



The Islamic University of Gaza

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Linear Algebra

MATH2341

Midterm Exam

Date: 09/04/2019

Student Name:	Question	1	2	3	4	Total
	Points	5	10	5	10	30
Student Number:	Score					

Solve the following questions

Question No. 1. Choose the correct answer for each of the following questions. For each question, only one answer is correct and only one answer is allowed.

- (a) (1 point) Suppose A is a 3×4 matrix and that $\vec{u}, \vec{v}$ are vectors in $\mathbb{R}^4$. If $A\vec{v} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ and

$$A(2\vec{u} - 3\vec{v}) = \begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix}, \text{ then what is } A\vec{u} ?$$

- (A) $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ (B) $\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$ (C) $\begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$ (D) $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ (E) $\begin{pmatrix} -1 \\ -2 \\ 0 \end{pmatrix}$

- (b) (1 point) Solve the system of equations below:

$$\begin{array}{rrrr} x_1 & + & x_2 & + & x_3 & = & 2 \\ x_1 & + & x_2 & + & 2x_3 & = & 1 \\ x_1 & + & 2x_2 & + & 4x_3 & = & -1 \end{array}$$

- (A) $\begin{pmatrix} -1 \\ 4 \\ -1 \end{pmatrix}$ (B) $\begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$ (C) $\begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}$ (D) $\begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$ (E) There is no solution.

- (c) (1 point) Let

$$\vec{a}_1 = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}, \vec{a}_2 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \vec{b} = \begin{pmatrix} h \\ 1 \\ 1 \end{pmatrix}$$

For which value(s) of h is $\vec{b}$ in the plane spanned by $\vec{a}_1$ and $\vec{a}_2$?

- (A) $-\frac{1}{2}$ (B) 1 (C) $\frac{3}{2}$ (D) $\frac{1}{2}$ and 3 (E) $-\frac{3}{2}$

- (d) (1 point) Suppose $T : \mathbb{R}^n \rightarrow \mathbb{R}^2$ is a linear transformation and suppose we are given vectors $\vec{u}$ and $\vec{v}$ such that $T(\vec{u}) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, T(\vec{v}) = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$. What is $T(-\vec{u} + 2\vec{v})$?

- (A) $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$ (B) $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ (C) $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$ (D) $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ (E) $\begin{pmatrix} -3 \\ -4 \end{pmatrix}$.

- (e) (1 point) Consider the following sets of vectors

$$(I) \left\{ \begin{pmatrix} 1 \\ 7 \\ 6 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 9 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix}, \begin{pmatrix} 4 \\ 1 \\ 8 \end{pmatrix} \right\}, \quad (II) \left\{ \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 8 \end{pmatrix}, \begin{pmatrix} 3 \\ 4 \\ 13 \end{pmatrix} \right\}, \quad (III) \left\{ \begin{pmatrix} -2 \\ 4 \\ 6 \end{pmatrix}, \begin{pmatrix} 3 \\ -6 \\ 9 \end{pmatrix} \right\}$$

Which of these sets is linearly dependent?

- (A) (I) only (B) (II) only (C) (III) only
(D) (I) and (II) only (E) (I) and (III) only (F) (II) and (III) only.

Question No. 2. TRUE/FALSE: For each statement below, please answer true or false.

- (a) (1 point) If A and B are two matrices of size $n \times n$, then

$$(A + B)(A + B) = A^2 + 2AB + B^2$$

☐ True ☐ False

- (b) (1 point) If A and B are matrices and AB is invertible, then A and B are invertible.

☐ True ☐ False

- (c) (1 point) If a system of linear equations has more variables than equations, then the system must have infinitely many solutions.

☐ True ☐ False

- (d) (1 point) The three vectors $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, and $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ span $\mathbb{R}^3$.

☐ True ☐ False

- (e) (1 point) The augmented matrix $\left(\begin{array}{ccc|c} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3 \end{array} \right)$ is in reduced row echelon form.

☐ True ☐ False

- (f) (1 point) If A is a 4×3 matrix and the solution set for $A\vec{x} = 0$ is a line, then A has 2 pivots.

☐ True ☐ False

- (g) (1 point) If A is an $m \times n$ matrix and $m > n$, then the linear transformation $T(\vec{x}) = A\vec{x}$ cannot be one-to-one.

☐ True ☐ False

- (h) (1 point) Let A be an $m \times n$ matrix (m rows and n columns) and $\vec{b}$ in $\mathbb{R}^m$. If A has fewer than m pivots then $A\vec{x} = \vec{b}$ has infinitely many solutions.

☐ True ☐ False

- (i) (1 point) If A is an $n \times n$ matrix and $A\vec{x} = \vec{b}$ has exactly one solution for some $\vec{b}$ in $\mathbb{R}^n$, then A is invertible.

☐ True ☐ False

- (j) (1 point) If A is an $n \times n$ matrix and $A\vec{e}_1 = A\vec{e}_2$, then A is not invertible.

☐ True ☐ False

Question No. 3.

- (a) (3 points) Solve the system of equations by putting an augmented matrix into reduced row echelon form. Clearly indicate which variables (if any) are free variables.

$$\begin{array}{ccccccccc} x_1 & +2 & x_2 & +2x_3 & - & x_4 & = & 4 \\ 2 & x_1 & +4 & x_2 & + & x_3 & -2x_4 & = & -1 \\ -x_1 & -2 & x_2 & - & x_3 & + & x_4 & = & -1 \end{array}$$

- (b) (2 points) Write the set of solutions to

$$\begin{array}{ccccccccc} x_1 & +2 & x_2 & +2x_3 & - & x_4 & = & 0 \\ 2 & x_1 & +4 & x_2 & + & x_3 & -2x_4 & = & 0 \\ -x_1 & -2 & x_2 & - & x_3 & + & x_4 & = & 0 \end{array}$$

in parametric vector form.

Question No. 4. Suppose A is a matrix that can be row reduced to the identity matrix I_4 by applying the following row operations (in the given order):

- 1) replacement $\xrightarrow{R_2 - 2R_1}$
- 2) scaling $\xrightarrow{2R_3}$
- 3) interchange $\xrightarrow{R_3 \leftrightarrow R_4}$

(a) (3 points) What are the matrices E_1 , E_2 , and E_3 ?

(b) (3 points) What are the matrices E_1^{-1} , E_2^{-1} , and E_3^{-1} ?

(c) (4 points) Use part(a) and part(b) above to write the matrices A and A^{-1} as a product of elementary matrices?

$A =$ _____

$A^{-1} =$ _____