



Student Name:..... Student No.:.....

Q1) [10 pt.] Classify the following sentences whether true or false.

- 1) If $f_1(x)$ and $f_2(x)$ are two functions then $f_1(x) * f_2(x)$ is $O(\max(f_1(x), f_2(x)))$
- 2) If $f(x)$ is $O(g(x))$ and $f(x)$ is $\Omega(g(x))$ then $f(x)$ is $\Theta(g(x))$
- 3) $f(x) = (x+1)/(x+2)$ is onto from $\mathbb{R}$ to $\mathbb{R}$
- 4) Two sets are called disjoint if their intersection is the empty set
- 5) The complexity of binary search algorithm is $O(\log(n))$
- 6) The set Z is the set formed by the union of the set Z^+ and the set $\{0\}$.
- 7) The number 1001 is a prime number
- 8) $\forall x (x^2 + 1 \geq 2)$ where the universe of discourse is Z^+
- 9) If $a|b$ and $b|c$ then $a|c$
- 10) $x+y = y+z$ is a proposition

(X)
(X)
(X)
(X)
(X)
(X)
(X)
(X)
(X)
(X)

10

Q2) [10 pt.] Select the correct Answer

- 1) $a^{x*y} \bmod (x*y) =$
a) x b) y c) a d) 1
- 2) if $A - B = \{1,3,6\}$, $B - A = \{2,5,8\}$ then
a) $A = \{1,3,6\}$, $B = \{2,3,5,8\}$
b) $A = \{1,3,6\}$, $B = \{2,4,5,8\}$
c) $A = \{1,3,5,6\}$, $B = \{2,3,4,5,8\}$
d) $A = \{1,3,4,6\}$, $B = \{2,4,5,8\}$
- 3) the inverse of 7 modulo 17 is
a) 12 b) 5 c) 6 d) 2
- 4) The case of not finding an element in a list of n integers using linear search called
a) Best case b) worst case c) average case d) none of the above
- 5) the number $(1234)_{16} = ()_8$
a) 2AC b) 11064 c) 042610 d) 46011
- 6) If $A = \{1,2,3,4,5\}$, $B = \{2,3,5,6,7\}$ then $\{1,2,3,4,5,6,7\}$ is called
a) $A \cup B$ b) $A \cap B$ c) $A - B$ d) $B - A$
- 7) If $U = \{1,2,3,4,5,6,7,8\}$ and the binary representation of $A = 1001 0110$ then $A =$
a) $\{1,4,6,7\}$ b) $\{2,3,5,8\}$ c) $\{1,2,3,4\}$ d) $\{5,1,6,2\}$
- 8) $A \cup (A \cap B) =$
a) Φ b) U c) A d) B
- 9) $\gcd(2*2*3*5*7, 2*3*3*5*7) =$
a) $2*3*5*7$ b) $2*2*3*3*5*7$ c) $2*2*3*3*5*5*7*7$ d) $2*3*5*7$
- 10) If $x_{n+1} = (3x_n + 5) \bmod 7$ and $x_0 = 2$ then $x_3 =$
a) 1 b) 4 c) 0 d) 3

X₁

X_{3 1}

X₂

10

Q3) [15 pt.]

a) [9 pt.] Write Pseudocode algorithm to implement the Binary Division with Remainder and determine Big Theta order.

procedure div-rem($a, b \in \mathbb{Z}^+$)

$n := \max(\text{length of } a \text{ in bits, length of } b \text{ in bits}) \quad \theta(1)$

for $i := n-1$ downto 0

—————→ n times

if $a \geq d \cdot 2^i$ then

⑥ $q_i := 1$
else $a := a - d \cdot 2^i$

$q_i := 0$, $r := a$

return q, r

{ q = quotient & r = remainder }

③ $\theta = n \theta(1) + \theta(1)$
 $= \theta(n) + \theta(1)$
 $f(n) = \theta(n)$

b) [6 pt.] Find as good big-O estimate as possible to the following

①.5 1. $(n^2 + 100 - 5n)(4n^3 + 70n + 3) \quad O(n^2) * O(n^3) = O(n^5)$

①.5 2. $3n^2 \log n + 5n + 1/n \quad O(n^2 \log n) + O(n) + O(n^{-1}) = O(n^2 \log n)$

①.5 3. $n^2 + 30n + 3 \log n!$
 $O(n^2) + O(\log n!) \quad \log n! < \log n^n = n \log n$
 $\therefore O(n^2)$

①.5 4. $25n \log n + 300 + (3n + 1)(2n + 5)$
 $O(n \log n) + O(1) + O(n^2) = O(n^2)$

Q4) [10 pt.] solve the following parts:

a) [5pt.] By logical equivalence rules, prove that $(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$ is a tautology

$$\neg[(p \rightarrow q) \wedge (q \rightarrow r)] \vee (p \rightarrow r) = \neg(p \rightarrow q) \vee \neg(q \rightarrow r) \vee (p \rightarrow r)$$

$$\neg(\neg p \vee q) \vee \neg(\neg q \vee r) \vee (\neg p \vee r) = (p \wedge \neg q) \vee (q \wedge \neg r) \vee (\neg p \vee r)$$

$$(p \wedge \neg q) \vee \neg p \vee (q \wedge \neg r) \vee r = [(p \vee \neg p) \wedge (\neg q \vee \neg p)] \vee [(q \vee r) \wedge (\neg r \vee r)]$$

$$[T \wedge (\neg q \vee \neg p)] \vee [(q \vee r) \wedge T] = \neg q \vee \neg p \vee q \vee r$$

$$\neg q \vee q \vee \neg p \vee r = T \vee \neg p \vee r = T$$

b) [5pt.] If It is rainy, then I will stay home.

1. Write the converse of this statement.

②.5 if I stay home, then it is rainy

2. Write the contrapositive of this statement.

②.5 if I don't stay home, then it is not rainy

sec. 1.5
Exercise
25

Q5 [15 pt.] From your knowledge about Proving methods, solve the following parts:

a) [5pt.] Prove that the sum of an irrational number and rational number is irrational using a proof by contradiction. suppose that r is rational and i is irrational and

$s = r + i$ is rational

① $s + (-r) = i$ is irrational

but $s + (-r) = \frac{p}{q} - \frac{t}{u} = \frac{pu - tq}{qu}$ $q, u \neq 0$ then $qu \neq 0$

② to be $s = r + i$ is rational should to be i is rational
but we suppose that i is irrational

② $\circ\circ$ ($s = r + i$ is rational) $\rightarrow$ i is rational

$\circ\circ$ i is irrational $\rightarrow$ $\boxed{s = r + i \text{ is irrational}}$

b) [5pt.] All animals with scales are dragons. Some animals which are not dragons have sharp claws. So there are animals without scales which have sharp claws. From the premises, prove the conclusion.

let $S(x) = x$ has scales $\wedge D(x) = x$ is a dragon $\wedge C(x) = x$ has sharp claws

1. $\forall x [S(x) \rightarrow D(x)]$ premise
2. $\exists x [\neg D(x) \wedge C(x)]$ premise
3. $\neg D(a) \wedge C(a)$ existential specification from (2)
4. $S(a) \rightarrow D(a)$ universal specification from (1)
5. $\neg D(a)$ from (3)
6. $\neg S(a)$ from (4)
7. $C(a)$ from (3)
8. $\neg S(a) \wedge C(a)$ from (6) $\wedge$ (7)
9. $\exists x [\neg S(x) \wedge C(x)]$ existential specification from (8)

c) [5pt.] Let

(i) n is an even integer.

(ii) $3n + 2$ is an even.

1. Prove that (i) $\rightarrow$ (ii).

2. Prove that (ii) $\rightarrow$ (i).

Contrapositive Hint
 $P \rightarrow Q = \neg Q \rightarrow \neg P$

① let (n) be an even Integer then $n = 2K$ $\wedge$ K is Integer, therefore,

②.5 $3n + 2 = 3(2K) + 2 = 6K + 2 = 2(3K + 1) = 2(j)$ and j is Integer
then $3n + 2$ is even $\#$

② if $3n + 2$ is an even then n is even by Contrapositive let n is an odd

$n = 2m + 1$ $\wedge$ $3n + 2 = 3(2m + 1) + 2 = 6m + 3 + 2 = 6m + 4 + 1$

②.5 $= 2(3m + 2) + 1 = 2(q) + 1$ and q is an Integer

$\circ\circ$ $3n + 2$ is an odd

from Contrapositive n is an odd $\rightarrow 3n + 2$ is an odd

$\circ\circ$ $3n + 2$ is an even $\rightarrow n$ is an even

Q6) [10 pt.]

a) [5 pt.] Let $A = \{a, b, c, d, e\}$ $B = \{1, a, b, 2, c\}$ $C = \{3, c, a, 1, f\}$

1. Write $A \cup B$.
2. Write $A - (B \cap C)$
3. Write $(A \cap C) \times (C - B)$
4. What is the cardinality of $P(A)$, power set of A ?
5. What is the cardinality of $A \times C$?

- ① 1- $A \cup B = \{a, b, c, d, e, 1, 2\}$
- ① 2- $A - (B \cap C) = A - \{1, a, c\} = \{b, d, e\}$
- ① 3- $\{a, c\} \times \{3, f\} = \{(a, 3), (a, f), (c, 3), (c, f)\}$
- ① 4- $|A| = 5$ then $|P(A)| = 2^5 = 32$
- ① 5- $|A \times C| = |A| \cdot |C| = 5 \times 5 = 25$

b) [3pt.] Let A and B sets that contain the letters of the words: MATHEMATICS, and APPLICATIONS respectively, use bit strings to find $A - B$ and intersection of these sets

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	
1	0	1	0	1	0	0	1	1	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	mathematics
1	0	1	0	0	0	0	0	1	0	0	1	0	1	1	1	0	0	1	1	0	0	0	0	0	0	applications
0	0	0	0	1	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	$A - B$ {e, h, m}
1	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	$A \cap B$ {a, c, i, s, t}

c) [2pt.] Let A and B be given two sets. Without using Venn diagrams, show that $\bar{A} \cap (A \cup B) = B - A$

$$\begin{aligned} \bar{A} \cap (A \cup B) &\stackrel{1/2}{=} (\bar{A} \cap A) \cup (\bar{A} \cap B) \stackrel{1/2}{=} \emptyset \cup (\bar{A} \cap B) \\ &\stackrel{1/2}{=} (\bar{A} \cap B) \stackrel{1/2}{=} B - A \end{aligned}$$

Q6) [10 pt.]

a) [3pt.] Find $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}^T \times \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}$

$$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 \times 2 + 1 \times 1 & 2 \times 0 + 1 \times 2 \\ 3 \times 3 + 4 \times 1 & 3 \times 0 + 4 \times 2 \end{bmatrix} \\ = \begin{bmatrix} 7 & 2 \\ 13 & 8 \end{bmatrix}$$

b) [4pt.] Let $a=1000$ and $b=5040$. Find $\gcd(a,b)$ by using Euclidean algorithm.

$$\gcd(1000, 5040) = \gcd(5040, 1000) = \gcd(5040 \bmod 1000, 1000) \\ = \gcd(1000, 40) = \gcd(1000 \bmod 40, 40) = \gcd(0, 40)$$

$$\therefore \gcd(1000, 5040) = 40$$

c) [3pt.] Find $\text{lcm}(30, 18)$

$$18 = 2 \times 3 \times 3 = 2^1 \times 3^2 \times 5^0$$

$$30 = 2 \times 3 \times 5 = 2^1 \times 3^1 \times 5^1$$

$$\therefore \text{lcm}(30, 18) = 2^1 \times 3^2 \times 5^1$$

$$= 2 \times 9 \times 5$$

$$= 90$$

Grade of Midterm Exam from 30 = $75/2.5 = 30$

Q1 (10)	Q2 (10)	Q3 (15)	Q4 (10)	Q5 (15)	Q6 (10)	Q7 (10)	Total 80
10	10	15	10	15	10	10	80

75 + 5 =

Good Luck

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