

St. Name:	Question	1	2	3	4	5	6	Total
St. Number:	Points	30	20	14	14	12	10	100
	Score							

Solve the following questions

Q.I) Choose the correct answer for each of the following questions.

1. Let $A = \begin{bmatrix} 1 & 0 \\ 0 & -3 \\ 1 & 0 \end{bmatrix}$.

What is the eigenvalues of $A^T A$?

- (A) $\lambda = -2$ and $\lambda = -9$ (D) $\lambda = -2$ and $\lambda = -4$
 (B) $\lambda = 9$ and $\lambda = 2$
 (C) $\lambda = 5$ and $\lambda = 9$

2. Which of the following is Linearly independent?

- (A) $\{e^{-t}, e^t, e^{2t}\}$ (C) $\{t, 1+t^2, 1+t+t^2\}$
 (B) $\{\sin^2 t, \cos 2t, 1\}$ (D) Non of the mentioned

3. Given that $A = \begin{pmatrix} 1 & 1 & 2 \\ 3 & 1 & 0 \\ -2 & 0 & 3 \end{pmatrix}$ has an inverse $A^{-1} = \begin{pmatrix} -\frac{3}{2} & \frac{3}{2} & \star \\ \frac{3}{2} & -\frac{1}{2} & \star \\ -1 & 1 & \star \end{pmatrix}$

Find the missing third column of A^{-1} .

- (A) $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ (B) $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ (C) $\begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}$ (D) $\begin{pmatrix} 3 \\ 1 \\ -10 \end{pmatrix}$

4. Let $A = \begin{bmatrix} r & 1 & 0 \\ 0 & 3 & 5 \\ -r & 0 & 3 \end{bmatrix}$.

What is the value of r such that $\text{trac } A = \det A$?

- (A) 2 (B) 3 (C) -1 (D) 0

5. Suppose that A and B are invertible matrices of the same size. Which of the following need not be invertible?

- (A) ABA^{-1} (B) $A^T B^T$ (C) $A^{-1}B$ (D) $AB + I$

6. Let $S = \left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\}$

Which of the following vectors belong to $\text{Span } S$?

- (A) $\begin{bmatrix} 0 & -4 & 3 \\ 0 & 0 & -1 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & -2 & 0 \\ 0 & 3 & 1 \end{bmatrix}$
 (B) $\begin{bmatrix} 3 & -2 & 0 \\ -3 & 3 & 1 \end{bmatrix}$ (D) Non of the mentioned

7. If A is 3×6 matrix, which of the following is impossible

- (a) Rows of A are linearly independent.
 (b) Row space of $A = \text{Columns space of } A^T$
 (c) The dimension of null space has at least 3
 (d) $\dim \text{Row}(A) < \dim \text{Col}(A)$.

8. Which of the following is **NOT** a subspace of the vector space $M_{2 \times 2}$

- (a) $W = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : \text{All matrices in } M_{2 \times 2} \text{ where } a + b + c = 0 \right\}$
 (b) $W = \left\{ \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} : \text{All matrices in } M_{2 \times 2} \text{ where } a - c = 0 \right\}$
 (c) $W = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : \text{All matrices in } M_{2 \times 2} \text{ where } a + b > 0 \right\}$
 (d) $W = \text{The set of all upper triangular matrices in } M_{2 \times 2}$.

9. Assume that A is row equivalent to B where

$$A = \begin{pmatrix} 1 & -3 & 4 & -1 & 9 \\ -2 & 6 & -6 & -1 & -10 \\ -3 & 9 & -6 & -6 & -3 \\ 3 & -9 & 4 & 9 & 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & -3 & 0 & 5 & -7 \\ 0 & 0 & 2 & -3 & 8 \\ 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Without calculation, find $\text{Rank}(A)$ and $\dim(\text{Nul}(A))$.

- (A) $\text{Rank}(A) = 2$ and $\dim(\text{Nul}(A)) = 3$.
 (B) $\text{Rank}(A) = 3$ and $\dim(\text{Nul}(A)) = 2$.
 (C) $\text{Rank}(A) = 2$ and $\dim(\text{Nul}(A)) = 2$.
 (D) $\text{Rank}(A) = 3$ and $\dim(\text{Nul}(A)) = 3$.

10. Let $A = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 1 \\ 1 & 3 & 0 \end{bmatrix}$

Which of the following vectors belong to $\text{Null } A$?

- (A) $\begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$ (B) $\begin{bmatrix} -2 \\ 2 \\ 2 \end{bmatrix}$ (C) $\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$ (D) $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

11. For which values of x is the matrix $A = \begin{bmatrix} 1 & 0 & -3 \\ 0 & x & 2 \\ 3 & -10 & x \end{bmatrix}$ **not invertible**?
- (A) -4 and 5 (B) -4 and -5 (C) -2 and -4 (D) 4 and -5
12. Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ linear transformation defined by $T \left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right) = \begin{bmatrix} 2x_1 - x_3 \\ 3x_2 + 2x_3 \end{bmatrix}$
then
- (A) T is onto
(B) T is one to one
(C) $\text{Rank } A = 1$
(D) The columns of A are linearly independent.
13. Let A, B, C be square invertible matrices satisfying $BA = B^{-1}C$. Assume that $\det B = 2$ and $\det C = 4$. Find a formula for A and calculate the determinant of A ?
- (A) $A = B^{-1}CB$, $\det A = 8$
(B) $A = (B^{-1})^2C$, $\det A = 1$
(C) $A = C$, $\det A = 4$
(D) $A = (B^{-1})^2C$, $\det A = 4$
14. If we have 5×5 matrices with the following characteristic polynomials. Which of them can be diagonalizable
- (A) $(t - 2)(t^2 - 4)(t^2 - 1)$
(B) $t(t^2 - 4)(t^2 + 2)$
(C) $(t^2 - 9)(t^2 + 10)$
(D) $t(t^2 + 3)^2$
15. Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$
Find all vectors $x \in \mathbb{R}^2$ that are both in $\text{Col } A$ and $\text{Null } A$?
- (A) Only the vector 0 (C) all multiples of $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
(B) all solutions of $Ax = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ (D) all multiples of $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$\text{Q.II) } A = \begin{pmatrix} 1 & 4 & 0 & 2 & -3 & 1 \\ 0 & -2 & 4 & 3 & 5 & 3 \\ 3 & 0 & 0 & -4 & 0 & 3 \\ 0 & 4 & -8 & -6 & -10 & -6 \\ 0 & 0 & 0 & 3 & 0 & 0 \end{pmatrix} \text{ row reduces to } U = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -\frac{3}{4} & 0 \\ 0 & 0 & 1 & 0 & \frac{7}{8} & \frac{3}{4} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

1. What is a basis for Col A ? What is the dimension of Col A ?

2. What is a basis for Nul A ? What is the dimension of Nul A ?

3. What is a basis for Row A ? What is the dimension of Row A ?

Q.III) Suppose $T: \mathbb{R}^3 \rightarrow \mathbb{R}^4$ is a linear transformation and that

$$T \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ 2 \\ 0 \end{pmatrix}, \quad T \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 0 \\ 4 \end{pmatrix}, \quad T \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \\ 6 \\ -2 \end{pmatrix}$$

1. What is $T \left(\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} \right)$

2. We can write $T(\mathbf{x}) = A\mathbf{x}$. What is A ?

Q.IV) 1. Let $A = \begin{bmatrix} 2 & 0 & 0 & -4 \\ 0 & 1 & 0 & 0 \\ 0 & 5 & -1 & 0 \\ 4 & 0 & 0 & 2 \end{bmatrix}$. Find the determinant of $A^T - 3I_4$?

2. If $\mathcal{B} = \left\{ \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$ be a basis and the $\mathcal{B}$ -coordinate vector of v is $[v]_{\mathcal{B}} = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$. Find v ?

Q.V) Find the eigenvalues of the matrix A and determine a basis for each eigenspace.

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 2 & 5 & 0 \\ 1 & -2 & -1 \end{bmatrix}$$

Find the dimensions of the eigenspaces of A.

Q.VI) Determine whether the following set is a basis for $\mathcal{M}_{2 \times 2}$

$$\left\{ \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \right\}$$