

Student Name:	Question	1	2	3	4	Total
	Points	10	10	15	15	50
Student Number:	Score					

Solve the following questions

Q.I) Choose the correct answer for each of the following questions.

1. If A and B are invertible matrices from the same size, then

- (a) $\text{rank } A = \text{rank } B$.
- (b) AB is invertible matrix.
- (c) The linear system $ABx = 0$ has only the zero solution.
- (d) All of the above.

2. If A is skew symmetric matrix of size $n \times n$, then ?

- (A) $A - A^T = 0$ (B) $A + A^T = 2A$ (C) $A - A^T = 2A$ (D) $A + A^T = I_n$

3. Determine the values of r for which $\mathbf{v} = \begin{bmatrix} 2 \\ r \\ -8 \end{bmatrix}$ is in the span of $\mathcal{S} = \left\{ \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \right\}$,

- (A) -6 (B) 3 (C) 2 (D) 1

4. Compute the vector found by rotating $u = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ by the angle $\theta = 270^\circ$.

- (A) $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$ (B) $\begin{bmatrix} -3 \\ -2 \end{bmatrix}$ (C) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ (D) $\begin{bmatrix} -2 \\ -3 \end{bmatrix}$

5. Which of the following is the inverse of the elementary matrix $E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(A) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{4} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -\frac{1}{4} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Q.II) Solve the linear system of equations

$$\begin{array}{rcccccl} & x_1 & -3x_2 & & +x_4 & = & 1 \\ x_1 & & & +3x_3 & -2x_4 & = & 4 \\ & x_2 & & +4x_3 & -x_4 & = & 3 \end{array}$$

Q.III) Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be a transformation defined by

$$T \left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right) = \begin{bmatrix} x_1 - x_3 \\ x_2 - 2x_3 \\ -x_1 + 2x_2 \end{bmatrix}$$

1. Show that T is linear Transformation.

2. Determine a generating set for the range of T .

3. Determine a generating set for the null space of T .

4. Is T onto?

5. Is T one-to-one?

Q.IV) Let A be a matrix with the reduced row echelon form $R = \begin{bmatrix} 1 & -1 & 0 & -2 & 0 & 2 \\ 0 & 0 & 1 & 3 & 0 & -4 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$,
 where A has the columns $\mathbf{a}_2 = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}$, $\mathbf{a}_3 = \begin{bmatrix} 1 \\ 6 \\ -2 \end{bmatrix}$, and $\mathbf{a}_5 = \begin{bmatrix} -1 \\ -7 \\ 3 \end{bmatrix}$

1. Find $\text{Rank}(A)$ and $\text{Nullity}(A)$.

2. Determine whether the vectors $\{a_2, a_3, a_5\}$ are linearly independent.

3. Find the matrix A .