

Q2/ show that $p \wedge (p \rightarrow q) \equiv q \wedge p$

$$\begin{aligned} p \wedge (\neg p \vee q) &\equiv (p \wedge q) \vee (p \wedge \neg p) \\ &\equiv p \wedge q \end{aligned}$$

(b) Suppose that we have two propositions:
it is below freezing, it is snowing.

(i) $p \rightarrow q$

(ii) $(p \vee q) \wedge (p \rightarrow \neg q)$

Q3/ Find as good big-O estimate as possible

(1) $(n^2 + 5n + 100)(4n^3 + 70n + 3)$

$$\begin{aligned} &4n^5 + 70n^3 + \dots \\ &\rightarrow O(n^5) \end{aligned}$$

(2) $3n^2 \log n + 5n + 48000$

$$O(n^2 \log n) \quad O(n) \quad O(1)$$

$$\downarrow$$

$$\rightarrow O(n^2 \log n)$$

(3) $n^2 + n + \log n!$

$$O(n^2) \quad O(n) \quad O(\log n!)$$

$$\rightarrow O(n^2)$$

$$\log n! < n \log n < n^2$$

$$\textcircled{4} \quad 25n \log n + 300 + (3n+1)(2n+5) \quad \boxed{2}$$

$$O(n \log n) \quad \quad \quad \textcircled{6n^2 + 15n + \dots}$$

$$O(n^2)$$

Since $n^2 > n \log n$

$$\Rightarrow O(n^2).$$

Q4 | ① $a = 2730$, $b = 570$ $\gcd(a, b)$
using Euclidean algorithm

$$\begin{aligned} \gcd(2730, 570) &= \gcd(2730 \bmod 570, 570) \\ &= \gcd(570, 450) = \gcd(570 \bmod 450, 450) \\ &= \gcd(450, 120) = \gcd(450 \bmod 120, 120) \\ &= \gcd(120, 90) = \gcd(120 \bmod 90, 90) \\ &= \gcd(90, 30) = \gcd(90 \bmod 30, 30) \\ &= \gcd(30, 0) = \boxed{30} \end{aligned}$$

② Find $\text{lcm}(15, 18)$

$$15 = 3 \times 5$$

$$18 = 2 \times 3 \times 3 = 2 \times 3^2$$

$$\text{lcm} = 2^{\max(0,1)} \times 3^{\max(1,2)} \times 5^{(0,1)}$$

$$= 2 \times 3^2 \times 5 = 90$$

③ $\gcd(732, 324)$ by Prime Factorization 3

$$\begin{aligned} 732 &= 2 \cdot 366 = 2 \cdot 2 \cdot 183 = 2 \cdot 2 \cdot 3 \cdot 61 \leftarrow \text{Prime from} \\ &= 2^2 \cdot 3 \cdot 61 \quad \downarrow \sqrt{61} = 7.8 \\ 324 &= 2 \cdot 162 = 2 \cdot 2 \cdot 81 = 2 \cdot 2 \cdot 3 \cdot 27 \\ &= 2 \cdot 2 \cdot 3 \cdot 3 \cdot 9 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \\ &= 2^2 \cdot 3^4 \end{aligned}$$

$$\begin{aligned} \gcd(732, 324) &= 2^{\min(2,2)} \cdot 3^{\min(1,4)} \cdot 61^{\min(1,0)} \\ &= 2^2 \cdot 3^1 \cdot 61^0 = 12 \end{aligned}$$

Q5 | (A) by Contradiction, if $m \cdot n$ is an odd integer^P
then both m and n ¹⁹ are odd integers

We suppose $P \wedge \neg Q$ that

$m \cdot n$ is an odd integer and $\neg (m \text{ and } n \text{ are odd integers})$

$\Rightarrow$ and m is an even or n is an even

if m is an even, so we can write it
as $2k$, k is integer.

$m \cdot n = 2kn = 2(kn) \leftarrow$ even number
which contradicts our supposition.

same if n is an even, we can write
it as $2t$, t is integer

$m \cdot n = 2tm = 2(tm) \leftarrow$ even.

we have contradiction

⑧ if a and b are rational, then ab is rational 4

By direct prove, suppose P is true

So a is rational and b is rational

we can write them as $\frac{x}{y}$, $\frac{w}{z}$ respectively

with x, y, w, z are integers and $y, z \neq 0$

$$ab = \frac{x \cdot w}{y \cdot z} \leftarrow \text{rational.}$$

⑨ All animals with scales are dragons. Some animals which are not dragons have sharp claws. So there are animals without scales which have sharp claws. From the premises prove the conclusion

Let $S(x)$: x is with scales, $D(x)$: x is dragon
 $C(x)$: x has sharp claws.

$$\textcircled{1} \forall x (S(x) \rightarrow D(x)) \quad \text{premise.}$$

$$\textcircled{2} \exists x (\neg D(x) \wedge C(x)) \quad \text{premise}$$

we need to conclude $\boxed{\exists x (\neg S(x) \wedge C(x))}$

$$\textcircled{3} S(p) \rightarrow D(p) \quad \text{Universal instantiation from } \textcircled{1}$$

$$\textcircled{4} \neg D(p) \wedge C(p) \quad \text{Existential instantiation from } \textcircled{2}$$

$$\textcircled{5} \neg D(p) \quad \text{simplification from } \textcircled{4}$$

$$\textcircled{6} \neg S(p) \quad \text{modus tollens from } \textcircled{3} \text{ and } \textcircled{5}$$

$$\textcircled{7} C(p) \quad \text{simplification from } \textcircled{4}$$

$$\textcircled{8} \neg S(p) \wedge C(p) \quad \text{Conjunction from } \textcircled{6}, \textcircled{7}$$

$$\textcircled{9} \exists x (\neg S(x) \wedge C(x)) \quad \text{Existential generalization from } \textcircled{8}$$

Q6) @ the negation of

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$$\neg (\exists x P(x) \rightarrow \forall x (P(x) \wedge Q(x)))$$

$$\neg (\neg \exists x P(x) \vee \forall x (P(x) \wedge Q(x)))$$

$$\exists x P(x) \wedge \neg [\forall x (P(x) \wedge Q(x))]$$

$$\exists x P(x) \wedge \exists x \neg (P(x) \wedge Q(x))$$

$$\exists x P(x) \wedge [\exists x \neg P(x) \vee \neg Q(x)]$$

$$\exists x P(x) \wedge \neg P(x) \vee$$

هم هنا
وطالع
مش
بس
بنياني
هو المطلوب

$$\exists x P(x) \wedge \exists x \neg P(x) \vee \exists x P(x) \wedge \exists x \neg Q(x)$$

$$\exists x (P(x) \wedge \neg P(x)) \vee \exists x P(x) \wedge \exists x \neg Q(x)$$

F

$$\exists x (P(x) \wedge \neg Q(x))$$

$$\exists x (Q(x) \rightarrow P(x))$$

(b) Using Bubble Sort, give the ordering of the list {5, 7, 4, 1, 3} 6

Procedure bubble sort ($a_1, \dots, a_n$: real numbers with $n \geq 2$)

for $i := 1$ to $n-1$

for $j := 1$ to $n-i$

if $a_j > a_{j+1}$ then interchange a_j and a_{j+1}

حسب المطلوب
تتبعها
بخطوطها
للتذكير

① {5, 7, 4, 1, 3} $\Rightarrow$ ② {5, 7, 4, 1, 3} $\Rightarrow$ {5, 4, 7, 1, 3}

③ {5, 4, 1, 7, 3} $\Rightarrow$ ④ {5, 4, 1, 3, 7}

من $j=1$ لـ $j=4$ وتقصص

⑤ {4, 5, 1, 3, 7} $\Rightarrow$ {4, 1, 5, 3, 7}

من $j=1$ لـ $j=3$

⑥ {4, 1, 3, 5, 7}

⑦ {1, 4, 3, 5, 7} $\Rightarrow$ {1, 3, 4, 5, 7}

من $j=1$ لـ $j=2$

⑧ {1, 3, 4, 5, 7}

من $j=1$ لـ $j=1$ وها حتمت بالامرة واحدة

اربعة مرة كما $i=4$ خارج بس مابل

Q7

A, B are sets, Find

7

② show that

$$\bar{A} \cap (A \cup B) = B - A$$

$$\bar{A} \cap (A \cup B) = B - A \quad \text{كلنا حتى نثبت أن}$$

$$\bar{A} \cap (A \cup B) \subseteq B - A$$

لازم نثبت أن

$$B - A \subseteq \bar{A} \cap (A \cup B) \quad \circ$$

$$\textcircled{1} \quad \bar{A} \cap (A \cup B) \subseteq B - A$$

$$\text{Let } x \in \bar{A} \cap (A \cup B)$$

$$x \in \bar{A} \text{ and } (x \in A \text{ or } x \in B)$$

$$\bullet x \in \bar{A} \text{ and } x \in A \Rightarrow x \in \emptyset \subseteq B - A$$

$$\bullet x \in \bar{A} \text{ and } x \in B \Rightarrow x \in B - A \subseteq B - A$$

$$\textcircled{2} \quad B - A \subseteq \bar{A} \cap (A \cup B)$$

$$x \in B \text{ and } x \notin A$$

$$\text{so } x \in \bar{A}$$

$$x \in \bar{A} \Rightarrow B - A \subseteq \bar{A} \cap (A \cup B)$$

⑥ A, B sets contains the letter of the words
COMPUTER and COMMUNICATION
respectively, find the union and intersection

$$A \cup B = \{C, O, M, P, U, T, E, R, N, I, A\}$$

$$A \cap B = \{C, O, M, U, T\}$$

Q8 Write an algorithm to add two matrices and what is the time complexity. 8

procedure addmatrices (A, B : $m \times n$ matrices)

$C := m \times n$ matrix $O(1)$

for $i := 1$ to m $O(m \times n)$

for $j := 1$ to n

$C_{ij} := a_{ij} + b_{ij}$

return C

the complexity is $O(m+n)$

(b) $A = \begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 2 \\ 3 & 4 \end{bmatrix}$

$$C = \begin{bmatrix} 8 & 6 \\ 4 & 9 \end{bmatrix}$$

(c) $A^T \times B$

$$A^T = \begin{bmatrix} 8 & 4 \\ 6 & 9 \end{bmatrix} \cdot \begin{bmatrix} 6 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 60 & 32 \\ 63 & 48 \end{bmatrix}$$

Q-Bonus

السؤال بدو
تغيريل

9

IYE - QDD - DRO - LYXEC

بما انو تشفير من خلال $f(x) = x + 10$

فك التشفير لازم نطرح 10 $x - 10 \pmod{26}$

XOU - GET - THE - BONUS

1	2	3	4	5	6	7	8	9
A	B	C	D	E	F	G	H	I

10	11	12	13	14	15	16	17
J	K	L	M	N	O	P	Q

18	19	20	21	22	23	24	25	26
R	S	T	U	V	W	X	Y	Z