



Name: -----

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Answer all following four Questions

Q1) Let $F(x,y)$ be the statement "x can help y", where the domain consists of all people in world. Use quantifiers and logical operators to express each of the statements:

(i) Everybody can help Ali.

$$\forall x F(x, \text{Ali})$$

(ii) Sara can help someone.

$$\exists y F(\text{Sara}, y)$$

(iii) Just one man can help both Ahmed and Omar.

$$\exists! x (F(x, \text{Ahmed}) \wedge F(x, \text{Omar}))$$

(iv) No one can help himself.

$$\neg \exists x F(x, x)$$

(v) Not everyone can help someone.

$$\neg \forall x \exists y F(x, y)$$

Q2) ^①All animals with scales are dragons. ^②Some animals which are not dragons have sharp claws. So, ^③there are animals without scales which have sharp claws. From the premises, prove the conclusion.

let $S(x) = x$ has scales & $D(x) = x$ is a dragon
 $C(x) = x$ has sharp claws

1. $\forall x [S(x) \rightarrow D(x)]$ ①

2. $\exists x [\neg D(x) \wedge C(x)]$ ②

3. $\neg D(a) \wedge C(a)$ $\rightarrow$ existential specification

4. $S(a) \rightarrow D(a)$ $\rightarrow$ universal specification

5. $\neg D(a)$ $\rightarrow$ from 3.

6. $\neg S(a)$ $\rightarrow$ from 4. and 5.

7. $C(a)$ $\rightarrow$ from 3.

8. $\neg S(a) \wedge C(a)$ $\rightarrow$ from 6. & 7.

9. $\exists x [\neg S(x) \wedge C(x)]$ $\rightarrow$ existential specification

$\rightarrow$ it is required ③

Q3) by contradiction prove that $\sqrt[3]{2}$ is irrational.

$\sqrt[3]{2}$ is rational (by Contradiction)

$$\sqrt[3]{2} = \frac{a}{b}, \quad a, b \in \mathbb{Z}, \quad b \neq 0$$

$$2 = \frac{a^3}{b^3} \Rightarrow b^3 = \frac{a^3}{2} \text{ or } a^3 = 2b^3, \quad a \text{ is even}$$

$$a \text{ is even} \Rightarrow a = 2K, \quad K \in \mathbb{Z}$$

$$2b^3 = (2K)^3 \Rightarrow b^3 = \frac{8K^3}{2} = 4K^3$$

$$\therefore b^3 \text{ is even} \Rightarrow b \text{ is even}$$

$\therefore a$ and b have common factor of two
by Contradiction $\sqrt[3]{2}$ is irrational

Q4) from your knowledge about the sets, answer the following two parts.

a) Let A and B be given two sets. With set builder notation, show that

$$\bar{A} \cap (A \cup B) = B - A$$

$$\begin{aligned} \bar{A} \cap (A \cup B) &= \{x \mid x \in (\neg A \wedge (A \vee B))\} \\ &= \{x \mid x \in ((\neg A \wedge A) \vee (\neg A \wedge B))\} \\ &= \{x \mid x \in (F \vee (\neg A \wedge B))\} \\ &= \{x \mid x \in (\neg A \wedge B)\} \\ &= \{x \mid x \in B \wedge x \notin A\} \\ &= B - A \end{aligned}$$

b) Let $A \times B = \{(1, 2), (2, 2), (3, 1), (1, 2), (1, 4), (2, 1), (2, 4), (3, 4)\}$, find the power of B , $P(B)$.

$$A = \{1, 2, 3\}, \quad B = \{1, 2, 4\}$$

therefore,

$$P(B) = \{\emptyset, \{1\}, \{2\}, \{4\}, \{1, 2\}, \{1, 4\}, \{2, 4\}, \{1, 2, 4\}\}$$

Q5) Write an algorithm to add two matrices together, and what is the time complexity?

Procedure matrixadd (A, B : matrices)

for $i := 1$ to m

for $j := 1$ to n

$$C_{ij} := a_{ij} + b_{ij} \quad \mathcal{O}(1)$$

return C { $C = [C_{ij}]$ is sum of A and B } $\mathcal{O}(1)$

$$\text{time} = \sum_{i=1}^m \sum_{j=1}^n \mathcal{O}(1) + \mathcal{O}(1) = \mathcal{O}(m \cdot n) = \underline{\underline{\mathcal{O}(n^2)}}$$

Q6) Answer the following parts.

a) Determine $A^{-1}B^T$ where $A = \begin{bmatrix} 2 & 4 \\ 1 & 5 \end{bmatrix}$, and $B = \begin{bmatrix} 6 & 2 \\ 3 & 4 \end{bmatrix}$

$$A^{-1} = \frac{1}{6} \begin{bmatrix} 5 & -4 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 5/6 & -4/6 \\ -1/6 & 2/6 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 6 & 3 \\ 2 & 4 \end{bmatrix}$$

$$A^{-1}B^T = \begin{bmatrix} 5/6 & -4/6 \\ -1/6 & 2/6 \end{bmatrix} \begin{bmatrix} 6 & 3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 11/3 & -1/6 \\ -1/3 & 5/6 \end{bmatrix}$$

b) Determine whether the function $f: \mathbb{R}^+ \rightarrow \mathbb{R}^+, f(x) = 3x^2 + 2$ is:

(i) One to one.

(ii) Onto

Justify your answer

ii) it is not Onto

the domain of x is $\mathbb{R}^+$

$$x > 0 \quad \therefore \quad x^2 > 0$$

$$3x^2 > 0 \quad \therefore \quad 3x^2 + 2 > 2$$

the range less than
Codomain

i) it is one to one
every image has a unique
pre-image