

Lectures on Linear Algebra

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Lecture 1

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Chapter 1 MATRICES, VECTORS, AND SYSTEMS OF LINEAR EQUATIONS

1.1 MATRICES AND VECTORS

- The Natural numbers $\mathbb{N} := \{1, 2, 3, 4, \dots\}$.
- The Integer numbers
 $\mathbb{Z} := \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$.
- The Rational numbers $\mathbb{Q} := \{\frac{a}{b} : a, b \in \mathbb{Z}; b \neq 0\}$.
- Elements that are not in $\mathbb{Q}$ became known as irrational numbers $\mathbb{I}\mathbb{Q}$, meaning that they are not ratios of integers.
- The Real numbers $\mathbb{R} = \mathbb{Q} \cup \mathbb{I}\mathbb{Q}$.
- $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$

1.1 MATRICES AND VECTORS

Definition

A matrix is a rectangular array of scalars. If the matrix has m rows and n columns, we say that the size of the matrix is m by n , written $m \times n$. The matrix is square if $m = n$. The scalar in the i th row and j th column is called the (i, j) -entry of the matrix.

General form of 3X3 matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = [a_{ij}]$$

Example

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & -8 \\ 5 & 0 & -5 \end{bmatrix}$$

1.1 MATRICES AND VECTORS

MATRIX SUMS AND SCALAR MULTIPLICATION

Let $A = [a_{ij}]$ and $B = [b_{ij}]$ and let $c \in \mathbb{R}$, then

① $A + B = [a_{ij}] + [b_{ij}] = [a_{ij} + b_{ij}]$

② $cA = [ca_{ij}]$

We call the $m \times n$ matrix O in which each entry is 0 the $m \times n$ zero matrix.

Example

Compute the matrices $A - B$, $2A$, and $A + 3B$, where

$$A = \begin{bmatrix} 3 & -1 & 1 \\ 3 & 0 & -2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 3 & 0 \\ 2 & -1 & 4 \end{bmatrix}$$

1.1 MATRICES AND VECTORS

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Properties of Matrix Addition and Scalar Multiplication

Let A, B , and C be $m \times n$ matrices, and let s and t be any scalars. Then

- (a) $A + B = B + A$. (commutative law)
- (b) $(A + B) + C = A + (B + C)$. (associative law)
- (c) $A + O = A$.
- (d) $A + (-A) = O$.
- (e) $(st)A = s(tA)$.
- (f) $s(A + B) = sA + sB$
- (g) $(s + t)A = sA + tA$

1.1 MATRICES AND VECTORS

MATRIX TRANSPOSES

The transpose of an $m \times n$ matrix A is the $n \times m$ matrix denoted by A^T whose (i, j) -entry is the (j, i) -entry of A , i.e. if $A = [a_{ij}]$ then $A^T = [a_{ji}]$.

Example

Let $A = \begin{bmatrix} 2 & -1 & 1 \\ 3 & 0 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 & 0 \\ 2 & -1 & 4 \end{bmatrix}$. Compute the following matrices:

- (a) A^T
- (b) $(3B)^T$
- (c) $(A + B)^T$

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Theorem (Properties of the Transpose)

Let A and B be $m \times n$ matrices, and let s be any scalar. Then

(a) $(A + B)^T = A^T + B^T$

(b) $(sA)^T = sA^T$

(c) $(A^T)^T = A.$

1.1 MATRICES AND VECTORS

VECTORS

- A matrix that has exactly one row is called a **row vector**, and a matrix that has exactly one column is called a **column vector**.
- The entries of a vector are called **components**. we denote the set of all column vectors with n components by $\mathbb{R}^n$. Occasionally, we identify a vector u in $\mathbb{R}^n$ with an n -tuple, $(u_1, u_2, \dots, u_n)$.
- In particular, the vector in $\mathbb{R}^n$ with all zero components is denoted by 0 and is called the zero vector. It satisfies $u + 0 = u$ and $0u = 0$ for every $u \in \mathbb{R}^n$.

1.1 MATRICES AND VECTORS

Example

Let $\mathbf{u} = \begin{bmatrix} 2 \\ -4 \\ 7 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 5 \\ 3 \\ 0 \end{bmatrix}$.

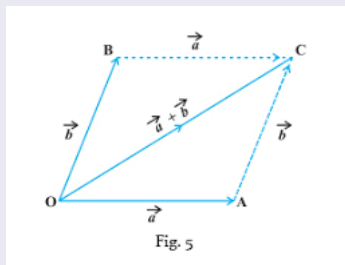
Then

$$\mathbf{u} + \mathbf{v} = \begin{bmatrix} 7 \\ -1 \\ 7 \end{bmatrix}, \quad \mathbf{u} - \mathbf{v} = \begin{bmatrix} -3 \\ -7 \\ 7 \end{bmatrix}, \quad \text{and} \quad 5\mathbf{v} = \begin{bmatrix} 25 \\ 15 \\ 0 \end{bmatrix}$$

1.1 MATRICES AND VECTORS

VECTOR ADDITION AND THE PARALLELOGRAM LAW

The sum $\vec{a} + \vec{b}$ is the arrow along the diagonal of the parallelogram.



1.1 MATRICES AND VECTORS

SCALAR MULTIPLICATION

If $\mathbf{v} = \begin{bmatrix} a \\ b \end{bmatrix}$, then

- 1) If $c > 0$, $c\mathbf{v}$ is a vector that points in the same direction as $\mathbf{v}$, and whose length is c times the length of $\mathbf{v}$.
- 2) If $c < 0$, $c\mathbf{v}$ points in the opposite direction from $\mathbf{v}$, and has length $|c|$ times the length of $\mathbf{v}$.
- 3) We call two vectors **parallel** if one of them is a scalar multiple of the other.

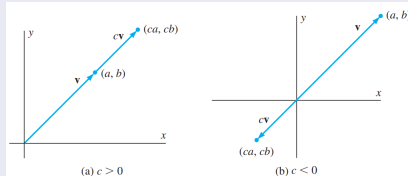


Figure 1.5 Scalar multiplication of vectors

1.1 MATRICES AND VECTORS

EXERCISES Pages 11 and 12

1-32, and 37-56.