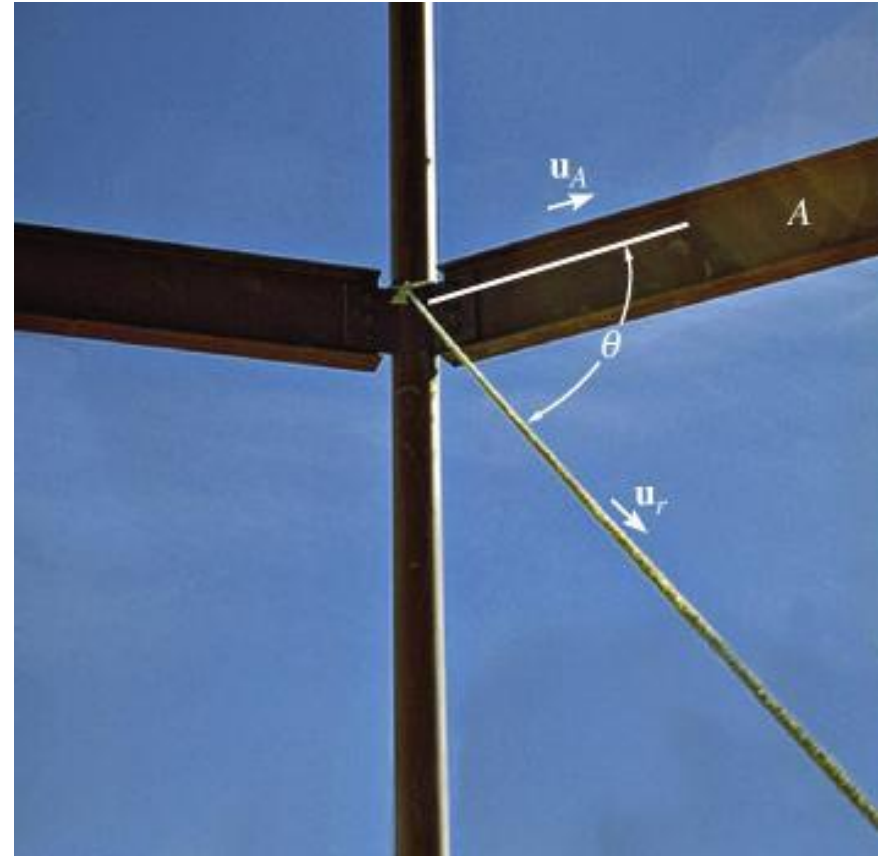


DOT PRODUCT

Today's Objective:

Students will be able to use the dot product to:

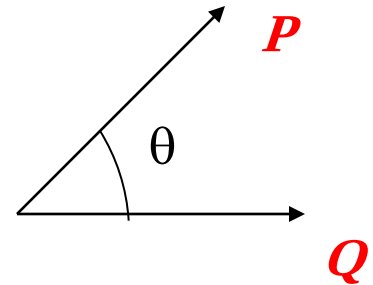
- a) determine an angle between two vectors, and,
- b) determine the projection of a vector along a specified line.



READING QUIZ

1. The dot product of two vectors $\mathbf{P}$ and $\mathbf{Q}$ is defined as:

- A) $P Q \cos \theta$ B) $P Q \sin \theta$
C) $P Q \tan \theta$ D) $P Q \sec \theta$



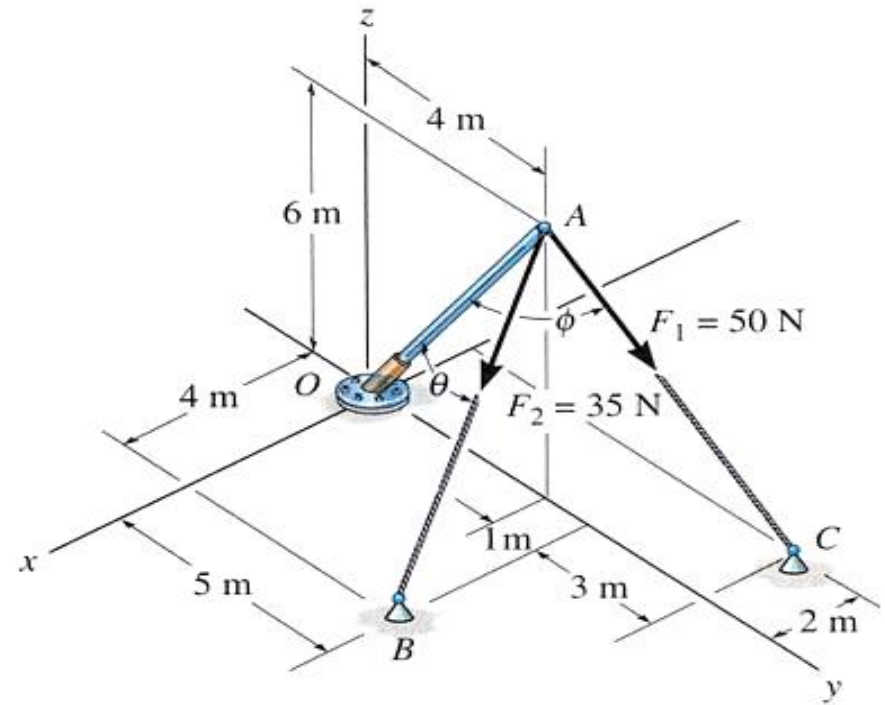
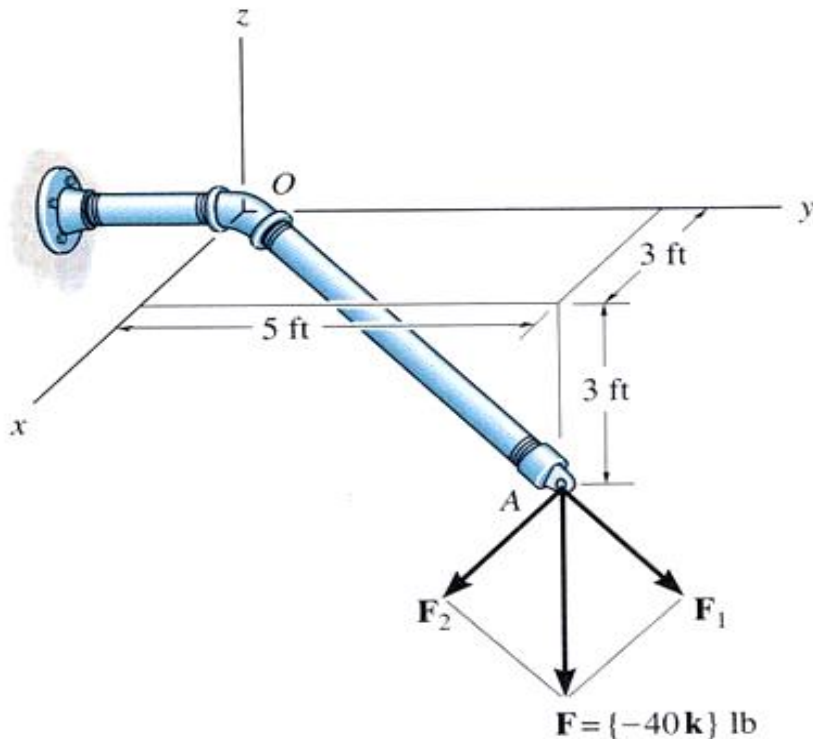
2. The dot product of two vectors results in a _____ quantity.

- A) scalar B) vector
C) complex D) zero



APPLICATIONS

For this geometry, can you determine angles between the pole and the cables?



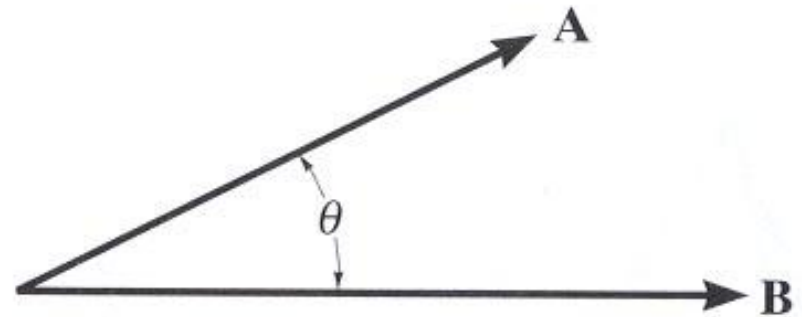
For force $\mathbf{F}$ at Point A, what component of it (F_1) acts along the pipe OA? What component (F_2) acts perpendicular to the pipe?

DEFINITION

The dot product of vectors $\mathbf{A}$ and $\mathbf{B}$ is defined as:

$$\mathbf{A} \cdot \mathbf{B} = A B \cos \theta.$$

Angle θ is the smallest angle between the two vectors and is always in a range of 0° to 180° .



Dot Product Characteristics:

1. The result of the dot product is a scalar (a positive or negative number).
2. The units of the dot product will be the product of the units of the $\mathbf{A}$ and $\mathbf{B}$ vectors.

DOT PRODUCT DEFINITION

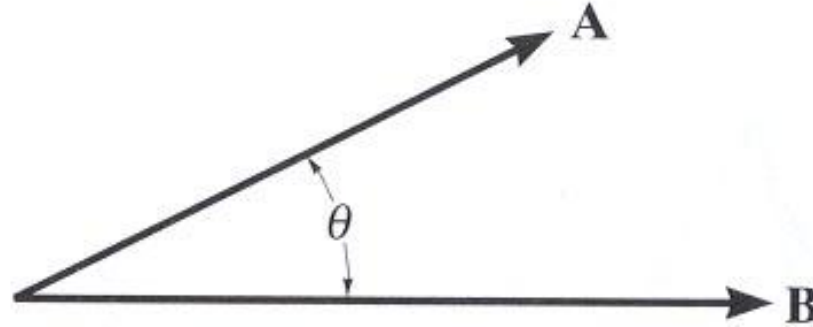
(continued)

Examples: $\mathbf{i} \bullet \mathbf{j} = 0$

$$\mathbf{i} \bullet \mathbf{i} = 1$$

$$\begin{aligned}\mathbf{A} \bullet \mathbf{B} &= (\mathbf{A}_x \mathbf{i} + \mathbf{A}_y \mathbf{j} + \mathbf{A}_z \mathbf{k}) \bullet (\mathbf{B}_x \mathbf{i} + \mathbf{B}_y \mathbf{j} + \mathbf{B}_z \mathbf{k}) \\ &= \mathbf{A}_x \mathbf{B}_x + \mathbf{A}_y \mathbf{B}_y + \mathbf{A}_z \mathbf{B}_z\end{aligned}$$

USING THE DOT PRODUCT TO DETERMINE THE ANGLE BETWEEN TWO VECTORS

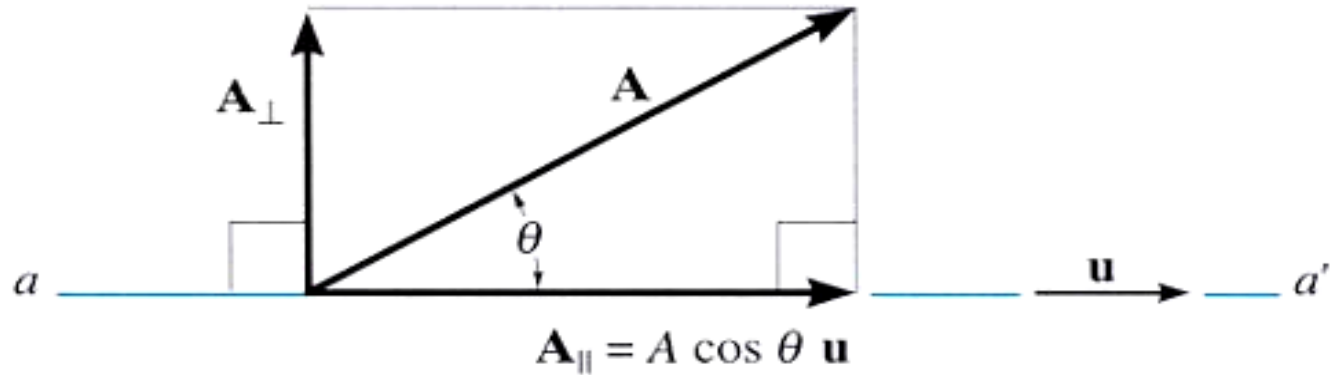


For the given two vectors in the Cartesian form, one can find the angle by

- a) Finding the dot product, $\mathbf{A} \cdot \mathbf{B} = (A_x B_x + A_y B_y + A_z B_z)$,
- b) Finding the magnitudes (A & B) of the vectors $\mathbf{A}$ & $\mathbf{B}$, and
- c) Using the definition of dot product and solving for θ , i.e.,

$$\theta = \cos^{-1} [(\mathbf{A} \cdot \mathbf{B}) / (A B)], \text{ where } 0^\circ \leq \theta \leq 180^\circ.$$

DETERMINING THE PROJECTION OF A VECTOR



You can determine the components of a vector parallel and perpendicular to a line using the dot product.

Steps:

1. Find the unit vector, $\mathbf{U}_{aa'}$ along line aa'
2. Find the scalar projection of $\mathbf{A}$ along line aa' by

$$A_{\parallel} = \mathbf{A} \cdot \mathbf{U} = A_x U_x + A_y U_y + A_z U_z$$

Dot Product Characteristics:

1. The result of the dot product is a scalar (a positive or negative number).
2. The units of the dot product will be the product of the units of the $\mathbf{A}$ and $\mathbf{B}$ vectors.

Laws of Operation

1. Commutative law:

$$\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$$

2. Multiplication by a scalar:

$$a(\mathbf{A} \cdot \mathbf{B}) = (a\mathbf{A}) \cdot \mathbf{B} = \mathbf{A} \cdot (a\mathbf{B}) = (\mathbf{A} \cdot \mathbf{B})a$$

3. Distributive law:

$$\mathbf{A} \cdot (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \cdot \mathbf{B}) + (\mathbf{A} \cdot \mathbf{D})$$

DETERMINING THE PROJECTION OF A VECTOR (continued)

3. If needed, the projection can be written as a vector, $\mathbf{A}_{//}$, by using the unit vector $\mathbf{U}_{aa'}$ and the magnitude found in step 2.

$$\mathbf{A}_{//} = A_{||} \mathbf{U}_{aa'}$$

4. The scalar and vector forms of the perpendicular component can easily be obtained by

$$A_{\perp} = (A^2 - A_{||}^2)^{1/2} \text{ and}$$

$$\mathbf{A}_{\perp} = \mathbf{A} - \mathbf{A}_{//}$$

(rearranging the vector sum of $\mathbf{A} = \mathbf{A}_{\perp} + \mathbf{A}_{//}$)

EXAMPLE

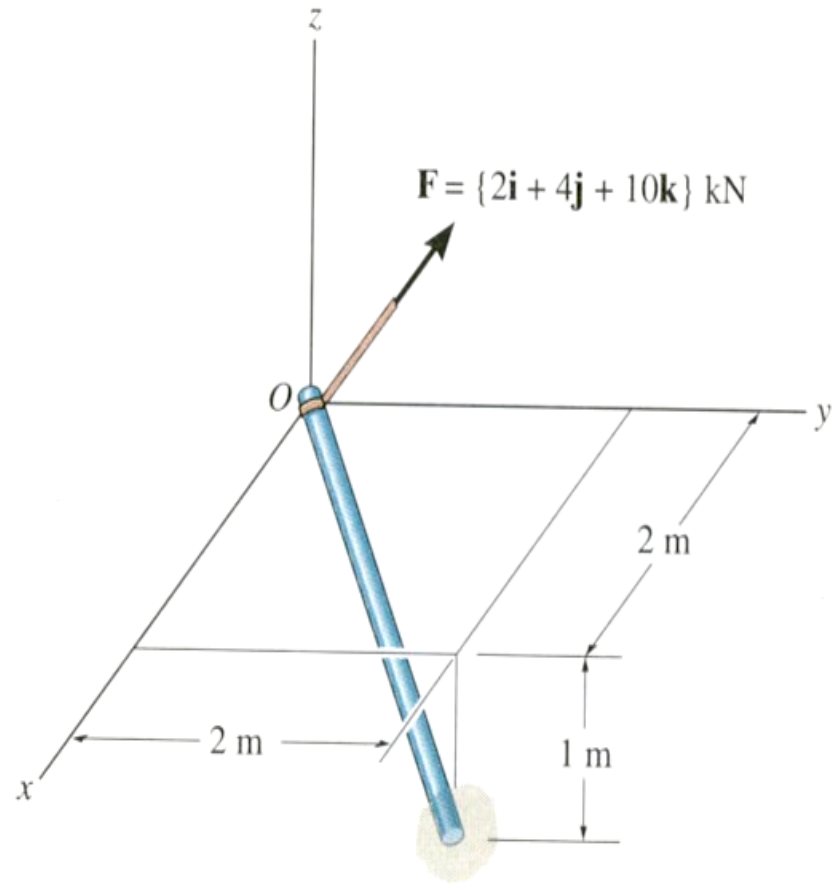
Given: Force $\mathbf{F}$ acting on a pole

Find: The angle between the force vector and the pole, and the magnitude of the projection of the force along the pole OA.

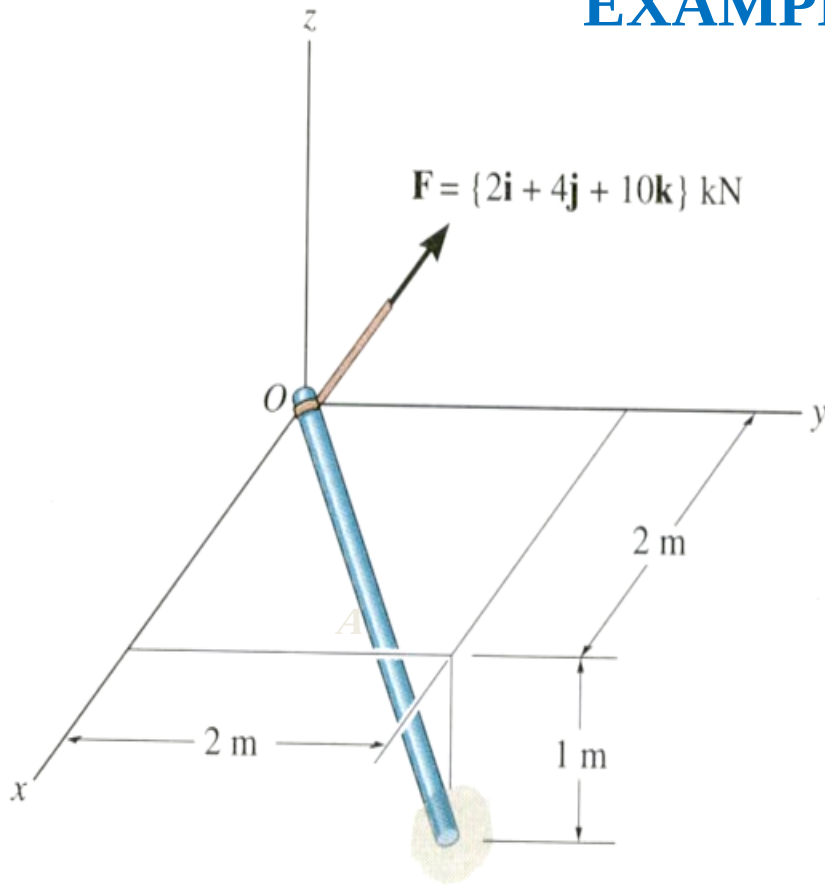
A

Plan:

1. Get $\mathbf{r}_{OA}$
2. $\theta = \cos^{-1}\{(\mathbf{F} \cdot \mathbf{r}_{OA})/(\|\mathbf{F}\| \|\mathbf{r}_{OA}\|)\}$
3. $F_{OA} = \mathbf{F} \cdot \mathbf{u}_{OA}$ or $F \cos \theta$



EXAMPLE (continued)



$$\mathbf{r}_{OA} = \{2 \mathbf{i} + 2 \mathbf{j} - 1 \mathbf{k}\} \text{ m}$$

$$r_{OA} = (2^2 + 2^2 + 1^2)^{1/2} = 3 \text{ m}$$

$$\mathbf{F} = \{2 \mathbf{i} + 4 \mathbf{j} + 10 \mathbf{k}\} \text{ kN}$$

$$F = (2^2 + 4^2 + 10^2)^{1/2} = 10.95 \text{ kN}$$

$$\mathbf{F} \cdot \mathbf{r}_{OA} = (2)(2) + (4)(2) + (10)(-1) = 2 \text{ kN} \cdot \text{m}$$

$$\theta = \cos^{-1}\{(\mathbf{F} \cdot \mathbf{r}_{OA})/(F r_{OA})\}$$

$$\theta = \cos^{-1}\{2/(10.95 * 3)\} = 86.5^\circ$$

$$\mathbf{u}_{OA} = \mathbf{r}_{OA}/r_{OA} = \{(2/3) \mathbf{i} + (2/3) \mathbf{j} - (1/3) \mathbf{k}\}$$

$$F_{OA} = \mathbf{F} \cdot \mathbf{u}_{OA} = (2)(2/3) + (4)(2/3) + (10)(-1/3) = 0.667 \text{ kN}$$

$$\text{Or } F_{OA} = F \cos \theta = 10.95 \cos(86.51^\circ) = 0.667 \text{ kN}$$

CONCEPT QUIZ

1. If a dot product of two non-zero vectors is 0, then the two vectors must be _____ to each other.
 - A) parallel (pointing in the same direction)
 - B) parallel (pointing in the opposite direction)
 - C) perpendicular
 - D) cannot be determined.

2. If a dot product of two non-zero vectors equals -1, then the vectors must be _____ to each other.
 - A) parallel (pointing in the same direction)
 - B) parallel (pointing in the opposite direction)
 - C) perpendicular
 - D) cannot be determined.

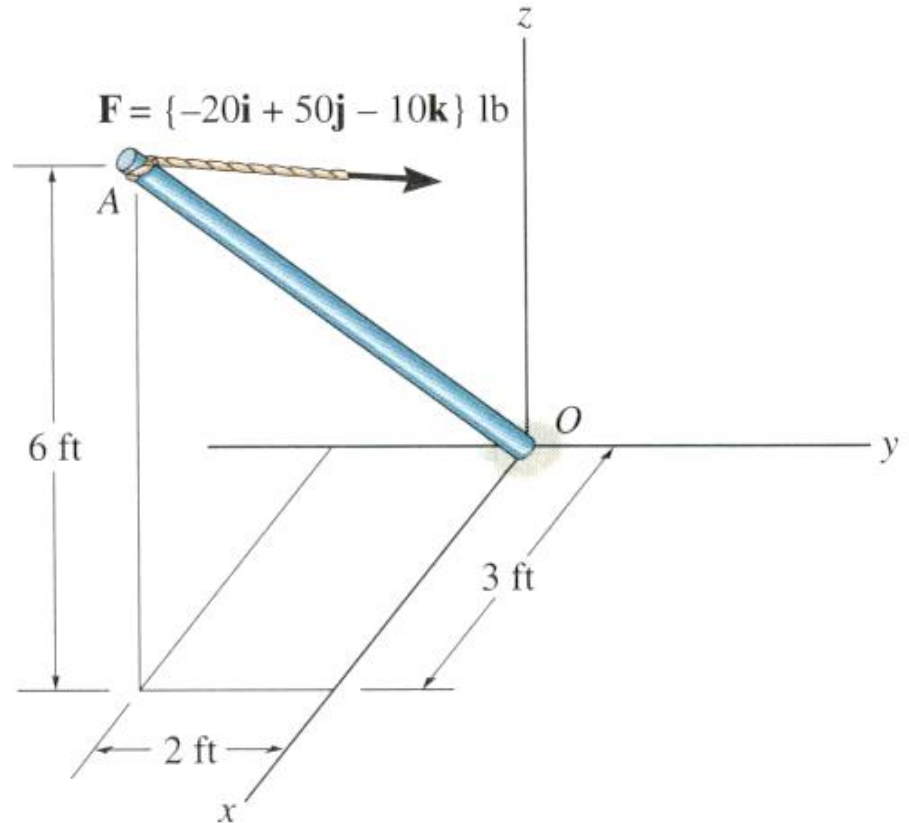
GROUP PROBLEM SOLVING

Given: The force acting on the pole.

Find: The angle between the force vector and the pole, and the magnitude of the projection of the force along the pole AO.

Plan:

1. Get $\mathbf{r}_{AO}$
2. $\theta = \cos^{-1}\{(\mathbf{F} \cdot \mathbf{r}_{AO})/(\|\mathbf{F}\| \|\mathbf{r}_{AO}\|)\}$
3. $F_{OA} = \mathbf{F} \cdot \mathbf{u}_{AO}$ or $F \cos \theta$



GROUP PROBLEM SOLVING (continued)

$$\mathbf{r}_{AO} = \{-3\mathbf{i} + 2\mathbf{j} - 6\mathbf{k}\} \text{ ft.}$$

$$r_{AO} = (3^2 + 2^2 + 6^2)^{1/2} = 7 \text{ ft.}$$

$$\mathbf{F} = \{-20\mathbf{i} + 50\mathbf{j} - 10\mathbf{k}\} \text{ lb}$$

$$F = (20^2 + 50^2 + 10^2)^{1/2} = 54.77 \text{ lb}$$

$$\begin{aligned}\mathbf{F} \cdot \mathbf{r}_{AO} &= (-20)(-3) + (50)(2) + (-10)(-6) \\ &= 220 \text{ lb} \cdot \text{ft}\end{aligned}$$

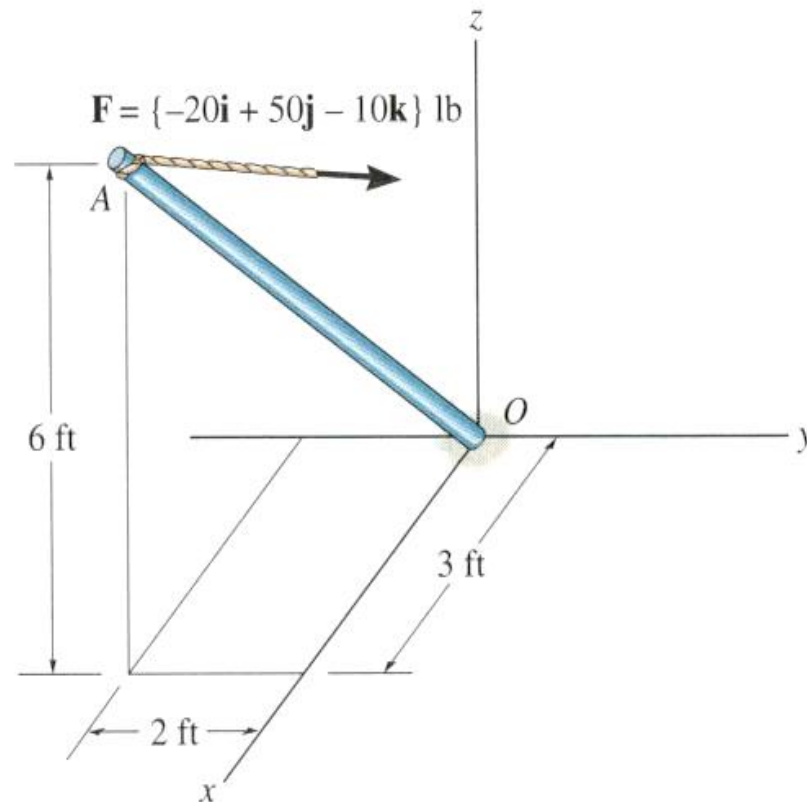
$$\theta = \cos^{-1}\{(\mathbf{F} \cdot \mathbf{r}_{AO})/(F r_{AO})\}$$

$$\theta = \cos^{-1}\{220/(54.77 \times 7)\} = 55.0^\circ$$

$$\mathbf{u}_{AO} = \mathbf{r}_{AO}/r_{AO} = \{(-3/7)\mathbf{i} + (2/7)\mathbf{j} - (6/7)\mathbf{k}\}$$

$$F_{AO} = \mathbf{F} \cdot \mathbf{u}_{AO} = (-20)(-3/7) + (50)(2/7) + (-10)(-6/7) = 31.4 \text{ lb}$$

$$\text{Or } F_{AO} = F \cos \theta = 54.77 \cos(55.0^\circ) = 31.4 \text{ lb}$$



ATTENTION QUIZ

1. The Dot product can be used to find all of the following except ____ .

- A) sum of two vectors
- B) angle between two vectors
- C) component of a vector parallel to another line
- D) component of a vector perpendicular to another line

2. Find the dot product of the two vectors $\mathbf{P}$ and $\mathbf{Q}$.

$$\mathbf{P} = \{5 \mathbf{i} + 2 \mathbf{j} + 3 \mathbf{k}\} \text{ m}$$

$$\mathbf{Q} = \{-2 \mathbf{i} + 5 \mathbf{j} + 4 \mathbf{k}\} \text{ m}$$

- A) -12 m
- B) 12 m
- C) 12 m²
- D) -12 m²
- E) 10 m²