

POSITION VECTORS & FORCE VECTORS

Today's Objectives:

Students will be able to :

- a) Represent a position vector in Cartesian coordinate form, from given geometry.
- b) Represent a force vector directed along a line.



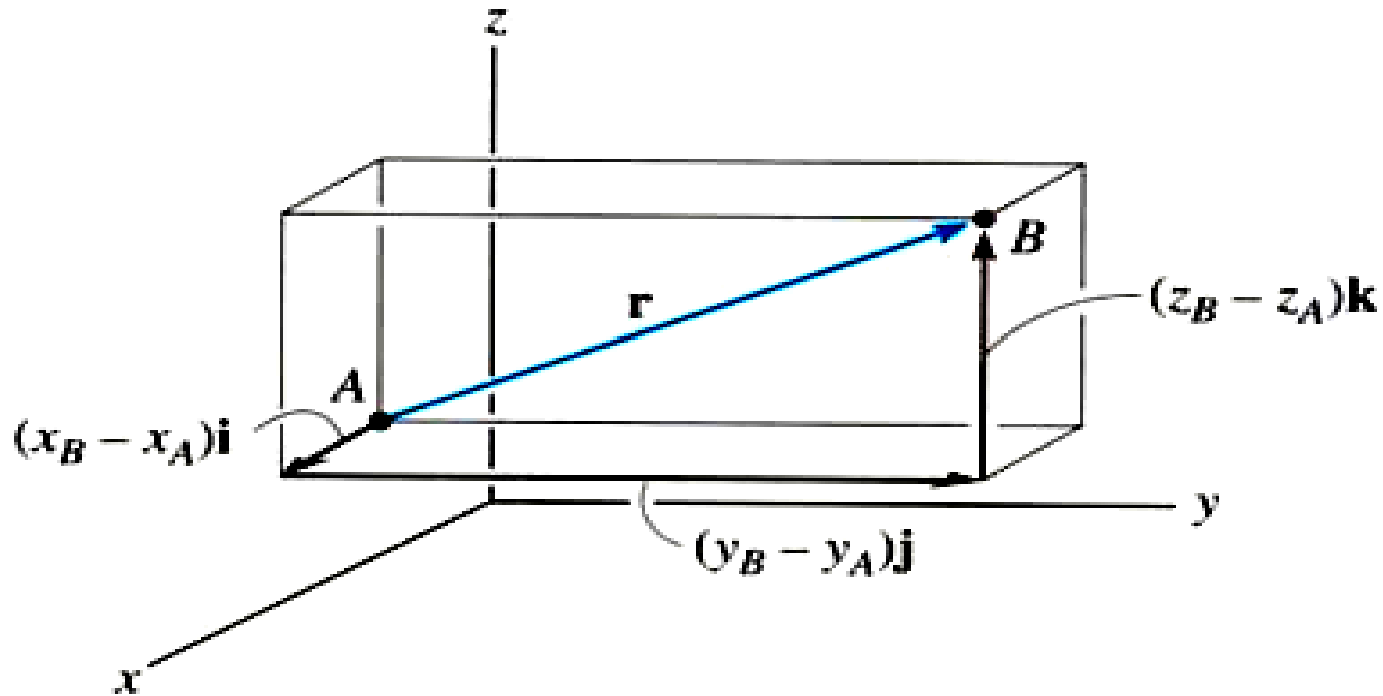
READING QUIZ

1. A position vector, $\mathbf{r}_{PQ}$, is obtained by
 - A) Coordinates of Q minus coordinates of P
 - B) Coordinates of P minus coordinates of Q
 - C) Coordinates of Q minus coordinates of the origin
 - D) Coordinates of the origin minus coordinates of P
2. A force of magnitude F , directed along a unit vector $\mathbf{U}$, is given by $\mathbf{F} = \underline{\hspace{2cm}}$.
 - A) $F (\mathbf{U})$
 - B) $\mathbf{U} / F$
 - C) $F / \mathbf{U}$
 - D) $F + \mathbf{U}$
 - E) $F - \mathbf{U}$



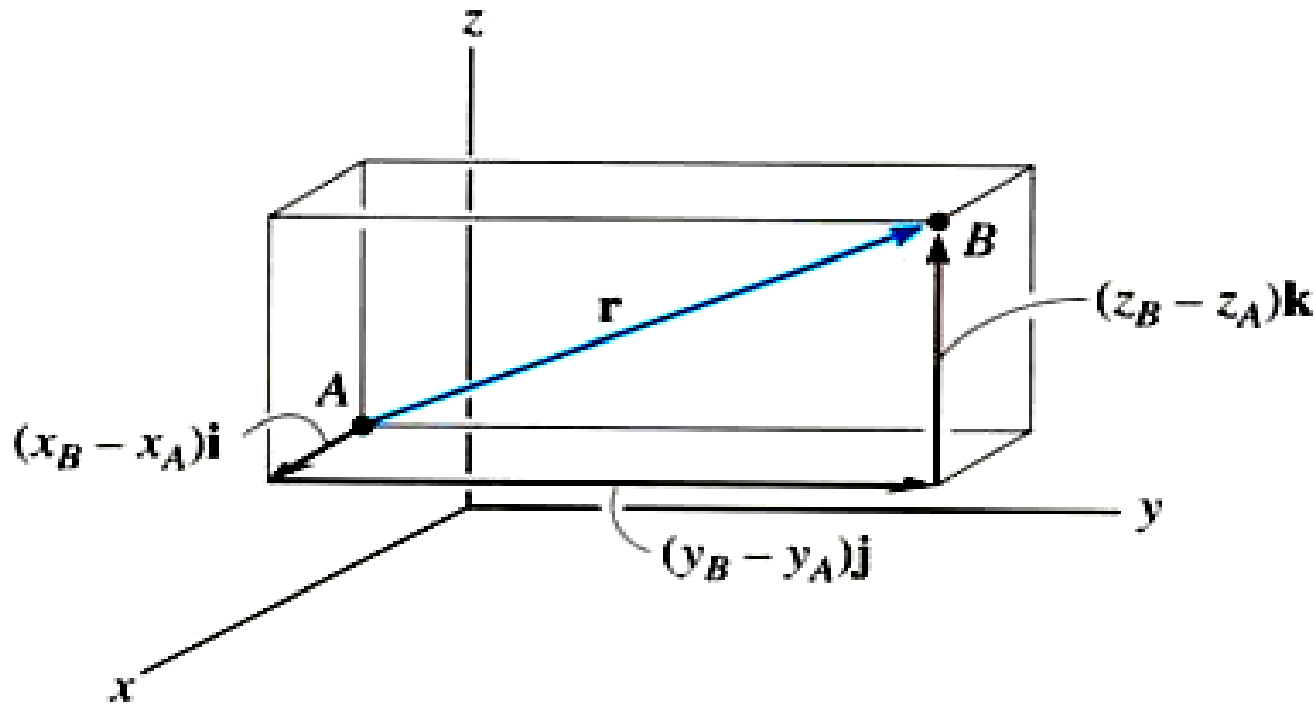
2.7 POSITION VECTOR

A position vector is defined as a fixed vector that locates a point in space relative to another point.



Consider two points, A & B, in 3-D space. Let their coordinates be (X_A, Y_A, Z_A) and (X_B, Y_B, Z_B) , respectively.

2.7 POSITION VECTOR



The position vector directed from A to B, $\mathbf{r}_{AB}$, is defined as

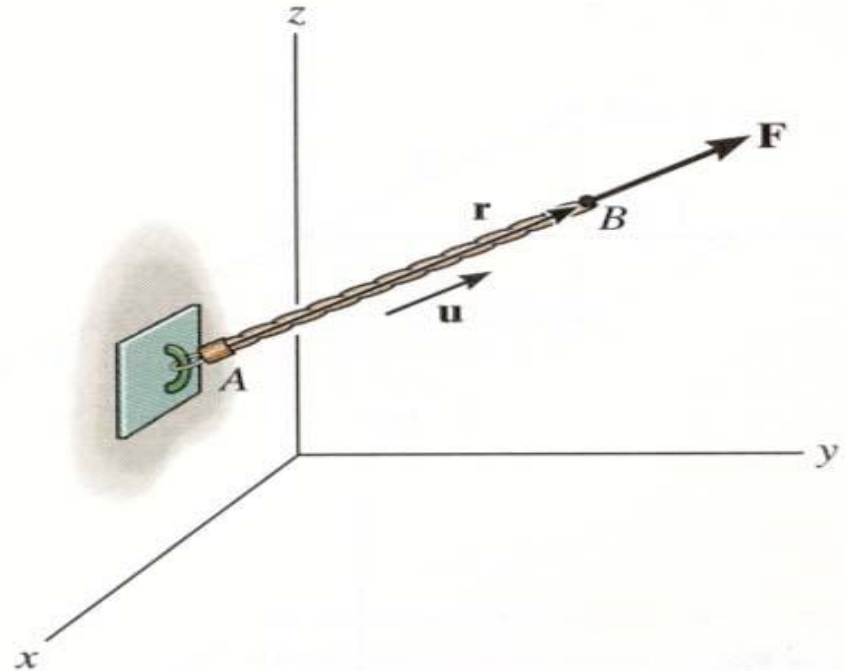
$$\mathbf{r}_{AB} = \{ (X_B - X_A) \mathbf{i} + (Y_B - Y_A) \mathbf{j} + (Z_B - Z_A) \mathbf{k} \} \text{m}$$

Note that B is the ending point and A is the starting point. So

ALWAYS subtract the “tail” coordinates from the “tip” coordinates!

2.8 FORCE VECTOR DIRECTED ALONG A LINE

If a force is directed along a line, then we can represent the force vector in Cartesian Coordinates by using a unit vector and the force magnitude. So we need to:



- Find the position vector, $\mathbf{r}_{AB}$, along two points on that line.
- Find the unit vector describing the line's direction,
 $\mathbf{u}_{AB} = (\mathbf{r}_{AB}/r_{AB})$.
- Multiply the unit vector by the magnitude of the force,
 $\mathbf{F} = F \mathbf{u}_{AB}$.

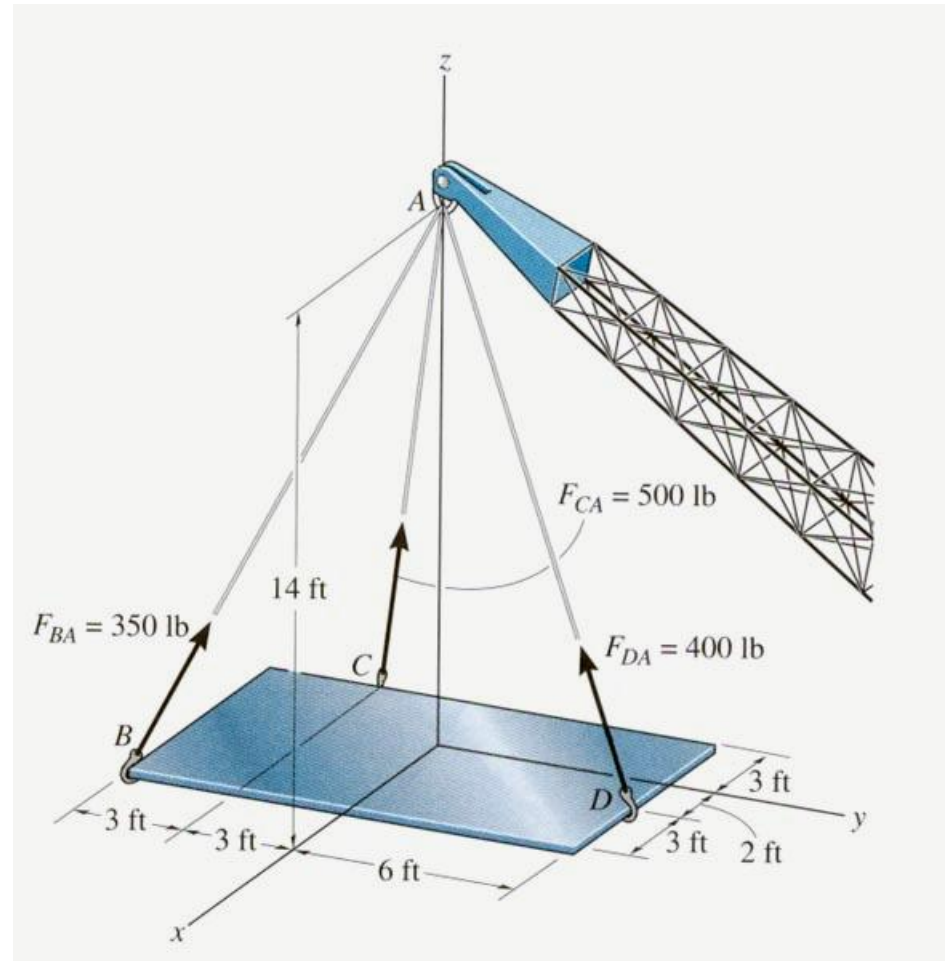
EXAMPLE

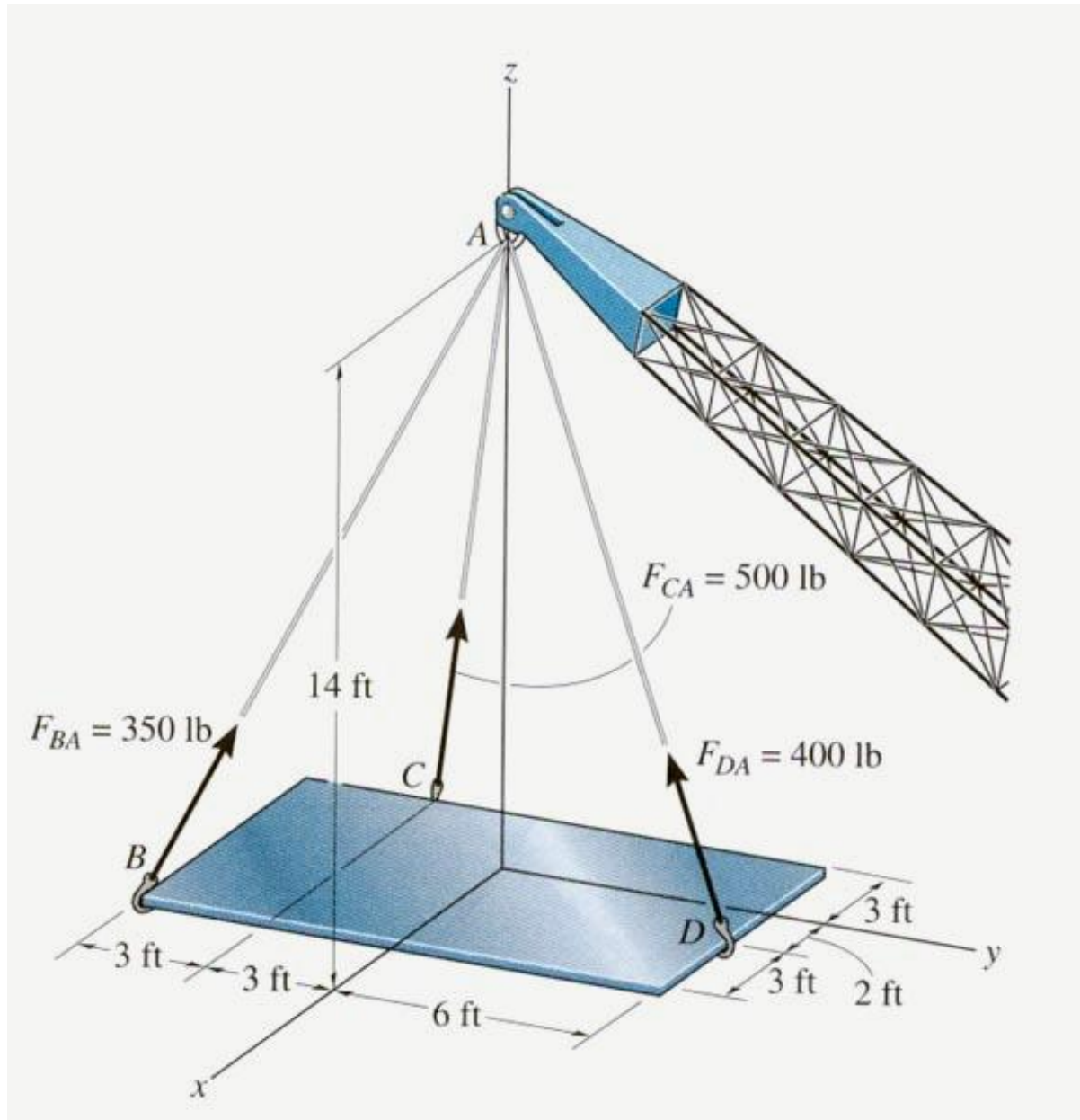
Given: 400 lb force along the cable DA.

Find: The force F_{DA} in the Cartesian vector form.

Plan:

1. Find the position vector $\mathbf{r}_{DA}$ and the unit vector $\mathbf{u}_{DA}$.
2. Obtain the force vector as $\mathbf{F}_{DA} = 400 \text{ lb } \mathbf{u}_{DA}$.





EXAMPLE (continued)

When relating D to A, we will have to go -2 ft in the x-direction, -6 ft in the y-direction, and +14 ft in the z-direction. Hence,

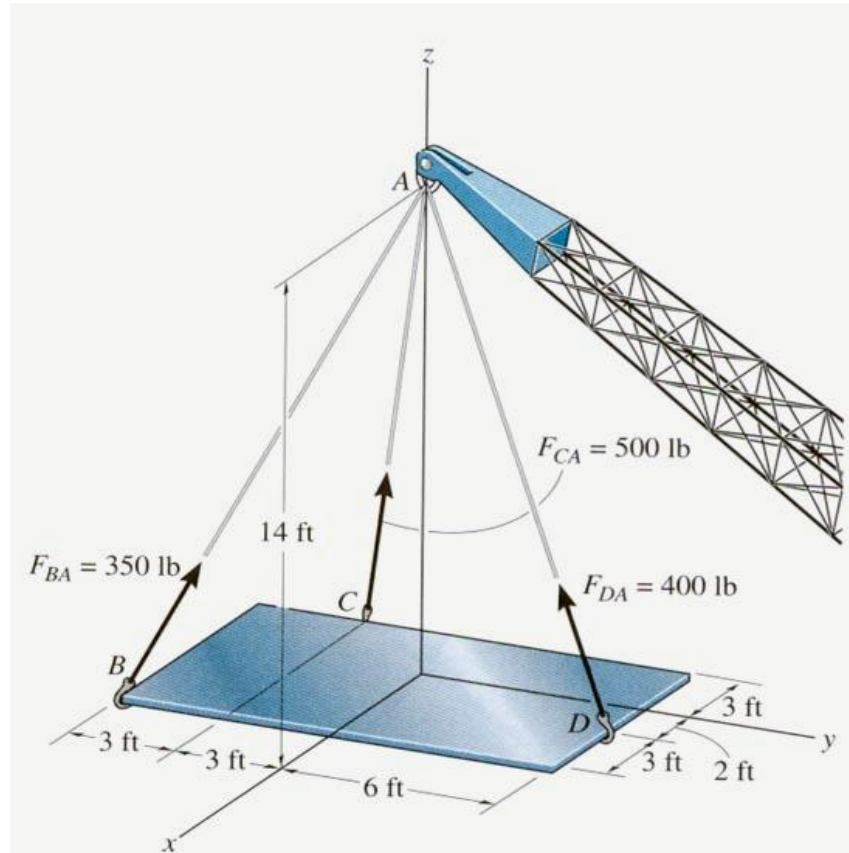
$$\mathbf{r}_{DA} = \{-2 \mathbf{i} - 6 \mathbf{j} + 14 \mathbf{k}\} \text{ ft.}$$

We can also find $\mathbf{r}_{DA}$ by subtracting the coordinates of D from the coordinates of A.

$$r_{DA} = (2^2 + 6^2 + 14^2)^{0.5} = 15.36 \text{ ft}$$

$$\mathbf{u}_{DA} = \mathbf{r}_{DA}/r_{DA} \text{ and } \mathbf{F}_{DA} = 400 \mathbf{u}_{DA} \text{ lb}$$

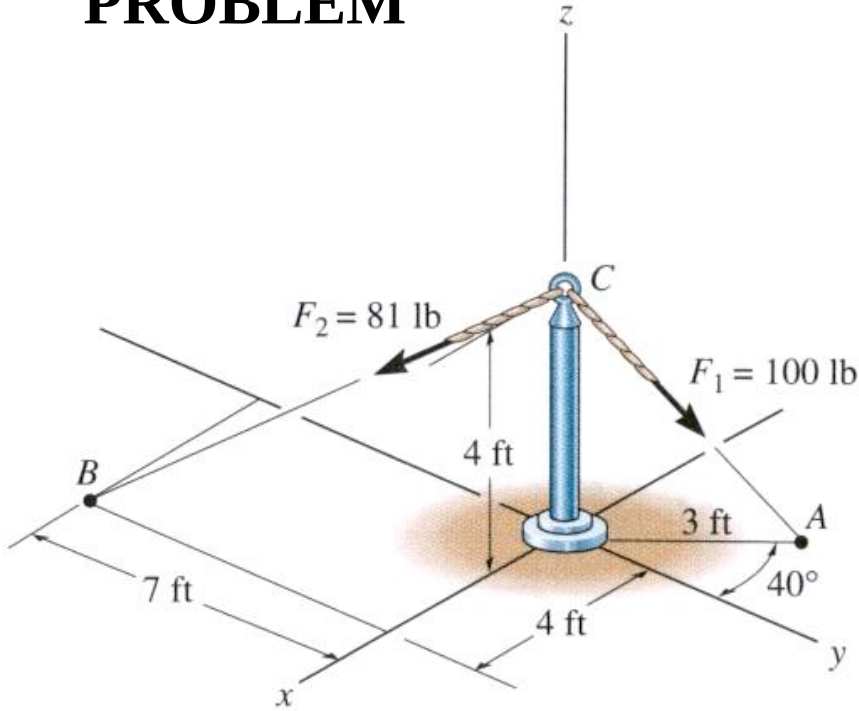
$$\begin{aligned} \mathbf{F}_{DA} &= 400\{(-2 \mathbf{i} - 6 \mathbf{j} + 14 \mathbf{k})/15.36\} \text{ lb} \\ &= \{-52.1 \mathbf{i} - 156 \mathbf{j} + 365 \mathbf{k}\} \text{ lb} \end{aligned}$$



PROBLEM

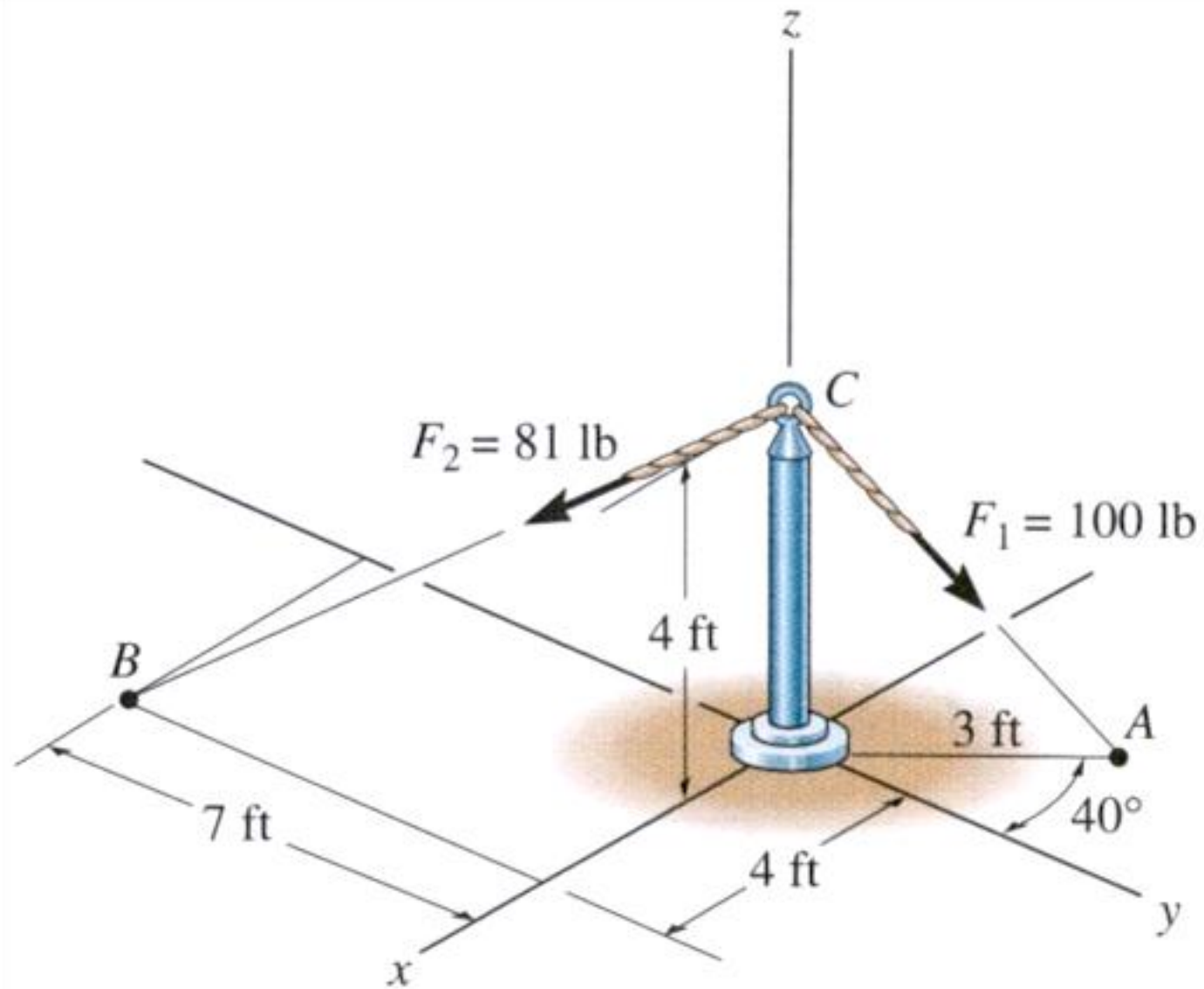
Given: Two forces are acting on a pipe as shown in the figure.

Find: The magnitude and the coordinate direction angles of the resultant force.



Plan:

- 1) Find the forces along CA and CB in the Cartesian vector form.
- 2) Add the two forces to get the resultant force, F_R .
- 3) Determine the magnitude and the coordinate angles of F_R .



PROBLEM SOLVING

$$\mathbf{F}_{CA} = 100 \text{ lb} \{ \mathbf{r}_{CA} / r_{CA} \}$$

$$\mathbf{F}_{CA} = 100 \text{ lb} (-3 \sin 40^\circ \mathbf{i} + 3 \cos 40^\circ \mathbf{j} - 4 \mathbf{k}) / 5$$

$$\mathbf{F}_{CA} = \{-38.57 \mathbf{i} + 45.96 \mathbf{j} - 80 \mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_{CB} = 81 \text{ lb} \{ \mathbf{r}_{CB} / r_{CB} \}$$

$$\mathbf{F}_{CB} = 81 \text{ lb} (4 \mathbf{i} - 7 \mathbf{j} - 4 \mathbf{k}) / 9$$

$$\mathbf{F}_{CB} = \{36 \mathbf{i} - 63 \mathbf{j} - 36 \mathbf{k}\} \text{ lb}$$

$$\mathbf{F}_R = \mathbf{F}_{CA} + \mathbf{F}_{CB} = \{-2.57 \mathbf{i} - 17.04 \mathbf{j} - 116 \mathbf{k}\} \text{ lb}$$

$$F_R = (2.57^2 + 17.04^2 + 116^2) = 117.3 \text{ lb} = 117 \text{ lb}$$

$$\alpha = \cos^{-1}(-2.57/117.3) = 91.3^\circ, \quad \beta = \cos^{-1}(-17.04/117.3) = 98.4^\circ$$

$$\gamma = \cos^{-1}(-116/117.3) = 172^\circ$$

