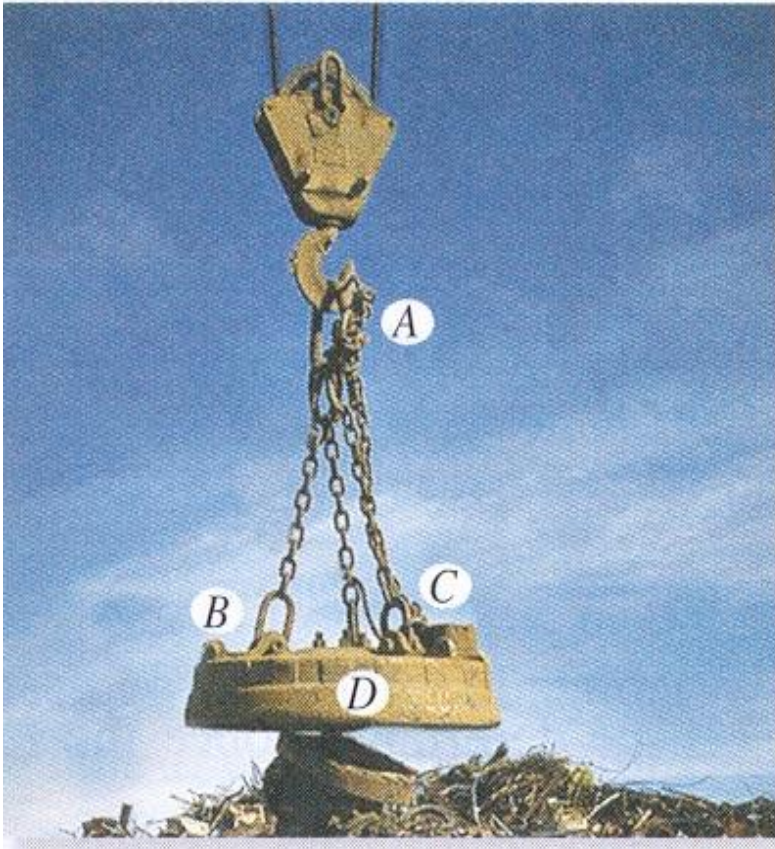


3.4 THREE-DIMENSIONAL FORCE SYSTEMS

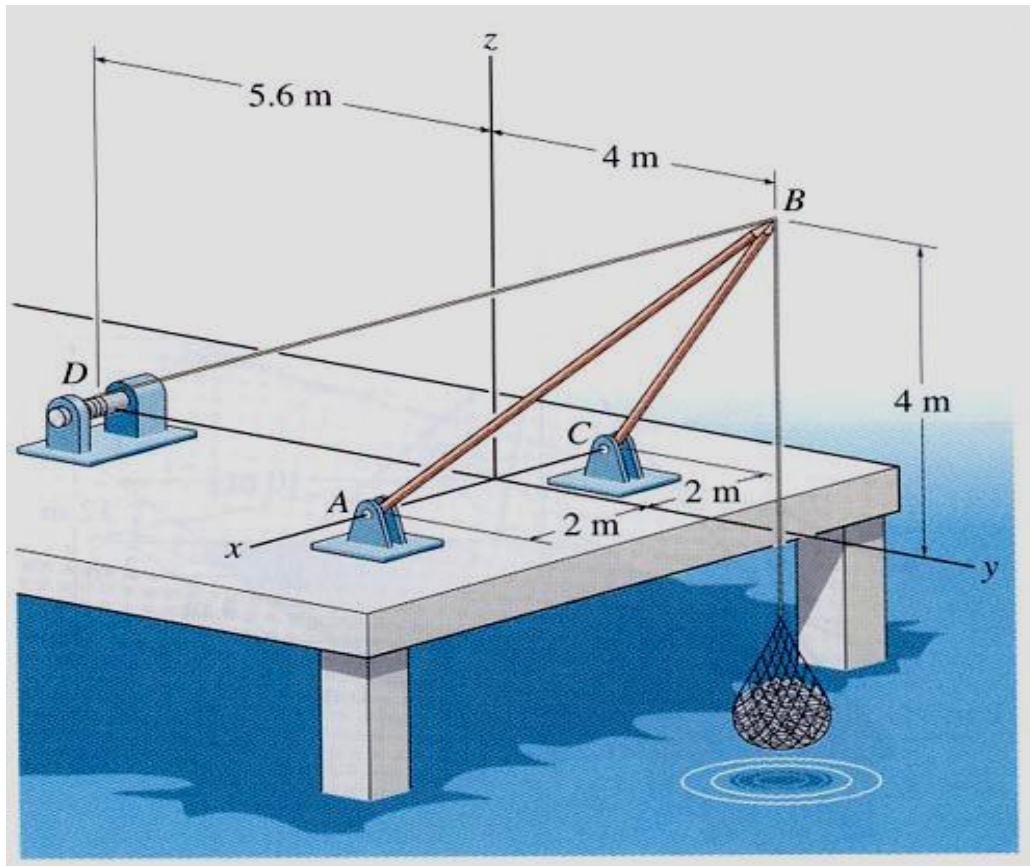


The weights of the electromagnet and the loads are given.

Can you determine the forces in the chains?

APPLICATIONS

(continued)



The shear leg derrick is to be designed to lift a maximum of 500 kg of fish.

THE EQUATIONS OF 3-D EQUILIBRIUM

When a particle is in equilibrium, the vector sum of all the forces acting on it must be zero ($\Sigma \mathbf{F} = 0$).

This equation can be written in terms of its x, y and z components. This form is written as follows.

$$(\Sigma F_x) \mathbf{i} + (\Sigma F_y) \mathbf{j} + (\Sigma F_z) \mathbf{k} = 0$$

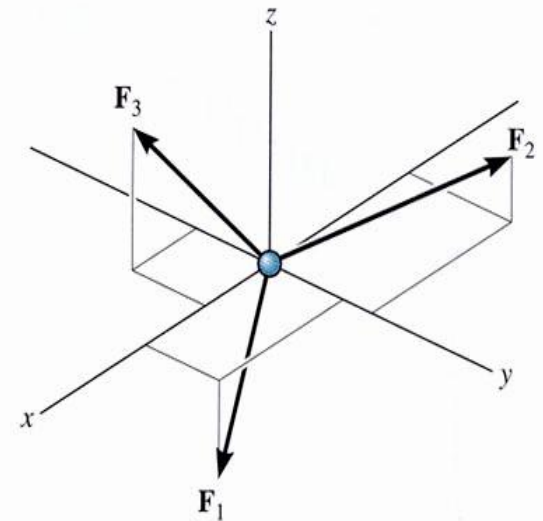
This vector equation will be satisfied only when

$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$

$$\Sigma F_z = 0$$

These equations are the three scalar equations of equilibrium. They are valid at any point in equilibrium and allow you to solve for up to three unknowns.



EXAMPLE #1

Given: F_1 , F_2 and F_3 .

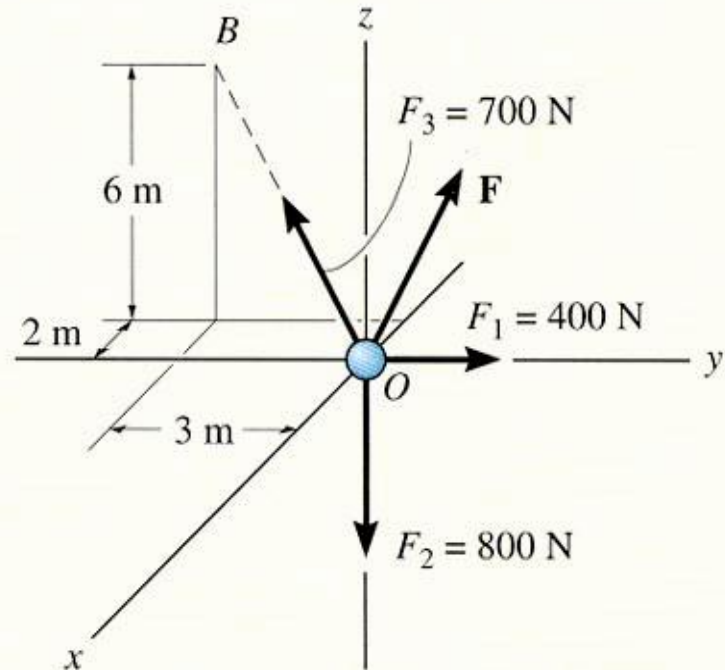
Find: The force F required to keep particle O in equilibrium.

Plan:

- 1) Draw a FBD of particle O.
- 2) Write the unknown force as

$$\mathbf{F} = \{F_x \mathbf{i} + F_y \mathbf{j} + F_z \mathbf{k}\} \text{ N}$$

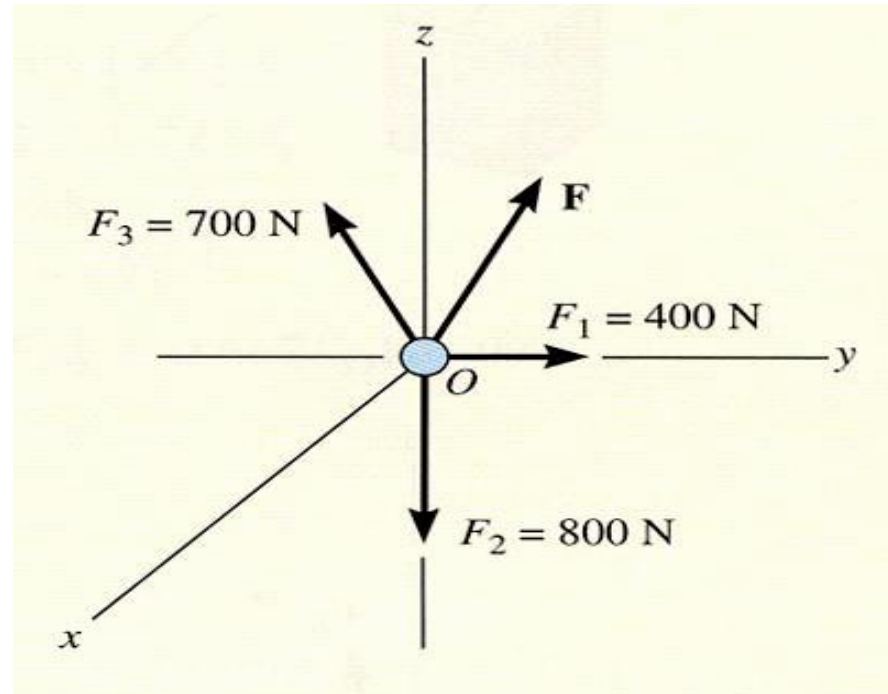
- 3) Write F_1 , F_2 and F_3 in Cartesian vector form.
- 4) Apply the three equilibrium equations to solve for the three unknowns F_x , F_y , and F_z .



EXAMPLE #1 (continued)

$$\mathbf{F}_1 = \{400 \mathbf{j}\} \text{ N}$$

$$\mathbf{F}_2 = \{-800 \mathbf{k}\} \text{ N}$$



$$\begin{aligned}\mathbf{F}_3 &= F_3 (\mathbf{r}_B / r_B) \\ &= 700 \text{ N } [(-2 \mathbf{i} - 3 \mathbf{j} + 6 \mathbf{k}) / (2^2 + 3^2 + 6^2)^{1/2}] \\ &= \{-200 \mathbf{i} - 300 \mathbf{j} + 600 \mathbf{k}\} \text{ N}\end{aligned}$$

EXAMPLE #1 (continued)

Equating the respective ***i***, ***j***, ***k*** components to zero, we have

$$\Sigma F_x = -200 + F_x = 0 ; \quad \text{solving gives } F_x = 200 \text{ N}$$

$$\Sigma F_y = 400 - 300 + F_y = 0 ; \quad \text{solving gives } F_y = -100 \text{ N}$$

$$\Sigma F_z = -800 + 600 + F_z = 0 ; \quad \text{solving gives } F_z = 200 \text{ N}$$

$$\text{Thus, } \mathbf{F} = \{200 \mathbf{i} - 100 \mathbf{j} + 200 \mathbf{k}\} \text{ N}$$

Using this force vector, you can determine the force's magnitude and coordinate direction angles as needed.

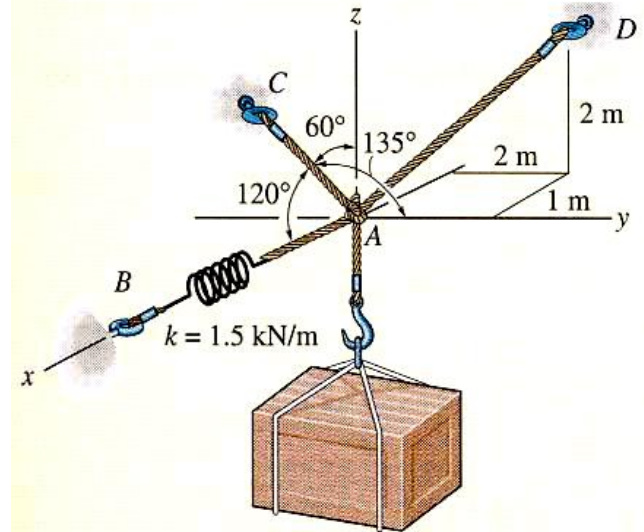
EXAMPLE #2

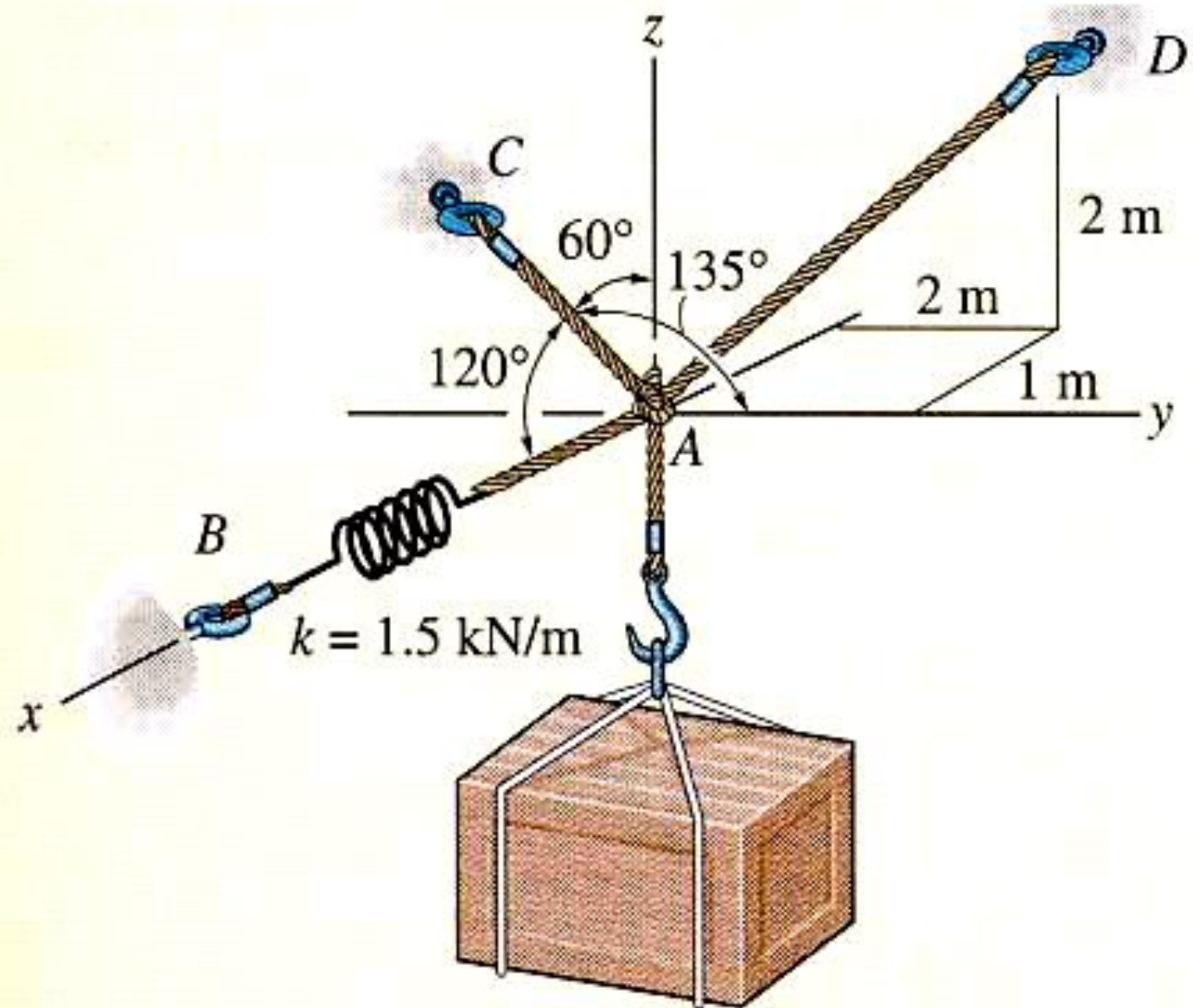
Given: A 100 Kg crate, as shown, is supported by three cords. One cord has a spring in it.

Find: Tension in cords AC and AD and the stretch of the spring.

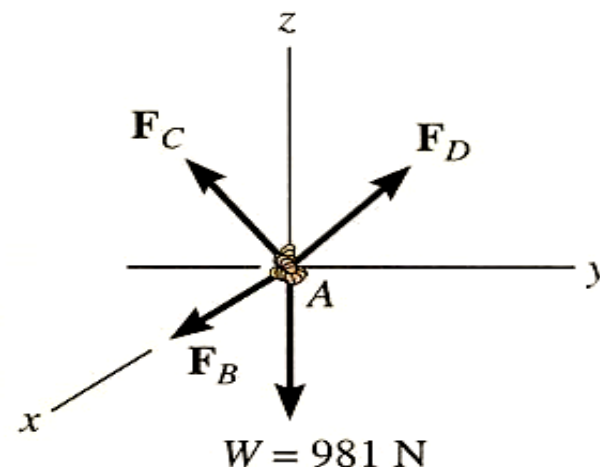
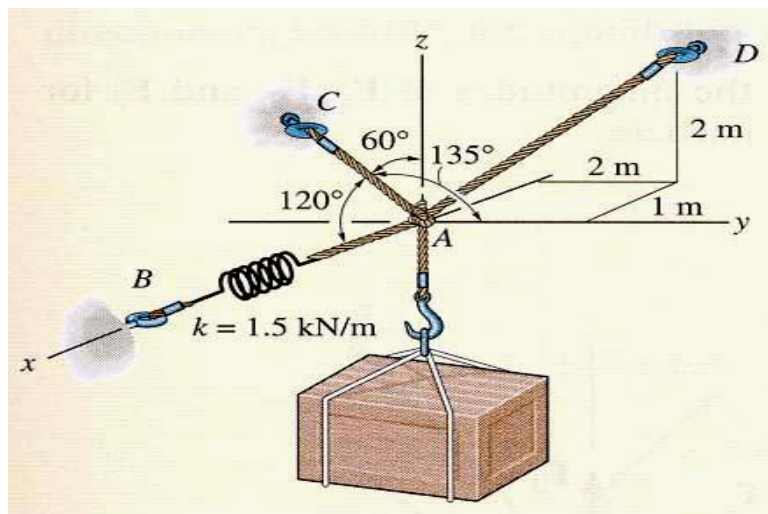
Plan:

- 1) Draw a free body diagram of Point A. Let the unknown force magnitudes be F_B , F_C , F_D .
- 2) Represent each force in the Cartesian vector form.
- 3) Apply equilibrium equations to solve for the three unknowns.
- 4) Find the spring stretch using $F_B = K * S$.





EXAMPLE #2 (continued)



FBD at A

$$\mathbf{F}_B = F_B \text{ N } \mathbf{i}$$

$$\begin{aligned} \mathbf{F}_C &= F_C \text{ N } (\cos 120^\circ \mathbf{i} + \cos 135^\circ \mathbf{j} + \cos 60^\circ \mathbf{k}) \\ &= \{-0.5 F_C \mathbf{i} - 0.707 F_C \mathbf{j} + 0.5 F_C \mathbf{k}\} \text{ N} \end{aligned}$$

$$\begin{aligned} \mathbf{F}_D &= F_D (\mathbf{r}_{AD}/r_{AD}) \\ &= F_D \text{ N } [(-1 \mathbf{i} + 2 \mathbf{j} + 2 \mathbf{k}) / (1^2 + 2^2 + 2^2)^{1/2}] \\ &= \{-0.3333 F_D \mathbf{i} + 0.667 F_D \mathbf{j} + 0.667 F_D \mathbf{k}\} \text{ N} \end{aligned}$$

EXAMPLE #2 (continued)

The weight is $\mathbf{W} = (-mg) \mathbf{k} = (-100 \text{ kg} * 9.81 \text{ m/sec}^2) \mathbf{k} = \{-981 \mathbf{k}\} \text{ N}$

Now equate the respective i, j, k components to zero.

$$\sum F_x = F_B - 0.5F_C - 0.333F_D = 0$$

$$\sum F_y = -0.707 F_C + 0.667 F_D = 0$$

$$\sum F_z = 0.5 F_C + 0.667 F_D - 981 \text{ N} = 0$$

Solving the three simultaneous equations yields

$$F_C = 813 \text{ N}$$

$$F_D = 862 \text{ N}$$

$$F_B = 693.7 \text{ N}$$

The spring stretch is (from $F = k * s$)

$$s = F_B / k = 693.7 \text{ N} / 1500 \text{ N/m} = 0.462 \text{ m}$$

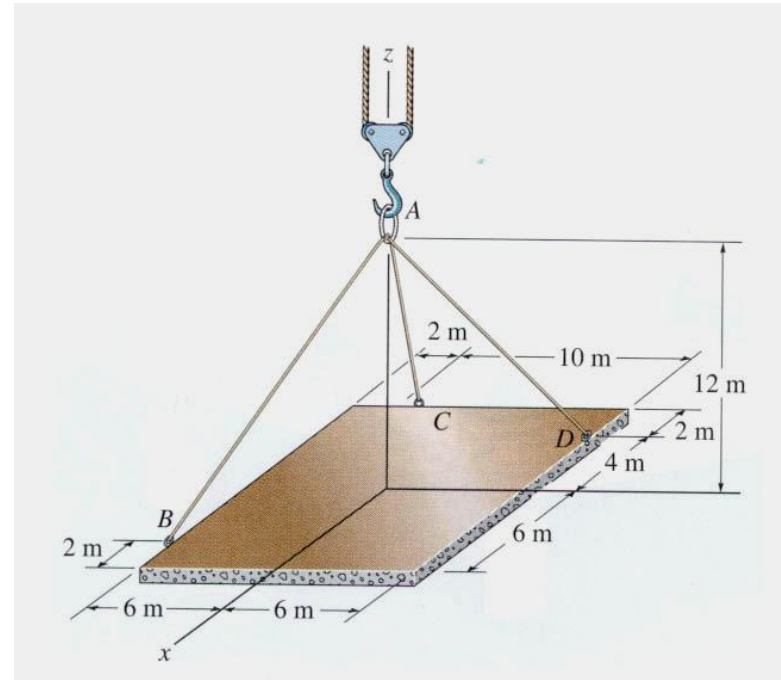
PROBLEM # 1

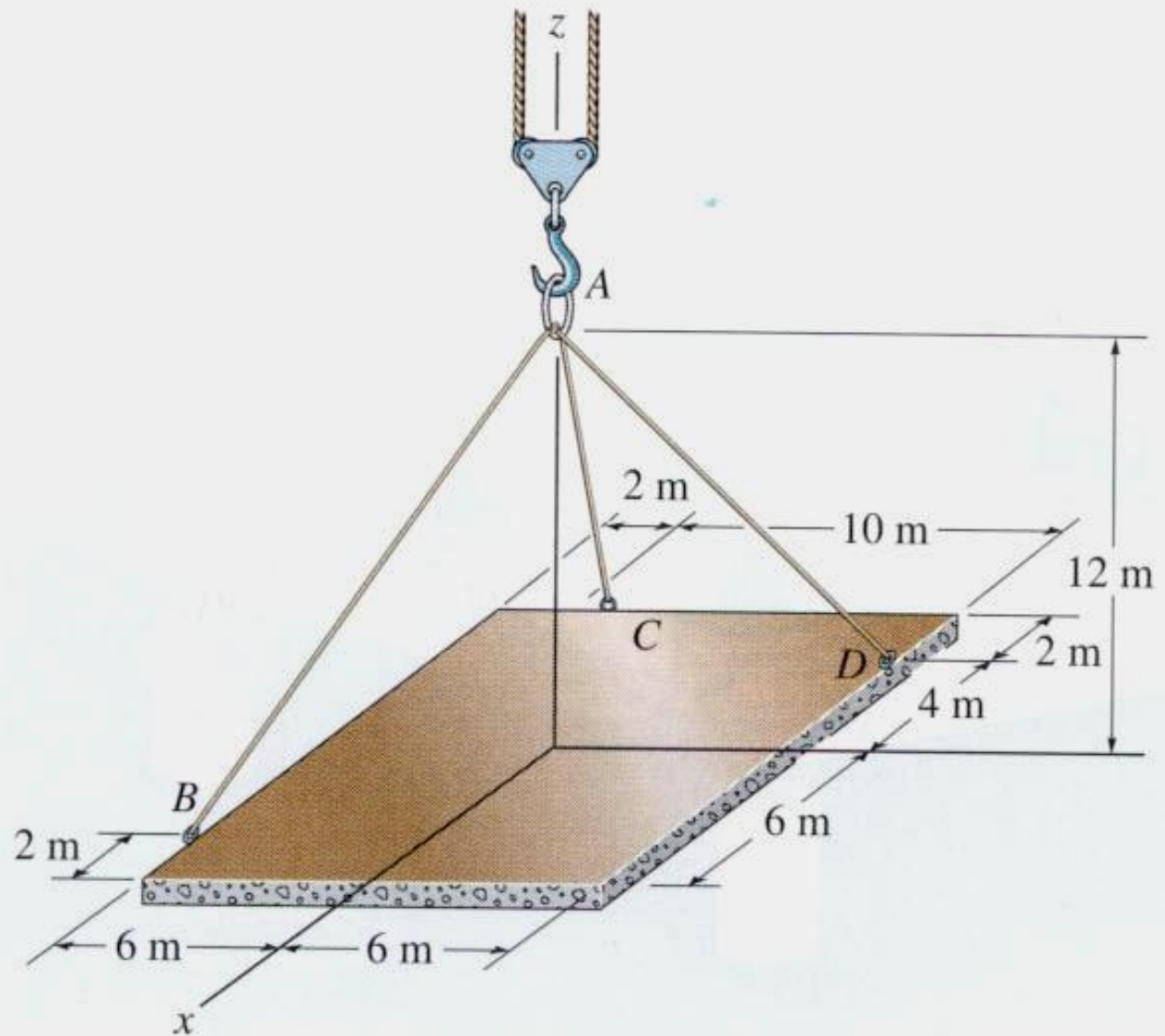
Given: A 150 Kg plate, as shown, is supported by three cables and is in equilibrium.

Find: Tension in each of the cables.

Plan:

- 1) Draw a free body diagram of Point A. Let the unknown force magnitudes be F_B , F_C , F_D .
- 2) Represent each force in the Cartesian vector form.
- 3) Apply equilibrium equations to solve for the three unknowns.





A 3D coordinate system with axes labeled x , y , and z . The z -axis is vertical, the x -axis points towards the bottom-left, and the y -axis points towards the right. A force vector $\mathbf{W}$ is shown as an orange arrow pointing upwards along the z -axis. Three other force vectors, $\mathbf{F}_B$, $\mathbf{F}_C$, and $\mathbf{F}_D$, are shown as red arrows originating from the same point. $\mathbf{F}_B$ points towards the bottom-left, $\mathbf{F}_C$ points towards the bottom-right, and $\mathbf{F}_D$ points towards the right, slightly above the y -axis.

$$\mathbf{F}_D = F_D(\mathbf{r}_{AD}/r_{AD}) = F_D(-4 \mathbf{i} + 6 \mathbf{j} - 12 \mathbf{k})\text{m}/(14 \text{ m})$$

PROBLEM # 1(continued)

The particle A is in equilibrium, hence

$$\mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D + \mathbf{W} = 0$$

Now equate the respective $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ components to zero (i.e., apply the three scalar equations of equilibrium).

$$\sum F_x = (4/14)F_B - (6/14)F_C - (4/14)F_D = 0$$

$$\sum F_y = (-6/14)F_B - (4/14)F_C + (6/14)F_D = 0$$

$$\sum F_z = (-12/14)F_B - (12/14)F_C - (12/14)F_D + 1472 = 0$$

Solving the three simultaneous equations gives

$$F_B = 858 \text{ N}$$

$$F_C = 0 \text{ N}$$

$$F_D = 858 \text{ N}$$