the first equation we get $t = \frac{\omega_f - \omega_i}{\alpha}$

Substituting for t into the second equation we obtain

$$\theta_f = \theta_i + \omega_i \left(\frac{\omega_f - \omega_i}{\alpha} \right) + \frac{1}{2} \alpha \left(\frac{\omega_f - \omega_i}{\alpha} \right)^2 \Rightarrow$$

$$\theta_f = \theta_i + \frac{\omega_i \omega_f - \omega_i^2 + \frac{1}{2} \omega_f^2 + \frac{1}{2} \omega_i^2 - \omega_i \omega_f}{\alpha} \Rightarrow$$

$$\theta_f = \theta_i + \frac{-\omega_i^2 + \omega_f^2}{2\alpha} \Rightarrow$$

$$\boxed{\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)}$$

These three equations are of the same forms as those for linear motion with the replacing:

$$x \rightarrow \theta, \quad v \rightarrow \omega, \text{ and } \quad a \rightarrow \alpha$$

Example 10.1 A wheel rotates with constant angular acceleration of 3.50 rad/s^2 . If the angular velocity is 2.00 rad/s at $t = 0$.

- a) Through what angle does the wheel rotate through in 2.00 s ?
- b) What is the angular velocity at $t = 2.00 \text{ s}$?

Solution a) Using the equation

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2} \alpha t^2 \quad \Rightarrow$$

$$\theta = 0 + 2.00 \times 2.00 + \frac{1}{2} \times 3.50 \times (2.00)^2 = 11.0 \text{ rad}$$

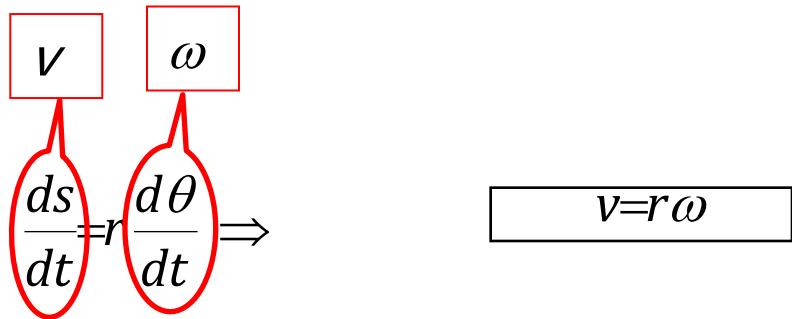
Note that this angle is equivalent to $\frac{11.0 \text{ rad}}{2\pi} = 1.75 \text{ revolution}$

b) Now Using the equation

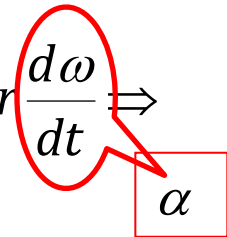
$$\omega_f = \omega_i + \alpha t \quad \Rightarrow \quad \omega = 2.00 + 3.50 \times 2.00 = 9.00 \text{ rad/s}$$

10.3 Angular and Translational Quantities

It is known that

$$s=r\theta \quad \Rightarrow \quad \frac{ds}{dt} = r \frac{d\theta}{dt} \quad \Rightarrow \quad v=r\omega$$


The tangential acceleration is defined as

$$a_t = \frac{dv}{dt} = r \frac{d\omega}{dt} \quad \Rightarrow \quad a_t = r\alpha$$


While the radial acceleration is defined as $a_r = \frac{v^2}{r} = r\omega^2$

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{r^2\alpha^2 + r^2\omega^4} \quad \Rightarrow \quad a = r\sqrt{\alpha^2 + \omega^4}$$

Remark: For a rigid body rotating about a fixed axis, every particle on the body has the same angular velocity and the same angular acceleration, while the linear velocity and the linear acceleration differ from point to point.

• Quick Quiz (1)

Two blocks are rotating with a turn-table of radius R . The first block is located at the rim of the table, while the second block is located at a distance $0.5R$ from the center of the table. If the table rotates with constant angular speed, the angular speeds of the two blocks are related as:

a) $\omega_1 = \omega_2$

b) $\omega_1 = 2\omega_2$

c) $\omega_1 = 0.5\omega_2$

d) $\omega_1 = 4\omega_2$

• Quick Quiz (2)

Referring to the previous question, the tangential speeds of the two blocks are related as:

a) $v_1 = v_2$

b) $v_1 = 2v_2$

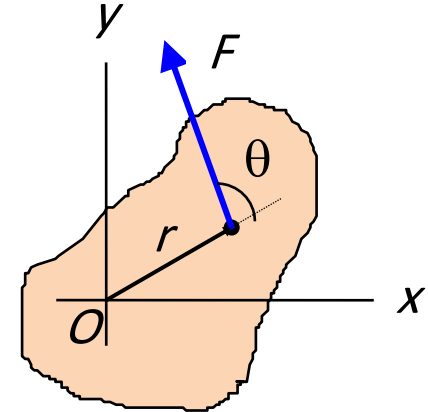
c) $v_1 = 0.5v_2$

d) $v_1 = 4v_2$

10.4 TORQUE

The tendency of a force to rotate an object about some axis is measured by a quantity called **torque**. Consider a force $\mathbf{F}$ acting on the particle P in a rigid body, as shown. The torque due to the force $\mathbf{F}$ is defined by the cross product of the vector $\mathbf{r}$ and the force vector $\mathbf{F}$, displacement

$$\vec{\tau} = \vec{r} \times \vec{F}$$



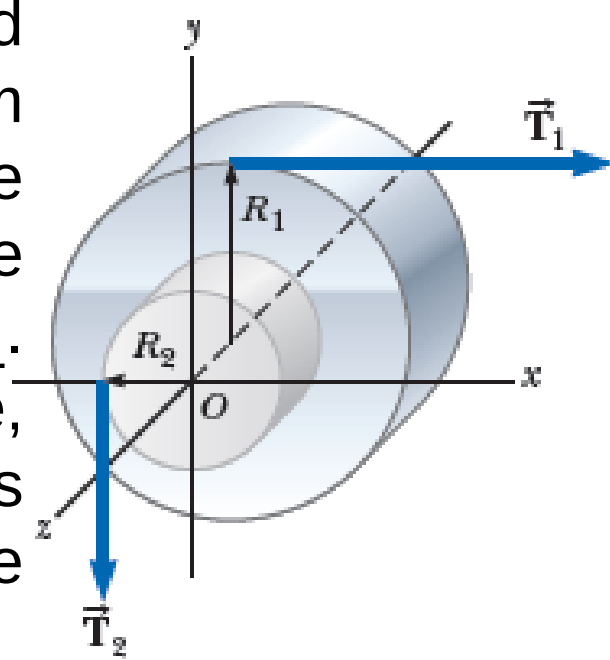
The torque is a vector quantity with magnitude

$$\tau = Fr \sin \theta = Fd$$

With $d = r \sin \theta$ is called the moment arm of the force and is defined as the perpendicular distance from the point of rotation to the line of action of the force.

The direction of τ is perpendicular to the plane of $\mathbf{r}$ and $\mathbf{F}$. It is positive if it tends to rotate the body counterclockwise, in the direction of increasing θ . It is negative if it tends to rotate the body clockwise.

Example 10.3 A one-piece cylinder is shaped as shown with a core section protruding from the larger drum. The cylinder is free to rotate about the z -axis. A rope wrapped around the drum, which has radius R_1 , exerts a force $\vec{T}_1$. A second rope wrapped around the core, which has radius R_2 , exerts a force $\vec{T}_2$, as shown. Find the net torque acting on the cylinder about the axis of rotation.



Solution It is clear that $\vec{T}_1$ tends to rotate the cylinder clockwise, while $\vec{T}_2$ tends to rotate it counterclockwise. The net torque is

$$\sum \tau = -\tau_1 + \tau_2 = -T_1 d_1 + T_2 d_2$$

But as it clear from the figure that the moment arm of $\vec{T}_1$ is R_1 , and that of $\vec{T}_2$ is R_2 . So we have

$$\sum \tau = -T_1 R_1 + T_2 R_2$$

10.5 Rigid Object Under a Net Torque

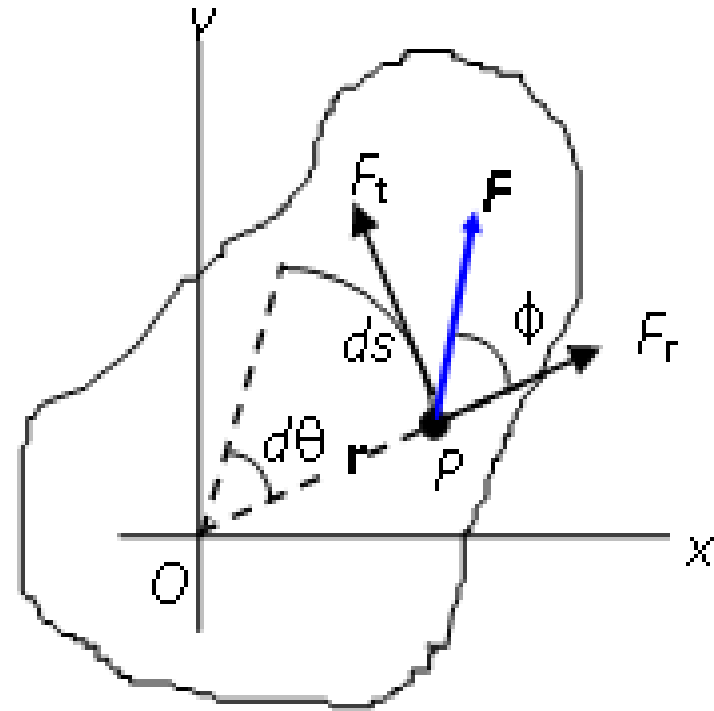
Consider a force $\mathbf{F}$ acting on a rigid body free to rotate about O . During the infinitesimal time dt , the element P of mass dm will move an infinitesimal distance ds along a circular path of radius r as the body rotates through an infinitesimal angle $d\theta$, where

$$ds = r d\theta$$

The torque acting on the body is $\tau = F_t r$

$$\text{But } F_t = M(a_{cm})_t \quad M(a_{cm})_t = \sum_{i=1}^n m_i a_{ti} \quad \text{and } a_t = r\alpha \quad \Rightarrow$$

$$\tau = \left(\sum_{i=1}^n m_i r_i^2 \right) \alpha = I \alpha \quad \text{with } I = \left(\sum_i m_i r_i^2 \right)$$



is called the **moment of inertia**. It represents the mass in all rotational equations. It measures the resistance of a body to changes in its rotational motion.

It depends on the masses of the particles making up the object and their distribution from the axis of rotation.

For a body of continuous mass distribution, the summation in Equation can be replaced by an integration over the body, i.e.,

$$I = \int r^2 dm$$

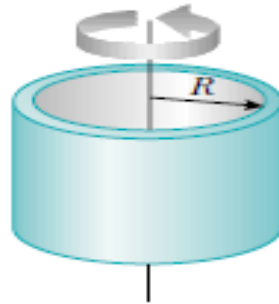
If there is more than one force doing torque on the body, the last equation can be generalized as

$$\boxed{\sum \tau_{ext} = I\alpha}$$

The last equation can be considered as Newton's second law for rotational motion.

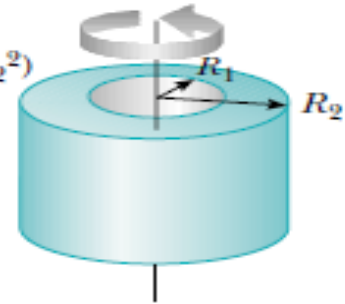
Moments of Inertia of Homogeneous Rigid Objects with Different Geometries

Hoop or thin cylindrical shell
 $I_{CM} = MR^2$

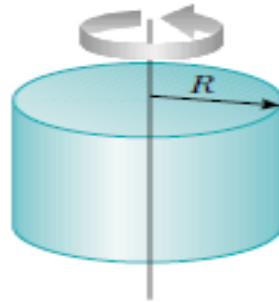


Hollow cylinder

$$I_{CM} = \frac{1}{2} M(R_1^2 + R_2^2)$$

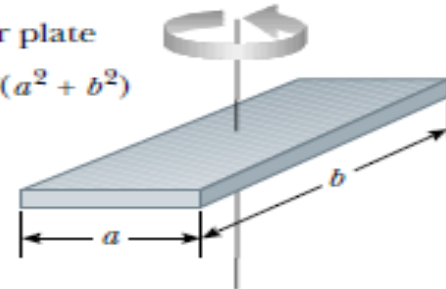


Solid cylinder or disk
 $I_{CM} = \frac{1}{2} MR^2$

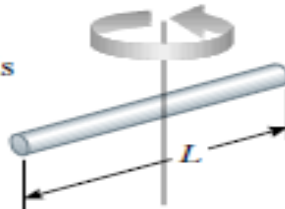


Rectangular plate

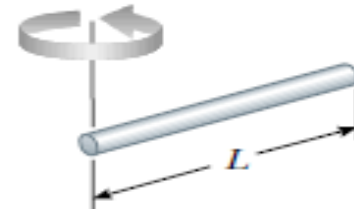
$$I_{CM} = \frac{1}{12} M(a^2 + b^2)$$



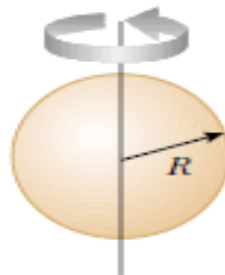
Long thin rod with rotation axis through center
 $I_{CM} = \frac{1}{12} ML^2$



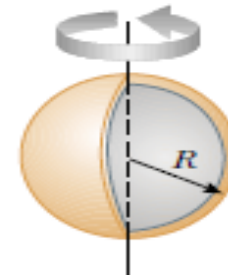
Long thin rod with rotation axis through end
 $I = \frac{1}{3} ML^2$



Solid sphere
 $I_{CM} = \frac{2}{5} MR^2$



Thin spherical shell
 $I_{CM} = \frac{2}{3} MR^2$

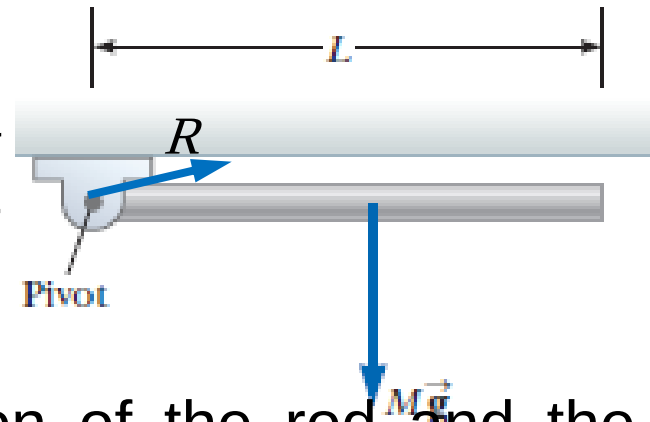


• Quick Quiz (3)

You turn off your electric drill and find that the time interval for the rotating bit to come to rest due to frictional torque in the drill is t . You replace the bit with a larger one such that the moment of inertia of the drill's entire rotating mechanism is doubled. When this larger bit is rotated at the same angular speed as the first and the drill is turned off, the frictional torque remains the same as that for the previous situation. What is the time interval for this second bit to come to rest?

- a) t b) $2t$ c) $0.5t$ d) Impossible to determine.

Example 10.4 A uniform rod of length L and mass M is attached at one end to a frictionless pivot and is free to rotate about it in the vertical plane as shown. The rod is released from rest in the horizontal position.



- What are the initial angular acceleration of the rod and the initial translational acceleration of its right end?
- Where can we place a penny on the rod that will stay in contact as both begin to fall?

Solution a) The torque acting on the rod is due only to Mg . The force acting on the rod by the pivot does no torque. Using

$$\sum \tau_{ext} = I\alpha \Rightarrow$$

Initially, when the rod is in the horizontal position, we have

$$Mg(L/2) = I\alpha_i \Rightarrow \alpha_i = \frac{MgL}{2I} = \frac{3g}{2L}$$

Using $a_t = r\alpha \Rightarrow a_{ti} = L\alpha_i = \frac{3}{2}g$

Note that even though that α is constant for all points of the rod, a_t varies from point to point. At the right end $a_t > g$.

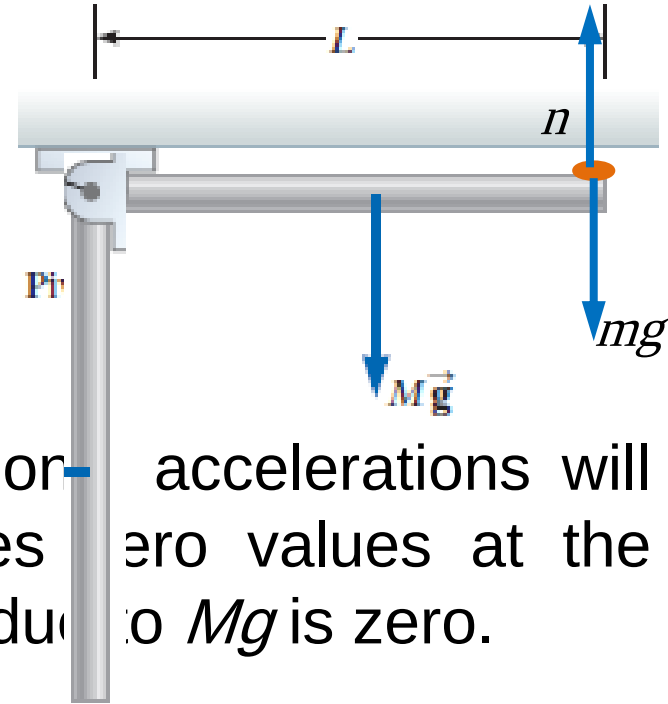
These values of the angular and translation accelerations will decrease as the rod rotates and reaches zero values at the vertical position. At this position the torque due to Mg is zero.

Now if a penny is placed on the rod at its right end and we have

$$\sum F_y = ma_t \Rightarrow mg - n = ma_t \Rightarrow$$

$$n = m(g - a_t) = -\frac{1}{3}mg \Rightarrow$$

This means that if a penny is placed on the rod at its right end it will not stay in contact.



(b) Now let the penny be a distance R from the pivot. Its tangential acceleration is

$$a_t = R\alpha = \frac{3}{2} \frac{Rg}{L}$$

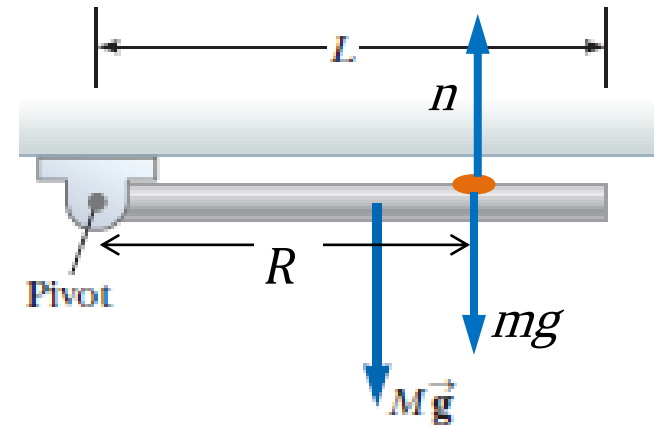
Now we have

$$\sum F_y = ma_t \Rightarrow mg - n = ma_t \Rightarrow$$

$$n = m(g - a_t) = mg \left(1 - \frac{3R}{2L} \right)$$

For the penny to stay on the rod $n > 0 \Rightarrow$

$$\frac{3R}{2L} < 1 \Rightarrow R < \frac{2}{3}L$$



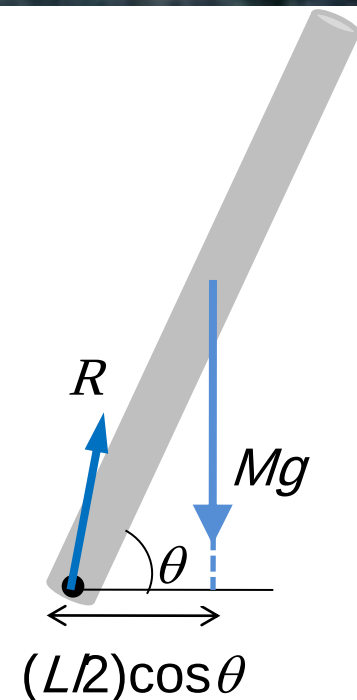
Example 10.5 When a tall smokestack falls over, it often breaks somewhere along its length before it hits the ground as shown. Why?

Solution Let us study the rotational motion of the stack when it makes an angle θ with the horizontal. Again the torque acting on the stack is due only to Mg . As we have done previously we obtain

$$Mg(L/2)\cos\theta = I\alpha \quad \Rightarrow \quad \alpha = \frac{3g}{2L}\cos\theta$$

$$\text{But } a_t = r\alpha = \frac{3g}{2L}r\cos\theta \quad \Rightarrow$$

It is clear, from the last equation that the tangential acceleration increases as r or θ increases.



This means that, each higher portion of the smokestack falls with a larger tangential acceleration than the portion below it and increases as the smokestack tips farther.

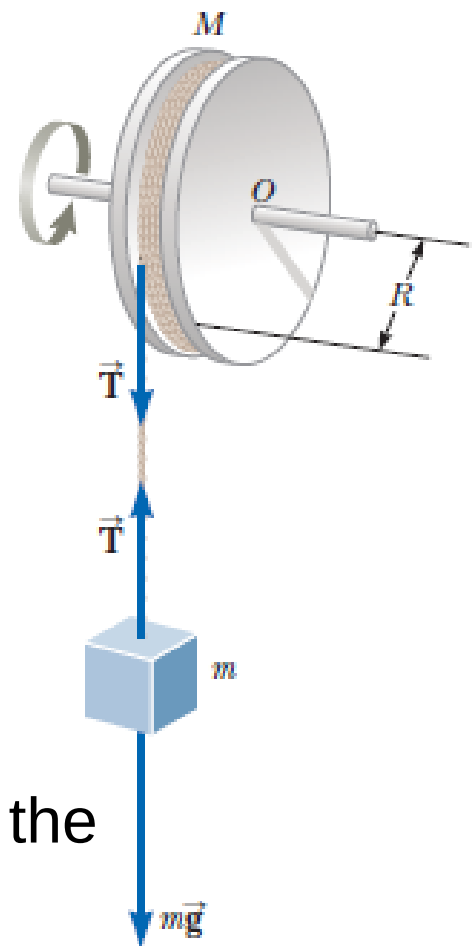
At some point, higher portions of the smokestack will have an acceleration greater than the acceleration due to gravity alone.



That can happen only if these portions are being pulled downward by a force in addition to the gravitational force. This force is the shear force from lower portions of the smokestack. Eventually, the shear force that provides this acceleration is greater than the smokestack can withstand, and the smokestack breaks.

The same thing happens with a tall tower of children's toy blocks. If one push such a tower over and it will come apart at some point before it strikes the floor.

Example 10.6 A wheel of radius R , mass M , and moment of inertia I is rotating on a frictionless, horizontal axle as shown. A light cord wrapped around the wheel and supports a body of mass m . Calculate the linear acceleration, a , of the body, the angular acceleration, α , of the wheel, and the tension, T , in the cord.



Solution For the wheel we apply

$$\sum \tau = I\alpha \Rightarrow TR = I\alpha \quad (1)$$

Note that the weight of the wheel and the force of the axle on it don't do any torque on the wheel. Why?

And for the mass we apply

$$\sum F = ma \Rightarrow mg - T = ma \quad (2)$$

The third equation is obtained from the relation between the angular and linear acceleration:

$$a = R\alpha \quad (3)$$

Using Eq. (1) & Eq.(3) $\Rightarrow$

$$T = I \frac{a}{R^2} \quad (4)$$

$$mg - T = ma$$

Now adding Eq. (2) & Eq.(4) $\Rightarrow$

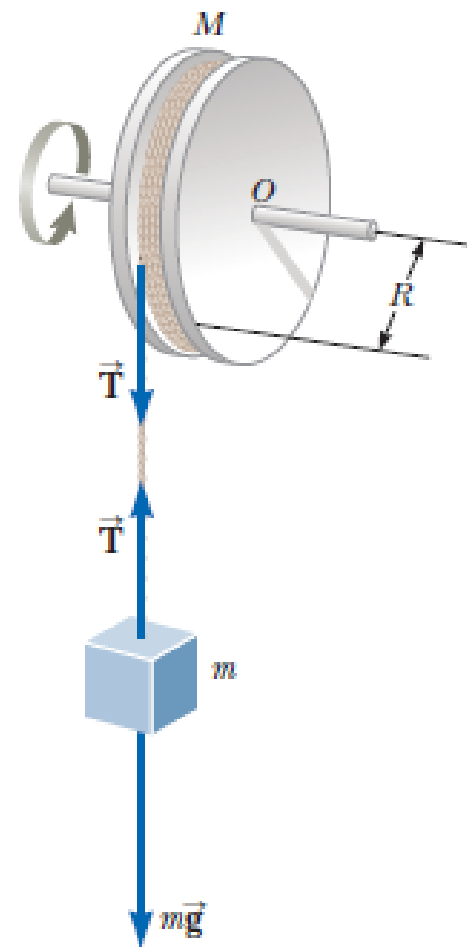
$$mg = \left(m + \frac{I}{R^2} \right) a \quad \Rightarrow \quad a = \frac{mg}{m + \left(I/R^2 \right)}$$

Substituting back in Eq.(4) $\Rightarrow$

$$T = \frac{mg}{1 + \left(mR^2/I \right)}$$

Note that if the wheel is very massive such that I becomes so large we have

$$a = \frac{mg}{\infty} = 0 \quad \text{and} \quad T = \frac{mg}{1 + \left(mR^2/\infty \right)} = mg$$



10.7 Rotational Kinetic Energy

Consider a rigid body consists of small particles, rotates about fixed axis with angular velocity ω .

The kinetic energy of the i th particle of mass m_i and linear speed v_i is given by

$$K_i = \frac{1}{2} m_i v_i^2$$

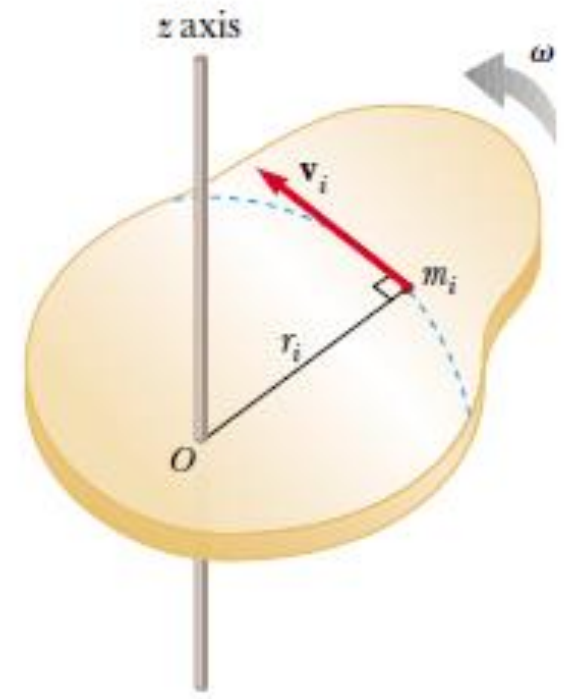
$$\text{But } v_i = r_i \omega \Rightarrow$$

$$K_i = \frac{1}{2} m_i r_i^2 \omega^2$$

The total kinetic energy of the rotating rigid body is the sum of the kinetic energies of the individual particles, i.e.,

$$K = \sum_i K_i = \frac{1}{2} \left(\sum_i m_i r_i^2 \right) \omega^2 = \frac{1}{2} I \omega^2$$

It is analogous to the formula $K = \frac{1}{2} m v^2$



• Quick Quiz (

A section of hollow pipe has inner radius r , outer radius R , mass M , and length L . The pipe is rotating about a central axis with angular speed ω . The rotational kinetic energy of the pipe is K .

a) The hollow pipe is replaced by a solid cylinder of the same mass and length. The rotational kinetic energy of the solid cylinder is K_s .

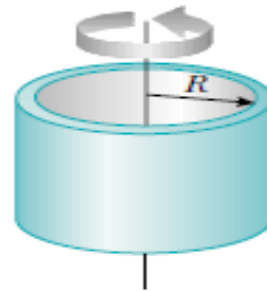
c) Both have the same rotational kinetic energy.

Example 10.10

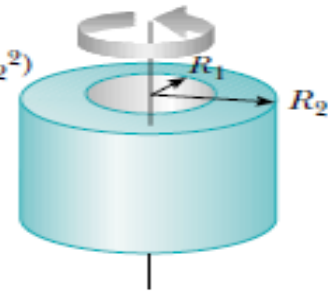
A uniform rod of mass M and length L is fastened to the wall at one end. A negligible mass block is attached to the other end. The rod is released from a horizontal position. The moment of inertia of the rod about the wall is I_w . The rotational kinetic energy of the system at the instant the rod is vertical is K . The angular speed of the rod at this instant is ω . The rotational kinetic energy of the system at the instant the rod is vertical is K_s .

Moments of Inertia of Homogeneous Rigid Objects with Different Geometries

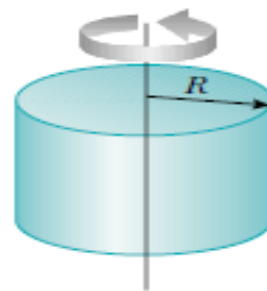
Hoop or thin cylindrical shell
 $I_{CM} = MR^2$



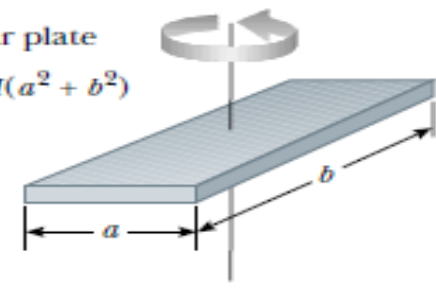
Hollow cylinder
 $I_{CM} = \frac{1}{2} M(R_1^2 + R_2^2)$



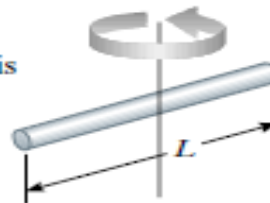
Solid cylinder or disk
 $I_{CM} = \frac{1}{2} MR^2$



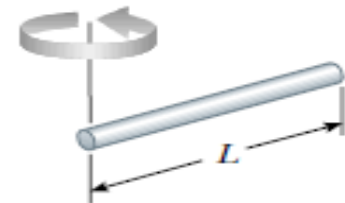
Rectangular plate
 $I_{CM} = \frac{1}{12} M(a^2 + b^2)$



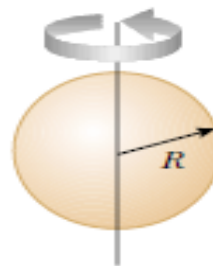
Long thin rod with rotation axis through center
 $I_{CM} = \frac{1}{12} ML^2$



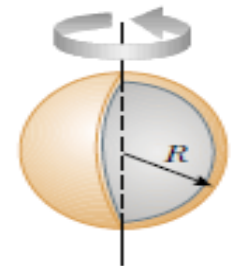
Long thin rod with rotation axis through end
 $I = \frac{1}{3} ML^2$



Solid sphere
 $I_{CM} = \frac{2}{5} MR^2$



Thin spherical shell
 $I_{CM} = \frac{2}{3} MR^2$



Solution To find the moment of inertia of the system we use the equation

$$I = \sum_i m_i r_i^2$$

a) When the axis of rotation is the y-axis, the spheres of mass m have no moment of inertia so we have

$$I = m(0) + m(0) + Ma^2 + Ma^2 = 2Ma^2$$

And for the kinetic energy we have

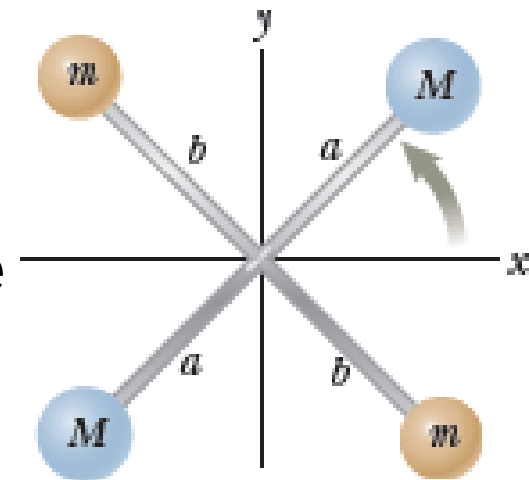
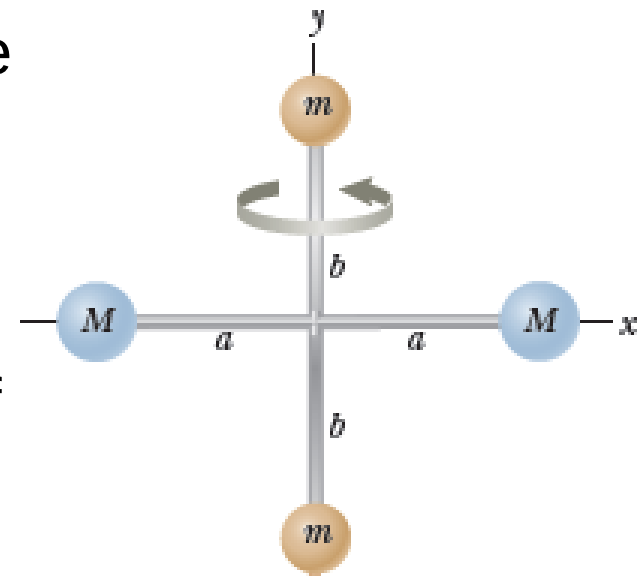
$$K = \frac{1}{2} I \omega^2 = Ma^2 \omega^2$$

b) When the axis of rotation is the z-axis, all the spheres have moment of inertia so we have

$$I = mb^2 + mb^2 + Ma^2 + Ma^2 = 2(Ma^2 + mb^2)$$

And for the kinetic energy we have

$$K = \frac{1}{2} I \omega^2 = (Ma^2 + mb^2) \omega^2$$



10.8 Energy Consideration in Rotational Motion

Let us consider again the situation in the figure shown

The work dW done by the force $\mathbf{F}$ is

$$dW = \mathbf{F} \cdot d\mathbf{s} = F_t ds = F_t (r d\theta)$$

But the net torque acting on the object is

$$\tau = F_t r \Rightarrow$$

$$dW = \tau d\theta$$

The total work W is then

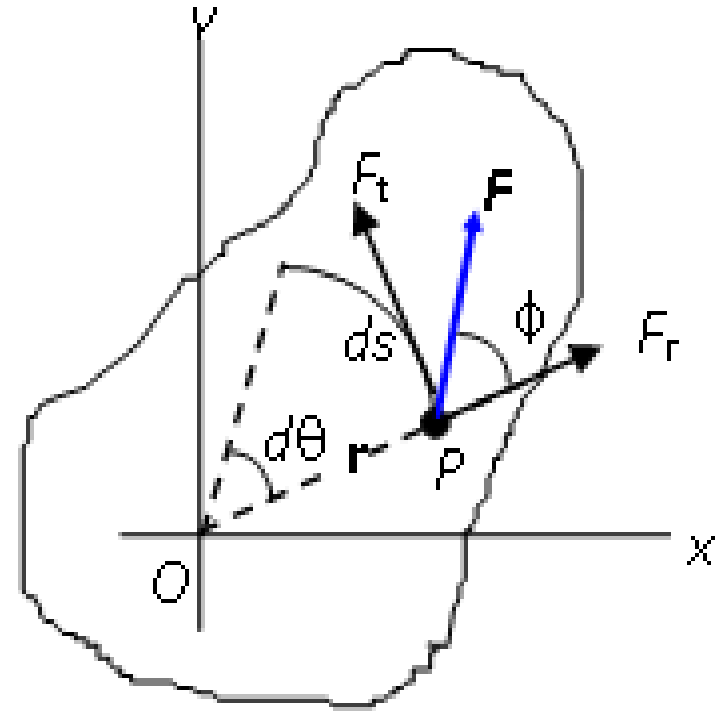
$$W = \int_{\theta_1}^{\theta_2} \tau d\theta$$

It is analogous to the formula

$$W = \int_{x_1}^{x_2} F dx$$

The power is now given as

$$P = \frac{dW}{dt} = \tau \frac{d\theta}{dt} = \tau \omega$$



Which is the rotational analogue of

$$P = Fv$$

$$\text{Now } \tau = I\alpha = I \frac{d\omega}{dt} \Rightarrow$$

$$P = \frac{dW}{dt} = \tau\omega = I\omega \frac{d\omega}{dt} \Rightarrow dW = I\omega d\omega \Rightarrow$$

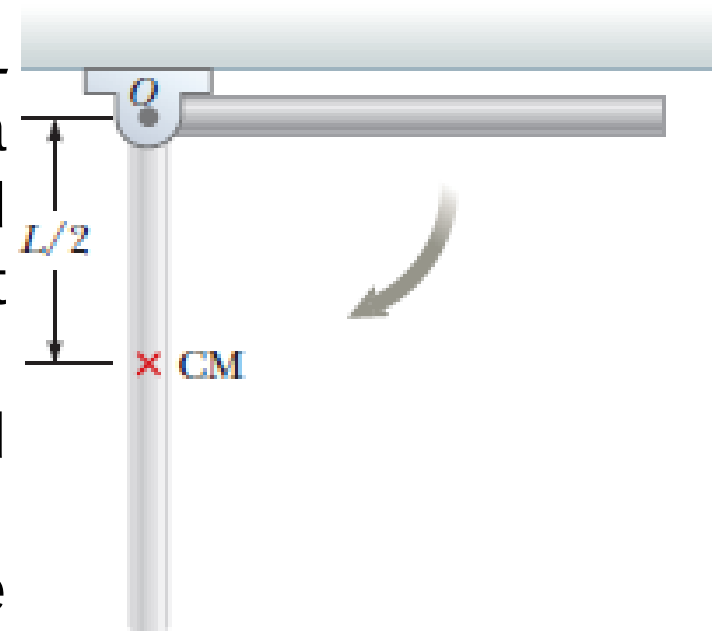
$$W = I \int_{\omega_i}^{\omega_f} \omega d\omega = \frac{1}{2} I (\omega_f^2 - \omega_i^2) = \Delta K \quad \text{with } K = \frac{1}{2} I \omega^2$$

In general, if a system involving rotating objects is isolated and no nonconservative forces act within the system, the isolated system model and the principle of conservation of mechanical energy can also be used to analyze the system, i.e.,

$$E_i = E_f$$

with K will include both translational and rotational kinetic energies.

Example 10.11 A uniform rod of length L and mass M is free to rotate on a frictionless pin passing through one end as shown. The rod is released from rest in the horizontal position.



- What is its angular speed when the rod reaches its lowest position?
- Determine the tangential speed of the center of mass and of the lowest point when the rod is in the vertical position.

Solution a) Considering the system to be the rod-earth system, and taking the horizontal position as the level of zero potential energy and use

$$E_i = E_f \Rightarrow$$

$$E_i = K_i + U_i = 0 \quad E_f = K_f + U_f = \frac{1}{2} I \omega^2 - Mg \frac{L}{2} \Rightarrow$$

$$\frac{1}{2}I\omega^2 - Mg(L/2) = 0 \quad \Rightarrow \quad \omega = \sqrt{\frac{MgL}{I}}$$

Knowing that the moment of inertial of the rod about its end is

$$I = \frac{1}{3}ML^2 \quad \Rightarrow \quad \omega = \sqrt{\frac{MgL}{I}} = \sqrt{\frac{3g}{L}}$$

b) To find the tangential speed we use

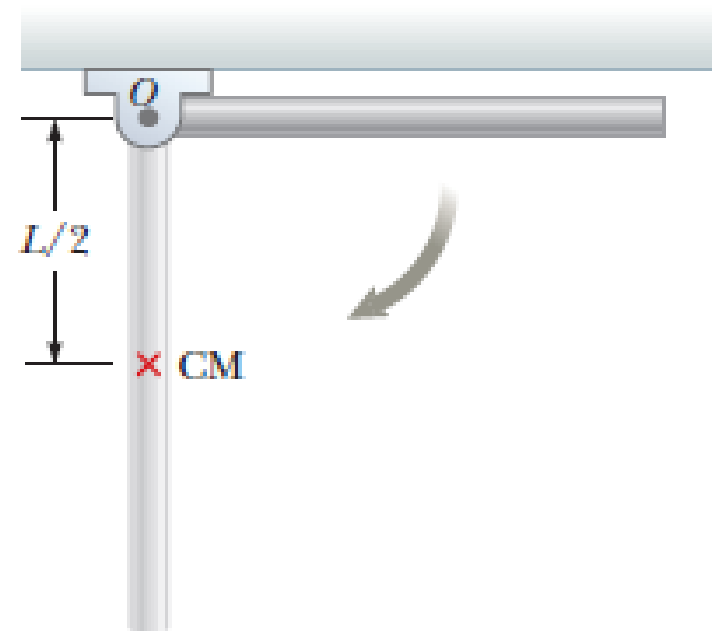
$$v = r\omega \quad \Rightarrow$$

For the center of mass we have

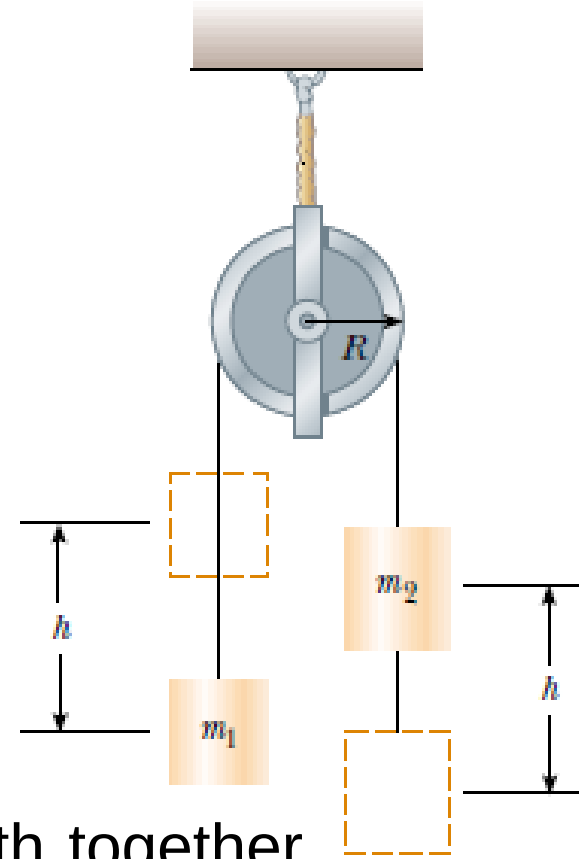
$$v = r\omega = (L/2)\omega = \frac{1}{2}\sqrt{3gL}$$

And for the lowest point we have

$$v = r\omega = L\omega = \sqrt{3gL}$$



Example 10.12 Two blocks having different masses m_1 and m_2 are connected by a string passing over a pulley having a radius R and a moment of inertia I about its axis of rotation, as shown. The string does not slip on the pulley, and the system is released from rest. Find the translational speeds of the blocks after m_2 descends through a distance h , and the angular velocity of the pulley at this time.



Solution Here the system consists of the earth together with the two blocks and the pulley. Applying the conservation of mechanical energy principle we get

$$E_i = E_f$$

$$\text{but } K_i = 0 \quad \text{and } K_f = \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 + \frac{1}{2}I\omega^2$$

Taking the zero potential energy level as the initial positions for each block we get

$$U_i = 0 \quad \text{and} \quad U_f = m_1 gh - m_2 gh$$

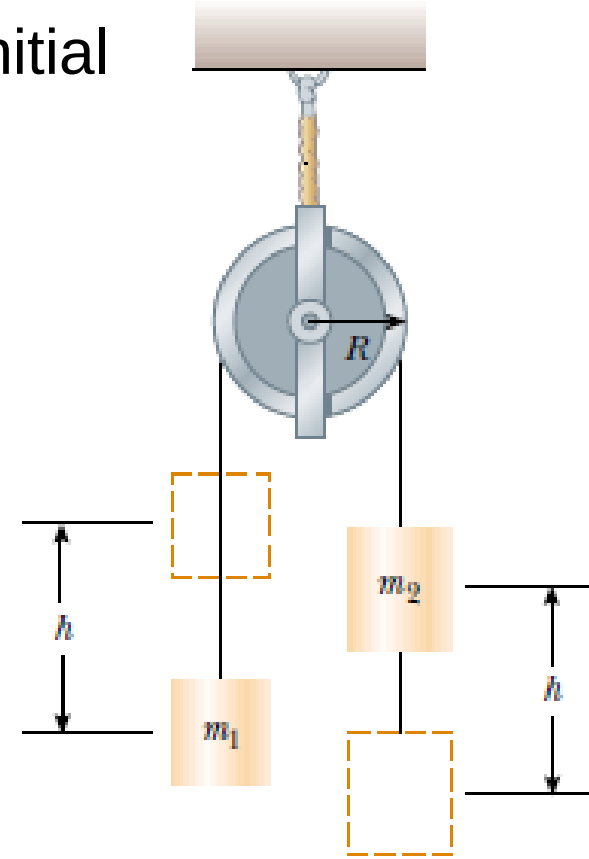
$$\frac{1}{2}m_1 v^2 + \frac{1}{2}m_2 v^2 + \frac{1}{2}I\omega^2 + m_1 gh - m_2 gh = 0 \Rightarrow$$

Knowing that $v = \omega r$ $\Rightarrow$

$$\frac{1}{2}m_1 v^2 + \frac{1}{2}m_2 v^2 + \frac{1}{2}\frac{I}{R^2}v^2 + m_1 gh - m_2 gh = 0 \Rightarrow$$

$$v^2 = \frac{2(m_1 - m_2)gh}{m_1 + m_2 + \frac{I}{R^2}} \Rightarrow v = \sqrt{\frac{2(m_1 - m_2)gh}{m_1 + m_2 + \frac{I}{R^2}}}$$

$$\omega = \frac{v}{R} = \sqrt{\frac{2(m_1 - m_2)gh}{(m_1 + m_2)R + \frac{I}{R}}}$$



Let us apply Newton's law for the pulley, i.e.,

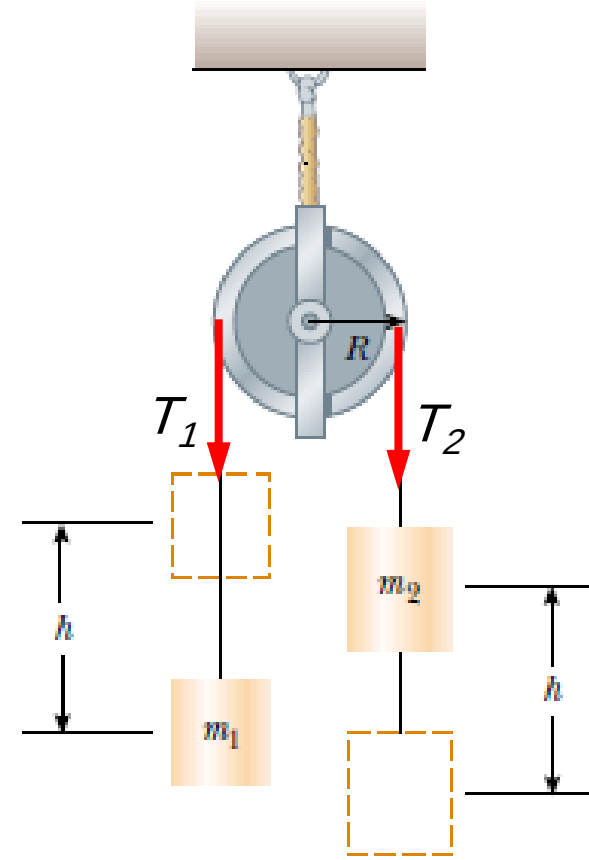
$$\sum \tau = I\alpha$$

$$T_2 R - T_1 R = I\alpha \quad \Rightarrow$$

$$(T_2 - T_1)R = I\alpha$$

It is clear now that for the pulley to rotate, the tensions should not be equal.

They are equal only if I is zero, which is the case when we neglect the mass of the pulley, as we did in chapter 5.



To summaries we have

Quantities	Translational	Rotational
Displacement	x	θ
Velocity	v	ω
Acceleration	a	α
Inertia	m	I
Causing motion	F	τ

Translational Motion

$$v = \frac{dx}{dt}, \quad a = \frac{dv}{dt}$$

$$a = \text{constant} \begin{cases} v_f = v_i + at \\ x_f = x_i + v_i t + \frac{1}{2}at^2 \\ v_f^2 = v_i^2 + 2a(x_f - x_i) \end{cases}$$

$$\sum F = ma$$

$$W = \int_{x_i}^{x_f} F_x dx, \quad K = \frac{1}{2}mv^2, \quad P = Fv$$

Rotational Motion

$$\omega = \frac{d\theta}{dt}, \quad \alpha = \frac{d\omega}{dt}$$

$$\alpha = \text{constant} \begin{cases} \omega_f = \omega_i + \alpha t \\ \theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 \\ \omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i) \end{cases}$$

$$\sum \tau = I\alpha$$

$$W = \int_{\theta_i}^{\theta_f} \tau d\theta, \quad K = \frac{1}{2}I\omega^2, \quad P = \tau\omega$$