

Islamic University of Gaza
Computer Engineering Dept.
ECOM3411: Discrete Math.
Dr. Mohammed Alhanjouri



Fall 2018-2019
Final exam
January 14, 2019
Exam duration: 120 minutes

Student Name \.....

Student No. \.....

- Try to solve all the following ten Questions.
- This exam should be solved without any book, or notes
- **Academic Integrity:** Plagiarism, cheating, and other forms of academic dishonesty are prohibited and may result in grade F for the course.

Q1 (12 pt.)	Q2 (10 pt.)	Q3 (13 pt.)	Q4 (12 pt.)	Q5 (10 pt.)	Q6 (15 pt.)	Q7 (8pt.)	Total/ 80

Q1) 1) [2 pt.] Using Euclidean algorithm, find gcd (124, 323). Use the result to find lcm (124, 323).

$$\begin{aligned}
 \gcd(124, 323) &= \gcd(124, 323 \bmod 124) = \gcd(124, 75) \\
 &= \gcd(75, 124 \bmod 75) = \gcd(49, 75 \bmod 49) = \gcd(26, 49 \bmod 26) \\
 &= \gcd(23, 26 \bmod 23) = \gcd(3, 23 \bmod 3) = \gcd(2, 3 \bmod 2) \\
 &= \gcd(1, 2 \bmod 1) = 1
 \end{aligned}$$

$$124 \times 323 = \gcd(124, 323) \times \text{lcm}(124, 323)$$

$$\text{LCM} = \frac{124 \times 323}{\gcd(124, 323)} = 40052$$

2) [2 pt.] List sequence of pseudorandom numbers that generated using the linear congruential generator $x_{n+1} = (3x_n + 2) \bmod 13$, where $x_0 = 1$

$$x_0 = 1$$

$$x_1 = (3 + 2) = 5$$

$$x_2 = (15 + 2) \equiv 4$$

$$x_3 = (12 + 2) \equiv 1$$

3) [6 pt.] Using the modular inverses, Find x in congruence: $89x \equiv 2 \pmod{232}$

$$232 = 2 \cdot 89 + 54$$

$$89 = 1 \cdot 54 + 35$$

$$54 = 1 \cdot 35 + 19$$

$$35 = 1 \cdot 19 + 16$$

$$19 = 1 \cdot 16 + 3$$

$$16 = 5 \cdot 3 + 1$$

$$3 = 2 \cdot 1 + 1$$

$$1 = 3 - 2 \cdot 1$$

$$= 3 - 2(16 - 5 \cdot 3) = -2 \cdot 16 + 11 \cdot 3$$

$$= -2 \cdot 16 + 11(19 - 1 \cdot 16) = 11 \cdot 19 - 13 \cdot 16$$

$$= 11 \cdot 19 - 13(35 - 1 \cdot 19) = -13 \cdot 35 + 24 \cdot 19$$

$$= -13 \cdot 35 + 24(54 - 1 \cdot 35) = 24 \cdot 54 - 37 \cdot 35$$

$$= 24 \cdot 54 - 37(89 - 1 \cdot 54) = -37 \cdot 89 + 61 \cdot 54$$

$$= -37 \cdot 89 + 61(232 - 2 \cdot 89)$$

$$= 61 \cdot 232 - 159 \cdot 89$$

or -159 inv. of $89 \pmod{232}$

$$x = (-159)(2) \pmod{232} = -318 \pmod{232}$$

$$x = -86 \pmod{232} = 146$$

4) [2 pt.] Decrypt the message ILZA DPZOLZ that were encrypted using the Caesar cipher
 $f(p) = (p+7) \pmod{26}$

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 a b c d e f g h i j k l m n o p q r s t u v w x y z
 t u v w x y z a b c d e f g h i j k l m n o p q r s

best wishes

Q2) 1) [5 pt.] Use mathematical induction to prove that 21 divides $4^{n+1} + 5^{2n-1}$ whenever n is a positive integer.

Basis step at $n=1$ 21 divides $4^2 + 5^1 = 16 + 5 = 21$ ✓

inductive step $4^{(k+1)+1} + 5^{2(k+1)-1} = 4^{k+2} + 5^{2k+1} = 4^{k+2} + 5^{2k+1}$

$$= 4 \cdot 4^{k+1} + 25 \cdot 5^{2k-1} = 4 \cdot 4^{k+1} + (21+4) \cdot 5^{2k+1}$$

$$= 4(4^{k+1} + 5^{2k+1}) + 21 \cdot 5^{2k+1}$$

$$= 4 \cdot 21A + 21 \cdot 5^{2k+1} = 21(4A + 5^{2k+1})$$

∴ 21 divides that

2) [5 pt.] Give a recursive algorithm for computing Fibonacci number at n position in sequence.

procedure Fibon(n)

begin

if $n \leq 1$ then

return n ;

else

return Fibon($n-1$) + Fibon($n-2$);

end

Q3) 1) [2 pt.] There are 38 different time periods during which classes at a university can be scheduled. If there are 677 different classes, how many different rooms will be needed?

$$\lceil 677/38 \rceil = 18$$

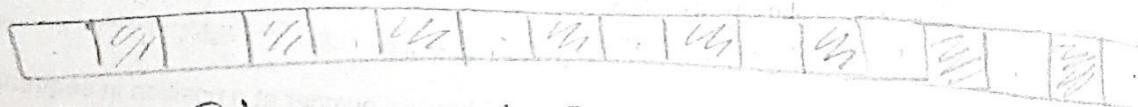
- 3) [5 pt.] Suppose that a department contains 10 men and 15 women. How many ways are there to form a committee with six members if it must have more women than men?

$$4w \text{ and } 2m = \frac{15!}{4!(15-4)!} * \frac{10!}{2!(10-2)!} = 1365 * 45 = 61425$$

$$5w \text{ and } 1m = \frac{15!}{5!(15-5)!} * \frac{10!}{1! * 10!} = 3003 * 10 = 30030$$

$$6w \text{ and } 0m = \frac{15!}{6!(15-6)!} = 5005 \quad 00 = \text{total} = 96460$$

- 4) [3 pt.] How many ways are there for eight men and five women to stand in a line so that no two women stand next to each other?



$$\text{for men } 8! = 40320$$

$$\text{for women } \frac{9!}{(9-5)!} = 15120 * 609,638,400$$

- 5) [3 pt.] If the 5th row of Pascal's triangle containing the binomial coefficients: 1 5 10 10 5 1
By VanderMonde's Identity, find the 3rd coefficient of 10th row in Pascal's triangle.

$$\binom{m+n}{r} = \sum_{j=0}^r \binom{m}{r-j} \binom{n}{j} \Rightarrow \binom{10}{2} = \sum_{j=0}^2 \binom{5}{2-j} \binom{5}{j}$$

$$= \binom{5}{2} \binom{5}{0} + \binom{5}{1} \binom{5}{1} + \binom{5}{0} \binom{5}{2}$$

$$= 10 * 1 + 5 * 5 + 1 * 10 = 45$$

- Q4) 1) [6 pt.] Assume that the population of the world in 2010 was 6.9 billion and is growing at the rate of 1.1% a year.

a) Set up a recurrence relation for the population of the world n years after 2010.

b) Find an explicit formula for the population of the world n years after 2010.

c) What will the population of the world be in 2030?

$$\text{a) } a_0 = 6.9, r = 1.1\% = 0.011 \rightarrow a_n = a_{n-1} + 0.011 a_{n-1}$$

$$\text{b) } a_n = 1.011 a_{n-1} = (1.011)^2 a_{n-2} = \dots = (1.011)^n a_0$$

$$\text{c) } a_{20} = (1.011)^{20} * 6.9$$

$$\approx 8.5876 \text{ billion}$$

2) [6 pt.] Solve these recurrence relations together with the initial conditions given.

$$a_{n+2} = -4a_{n+1} + 5a_n \text{ for } n \geq 0, a_0 = 2, a_1 = 8$$

$$r^2 + 4r - 5 = 0 \rightarrow (r-1)(r+5) = 0 \quad r = \begin{cases} 1 \\ -5 \end{cases}$$

$$a_n = \alpha_1 (1)^n + \alpha_2 (-5)^n$$

$$\text{at } a_0 = 2 = \alpha_1 + \alpha_2$$

$$a_1 = 8 = \alpha_1 + 5\alpha_2$$

$$\left. \begin{array}{l} 2 = \alpha_1 + \alpha_2 \\ 8 = \alpha_1 + 5\alpha_2 \end{array} \right\} \begin{array}{l} 6 = -6\alpha_2 \rightarrow \alpha_2 = -1 \\ \text{and } \alpha_1 = 3 \end{array}$$

$$a_n = 3 - (-5)^n$$

Quiz

Q5) 1) [5 pt.] If A is set of students and B is set of books in the school library. Let $R_1 \{(a,b) : \text{student } a \text{ is required to read book } b \text{ in a course}\}$, and $R_2 \{(a,b) : \text{student } a \text{ has read book } b\}$. Describe the relations:

a) $R_1 \cup R_2$

b) $R_1 \cap R_2$

c) $R_1 \oplus R_2$

$$\{(a,b) \mid a \text{ is required to read or has read } B\}$$

$$\{(a,b) \mid a \text{ is required to read and has read } B\}$$

$$\{(a,b) \mid \text{either } a \text{ is required to read } b \text{ but has not read it OR } a \text{ has read } b \text{ but is not required to}\}$$

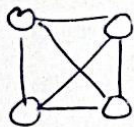
2) [5 pt.] Represent the relations on $\{1, 2, 3\}$ with a matrix and diagram, then find smallest superset to get equivalent relation. $R = \{(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)\}$

$$\text{reflexive} \rightarrow R = \{(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)\}$$

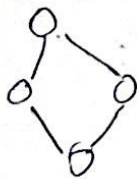
$$\text{symmetric} \rightarrow R = \{(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3), (2, 1), (3, 2), (3, 1)\}$$

$$\text{transitive} \rightarrow R = \{(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3), (2, 1), (3, 1), (3, 2)\}$$

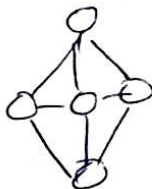
Q6) 1) [5 pt.] Draw then find the degree sequence of each of the following graphs. □
 a) K_4 b) C_4 c) W_4 d) $K_{2,3}$ e) Q_3 □



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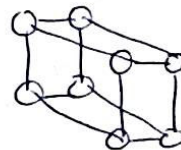
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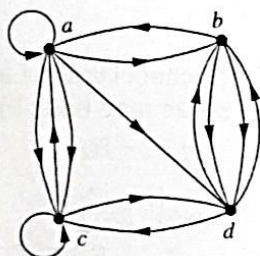


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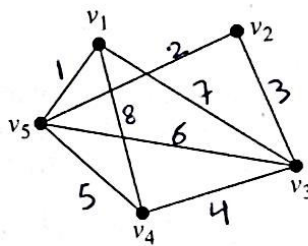


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2) [6 pt.] Represent graph G1 by adjacency matrix and graph G2 by incidence matrix



G1

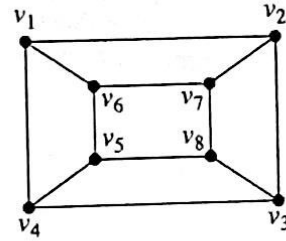
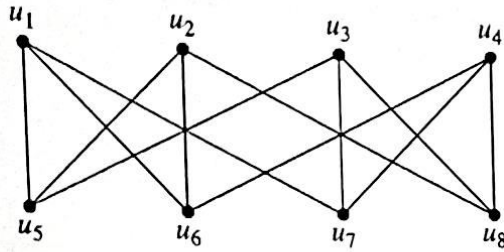


G2

$$\begin{matrix} & a & b & c & d \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 11 & 11 & 21 & 10 \\ 11 & 00 & 00 & 22 \\ 12 & 00 & 11 & 11 \\ 01 & 22 & 11 & 00 \end{bmatrix} \end{matrix}$$

$$\begin{matrix} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{matrix}$$

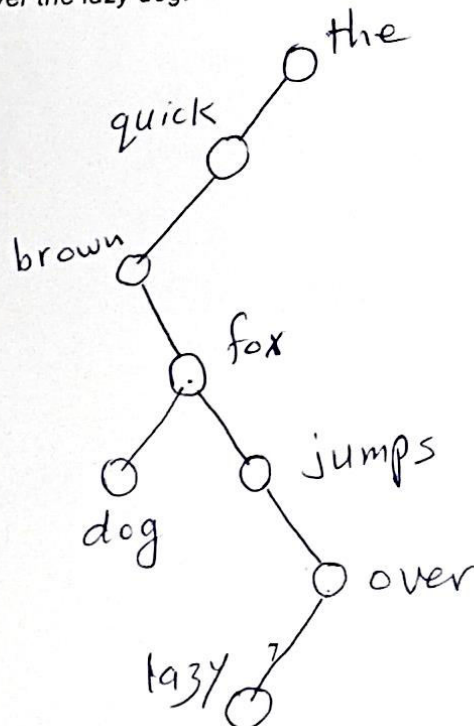
3) [4 pt.] Determine if the following two graphs G_1 and G_2 are isomorphic or not. Explain your answer.



	G_1	G_2
nodes	8	8
edges	12	12
$\deg(3)$	8	8
cycle(4)	6	6
cycle(5)	0	0
cycle(6)	4	4

yes

Q7) 1) [4 pt.] Using alphabetical order, build a binary search tree for the words in the sentence "The quick brown fox jumps over the lazy dog."



2) [4 pt.]. If you have 101 coins, one of which is a lighter counterfeit, and a free-beam balance, what is the number of coins that should be holding on each side to extract the lighter counterfeit with minimum steps? How much the minimum steps?

$$101 \bmod 3 = 2$$

$$\therefore \lceil 101/3 \rceil = 34 \text{ coins on each side}$$

$$h = \lceil \log_m 101 \rceil = \frac{\log 101}{\log 3} = \lceil 4.2 \rceil \\ = 5$$