



Name: ----- Student No.:-----

Q1) [4 points] By Direct proof, prove that between every two rational numbers there is an irrational number.

let two rational numbers : $\frac{a}{b}$ & $\frac{c}{b}$ a, b, c + integers

assume $a < c$

so $\frac{a+1}{b} \leq \frac{c}{b}$ we can note $x = \frac{a + \frac{\sqrt{2}}{2}}{b}$

is between our assumed numbers

$a \leq x \leq c$ because $\frac{\sqrt{2}}{2} < 1$

and $\sqrt{2}$ is irrational $\Rightarrow x$ is irrational between two

Q2) [4 points] Show that the premises "A student in this class has not read the book", ^① and "Everyone in this class passed the first exam" imply the conclusion "Someone who passed the first exam has not read the book" ^③

let $C(x)$ is "x is in this class"

$B(x)$ is "x has read the book"

$P(x)$ is "x passed the first exam"

conclusion is $\exists x (P(x) \wedge \neg B(x))$ required

1. $\exists x (C(x) \wedge \neg B(x))$ premise ①
2. $\forall x (C(x) \rightarrow P(x))$ premise ②
3. $C(a) \wedge \neg B(a)$ from ①
4. $C(a)$
5. $C(a) \rightarrow P(a)$ from ②
6. $P(a)$ from ④ & ⑤
7. $\neg B(a)$ from ③ & ④
8. $P(a) \wedge \neg B(a)$ from ⑥ & ⑦
9. $\exists x (P(x) \wedge \neg B(x))$ required as a conclusion

Q3) [3 points] By using Manipulations of summation find $\sum_{k=0}^{10} 3^{k+3}$

$$= \sum_{k=3}^{13} 3^k = \sum_{k=0}^{13} 3^k - \sum_{k=0}^2 3^k$$

$$= \frac{(r^{n+1} - 1)}{r - 1} = \frac{(3^{14} - 1)}{3 - 1} - \frac{(3^3 - 1)}{3 - 1}$$

$$= 2,391,484 - 13$$

$$= \underline{\underline{2,391,471}}$$

Q4) [4 points] By Mutual subsets, show that $A \times (B - C) = (A \times B) - (A \times C)$

$$A \times (B - C) \subseteq (A \times B) - (A \times C)$$

let $(x, y) \in A \times (B - C)$ i.e. $x \in A$ & $y \in (B - C)$

$\therefore (x, y) \in (A \times B)$ & $(x, y) \notin (A \times C)$ $y \in B, y \notin C$

So $(x, y) \in (A \times B) - (A \times C)$

it means that $A \times (B - C) \subseteq (A \times B) - (A \times C)$ (1)

$$(A \times B) - (A \times C) \subseteq A \times (B - C)$$

let $(x, y) \in (A \times B) - (A \times C)$

$(x, y) \in A \times B$ and $\notin (A \times C)$

$x \in A$

$y \in B$

$x \in A$ while $(x, y) \notin (A \times C)$
 $\therefore y \notin C$

this implies that $y \in B \setminus C \Rightarrow y \in (B - C)$
 and $x \in A$

$\therefore (x, y) \in A \times (B - C)$

+ means that $(A \times B) - (A \times C) \subseteq A \times (B - C)$ (2)

$\therefore A \times (B - C) = (A \times B) - (A \times C)$ from (1), (2)

$$f: Y \rightarrow Z, \quad f^{-1}: Z \rightarrow Y, \quad g: X \rightarrow Y, \quad g^{-1}: Y \rightarrow X$$

Q5) [3 points] Suppose that f is an invertible function from Y to Z and g is an invertible function from X to Y . Show that the inverse of the composition $f \circ g$ is given by $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

$$(f \circ g) \equiv (f: Y \rightarrow Z) \circ (g: X \rightarrow Y) \Rightarrow (f \circ g): X \rightarrow Z$$

$$(f \circ g)^{-1}: Z \rightarrow X \quad \rightarrow \textcircled{1}$$

$$g^{-1} \circ f^{-1} \equiv (g^{-1}: Y \rightarrow X) \circ (f^{-1}: Z \rightarrow Y) \Rightarrow (g^{-1} \circ f^{-1}): Z \rightarrow X \quad \textcircled{2}$$

because f and g invertible also f^{-1} & g^{-1}
 $\therefore$ all f, g, f^{-1}, g^{-1} functions are one to one

It means from $\textcircled{1}$ and $\textcircled{2}$

each element in Z has one image in X
 and each element in Z has one pre-image in Z .

Q6) [2 points] If $(AB)^T = \begin{bmatrix} 1 & 2 \\ 11 & 18 \end{bmatrix}$, and $B = \begin{bmatrix} 0 & 4 \\ 1 & 3 \end{bmatrix}$, find A

$$AB = \begin{bmatrix} 1 & 11 \\ 2 & 18 \end{bmatrix} = A \begin{bmatrix} 0 & 4 \\ 1 & 3 \end{bmatrix}$$

$$\therefore A = \begin{bmatrix} 1 & 11 \\ 2 & 18 \end{bmatrix} \begin{bmatrix} 0 & 4 \\ 1 & 3 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 1 & 11 \\ 2 & 18 \end{bmatrix} \begin{bmatrix} \frac{3}{-4} & \frac{-4}{-4} \\ \frac{-1}{-4} & \frac{0}{-4} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 11 \\ 2 & 18 \end{bmatrix} \begin{bmatrix} -3/4 & 1 \\ 1/4 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$$

$$f^{-1} \neq \frac{1}{f}$$

$$f \circ g \neq g \circ f$$

x(c)

(2)