



Islamic University of Gaza
Department of Math.
First Semester 26 /12/2016

Calculus B
Final Exam
Time: 2hours

	الاسم	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Total
	الرقم									
	المدرس	12	12	12	10	10	12	12	20	100

Solve the following questions:

[Q1] (a) Find y' for the following functions

(i) $y = x^3 \sin^{-1} x + 3^{\sin x}$.

(ii) $y = (1-x) \coth^{-1} \sqrt{x}$.

(b) Find an equation for the line tangent to the curve $x = \cos t$, $y = 1 + \sin t$ at

$t = \frac{\pi}{2}$.

[Q2] Evaluate the following integrals:-

(a) $\int \frac{dx}{x \sqrt{1 + (\ln x)^2}}$

(b) $\int_0^{\infty} x^2 e^{-x} dx$

[Q3] (a) Discuss the convergence of the improper integral $\int_1^{\infty} \frac{3-2e^{-x}}{x^2} dx$.

(b) Find the following limits

(i) $\lim_{x \rightarrow \infty} (e + x^3)^{\frac{1}{\ln x}}$

(ii) $\lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \csc x \right)$

[Q4] Test the convergence of the following series

a) $\sum_{n=4}^{\infty} \left(1 + \frac{1}{n}\right)^{n^2}$

b) $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n} + 3}{n + 3}$

[Q5] Find the interval and radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^{2n+1}}{2n+1}$$

[Q6](a) Find the Maclaurin series of the function $f(x) = \cosh 2x$.

(b) Find the vertices, foci, center of the conic section

$$9x^2 + 16y^2 + 54x - 64y + 1 = 0$$

[Q7](a) Discuss the symmetry, then sketch the graph of $r = 1 + 2 \sin \theta$.

(b) Replace the following polar equation by equivalent Cartesian equation

$$r = \frac{2}{2 \sin \theta + 3 \cos \theta}$$

(c) Find the length of the curve $x = 2 \cos \theta$, $y = 2 \sin \theta$, $0 \leq \theta \leq \frac{\pi}{2}$

[Q8] Choose the correct answer for each of the following statements:-

1) If $y = e^{-\frac{1}{2 \ln x}}$, then $y' =$

- a) $2x$ b) $-2x$ c) $\frac{1}{2x}$ d) $-\frac{1}{2x}$

2) The exact value of $x^{-\frac{1}{\log_3 x}}$ is

- a) 3 b) $\frac{1}{3}$ c) $\ln 3$ d) e^{-3}

3) The sum of the series $\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$ is

- a) 2 b) $\frac{1}{2}$ c) 3 d) $\frac{2}{3}$

4) The polar equation $\phi = \frac{\pi}{4}$ represents

- a) a circle b) a line through the origin c) a vertical line d) a plane

5) The polar equation $x = 3 \cos \theta$, $y = 4 \sin \theta$, $\theta \in [0, 2\pi]$ represents

- a) a hyperbola b) an ellipse c) a parabola d) a cardioid

6) The series $\sum_{n=1}^{\infty} (-1)^n \left(\frac{1}{n}\right)$

- a) converges absolutely b) converges c) converges conditionally d) diverges

7) The polar equation for the line $x = 2$ is

- a) $r = 2 \csc \theta$ b) $r = 2 \sec \theta$ c) $r = 2 \sin \theta$ d) $r = 2 \cos \theta$

8) If $-\log_x \left(\frac{1}{32}\right) = 5$ then $x =$

- a) $\frac{1}{2}$ b) 2 c) 5 d) $\frac{1}{5}$

9) Consider the series $\sum_{n=1}^{\infty} a_n$, $a_n > 0$, such that $\lim_{n \rightarrow \infty} a_n = 0$, then the series

- a) converges b) diverges c) may converges or diverges d) neither of the above

10) If $f(x) = \log_x e$, then $f^{-1}(x) =$

- a) e^{-x} b) e^x c) $\ln x$ d) $\frac{1}{\ln x}$

Question	1	2	3	4	5	6	7	8	9	10
Answer										

Good Luck