



Student Name:.....

Student No.:.....

Q1 (26pt.)	Q2 (9pt.)	Q3 (6pt.)	Q4 (7pt.)	Q5 (10pt.)	Q6 (7pt.)	Q7 (7pt.)	Q8 (8pt.)	Bonus / 5	Total/ 80

- Try to solve all the following eight Questions.
- This exam should be solved without any calculator, book, or notes
- **Academic Integrity:** Plagiarism, cheating, and other forms of academic dishonesty are prohibited and may result in grade F for the course.

Q1) [26 pt.] Classify the following sentences whether true or false. There is no need to justify your answers on this problem.

- () 1. If the implication $\neg P \rightarrow Q$ is true, then $\neg Q \rightarrow P$.
- () 2. If A or B is true, and B is true, then A must be true.
- () 3. $\gcd(n+1, 2n+1) = 1$.
- () 4. For all $n > 1$, $2^n - 1$ is prime.
- () 5. For all odd n , $\gcd(n, n-2) = 1$.
- () 6. For all a, m , $a^{m-1} \equiv 1 \pmod{p}$.
- () 7. For all $a, m = pq$, where p and q are prime, $a^{(p-1)(q-1)} \equiv 1 \pmod{m}$.
- () 8. For all $a, m = pq$, where $\gcd(a, m) = 1$ and p and q are prime, $a^{(p-1)(q-1)} \equiv 1 \pmod{m}$.
- () 9. Number 5 has an inverse mod 12?
- () 10. Let p be prime; then we're guaranteed that $x^{p-1} \equiv 1 \pmod{p}$ for all $x \in \mathbb{N}$.
- () 11. Let $p \in \mathbb{N}$ be such that $x^{p-1} \equiv 1 \pmod{p}$ holds for every $x \in \mathbb{N}$ with $\gcd(x, p) = 1$; then p is guaranteed to be prime.
- () 12. Let p be prime and suppose $x \in \mathbb{N}$ satisfies $x \not\equiv 0 \pmod{p}$; then we're guaranteed that $x^{p^2-p} \equiv 1 \pmod{p^2}$.
- () 13. Let S, T be arbitrary sets; then we're guaranteed that $|S \cup T| = |S| + |T|$.
- () 14. If the implication $P \rightarrow Q$ is true, then its converse is guaranteed to be true, too.
- () 15. $\forall w \in \mathbb{Z}, \exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}, \exists z \in \mathbb{Z}: w+x = y+z$.
- () 16. $\exists x \in \mathbb{N}: \forall p \in \mathbb{Z}: p > 5 \Rightarrow x^2 \equiv 1 \pmod{p}$.
- () 17. $\forall p \in \mathbb{Z}, p > 5 \Rightarrow \exists x \in \mathbb{N}, x^2 \equiv 1 \pmod{p}$.
- () 18. If m is any natural number satisfying $m \equiv 1 \pmod{2}$, then the equation $2048x \equiv 1 \pmod{m}$ is guaranteed to have a solution for x .
- () 19. The propositions $P \Rightarrow Q$ and $Q \Rightarrow P$ are logically equivalent.
- () 20. The proposition $Q \vee P \vee (P \Rightarrow Q)$ is guaranteed to be true, no matter what the truth values of P and Q may be.
- () 21. If we are told that the proposition P is true, we are entitled to conclude that $((P \Rightarrow Q) \Rightarrow P) \Rightarrow Q$ must also be true.
- () 22. $\forall n \in \mathbb{N}, (x^2 - x = 0) \vee (x \geq 2)$.
- () 23. $\forall x \in \mathbb{N}, \exists y \in \mathbb{N}, \forall z \in \mathbb{N}, x+y = z$.
- () 24. $\forall x \in \mathbb{N}, \exists y \in \mathbb{N}, x^2 = y$.
- () 25. $(\forall x \in \mathbb{N}, \exists y \in \mathbb{N}, x < y) \equiv (\exists y \in \mathbb{N}, \forall x \in \mathbb{N}, x < y)$.
- () 26. Define the relation $\prec$ so that $n \prec m$ iff n divides m and $n < m$. Then the relation $\prec$ is a well-ordering on $\mathbb{N}$.

Q2) [9 pt.]

a) By using only the logical identities, show that $p \wedge (p \rightarrow q) \equiv q \wedge p$

b) Suppose that we have two propositions: it is below freezing, it is snowing. Write the following sentences as logical connective

i- That it is below freezing is necessary and sufficient for it to be snowing.

ii- It is either below freezing or it is snowing, but it is not snowing if it is below freezing.

Q3) [6 pt.] Find as good big-O estimate as possible to the following

1. $(n^2 + 5n + 100)(4n^3 + 70n + 3)$

2. $3n^2 \log n + 5n + 48000$

3. $n^2 + n + \log n!$

4. $25n \log n + 300 + (3n + 1)(2n + 5)$

Q4) [7 pt.] As an application of primes, try to solve the following problems

I- Let $a=2730$ and $b=570$. Find $\gcd(a,b)$ by using Euclidean algorithm.

II- Find $\text{lcm}(15,18)$

III- Find $\gcd(732,324)$ by Prime factorization

Q5) [10 pt.] From your knowledge with the proving methods, prove the following hypothesis:

A) Prove by contradiction that if $m.n$ is an odd integer, then both m and n are odd integers.

B) Prove that if a and b are rational, then ab is rational.

- C) All animals with scales are dragons. Some animals which are not dragons have sharp claws. So there are animals without scales which have sharp claws. From the premises, prove the conclusion.

Q6) [3 pt.] Try to find the right answer for the parts below:

- a) What is the negation of the statement: $\exists x P(x) \rightarrow \forall x (P(x) \wedge Q(x))$. Simplify the answer as possible.
- b) By using Bubble Sort algorithm step by step, give the ordering of the list $\{5, 7, 4, 1, 3\}$

Q7) [7 pt.] For A and B are sets, find:

- a) Let A and B be given two sets. Without using Venn diagrams, show that $\bar{A} \cap (A \cup B) = B - A$

- b) Let A and B sets that contain the letters of the words: COMPUTER, and COMMUNICATION respectively, use bit strings to find the union and intersection of these sets

Q8) [8 pt.]

- a) Write an algorithm to add two matrices together, and what is the time complexity?

- b) Check your algorithm by calculate the summation of the matrices below:

$$A = \begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix}, B = \begin{bmatrix} 6 & 2 \\ 3 & 4 \end{bmatrix}$$

- c) Determine $A^T \cdot B$

Q-Bonus) [5 pt.] this message for you, but to guarantee that you only one to read it, it is encrypted by Caesar's cipher function of $f(x) = x + 10$.
The message is: JYE-QYD-DRO-LYXEC

Good Luck

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