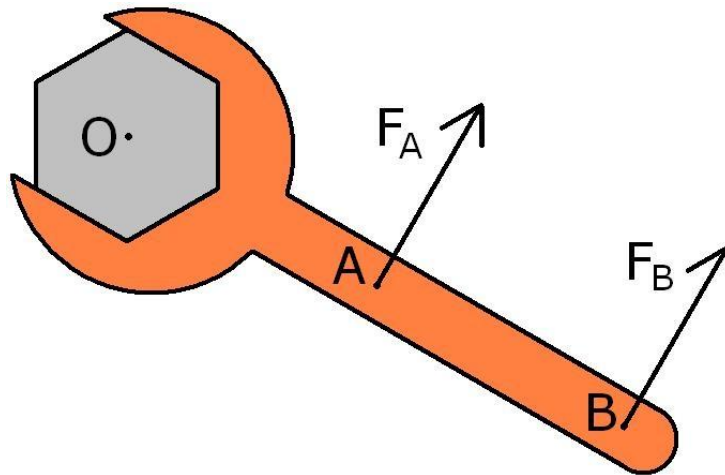


Chapter 4

Force System Resultant

Moment of a Force



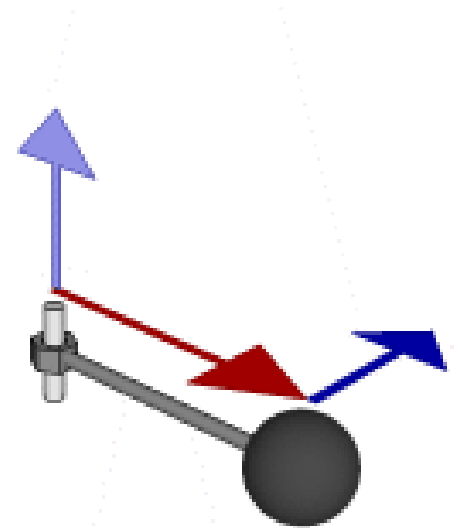
MOMENT OF A FORCE SCALAR FORMULATION, CROSS PRODUCT, MOMENT OF A FORCE VECTOR FORMULATION, & PRINCIPLE OF MOMENTS

Today's Objectives :

Students will be able to:

- a) understand and define moment, and,
- b) determine moments of a force in 2-D and 3-D cases.

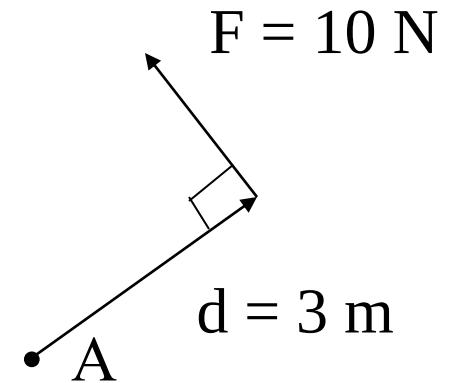
$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$
$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$



READING QUIZ

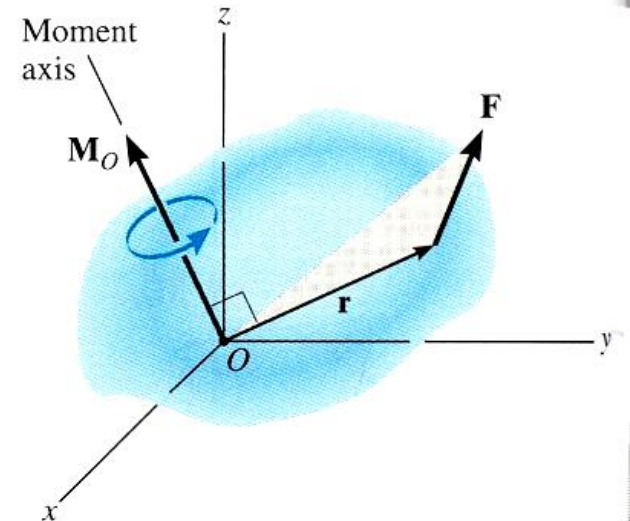
1. What is the moment of the 10 N force about point A (M_A)?

- A) 10 N·m B) 30 N·m C) 13 N·m
D) (10/3) N·m E) 7 N·m

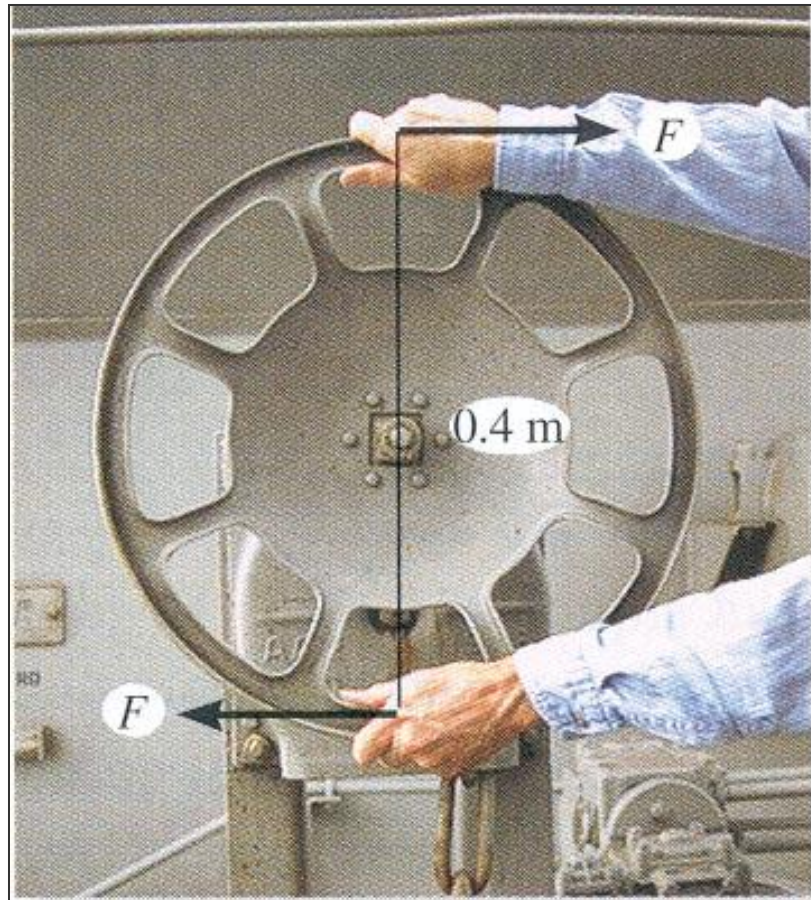


2. Moment of force $\mathbf{F}$ about point O is defined as $\mathbf{M}_O =$ _____ .

- A) $\mathbf{r} \times \mathbf{F}$ B) $\mathbf{F} \times \mathbf{r}$
C) $\mathbf{r} \cdot \mathbf{F}$ D) $\mathbf{r} * \mathbf{F}$



APPLICATIONS

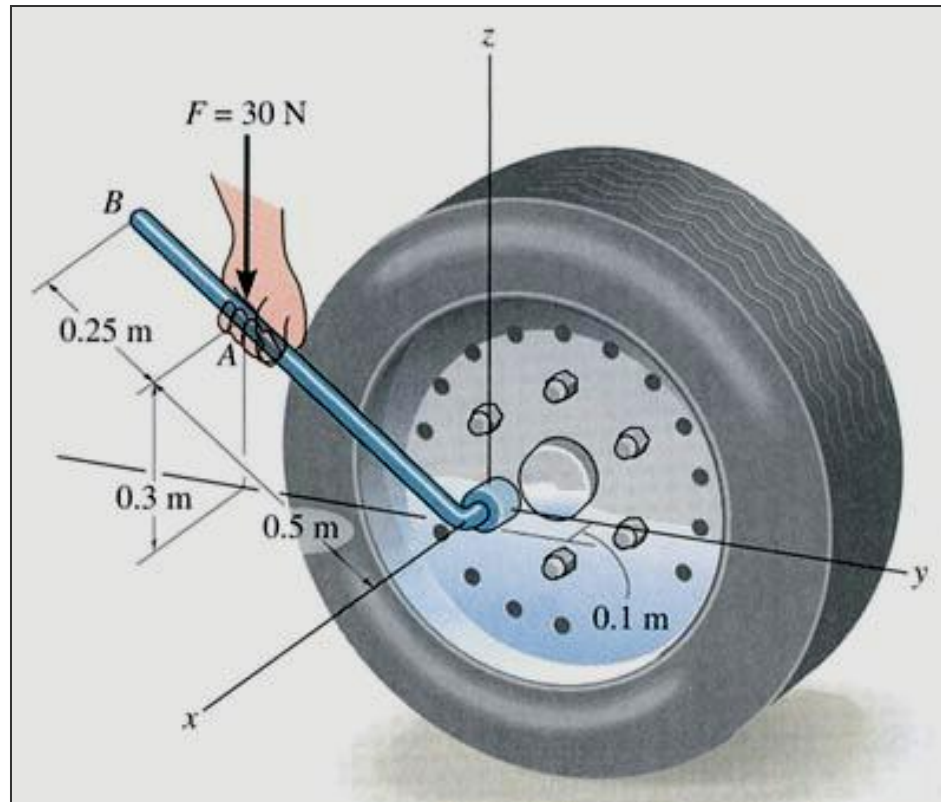


What is the net effect of the two forces on the wheel?



APPLICATIONS

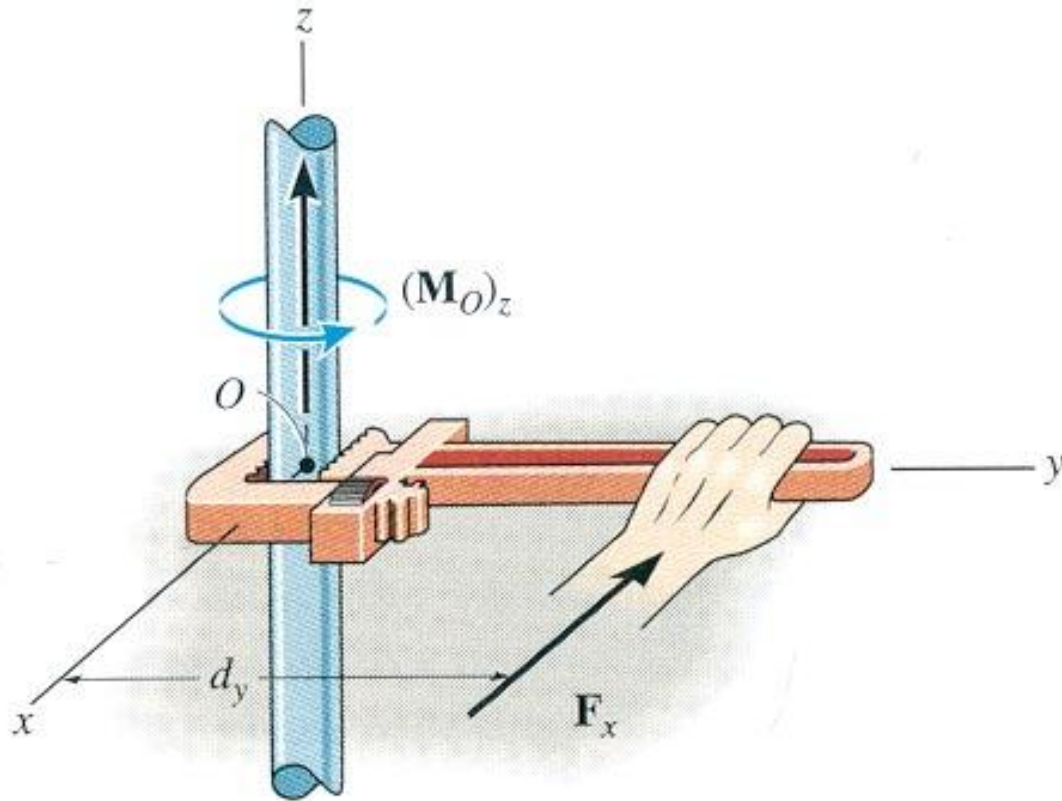
(continued)



What is the effect of the 30 N force on the lug nut?



4.1 MOMENT OF A FORCE-SCALAR FORMULATION,

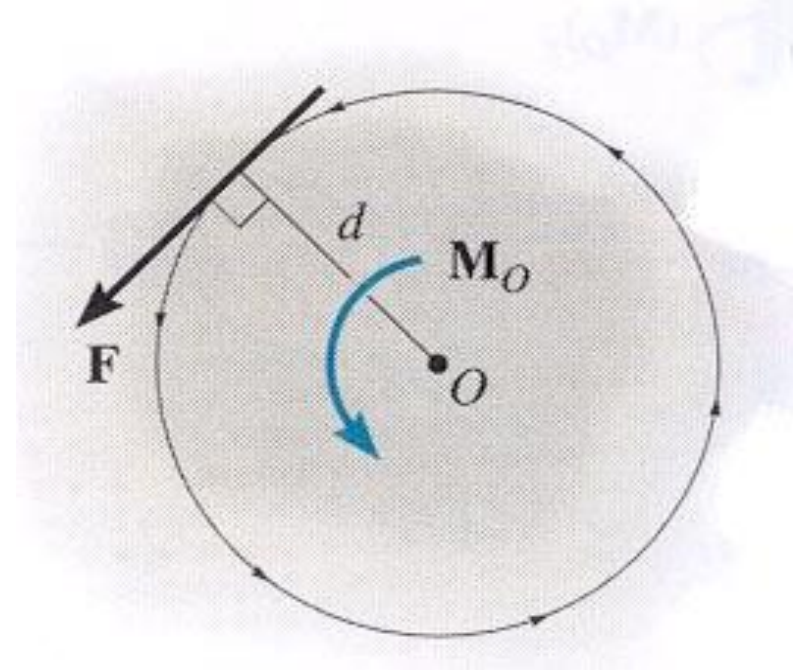


The moment of a force about a point provides a measure of the tendency for rotation (sometimes called a torque).

MOMENT OF A FORCE - SCALAR FORMULATION

In the 2-D case,
the magnitude of the
moment is :

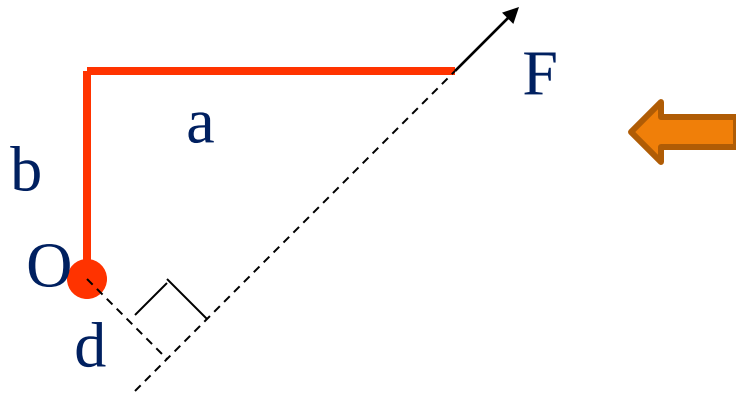
$$M_o = F d$$



As shown, d is the perpendicular distance from point O to the line of action of the force.

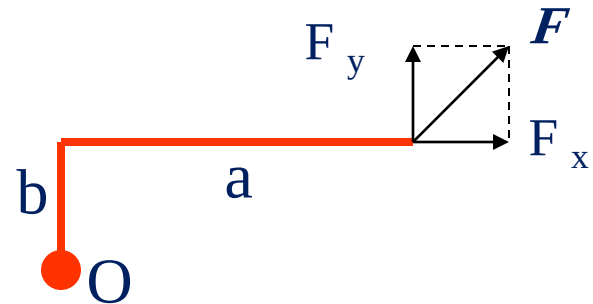
In 2-D, the direction of M_o is either clockwise or counter-clockwise depending on the tendency for rotation.

MOMENT OF A FORCE - SCALAR FORMULATION



For example, $M_O = F d$ and the direction is counter-clockwise.

Often it is easier to determine M_O by using the components of F as shown.



Using this approach, $M_O = (F_Y a) - (F_X b)$.

Note the different signs on the terms!

The typical sign convention for a moment in 2-D is that counter-clockwise is considered positive.

We can determine the direction of rotation by imagining the body pinned at **O** and deciding which way the body would rotate because of the force.

4.2 CROSS PRODUCT

Moments in 3-D can be calculated using scalar (2-D) approach but it can be difficult and time consuming. Thus, it is often easier to use a mathematical approach called the vector cross product.

The moment of a force will be formulated using Cartesian vectors in the next section. Before doing this, however, it is first necessary to expand our knowledge of vector algebra and introduce the cross-product method of vector multiplication.

4.2 CROSS PRODUCT

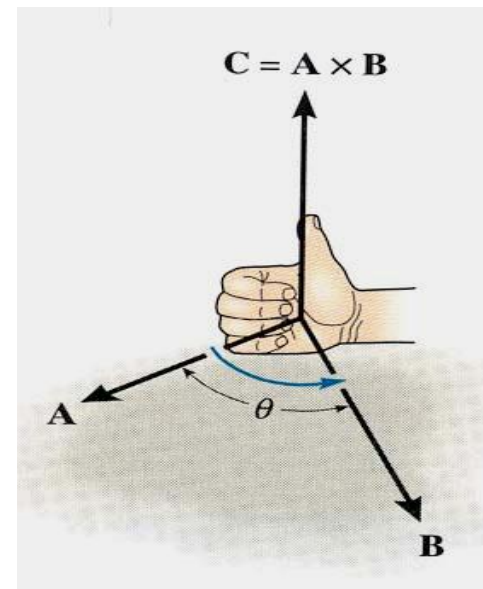
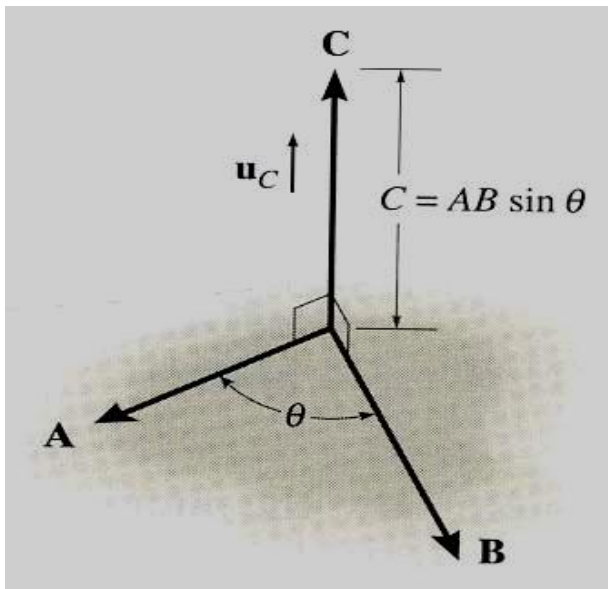
The cross product of two vectors **A** & **B** results in another vector **C**,

$$\text{i.e., } \mathbf{C} = \mathbf{A} \times \mathbf{B}.$$

The magnitude and direction of the resulting vector can be written as

$$\mathbf{C} = \mathbf{A} \times \mathbf{B} = AB \sin \theta \mathbf{U}_C$$

Here $\mathbf{U}_C$ is the unit vector perpendicular to both A and B vectors as shown (or to the plane containing the A and B vectors).

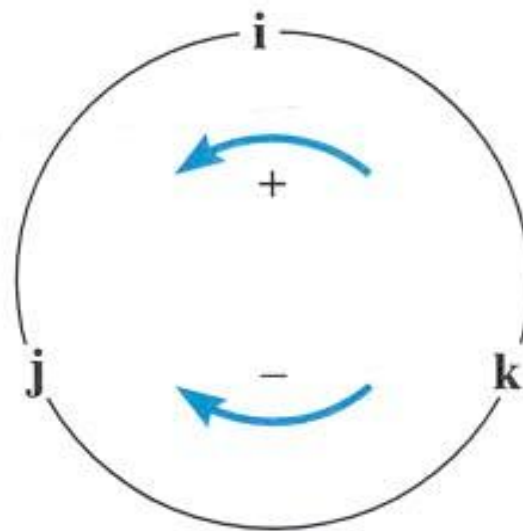
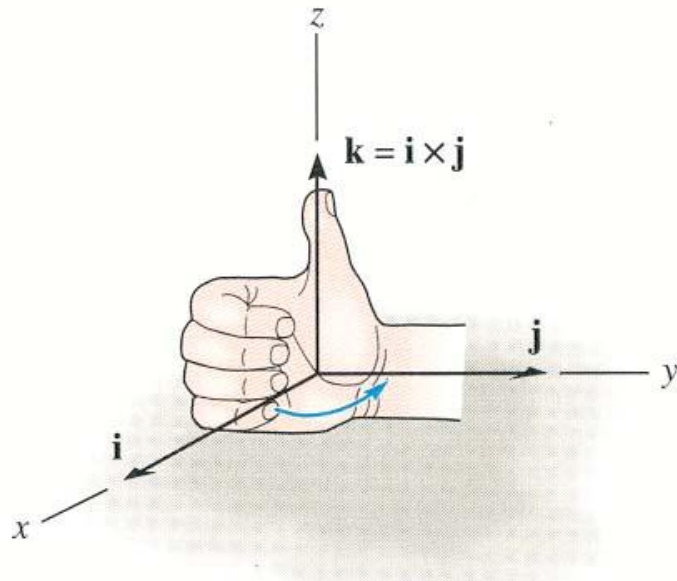


CROSS PRODUCT

The right hand rule is a useful tool for determining the direction of the vector resulting from a cross product.

For example: $\mathbf{i} \times \mathbf{j} = \mathbf{k}$

Note that a vector crossed into itself is zero, e.g., $\mathbf{i} \times \mathbf{i} = \mathbf{0}$



CROSS PRODUCT

Laws of Operation

1. The commutative law is *not* valid; i.e.,

$$\mathbf{A} \times \mathbf{B} \neq \mathbf{B} \times \mathbf{A}$$

Rather,

$$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$$

This is shown in Fig. 4–9 by using the right-hand rule. The cross product $\mathbf{B} \times \mathbf{A}$ yields a vector that acts in the opposite direction to $\mathbf{C}$; i.e., $\mathbf{B} \times \mathbf{A} = -\mathbf{C}$.

2. Multiplication by a scalar:

$$a(\mathbf{A} \times \mathbf{B}) = (a\mathbf{A}) \times \mathbf{B} = \mathbf{A} \times (a\mathbf{B}) = (\mathbf{A} \times \mathbf{B})a$$

This property is easily shown since the magnitude of the resultant vector ($|a|AB \sin \theta$) and its direction are the same in each case.

3. The distributive law:

$$\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{D})$$

The proof of this identity is left as an exercise (see Prob. 4–1). It is important to note that *proper order* of the cross products must be maintained, since they are not commutative.

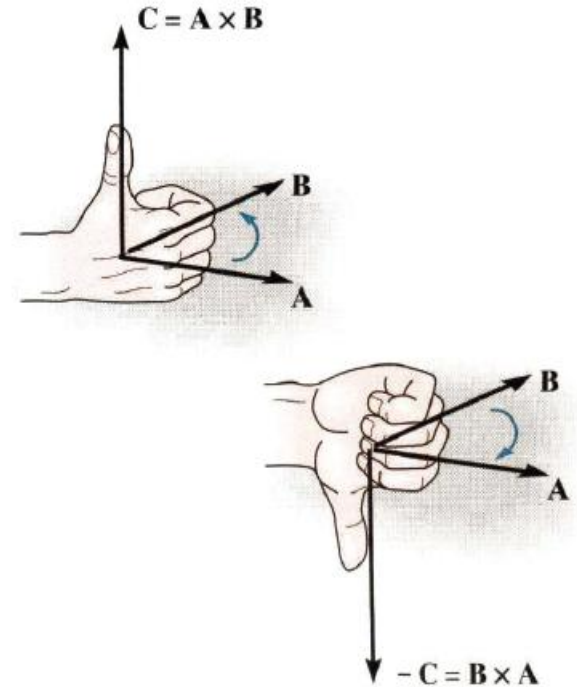


Fig. 4–9

z

CROSS PRODUCT

Consider now the cross product of two general vectors **A** and **B** which are expressed in Cartesian vector form. We have

$$\begin{aligned}\mathbf{A} \times \mathbf{B} &= (A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k}) \times (B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k}) \\ &= A_x B_x (\mathbf{i} \times \mathbf{i}) + A_x B_y (\mathbf{i} \times \mathbf{j}) + A_x B_z (\mathbf{i} \times \mathbf{k}) \\ &\quad + A_y B_x (\mathbf{j} \times \mathbf{i}) + A_y B_y (\mathbf{j} \times \mathbf{j}) + A_y B_z (\mathbf{j} \times \mathbf{k}) \\ &\quad + A_z B_x (\mathbf{k} \times \mathbf{i}) + A_z B_y (\mathbf{k} \times \mathbf{j}) + A_z B_z (\mathbf{k} \times \mathbf{k})\end{aligned}$$

Carrying out the cross-product operations and combining terms yields

$$\mathbf{A} \times \mathbf{B} = (A_y B_z - A_z B_y) \mathbf{i} - (A_x B_z - A_z B_x) \mathbf{j} + (A_x B_y - A_y B_x) \mathbf{k} \quad (4-4)$$

This equation may also be written in a more compact determinant form as

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

CROSS PRODUCT

Each component can be determined using 2×2 determinants.

For element **i**:

$$\begin{vmatrix} \textcircled{\mathbf{i}} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \mathbf{i}(A_y B_z - A_z B_y)$$

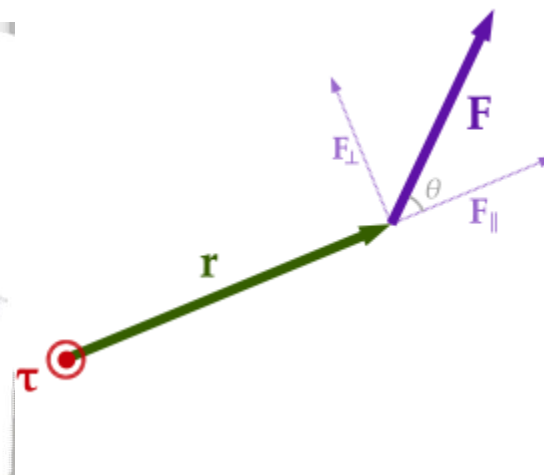
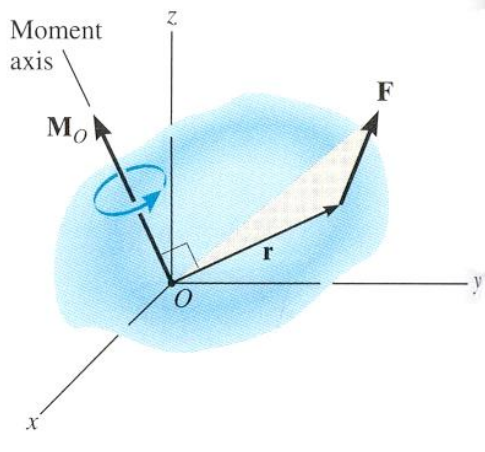
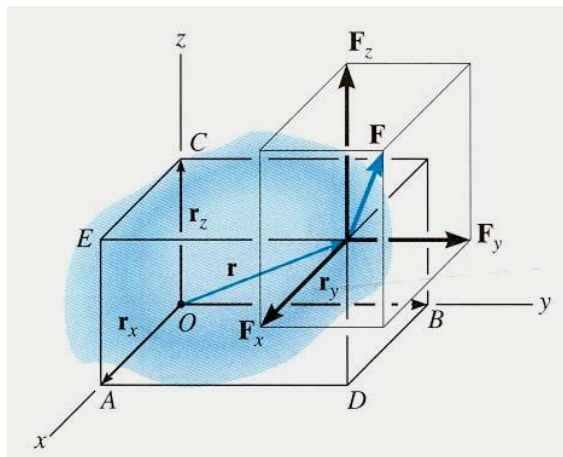
For element **j**:

$$\begin{vmatrix} \mathbf{i} & \textcircled{\mathbf{j}} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = -\mathbf{j}(A_x B_z - A_z B_x)$$

For element **k**:

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \textcircled{\mathbf{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \mathbf{k}(A_x B_y - A_y B_x)$$

4.3 MOMENT OF A FORCE – VECTOR FORMULATION

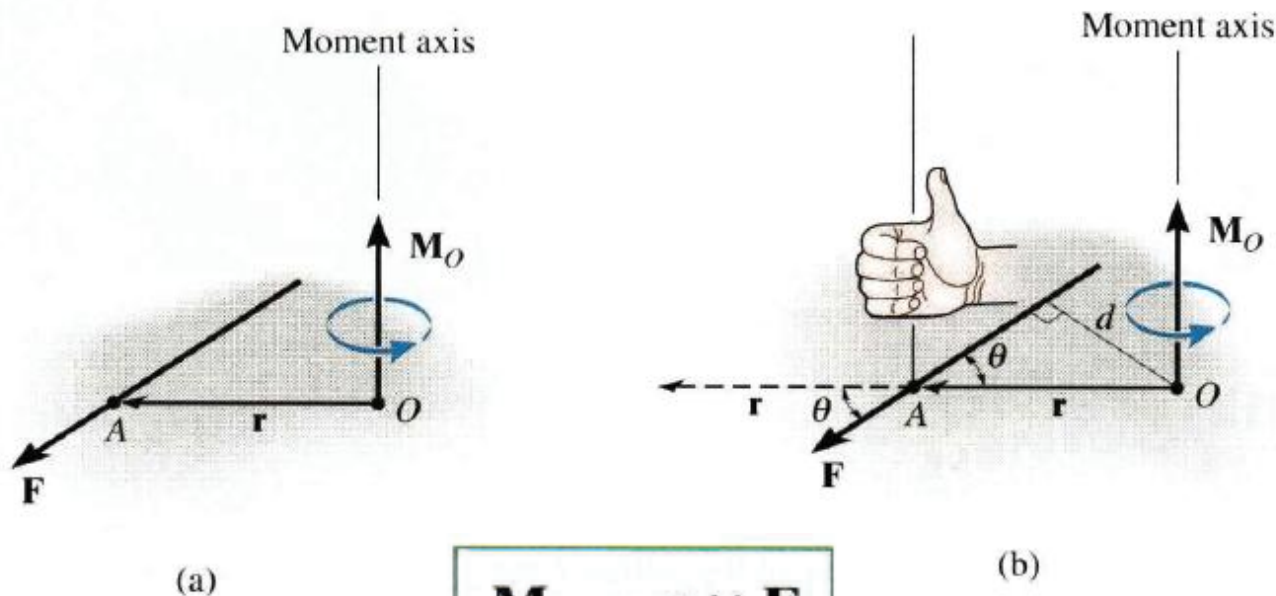


Moments in 3-D can be calculated using scalar (2-D) approach but it can be difficult and time consuming. Thus, it is often easier to use a mathematical approach called the vector cross product.

Using the vector cross product, $\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$.

Here $\mathbf{r}$ is the position vector from point O to any point on the line of action of $\mathbf{F}$.

4.3 MOMENT OF A FORCE – VECTOR FORMULATION



$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

Magnitude. $M_O = rF \sin \theta = F (r \sin \theta) = Fd$

Direction. The direction and sense of $\mathbf{M}_O$ in Eq. 4–6 are determined by the right-hand rule as it applies to the cross product.

MOMENT OF A FORCE – VECTOR FORMULATION

So, using the cross product, a moment can be expressed as

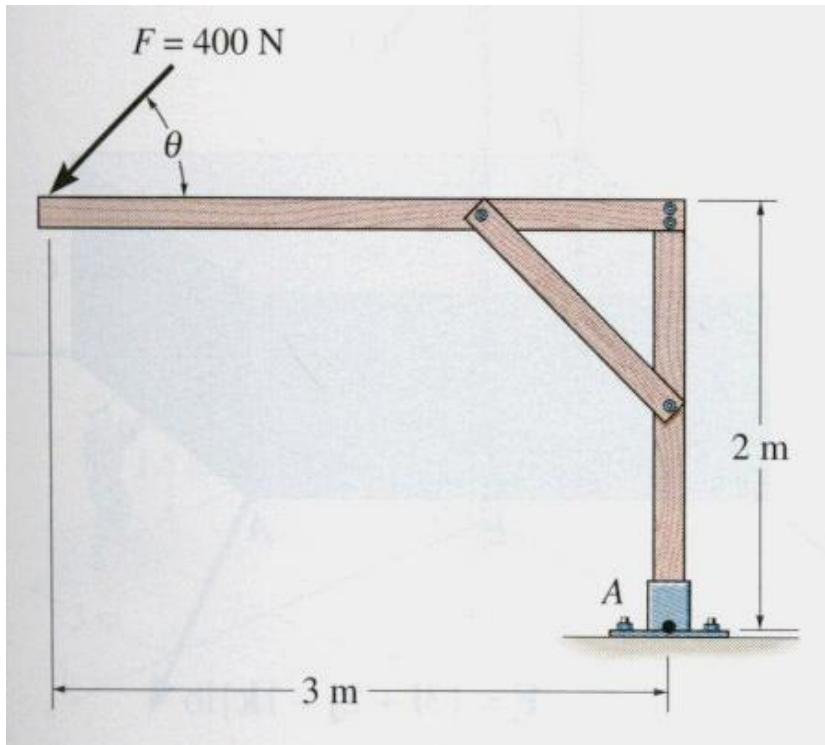
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

By expanding the above equation using 2×2 determinants (see Section 4.2), we get (sample units are N - m or lb - ft)

$$\mathbf{M}_O = (r_y F_z - r_z F_y) \mathbf{i} - (r_x F_z - r_z F_x) \mathbf{j} + (r_x F_y - r_y F_x) \mathbf{k}$$

The physical meaning of the above equation becomes evident by considering the force components separately and using a 2-D formulation.

EXAMPLE #1



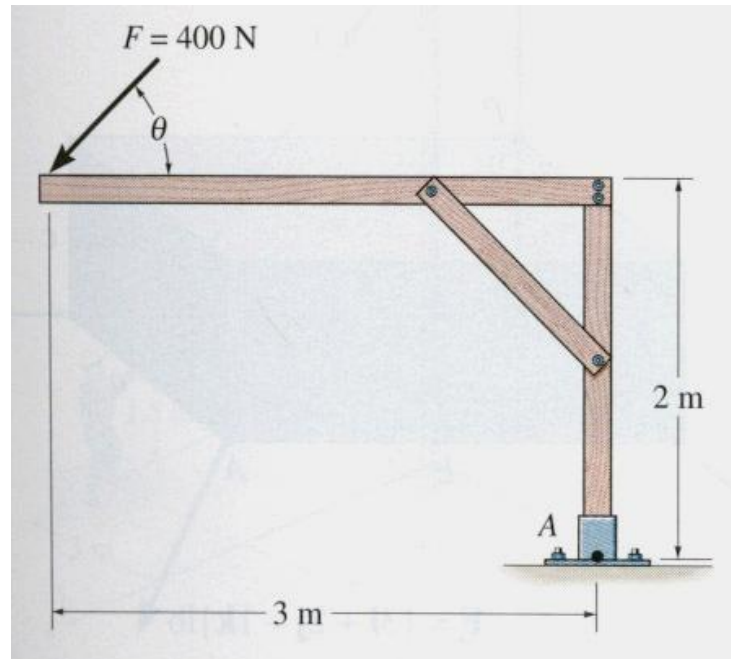
Given: A 400 N force is applied to the frame and $\theta = 20^\circ$.

Find: The moment of the force at A.

Plan:

- 1) Resolve the force along x and y axes.
- 2) Determine M_A using scalar analysis.

EXAMPLE #1 (continued)



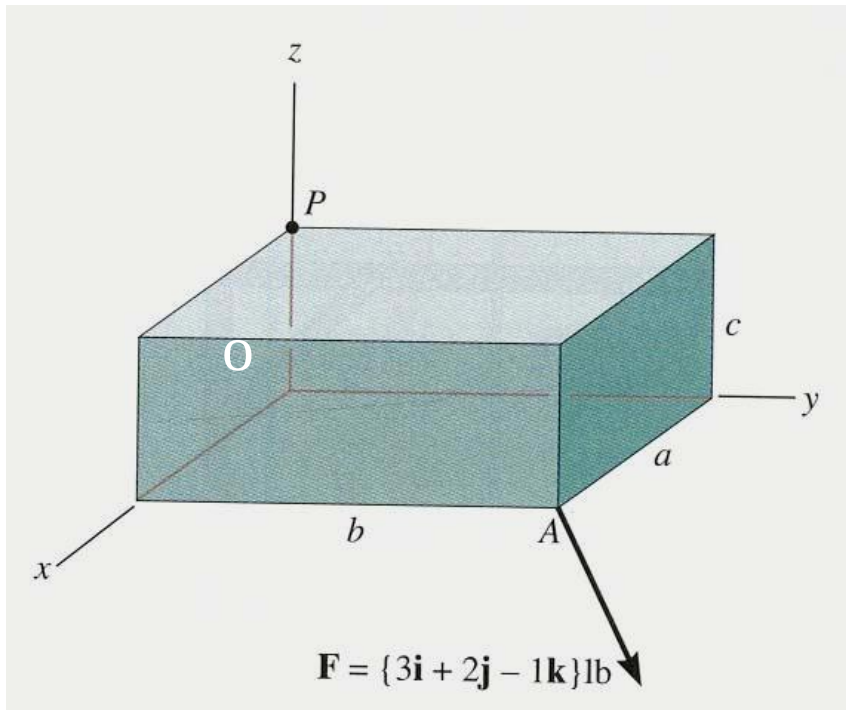
Solution

$$+ \uparrow F_y = -400 \cos 20^\circ \text{ N}$$

$$+ \rightarrow F_x = -400 \sin 20^\circ \text{ N}$$

$$\begin{aligned} + M_A &= \{(400 \cos 20^\circ)(2) + (400 \sin 20^\circ)(3)\} \text{ N}\cdot\text{m} \\ &= 1160 \text{ N}\cdot\text{m} \end{aligned}$$

EXAMPLE # 2



Given: $a = 3$ in, $b = 6$ in and $c = 2$ in.

Find: Moment of $\mathbf{F}$ about point O.

Plan:

1) Find $\mathbf{r}_{OA}$.

2) Determine $\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F}$.

Solution

$$\mathbf{r}_{OA} = \{3 \mathbf{i} + 6 \mathbf{j} - 0 \mathbf{k}\} \text{ in}$$

$$\begin{aligned} \mathbf{M}_O &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 6 & 0 \\ 3 & 2 & -1 \end{vmatrix} = [\{6(-1) - 0(2)\} \mathbf{i} - \{3(-1) - 0(3)\} \mathbf{j} + \{3(2) - 6(3)\} \mathbf{k}] \text{ lb}\cdot\text{in} \\ &= \{-6 \mathbf{i} + 3 \mathbf{j} - 12 \mathbf{k}\} \text{ lb}\cdot\text{in} \end{aligned}$$

CONCEPT QUIZ

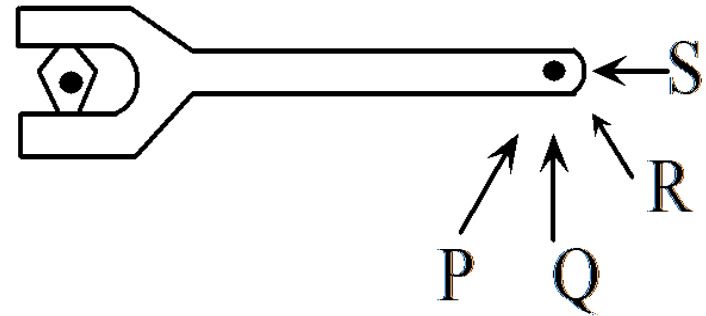
1. If a force of magnitude F can be applied in four different 2-D configurations (P,Q,R, & S), select the cases resulting in the maximum and minimum torque values on the nut. (Max, Min).

A) (Q, P)

B) (R, S)

C) (P, R)

D) (Q, S)



2. If $\mathbf{M} = \mathbf{r} \times \mathbf{F}$, then what will be the value of $\mathbf{M} \cdot \mathbf{r}$?

A) 0

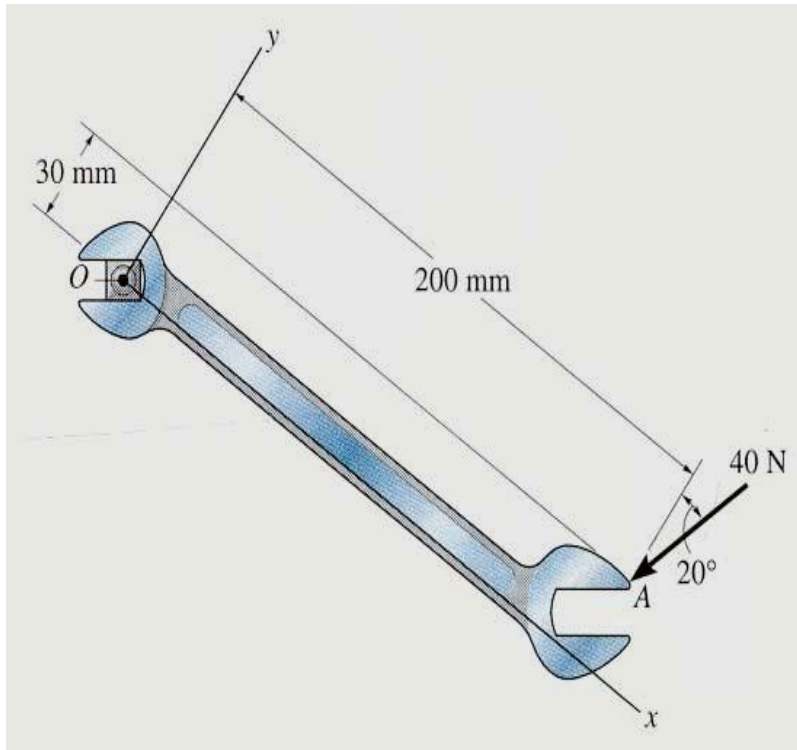
B) 1

C) $r^2 F$

D) None of the above.



PROBLEM # 1



Given: A 40 N force is applied to the wrench.

Find: The moment of the force at O.

Plan: 1) Resolve the force along x and y axes.

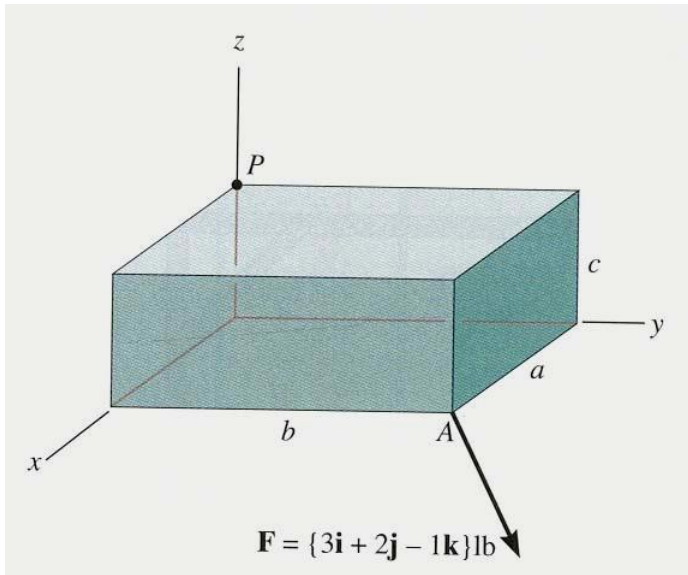
2) Determine M_O using scalar analysis.

Solution:

$$+ \uparrow F_y = -40 \cos 20^\circ \text{ N}$$
$$+ \rightarrow F_x = -40 \sin 20^\circ \text{ N}$$

$$+ \curvearrowright M_O = \{-(40 \cos 20^\circ)(200) + (40 \sin 20^\circ)(30)\} \text{ N}\cdot\text{mm}$$
$$= -7107 \text{ N}\cdot\text{mm} = -7.11 \text{ N}\cdot\text{m}$$

PROBLEM SOLVING



Given: $a = 3$ in , $b = 6$ in and $c = 2$ in

Find: Moment of F about point P

Plan: 1) Find $\mathbf{r}_{PA}$.

2) Determine $\mathbf{M}_P = \mathbf{r}_{PA} \times \mathbf{F}$

Solution: $\mathbf{r}_{PA} = \{ 3 \mathbf{i} + 6 \mathbf{j} - 2 \mathbf{k} \}$ in

$$\mathbf{M}_P = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 6 & -2 \\ 3 & 2 & -1 \end{vmatrix} = \{ -2 \mathbf{i} - 3 \mathbf{j} - 12 \mathbf{k} \} \text{ lb} \cdot \text{in}$$

ATTENTION QUIZ



1. Using the CCW direction as positive, the net moment of the two forces about point P is

- A) $10 \text{ N} \cdot \text{m}$ B) $20 \text{ N} \cdot \text{m}$ C) $-20 \text{ N} \cdot \text{m}$
D) $40 \text{ N} \cdot \text{m}$ E) $-40 \text{ N} \cdot \text{m}$

2. If $\mathbf{r} = \{ 5 \mathbf{j} \}$ m and $\mathbf{F} = \{ 10 \mathbf{k} \}$ N, the moment

$\mathbf{r} \times \mathbf{F}$ equals $\{ \text{_____} \}$ N·m.

- A) $50 \mathbf{i}$ B) $50 \mathbf{j}$ C) $-50 \mathbf{i}$
D) $-50 \mathbf{j}$ E) 0

