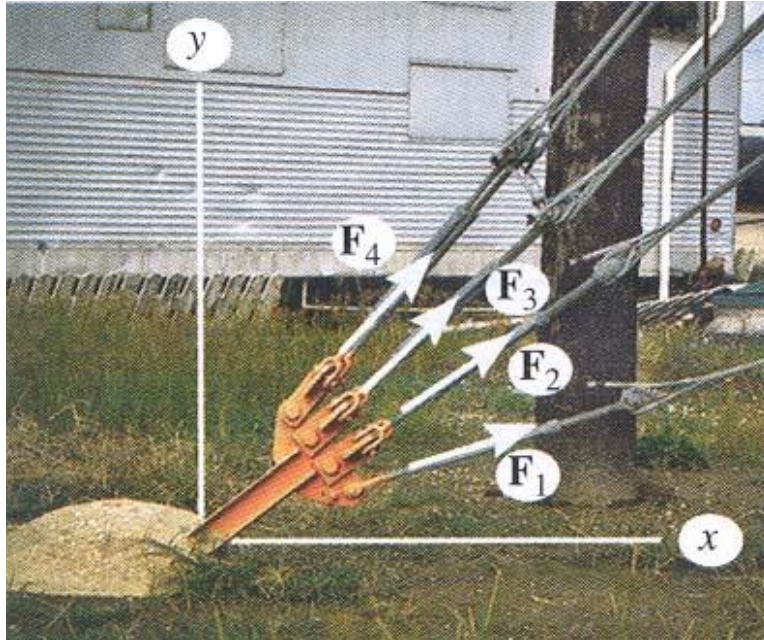


Chapter Two

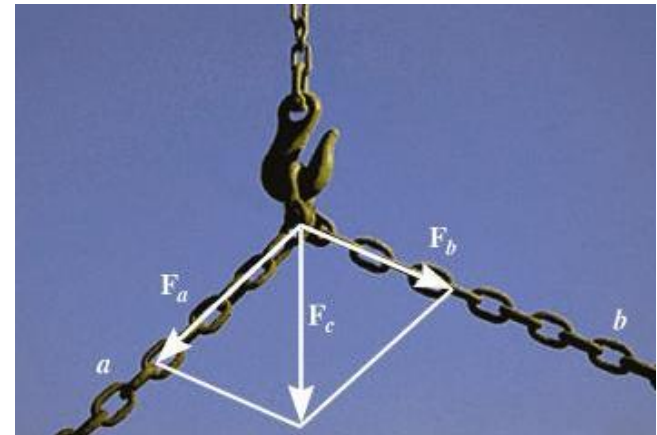
Force Vectors

APPLICATION OF VECTOR ADDITION



There are four concurrent cable forces acting on the bracket.

How do you determine the resultant force acting on the bracket ?



2.1 SCALARS AND VECTORS

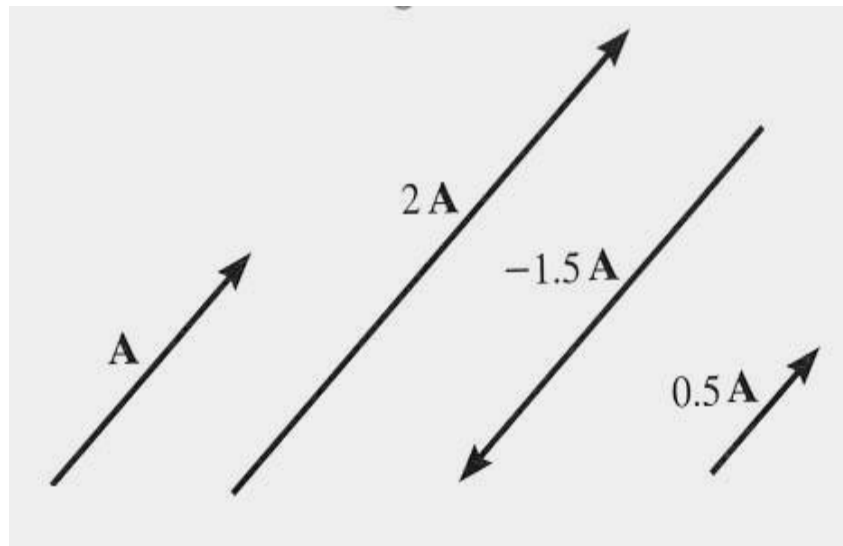
Scalars

Vectors

Examples:	mass, volume	force, velocity
Characteristics:	It has a magnitude (positive or negative)	It has a magnitude and direction
Addition rule:	Simple arithmetic	Parallelogram law
Special Notation:	None	Bold font, a line, an arrow or a “carrot”

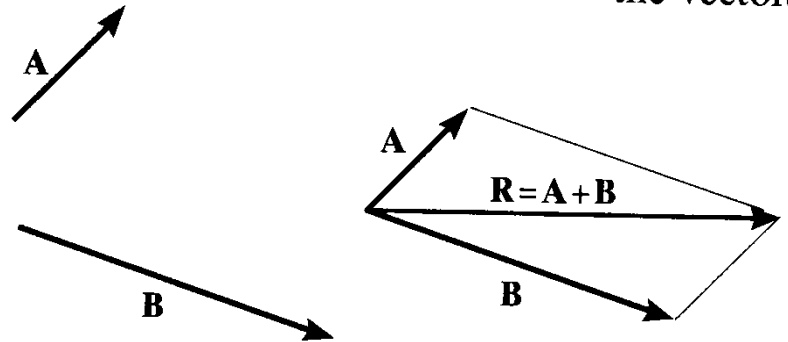
2.2 VECTOR OPERATIONS

Multiplication and Division of a vector by a scalar

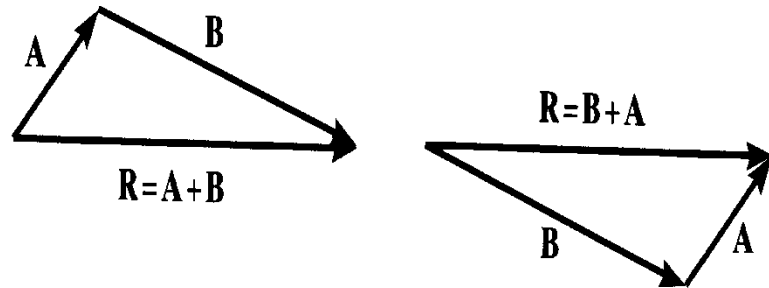


Vector Addition and Subtraction

Parallelogram Law:

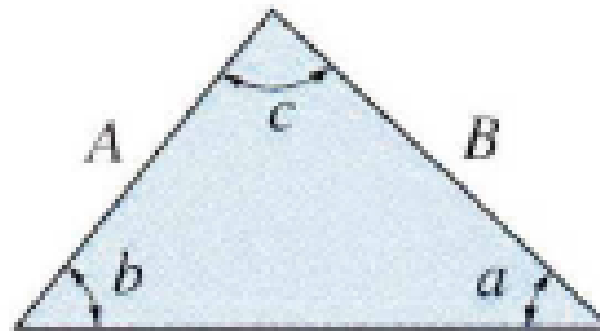


Triangle method
(always 'tip to tail'):



Vector subtraction $R' = A - B = A + (-B)$

Sine law



Sine law:

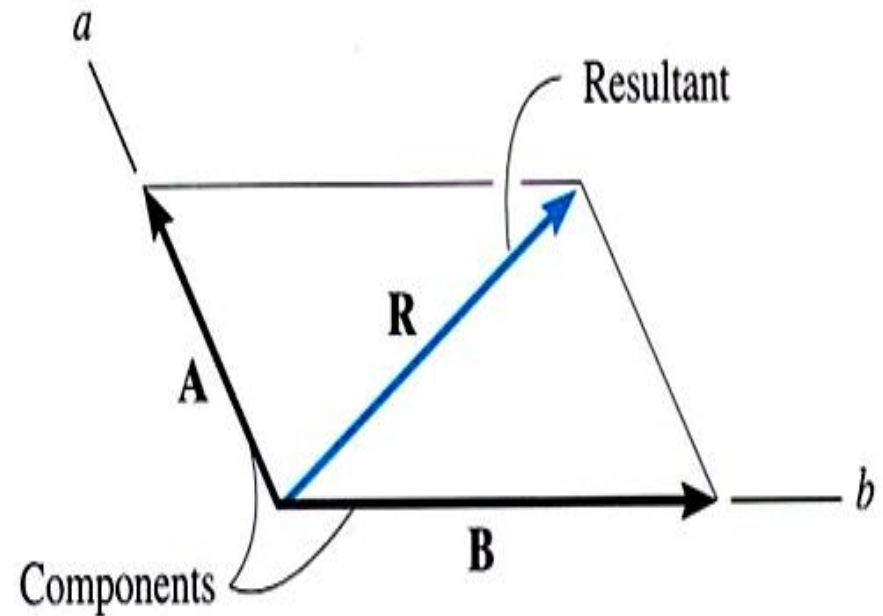
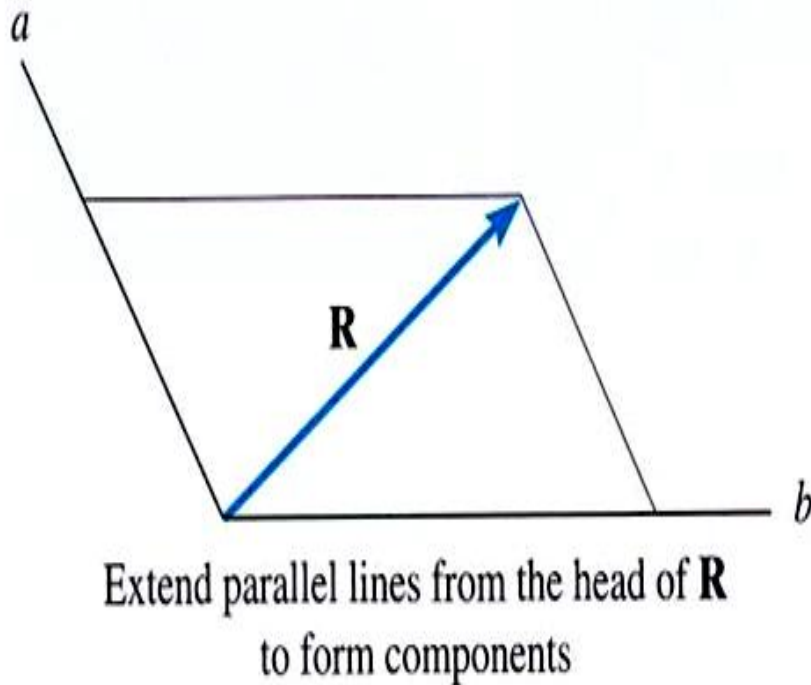
$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

Cosine law:

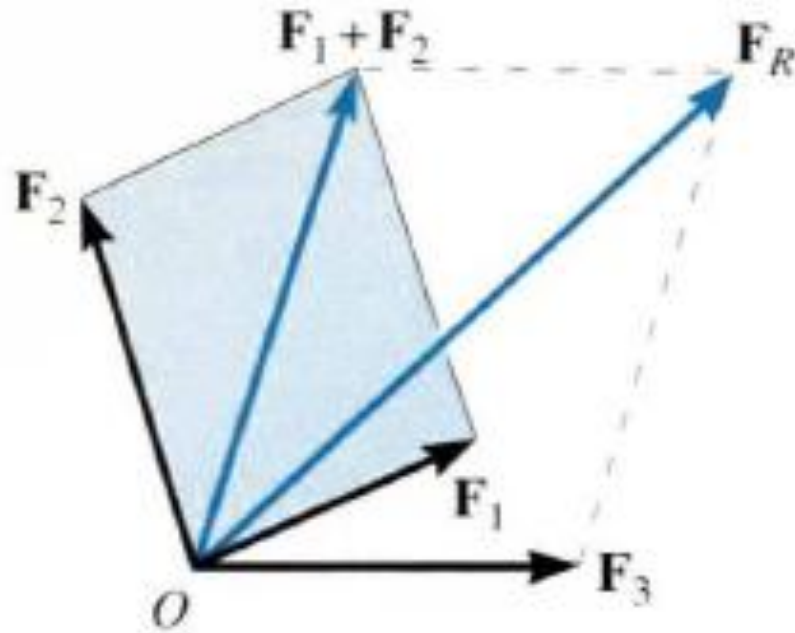
$$C = \sqrt{A^2 + B^2 - 2AB \cos c}$$

RESOLUTION OF A VECTOR

“Resolution” of a vector is breaking up a vector into components. It is kind of like using the parallelogram law in reverse.

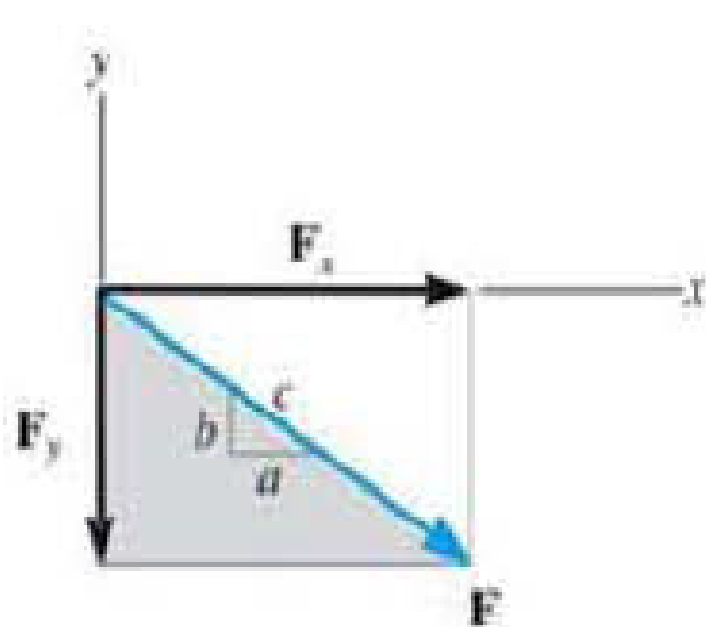


2.3 Vector Addition of Forces

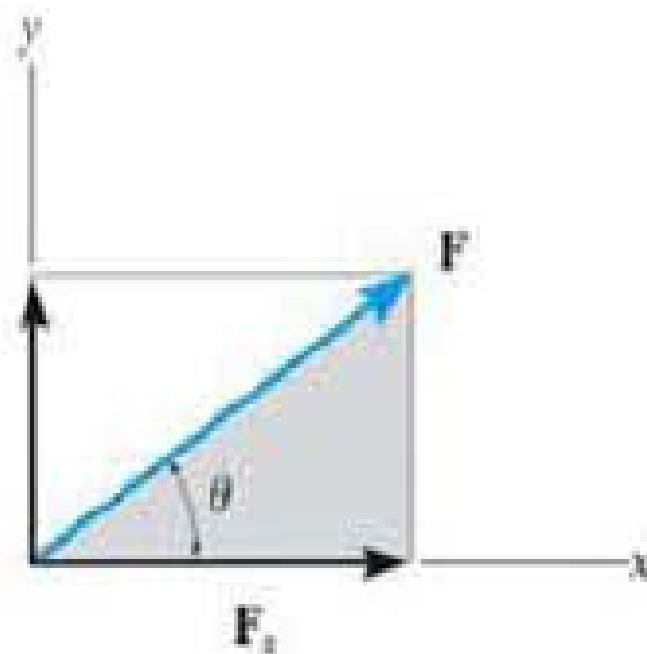


Scalar Notation. The rectangular components of force $\mathbf{F}$ shown in Fig. 2-15a are found using the parallelogram law, so that $\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$. Because these components form a right triangle, their magnitudes can be determined from

$$F_x = F \cos \theta \quad \text{and} \quad F_y = F \sin \theta$$

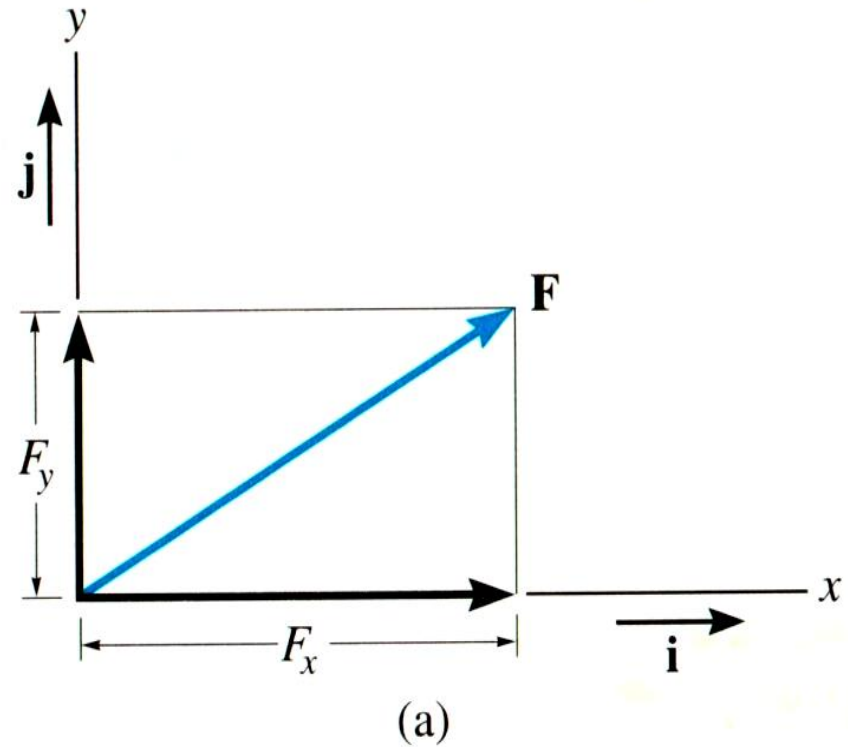


(b)



VECTOR NOTATION

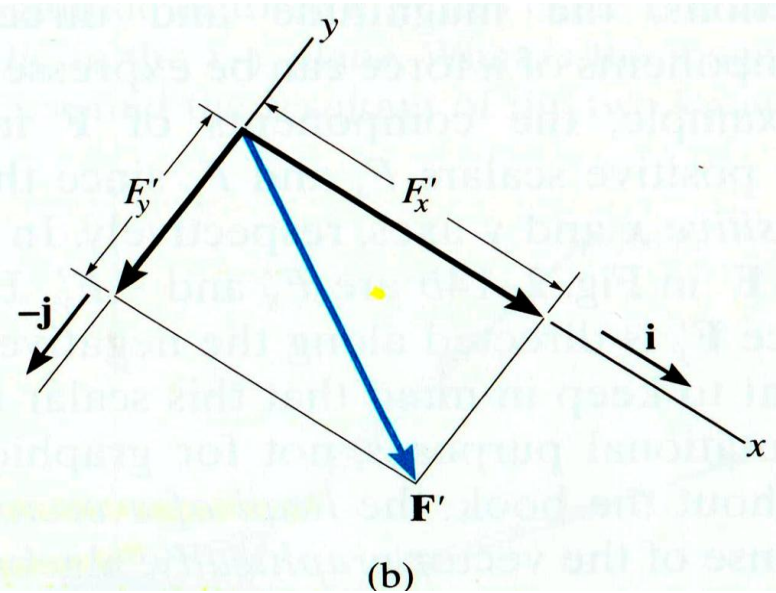
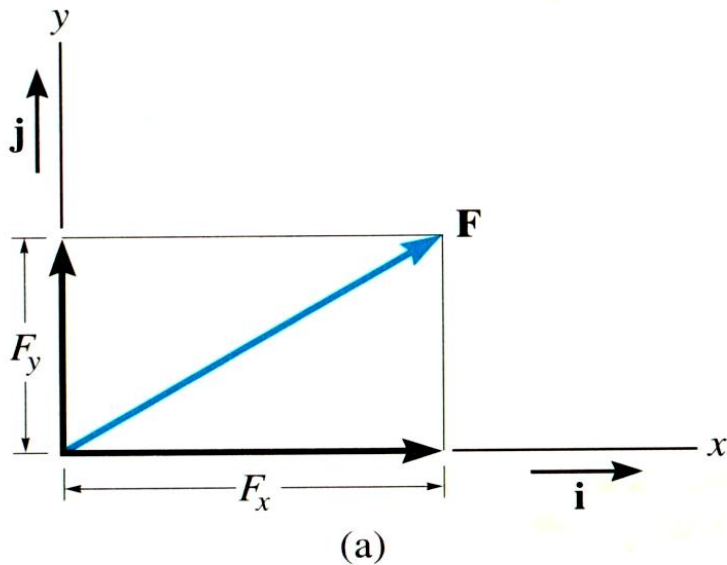
- We ‘resolve’ vectors into components using the x and y axes system.
- Each component of the vector is shown as a magnitude and a direction.



- The directions are based on the x and y axes. We use the “unit vectors” $\mathbf{i}$ and $\mathbf{j}$ to designate the x and y axes.

For example,

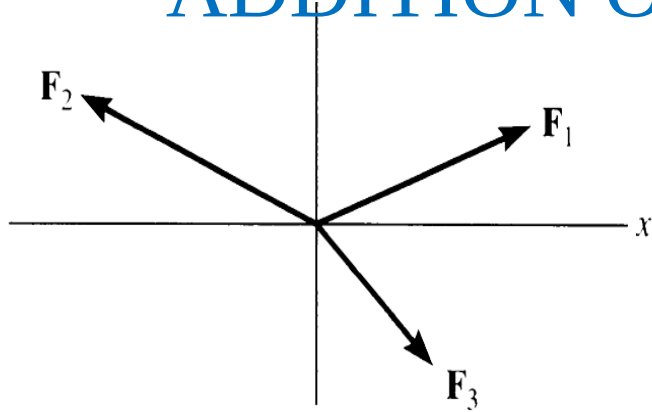
$$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j} \quad \text{or} \quad \mathbf{F}' = F'_x \mathbf{i} + F'_y \mathbf{j}$$



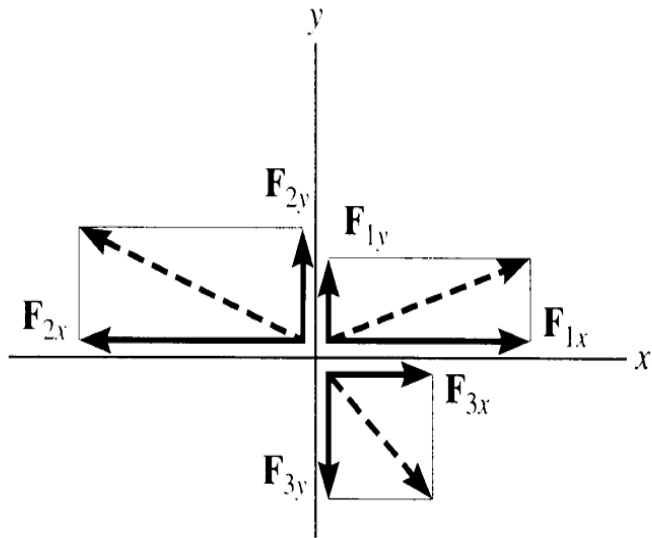
The x and y axes are always perpendicular to each other. Together, they can be directed at any inclination.



ADDITION OF SEVERAL VECTORS



(a)

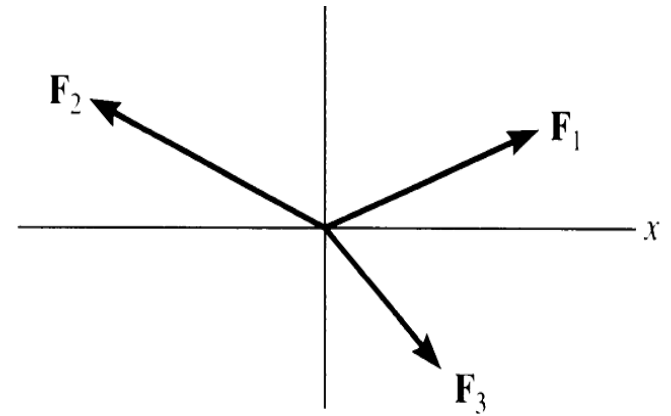


(b)

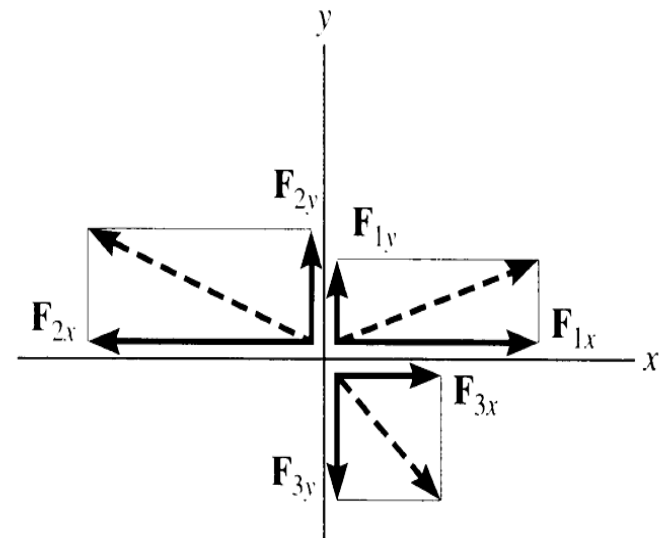
- Step 1 is to resolve each force into its components
- Step 2 is to add all the x components together and add all the y components together. These two totals become the resultant vector.
- Step 3 is to find the magnitude and angle of the resultant vector.

Example of this process,

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 \\ &= F_{1x}\mathbf{i} + F_{1y}\mathbf{j} - F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{3x}\mathbf{i} - F_{3y}\mathbf{j} \\ &= (F_{1x} - F_{2x} + F_{3x})\mathbf{i} + (F_{1y} + F_{2y} - F_{3y})\mathbf{j} \\ &= (F_{Rx})\mathbf{i} + (F_{Ry})\mathbf{j}\end{aligned}$$

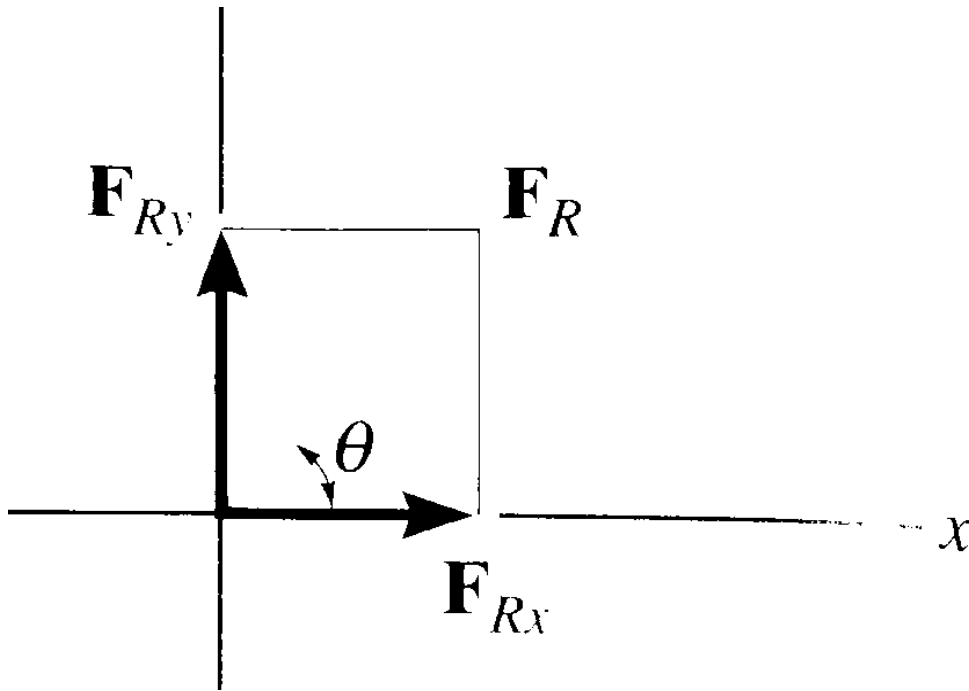


(a)



(b)

You can also represent a 2-D vector with a magnitude and angle.

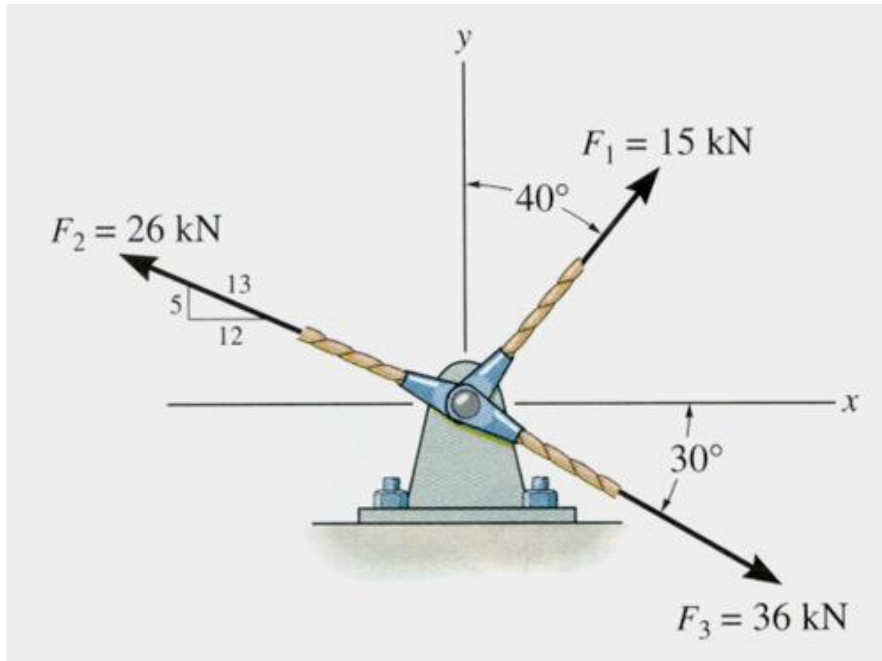


$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

$$\theta = \tan^{-1} \left| \frac{F_{Ry}}{F_{Rx}} \right|$$



EXAMPLE



Given: Three concurrent forces acting on a bracket.

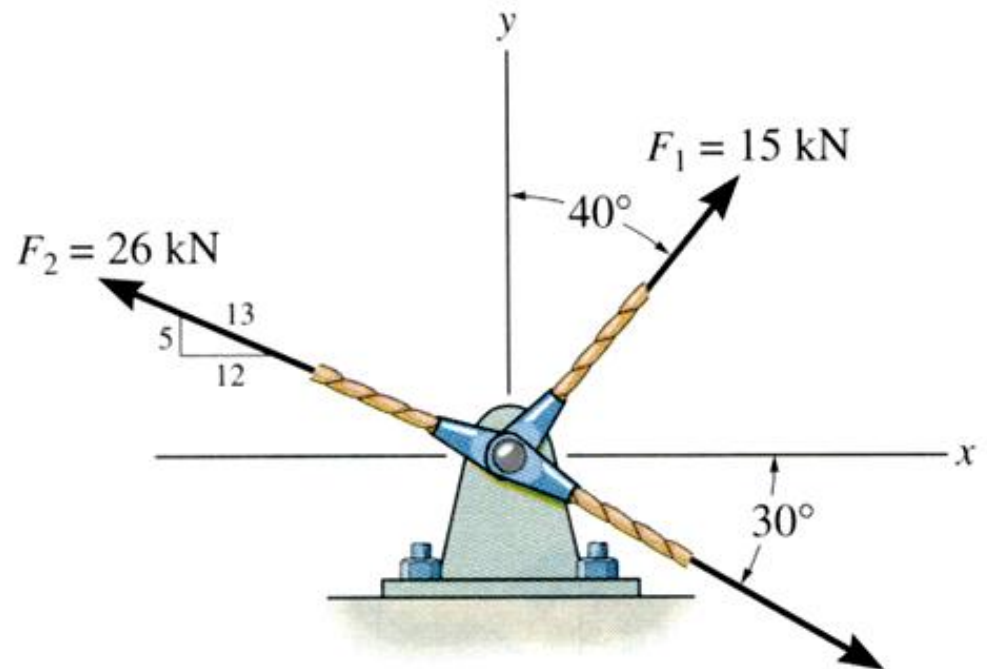
Find: The magnitude and angle of the resultant force.

Plan:

- Resolve the forces in their x-y components.
- Add the respective components to get the resultant vector.
- Find magnitude and angle from the resultant components.



EXAMPLE (continued)



$$\mathbf{F}_1 = \{ 15 \sin 40^\circ \mathbf{i} + 15 \cos 40^\circ \mathbf{j} \} \text{ kN}$$

$$= \{ 9.642 \mathbf{i} + 11.49 \mathbf{j} \} \text{ kN}$$

$$\mathbf{F}_2 = \{ -(12/13)26 \mathbf{i} + (5/13)26 \mathbf{j} \} \text{ kN}$$

$$= \{ -24 \mathbf{i} + 10 \mathbf{j} \} \text{ kN}$$

$$\mathbf{F}_3 = \{ 36 \cos 30^\circ \mathbf{i} - 36 \sin 30^\circ \mathbf{j} \} \text{ kN}$$

$$= \{ 31.18 \mathbf{i} - 18 \mathbf{j} \} \text{ kN}$$



EXAMPLE

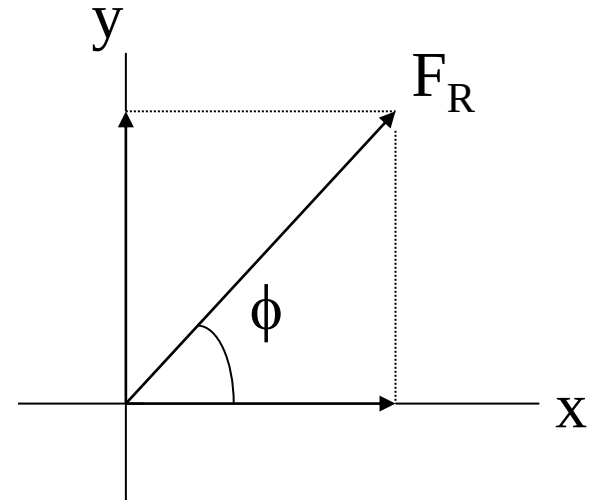
(continued)

Summing up all the $\mathbf{i}$ and $\mathbf{j}$ components respectively, we get,

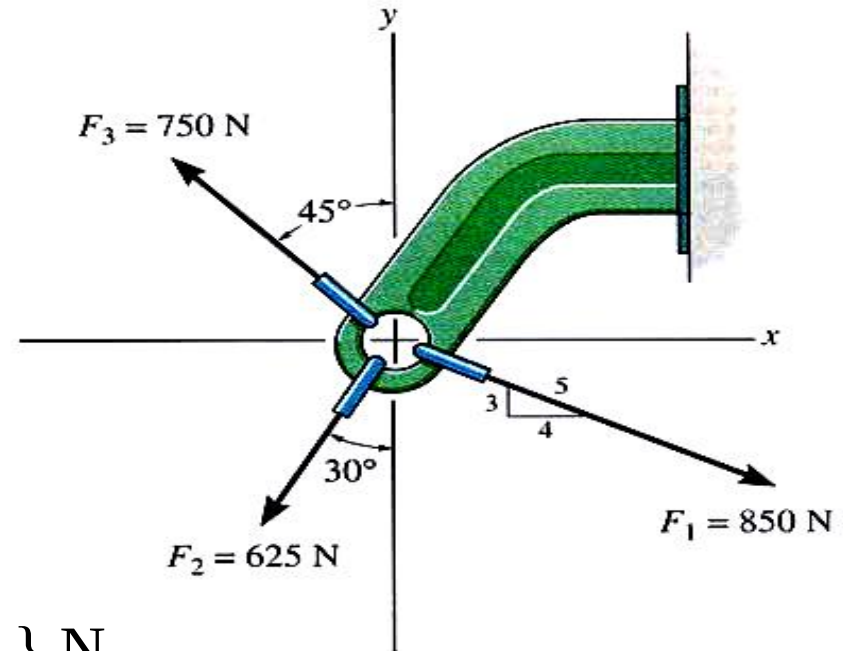
$$\begin{aligned}\mathbf{F}_R &= \{ (9.642 - 24 + 31.18) \mathbf{i} + (11.49 + 10 - 18) \mathbf{j} \} \text{ kN} \\ &= \{ 16.82 \mathbf{i} + 3.49 \mathbf{j} \} \text{ kN}\end{aligned}$$

$$F_R = ((16.82)^2 + (3.49)^2)^{1/2} = 17.2 \text{ kN}$$

$$\phi = \tan^{-1}(3.49/16.82) = 11.7^\circ$$



GROUP PROBLEM SOLVING (continued)



$$\begin{aligned} \mathbf{F}_1 &= \{ (4/5) 850 \mathbf{i} - (3/5) 850 \mathbf{j} \} \text{ N} \\ &= \{ 680 \mathbf{i} - 510 \mathbf{j} \} \text{ N} \end{aligned}$$

$$\begin{aligned} \mathbf{F}_2 &= \{ -625 \sin(30^\circ) \mathbf{i} - 625 \cos(30^\circ) \mathbf{j} \} \text{ N} \\ &= \{ -312.5 \mathbf{i} - 541.3 \mathbf{j} \} \text{ N} \end{aligned}$$

$$\begin{aligned} \mathbf{F}_3 &= \{ -750 \sin(45^\circ) \mathbf{i} + 750 \cos(45^\circ) \mathbf{j} \} \text{ N} \\ &= \{ -530.3 \mathbf{i} + 530.3 \mathbf{j} \} \text{ N} \end{aligned}$$



GROUP PROBLEM SOLVING (continued)

Summing up all the *i* and *j* components respectively, we get,

$$\begin{aligned} \mathbf{F}_R &= \{ (680 - 312.5 - 530.3) \mathbf{i} + (-510 - 541.3 + 530.3) \mathbf{j} \} \text{ N} \\ &= \{ -162.8 \mathbf{i} - 521 \mathbf{j} \} \text{ N} \end{aligned}$$

$$F_R = ((162.8)^2 + (521)^2)^{1/2} = 546 \text{ N}$$

$$\phi = \tan^{-1}(521/162.8) = 72.64^\circ \quad \text{or}$$

$$\text{From Positive x axis } \theta = 180 + 72.64 = 253^\circ$$

