

4.4 PRINCIPAL OF MOMENT (*Varignon's theorem*)

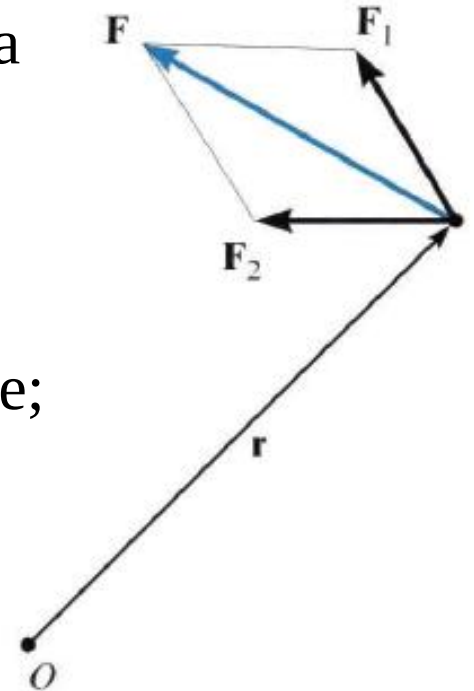
Principal of moments states that the moment of a force about a point is equal to the sum of moments of the force's components about the point .

To show this consider the force $\mathbf{F}$ shown in figure;

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$$

The moment $\mathbf{M}_O$ is then can be written as;

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2 = \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2) = \mathbf{r} \times \mathbf{F}$$



This concept has important applications since it is often easier to determine the moments of the force's components rather than the moment of the force itself.

EXAMPLE 4-6

A 200-N force acts on the bracket shown in Fig. 4-19*a*. Determine the moment of the force about point A.

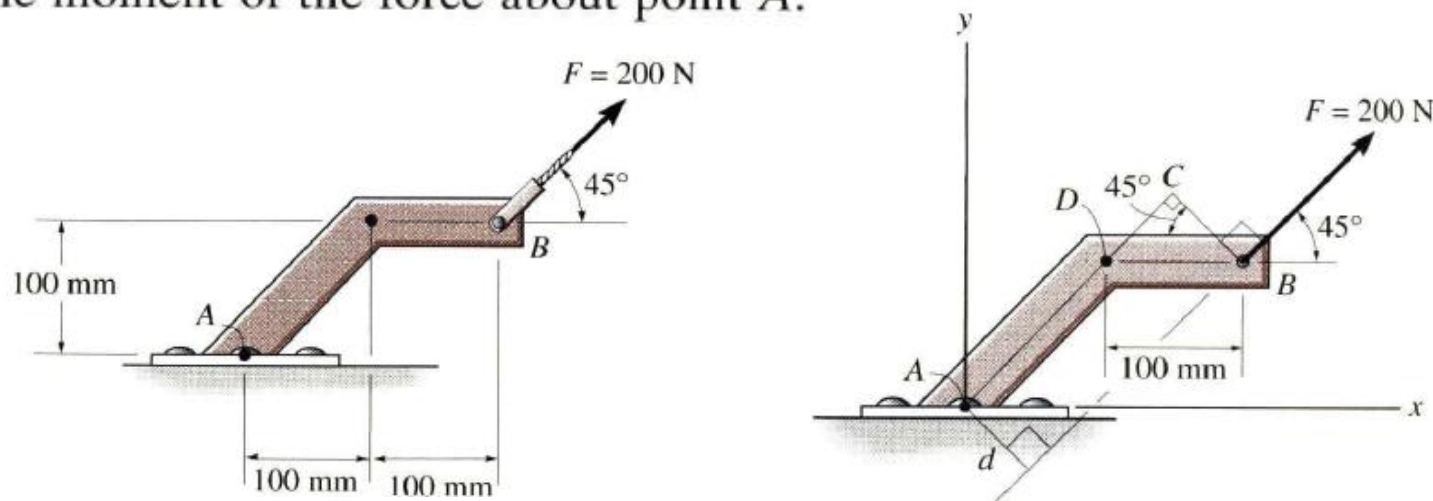


Fig. 4-19

Solution I

The moment arm d can be found by trigonometry, using the construction shown in Fig. 4-19*b*. From the right triangle BCD ,

$$CB = d = 100 \cos 45^\circ = 70.71 \text{ mm} = 0.07071 \text{ m}$$

Thus, $M_A = Fd = 200 \text{ N}(0.07071 \text{ m}) = 14.1 \text{ N}\cdot\text{m} \curvearrowright$

Hence, reporting the moment as a Cartesian vector, we have

$$\mathbf{M}_A = \{14.1\mathbf{k}\} \text{ N}\cdot\text{m}$$

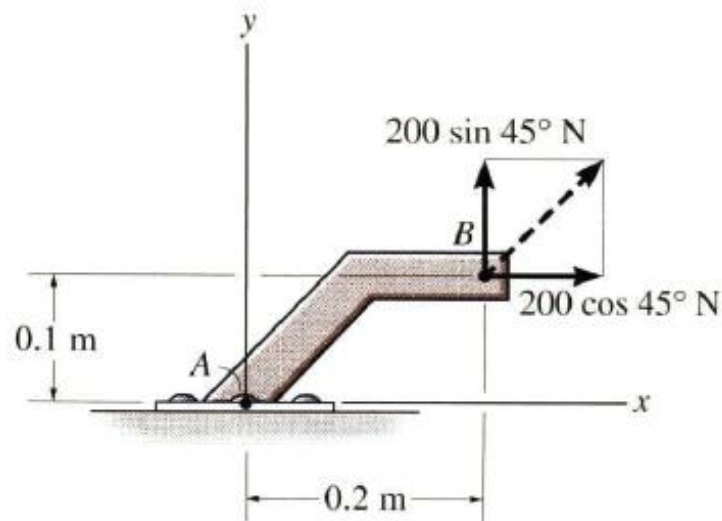
Solution II

The 200-N force may be resolved into x and y components, as shown in Fig. 4–19c. In accordance with the principle of moments, the moment of $\mathbf{F}$ computed about point A is equivalent to the sum of the moments produced by the two force components. Assuming counterclockwise rotation as positive, i.e., in the $+\mathbf{k}$ direction, we can apply Eq. 4–2 ($M_A = \Sigma Fd$), in which case

$$\begin{aligned}\downarrow + M_A &= (200 \sin 45^\circ \text{ N})(0.20 \text{ m}) - (200 \cos 45^\circ \text{ N})(0.10 \text{ m}) \\ &= 14.1 \text{ N}\cdot\text{m} \nearrow\end{aligned}$$

Thus,

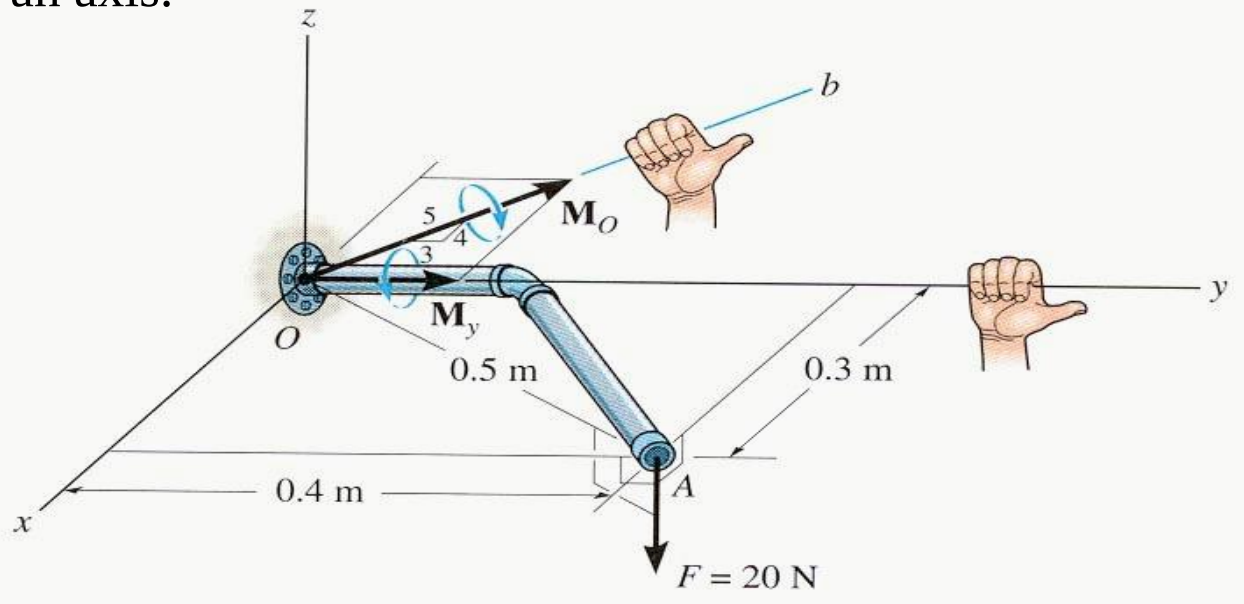
$$\mathbf{M}_A = \{14.1 \mathbf{k}\} \text{ N}\cdot\text{m}$$



By comparison, it is seen that Solution II provides a more *convenient method* for analysis than Solution I since the moment arm for each component force is easier to establish.

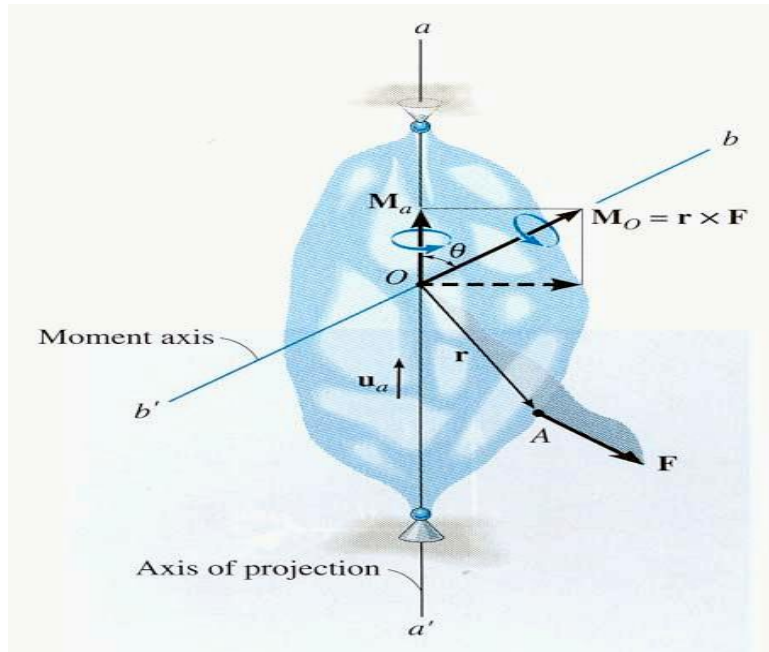
4.5 MOMENT ABOUT AN AXIS - Scalar analysis

Recall that the moment of a force about any point A is $M_A = F d_A$ where d_A is the perpendicular (or shortest) distance from the point to the force's line of action. This concept can be extended to find the moment of a force about an axis.



In the figure above, the moment about the y-axis would be $M_y = 20 (0.3) = 6 \text{ N}\cdot\text{m}$. However, this calculation is not always trivial and vector analysis may be preferable.

VECTOR ANALYSIS



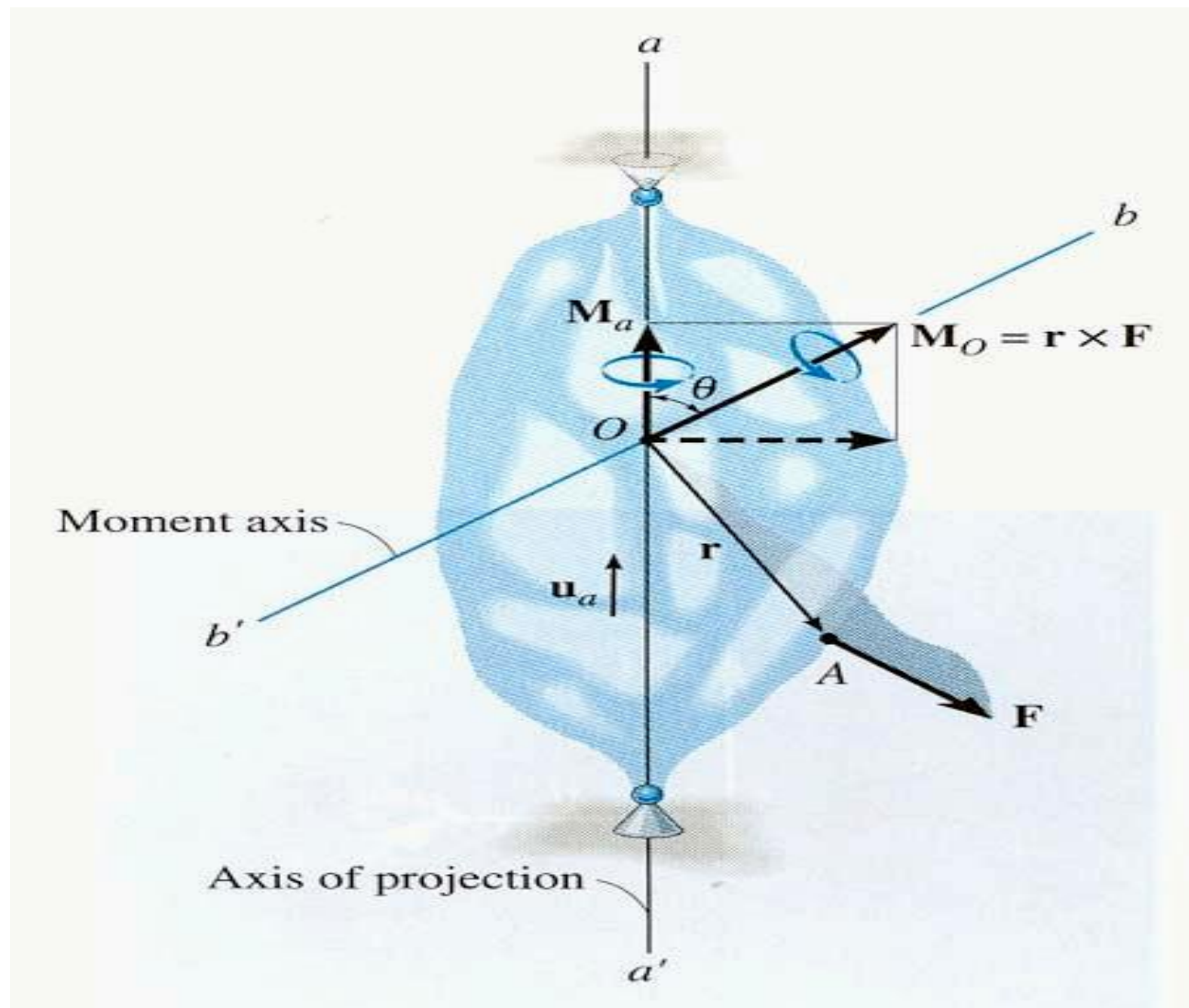
Our goal is to find the moment of $\mathbf{F}$ (the tendency to rotate the body) about the axis $a'-a$.

First compute the moment of $\mathbf{F}$ about any arbitrary point O that lies on the $a'-a$ axis using the cross product.

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

Now, find the component of $\mathbf{M}_O$ along the axis $a'-a$ using the dot product.

$$M_a = \mathbf{u}_a \cdot \mathbf{M}_O$$

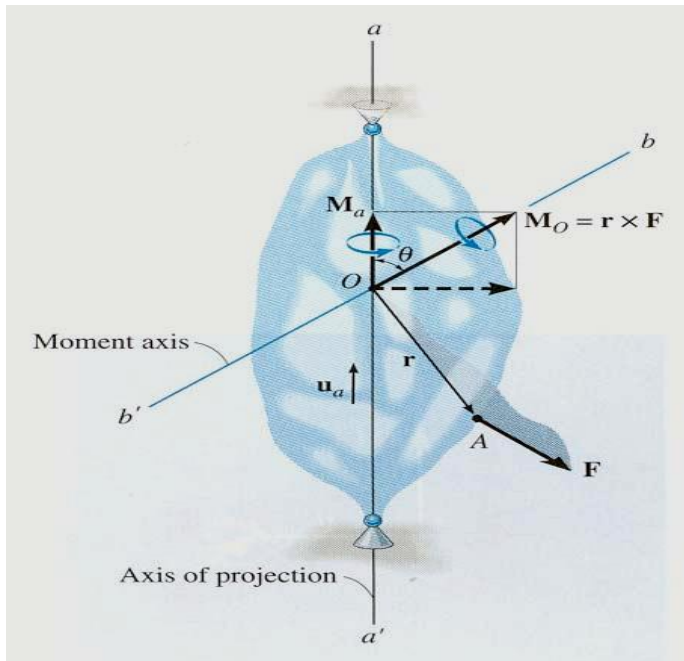


VECTOR ANALYSIS (continued)

M_a can also be obtained as

$$M_a = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

The above equation is also called the triple scalar product.



In the this equation,

$\mathbf{u}_a$ represents the unit vector along the axis a'-a axis,

$\mathbf{r}$ is the position vector from any point on the a'-a axis to any point A on the line of action of the force, and

$\mathbf{F}$ is the force vector.

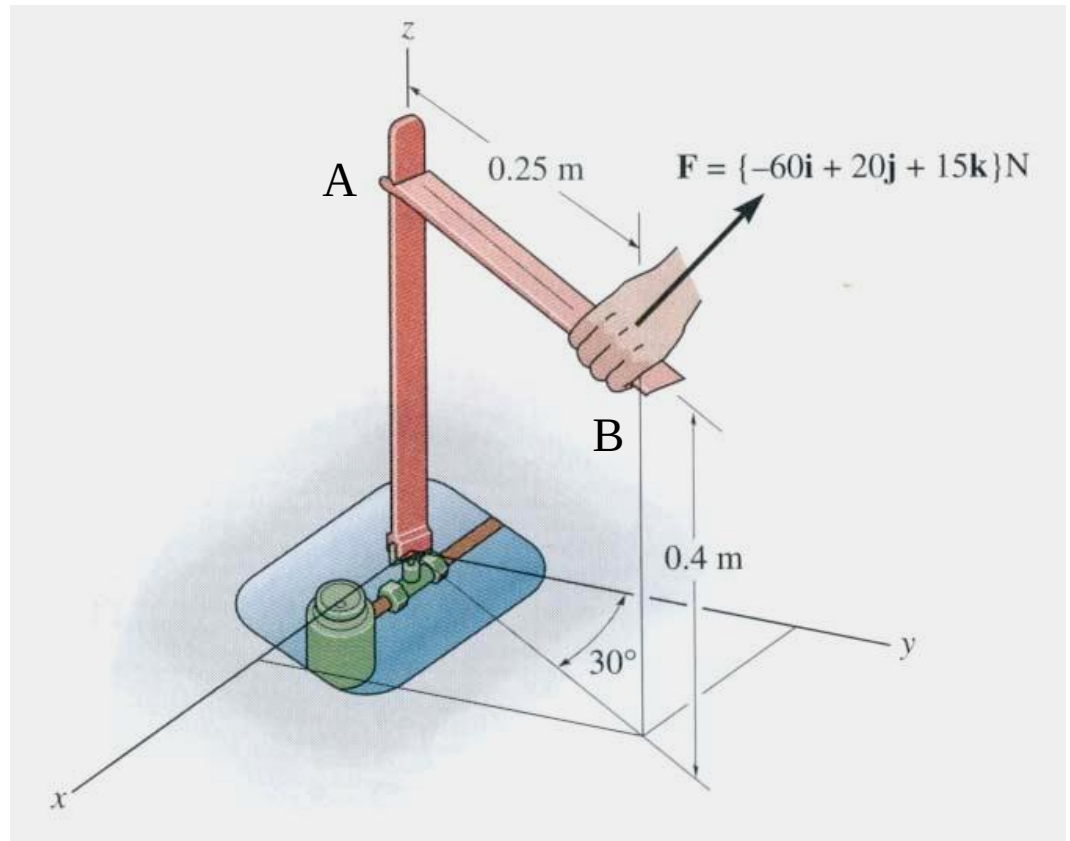
EXAMPLE

Given: A force is applied to the tool to open a gas valve.

Find: The magnitude of the moment of this force about the z axis of the value.

Plan:

- 1) We need to use $M_z = \mathbf{u} \cdot (\mathbf{r} \times \mathbf{F})$.
- 2) Note that $\mathbf{u} = 1 \mathbf{k}$.
- 3) The vector $\mathbf{r}$ is the position vector from A to B.
- 4) Force $\mathbf{F}$ is already given in Cartesian vector form.



EXAMPLE (continued)

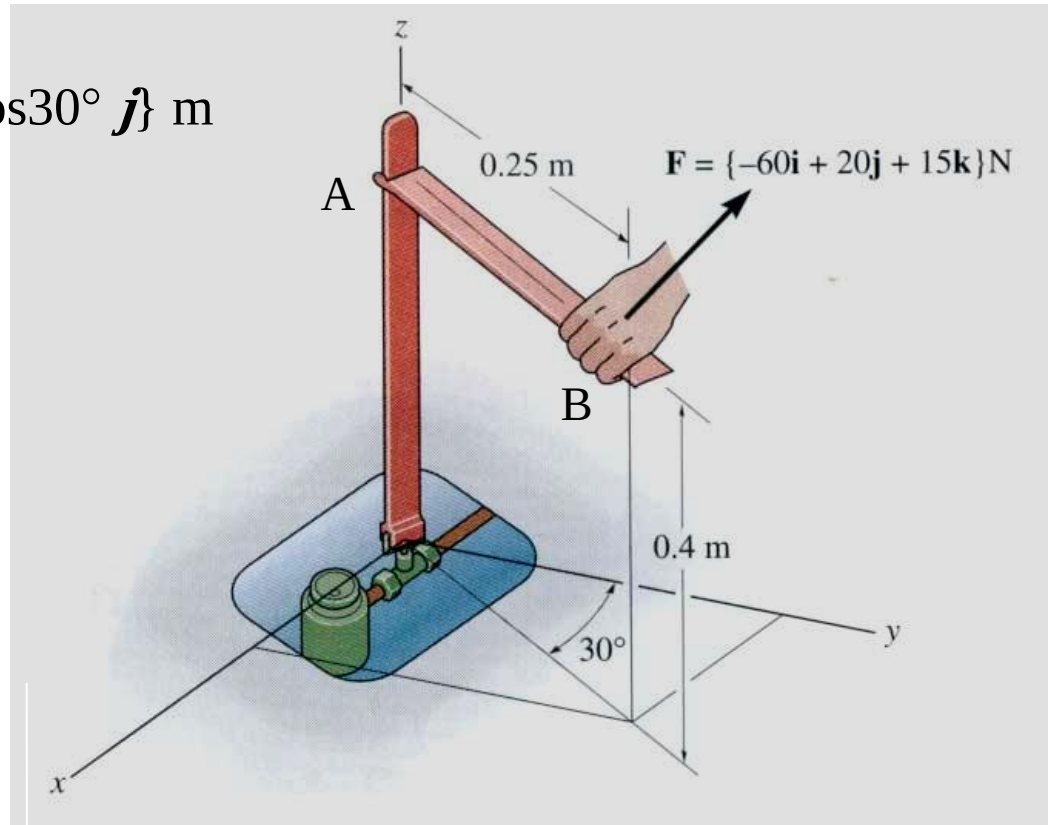
$$\mathbf{u} = 1 \mathbf{k}$$

$$\mathbf{r}_{AB} = \{0.25 \sin 30^\circ \mathbf{i} + 0.25 \cos 30^\circ \mathbf{j}\} \text{ m}$$

$$= \{0.125 \mathbf{i} + 0.2165 \mathbf{j}\} \text{ m}$$

$$\mathbf{F} = \{-60 \mathbf{i} + 20 \mathbf{j} + 15 \mathbf{k}\} \text{ N}$$

$$M_z = \mathbf{u} \cdot (\mathbf{r}_{AB} \times \mathbf{F})$$

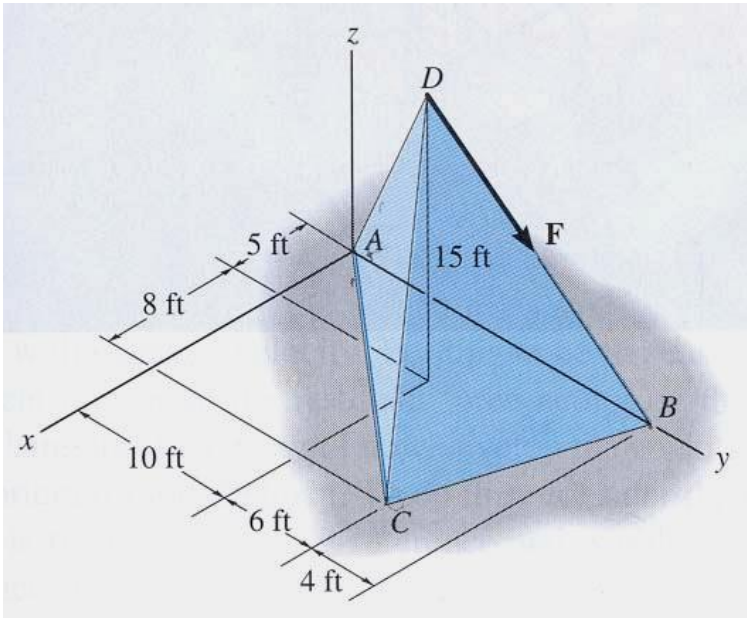


$$M_z =$$

$$= 1\{0.125(20) - 0.2165(-60)\} \text{ N}\cdot\text{m}$$

$$= 15.5 \text{ N}\cdot\text{m}$$

GROUP PROBLEM SOLVING



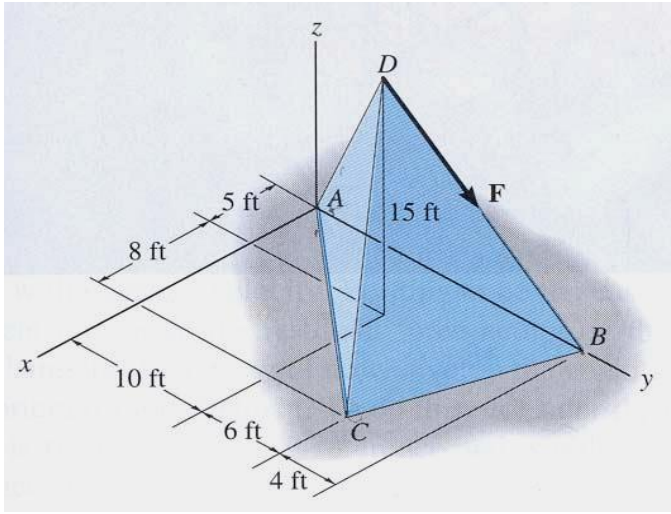
Given: A force of 80 lb acts along the edge DB.

Find: The magnitude of the moment of this force about the axis AC.

Plan:

- 1) We need to use $M_{AC} = \mathbf{u}_{AC} \cdot (\mathbf{r}_{AB} \times \mathbf{F}_{DB})$
- 2) Find $\mathbf{u}_{AC} = \mathbf{r}_{AC} / r_{AC}$
- 3) Find $\mathbf{F}_{DB} = 80 \text{ lb } \mathbf{u}_{DB} = 80 \text{ lb } (\mathbf{r}_{DB} / r_{DB})$
- 4) Complete the triple scalar product.

SOLUTION



$$\mathbf{r}_{AB} = \{ 20 \mathbf{j} \} \text{ ft}$$

$$\mathbf{r}_{AC} = \{ 13 \mathbf{i} + 16 \mathbf{j} \} \text{ ft}$$

$$\mathbf{r}_{DB} = \{ -5 \mathbf{i} + 10 \mathbf{j} - 15 \mathbf{k} \} \text{ ft}$$

$$\mathbf{u}_{AC} = (13 \mathbf{i} + 16 \mathbf{j}) \text{ ft} / (13^2 + 16^2)^{1/2} \text{ ft}$$

$$= 0.6306 \mathbf{i} + 0.7761 \mathbf{j}$$

$$\mathbf{F}_{DB} = 80 \{ \mathbf{r}_{DB} / (5^2 + 10^2 + 15^2)^{1/2} \} \text{ lb}$$

$$= \{ -21.38 \mathbf{i} + 42.76 \mathbf{j} - 64.14 \mathbf{k} \} \text{ lb}$$

Solution (continued)

Now find the triple product, $M_{AC} = \mathbf{u}_{AC} \cdot (\mathbf{r}_{AB} \times \mathbf{F}_{DB})$

$$M_{AC} = \begin{vmatrix} 0.6306 & 0.7706 & 0.00 \\ 0.0 & 20.0 & 0.00 \\ -21.38 & 42.76 & -64.14 \end{vmatrix} \text{ lb}\cdot\text{ft}$$

$$\begin{aligned} M_{AC} &= 0.6306 \{20 (-64.14) - 0 - 0.7706 (0 - 0)\} \text{ lb}\cdot\text{ft} \\ &= -809 \text{ lb}\cdot\text{ft} \end{aligned}$$

The negative sign indicates that the sense of M_{AC} is opposite to that of $\mathbf{u}_{AC}$