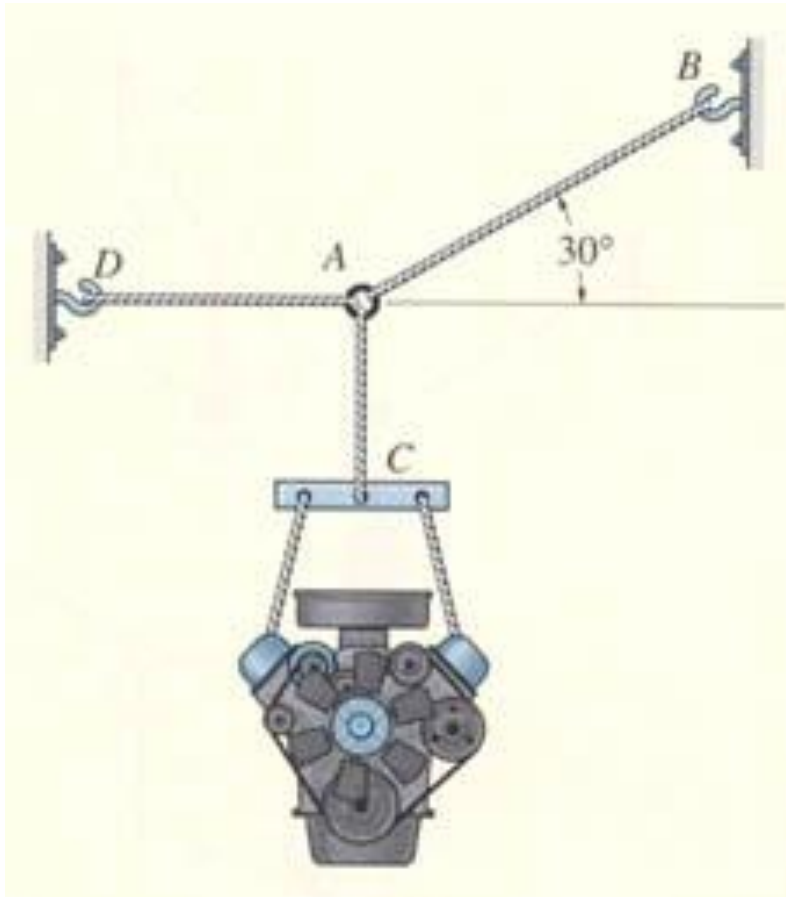


### 3.3 COPLANAR FORCE SYSTEMS



This is an example of a 2-D or coplanar force system. If the whole assembly is in equilibrium, then particle A is also in equilibrium.

To determine the tensions in the cables for a given weight of the engine, we need to learn how to draw a free body diagram and apply equations of equilibrium.

### 3.3 COPLANAR FORCE SYSTEMS

If a particle is subjected to a system of coplanar forces that lie in the  $x$ - $y$  plane, then each force can be resolved into its  $i$  and  $j$  components. For equilibrium,

$$\Sigma \mathbf{F} = \mathbf{0}$$

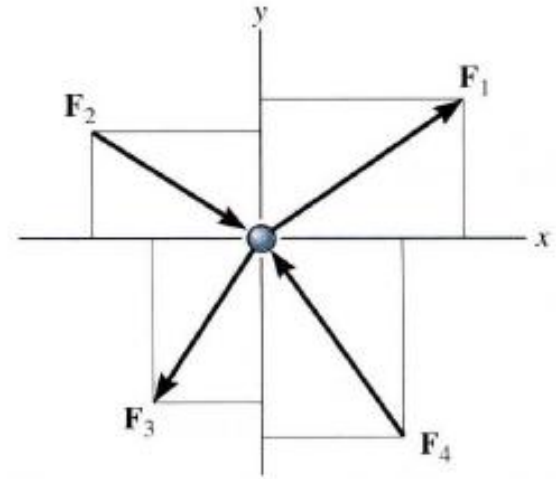
$$\Sigma F_x \mathbf{i} + \Sigma F_y \mathbf{j} = \mathbf{0}$$

For this vector equation to be satisfied, both the  $x$  and  $y$  components must be equal to zero.

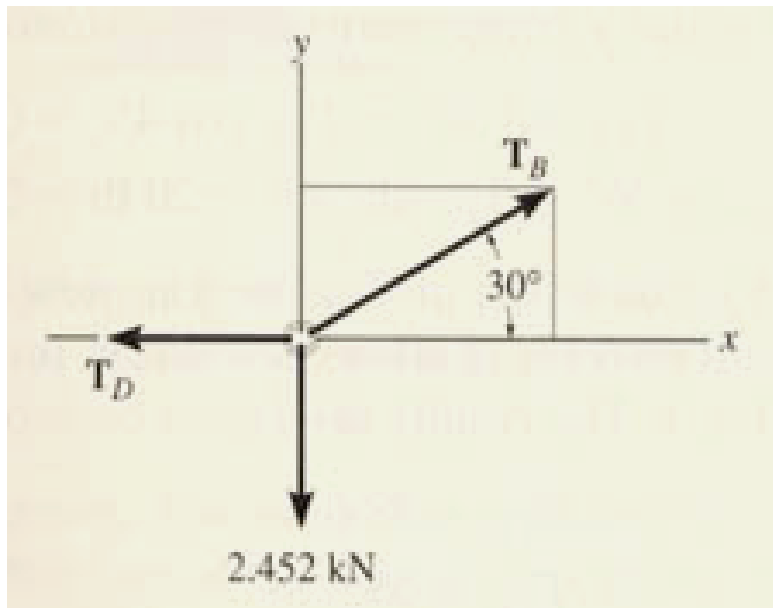
Hence,

$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$



## EQUATIONS OF 2-D EQUILIBRIUM



Since particle A is in equilibrium, the net force at A is zero.

$$\text{So } \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} = 0$$

$$\text{or } \Sigma \mathbf{F} = 0$$

In general, for a particle in equilibrium,  $\Sigma \mathbf{F} = 0$  or

$$\Sigma F_x \mathbf{i} + \Sigma F_y \mathbf{j} = 0 = 0 \mathbf{i} + 0 \mathbf{j} \quad (\text{A vector equation})$$

Or, written in a scalar form,

$$\Sigma F_x = 0 \quad \text{and} \quad \Sigma F_y = 0$$

These are two scalar equations of equilibrium (E of E).

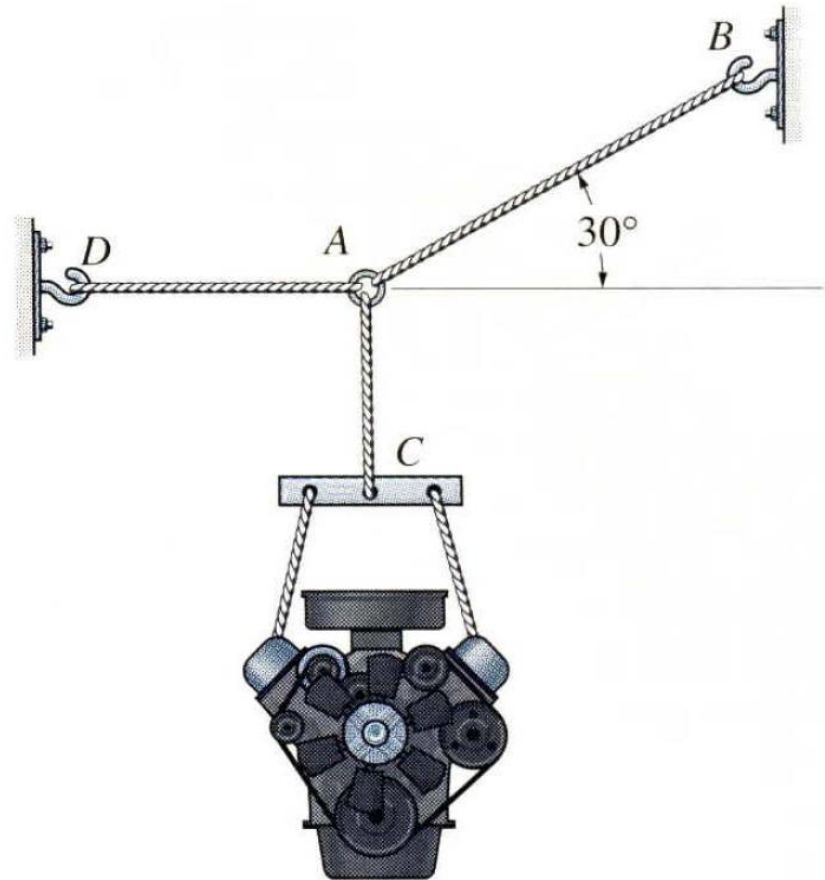
They can be used to solve for up to two unknowns.

## EXAMPLE 3.1

Determine the tension in cables AB and AC for the equilibrium of the 250-kg engine shown in fig.

Plan:

- Draw FBD for ring A
- Write E of E
- Solve for unknown forces



$$\begin{aligned} \text{weight } (250 \text{ kg})(9.81 \text{ m/s}^2) \\ = 2.452 \text{ kN} \end{aligned}$$

### EXAMPLE 3.1 *cont.*

Write the scalar E of E:

$$+ \rightarrow \Sigma F_x = T_B \cos 30^\circ - T_D = 0$$

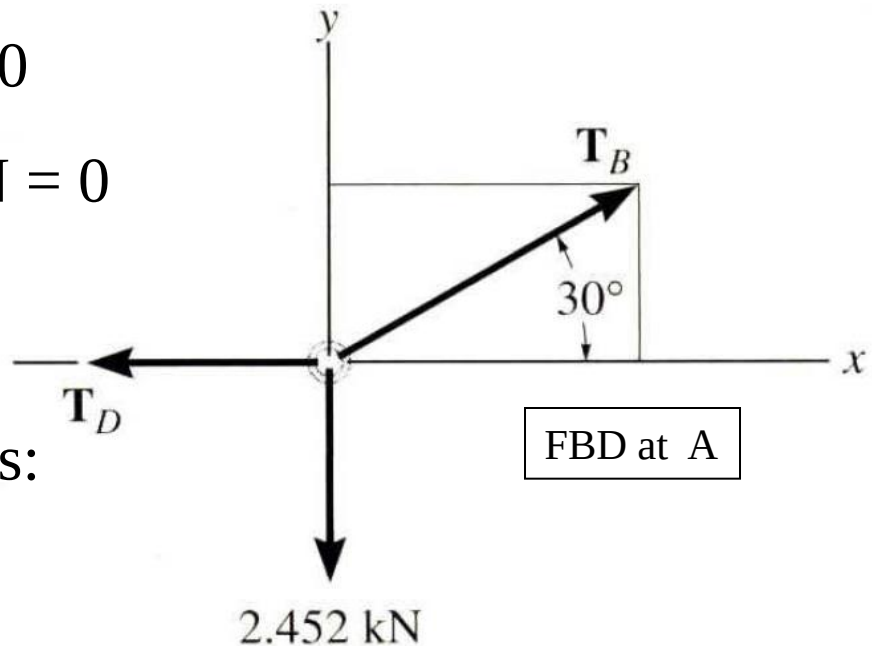
$$+ \uparrow \Sigma F_y = T_B \sin 30^\circ - 2.452 \text{ kN} = 0$$

Solving the second equation gives:

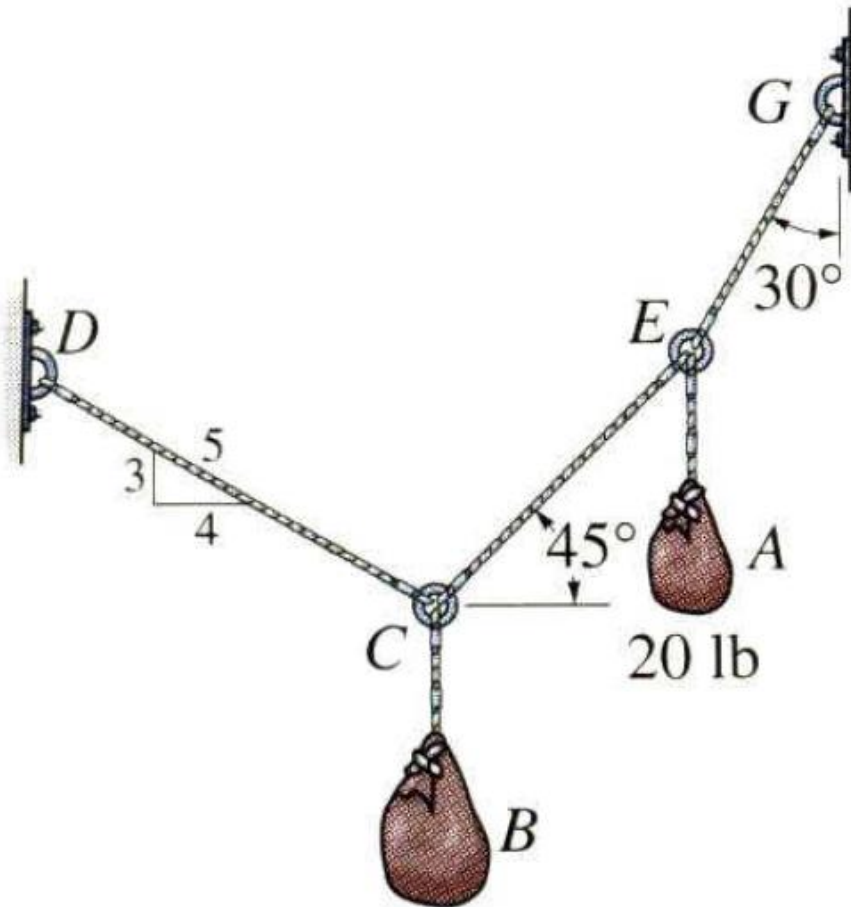
$$T_B = 4.90 \text{ kN}$$

From the first equation, we get:

$$T_D = 4.25 \text{ kN}$$



## EXAMPLE 3.2



**Given:** Sack A weighs 20 lb. and geometry is as shown.

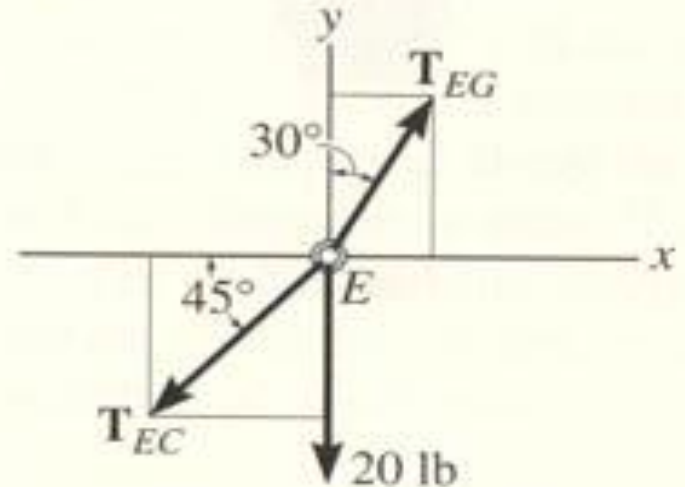
**Find:** Forces in the cables and weight of sack B.

**Plan:**

1. Draw a FBD for Point E.
2. Apply EofE at Point E to solve for the unknowns ( $T_{EG}$  &  $T_{EC}$ ).
3. Repeat this process at C.

### EXAMPLE 3.2 (continued)

A FBD at E should look like the one to the left. Note the assumed directions for the two cable tensions.



The scalar E-o-f-E are:

$$+ \rightarrow \quad \Sigma F_x = T_{EG} \sin 30^\circ - T_{EC} \cos 45^\circ = 0$$

$$+ \uparrow \quad \Sigma F_y = T_{EG} \cos 30^\circ - T_{EC} \sin 45^\circ - 20 \text{ lbs} = 0$$

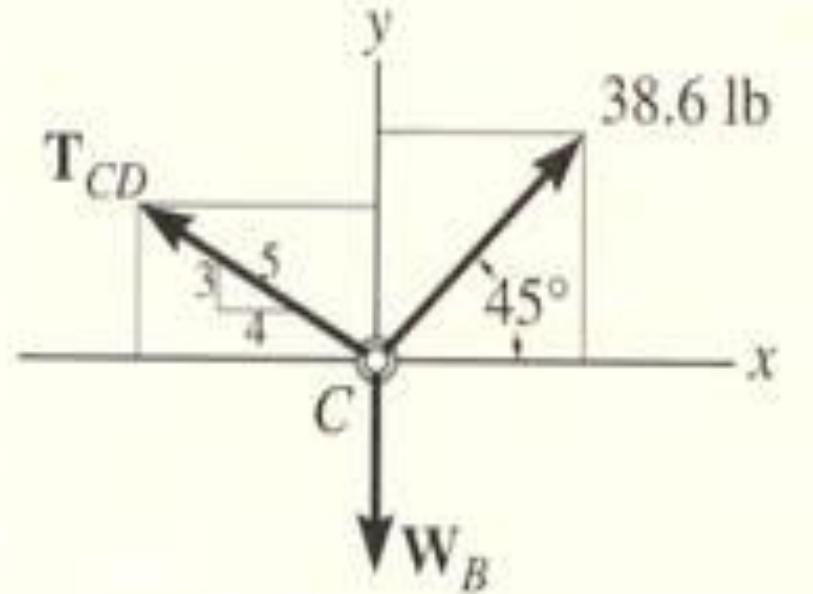
Solving these two simultaneous equations for the two unknowns yields:

$$T_{EC} = 38.6 \text{ lb}$$

$$T_{EG} = 54.6 \text{ lb}$$

### EXAMPLE 3.2 (continued)

Now move on to ring C.  
A FBD for C should look like the one to the left.



The scalar E-of-E are:

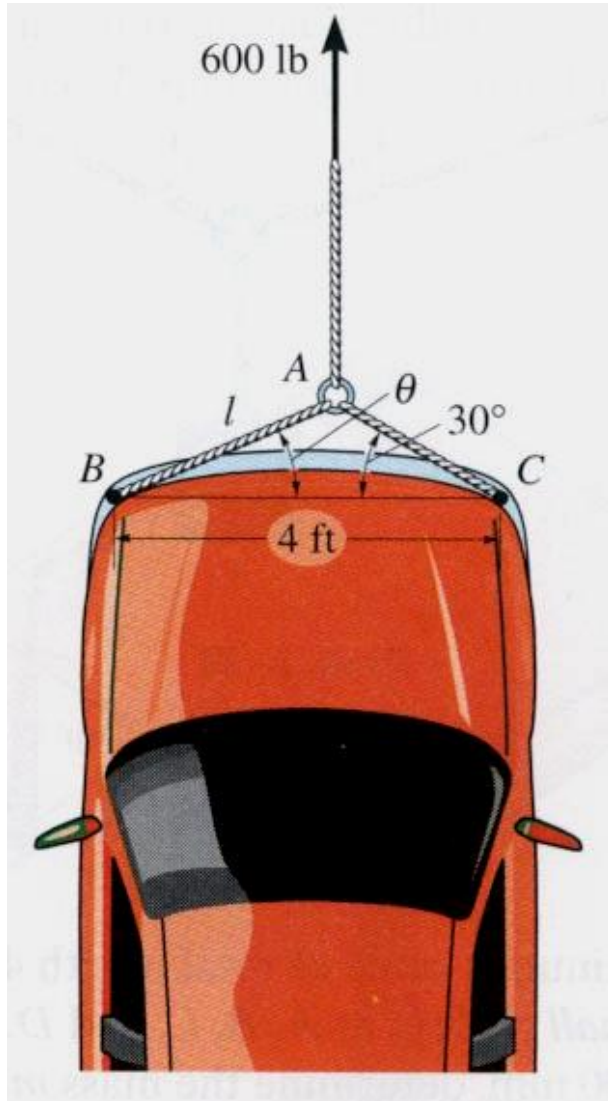
$$+ \rightarrow \Sigma F_x = 38.64 \cos 45^\circ - (4/5) T_{CD} = 0$$

$$+ \uparrow \Sigma F_y = (3/5) T_{CD} + 38.64 \sin 45^\circ - W_B = 0$$

Solving the first equation and then the second yields

$$T_{CD} = 34.2 \text{ lb} \quad \text{and} \quad W_B = 47.8 \text{ lb} .$$

## PROBLEM SOLVING



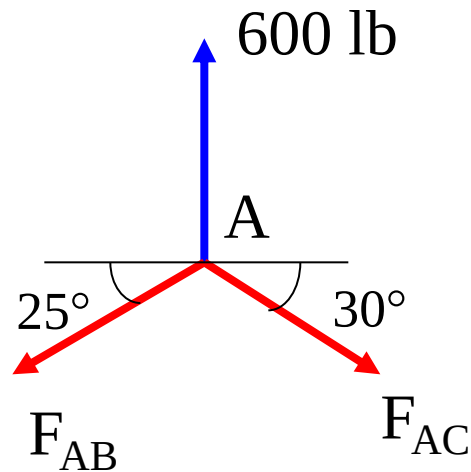
**Given:** The car is towed at constant speed by the 600 lb force and the angle  $\theta$  is  $25^\circ$ .

**Find:** The forces in the ropes AB and AC.

### Plan:

1. Draw a FBD for point A.
2. Apply the E-of-E to solve for the forces in ropes AB and AC.

## PROBLEM SOLVING (continued)



FBD at point A

Applying the scalar E-of-E at A, we get;

$$+ \rightarrow \sum F_x = F_{AC} \cos 30^\circ - F_{AB} \cos 25^\circ = 0$$

$$+ \rightarrow \sum F_y = -F_{AC} \sin 30^\circ - F_{AB} \sin 25^\circ + 600 = 0$$

Solving the above equations, we get;

$$F_{AB} = 634 \text{ lb}$$

$$F_{AC} = 664 \text{ lb}$$