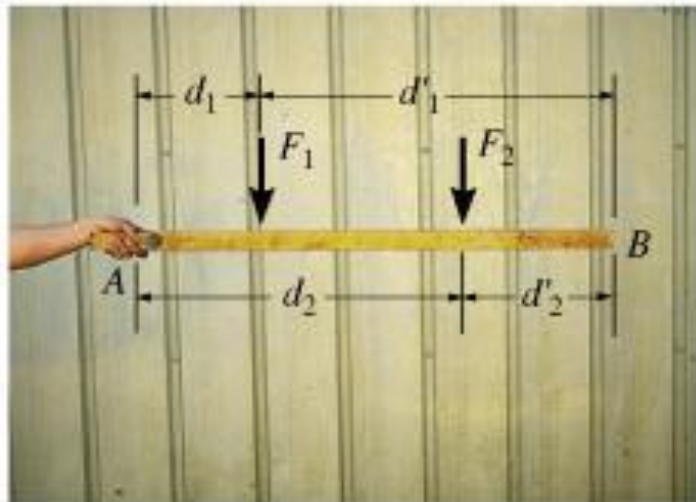


EQUIVALENT SYSTEMS, RESULTANTS OF FORCE AND COUPLE SYSTEM, & FURTHER REDUCTION OF A FORCE AND COUPLE SYSTEM

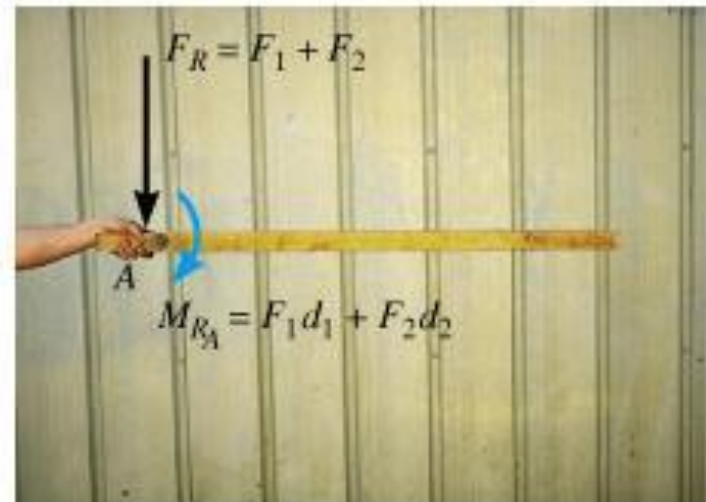
Today's Objectives:

Students will be able to:

- Determine the effect of moving a force.
- Find an equivalent force-couple system for a system of forces and couples.



=



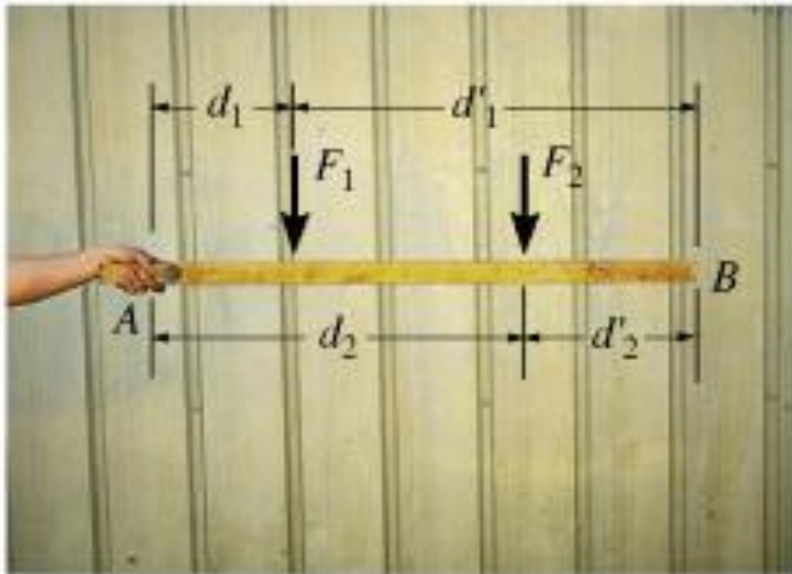
READING QUIZ

1. A general system of forces and couple moments acting on a rigid body can be reduced to a ____ .
 - 1) single force.
 - 2) single moment.
 - 3) single force and two moments.
 - 4) single force and a single moment.

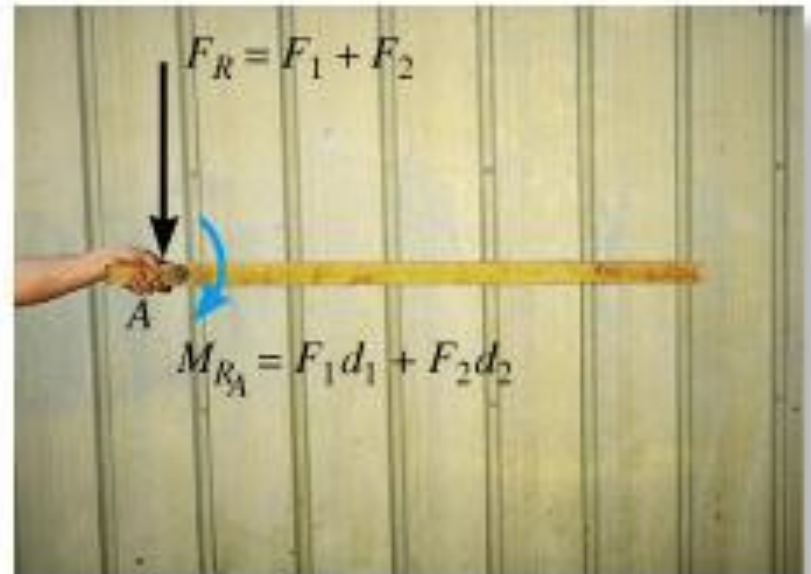
2. The original force and couple system and an equivalent force-couple system have the same _____ effect on a body.
 - 1) internal
 - 2) external
 - 3) internal and external
 - 4) microscopic



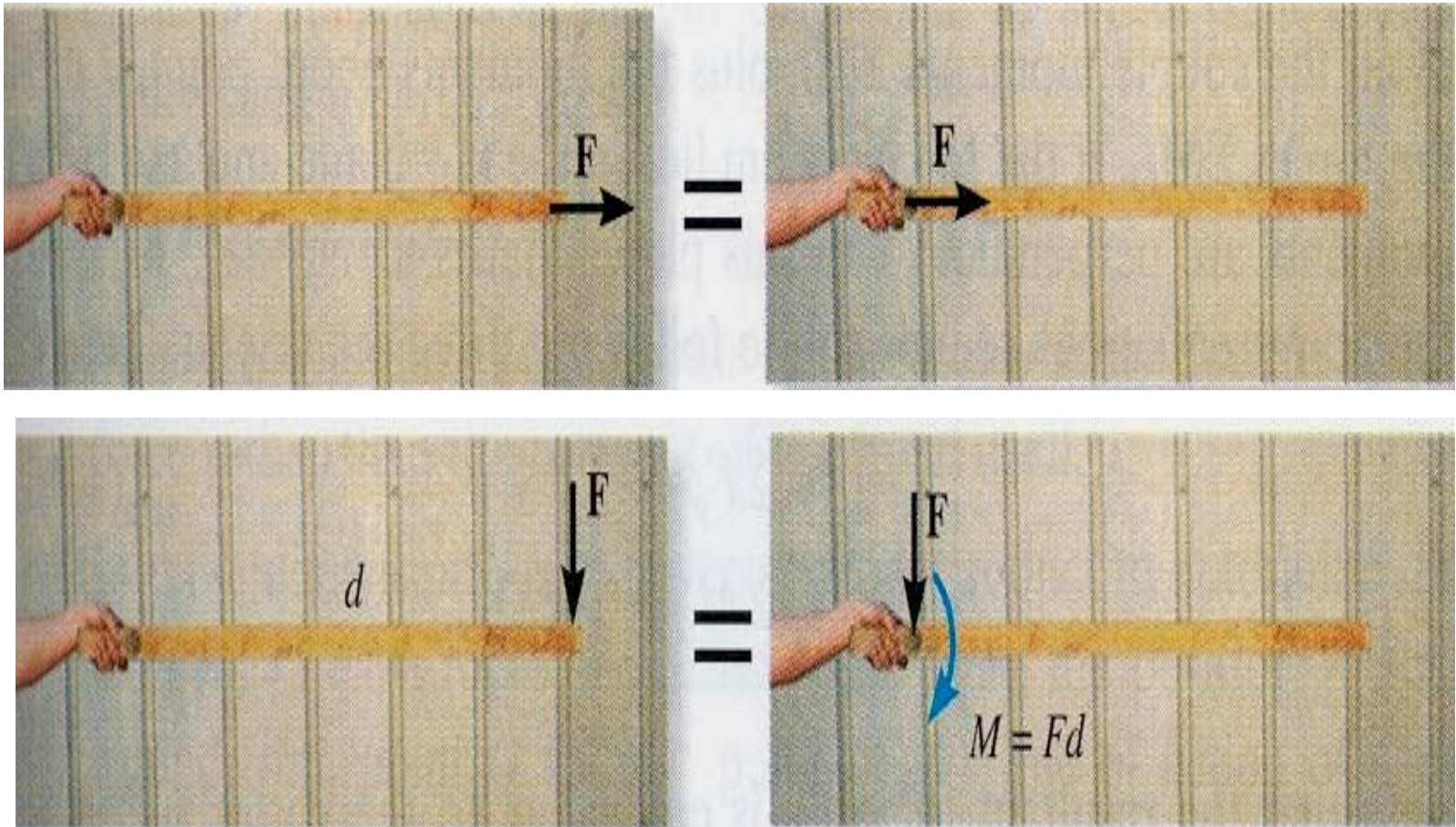
4.7 AN EQUIVALENT SYSTEM



=



APPLICATIONS

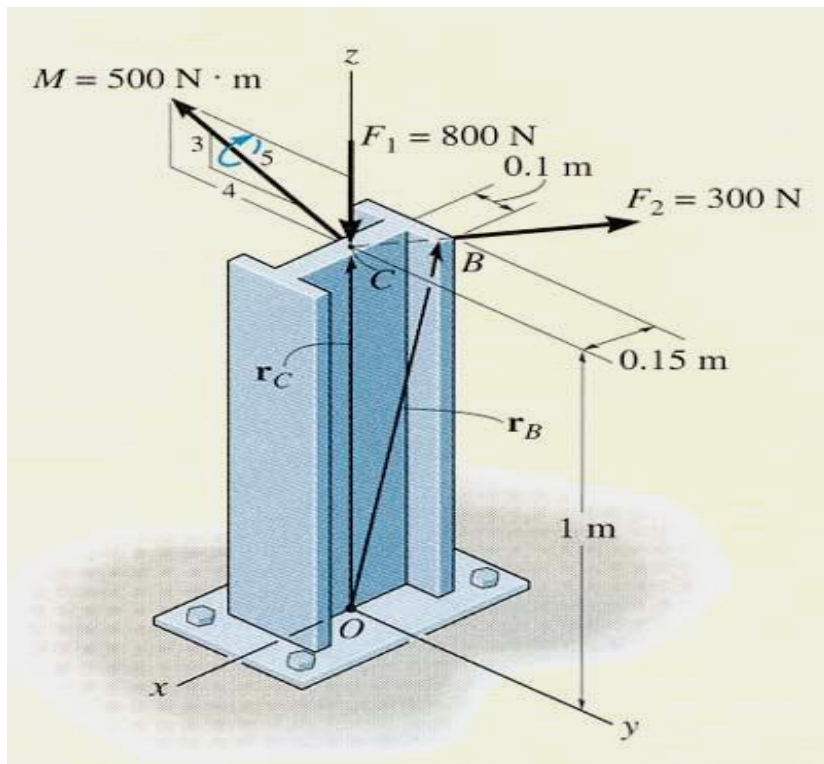


What is the resultant effect on the person's hand when the force is applied in four different ways ?

APPLICATIONS (continued)

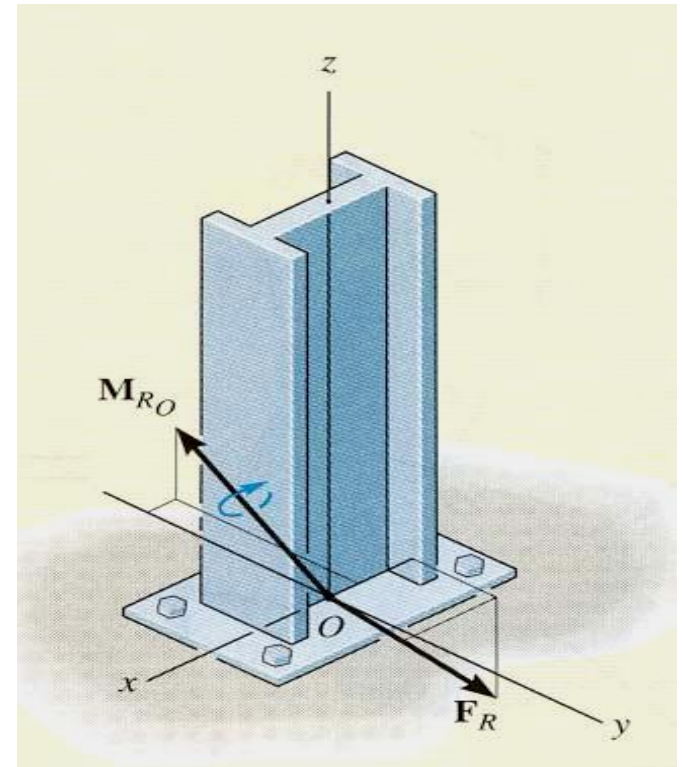
Several forces and a couple moment are acting on this vertical section of an I-beam.

Can you replace them with just one force and one couple moment at point O that will have the same external effect? If yes, how will you do that?

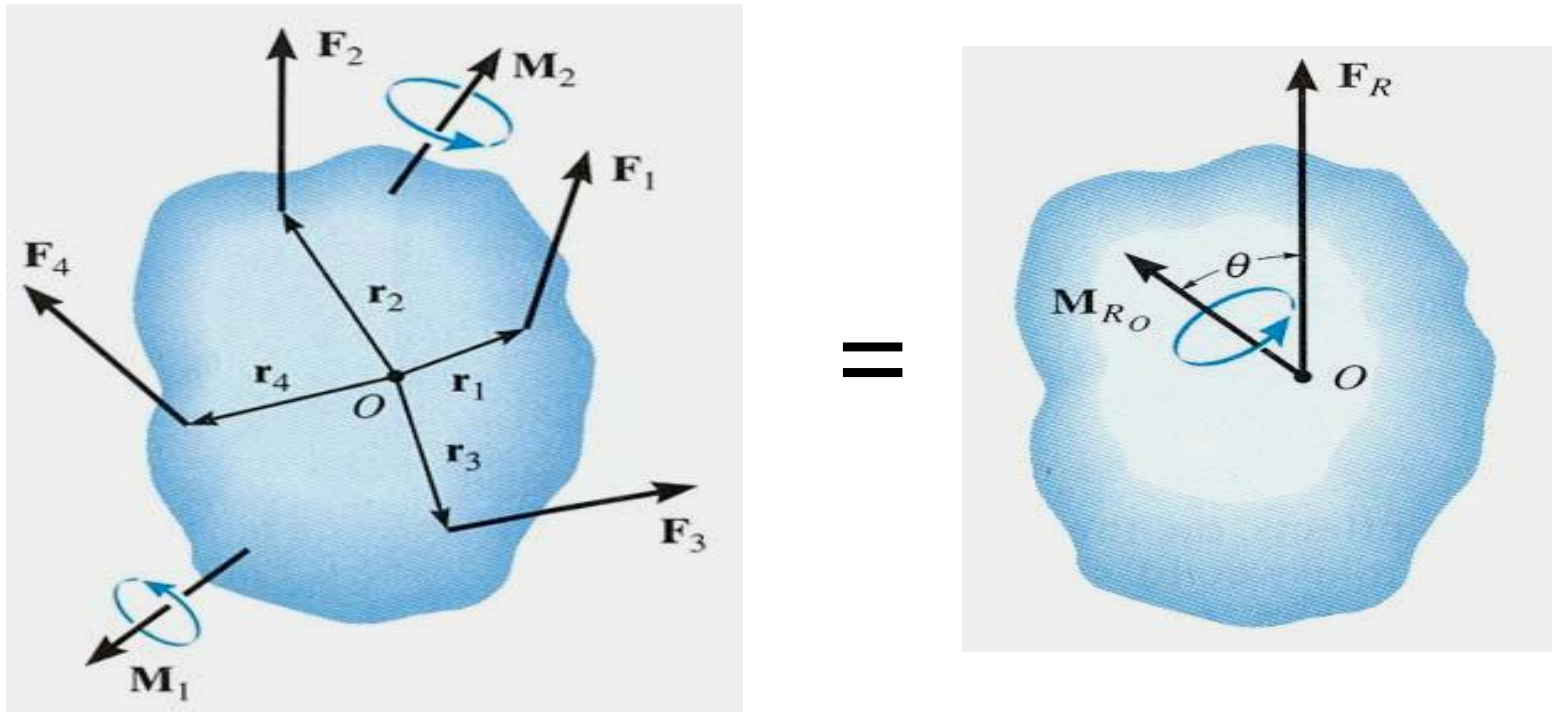


??

=



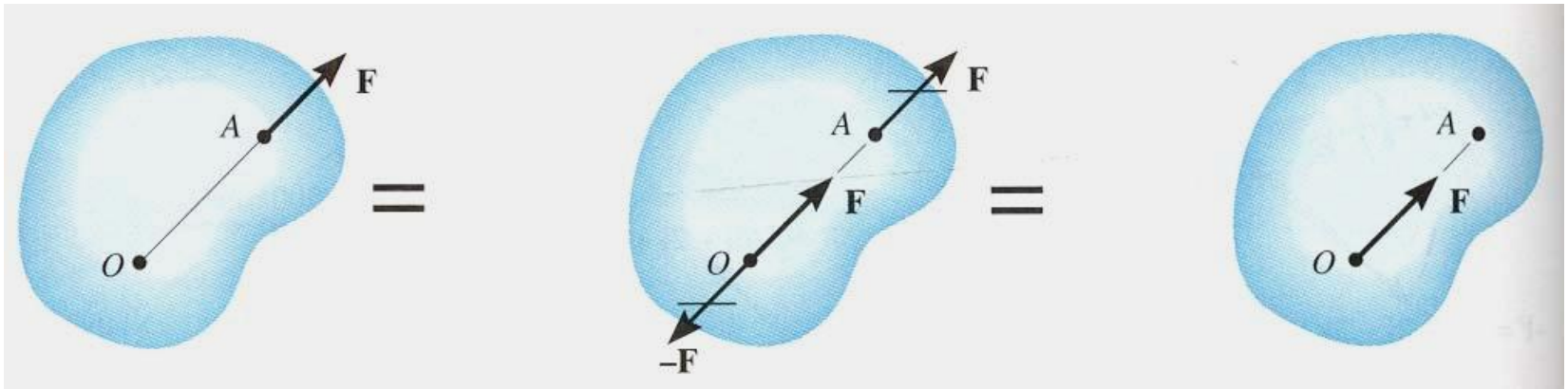
AN EQUIVALENT SYSTEM



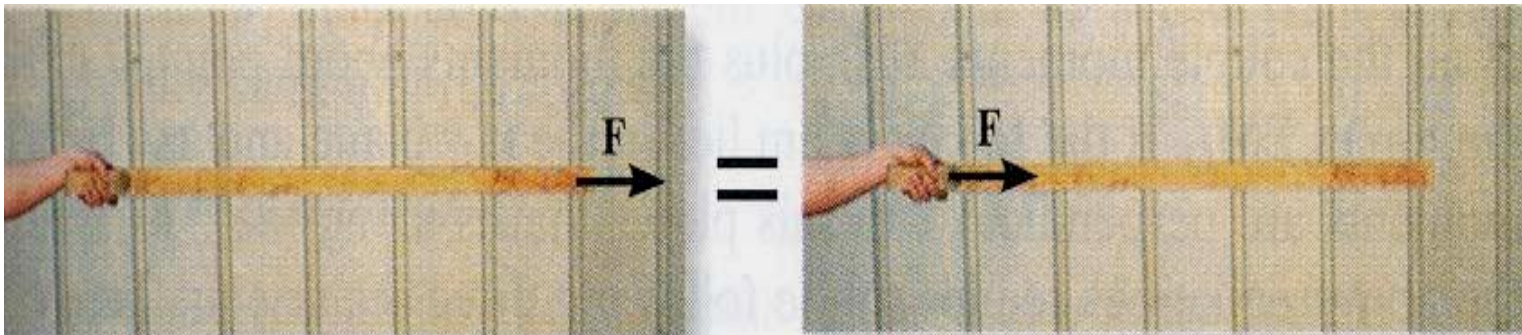
When a number of forces and couple moments are acting on a body, it is easier to understand their overall effect on the body if they are combined into a single force and couple moment having the same external effect

The two force and couple systems are called equivalent systems since they have the same external effect on the body.

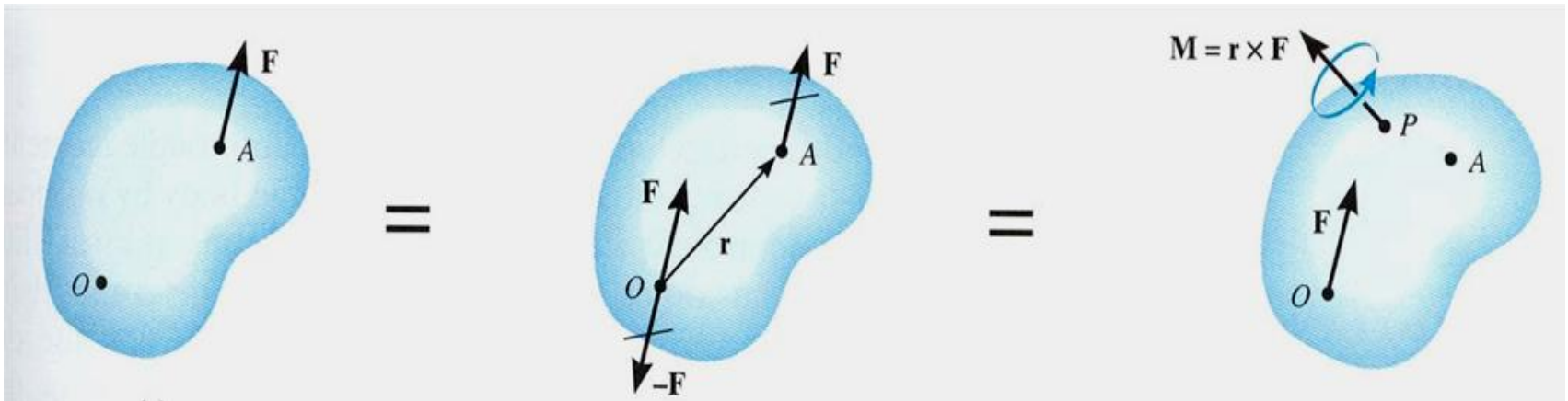
MOVING A FORCE ON ITS LINE OF ACTION



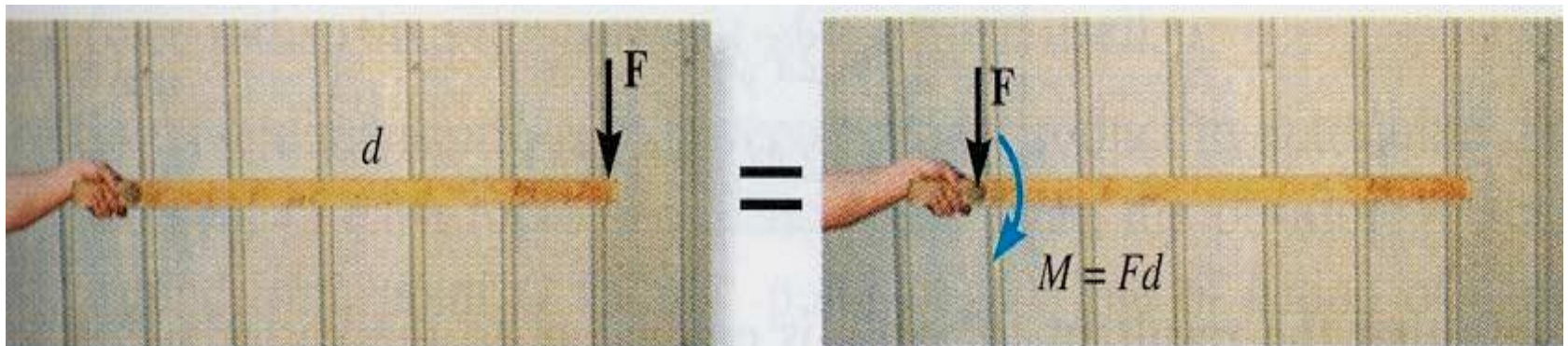
Moving a force from A to O , when both points are on the vectors' line of action, does not change the external effect. Hence, a force vector is called a sliding vector. (But the internal effect of the force on the body does depend on where the force is applied).



MOVING A FORCE OFF OF ITS LINE OF ACTION

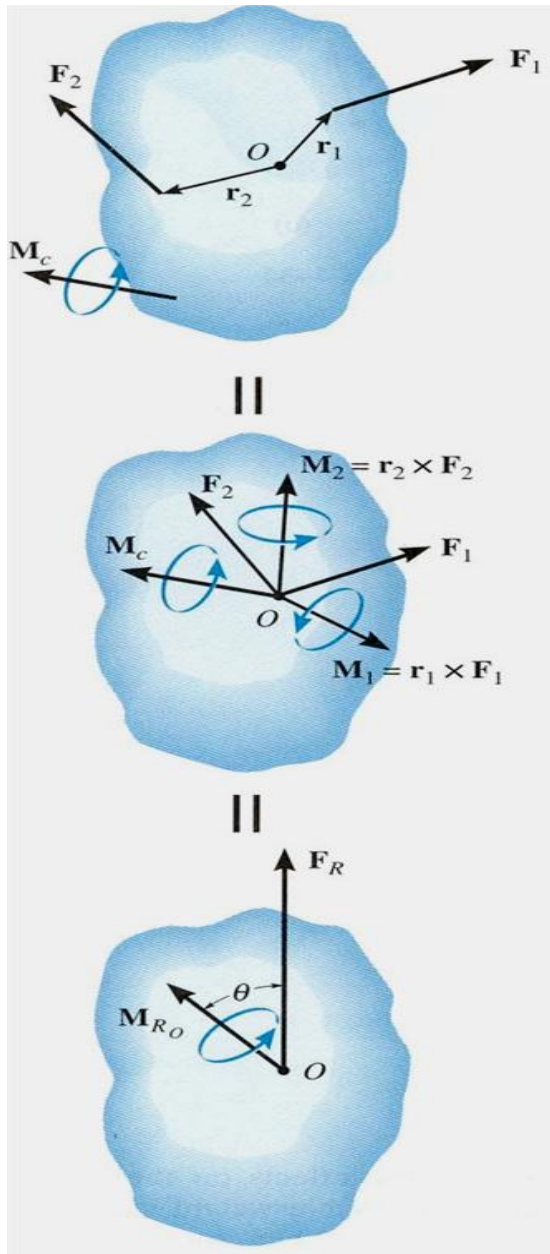


Moving a force from point A to O (as shown above) requires creating an additional couple moment. Since this new couple moment is a “free” vector, it can be applied at any point P on the body.



(Section 4.8)

RESULTANTS OF A FORCE AND COUPLE SYSTEM

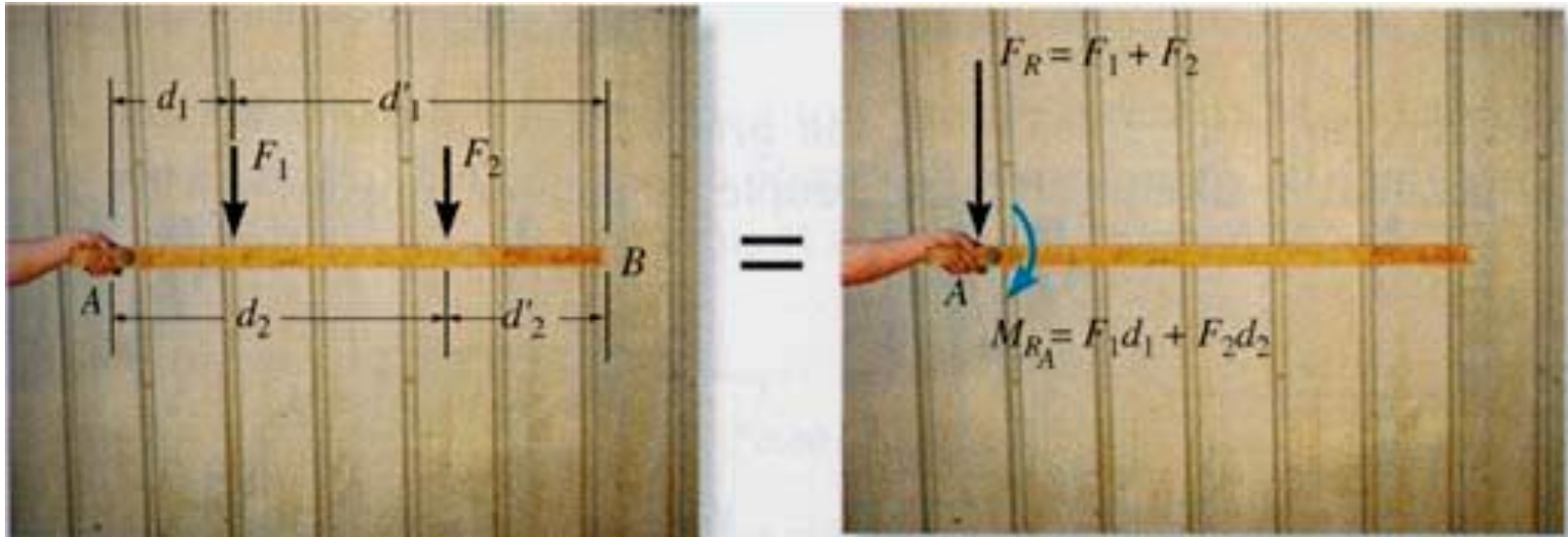


When several forces and couple moments act on a body, you can move each force and its associated couple moment to a common point O .

Now you can add all the forces and couple moments together and find one resultant force-couple moment pair.

$$\mathbf{F}_R = \Sigma \mathbf{F}$$
$$\mathbf{M}_{RO} = \Sigma \mathbf{M}_c + \Sigma \mathbf{M}_O$$

RESULTANT OF A FORCE AND COUPLE SYSTEM (continued)

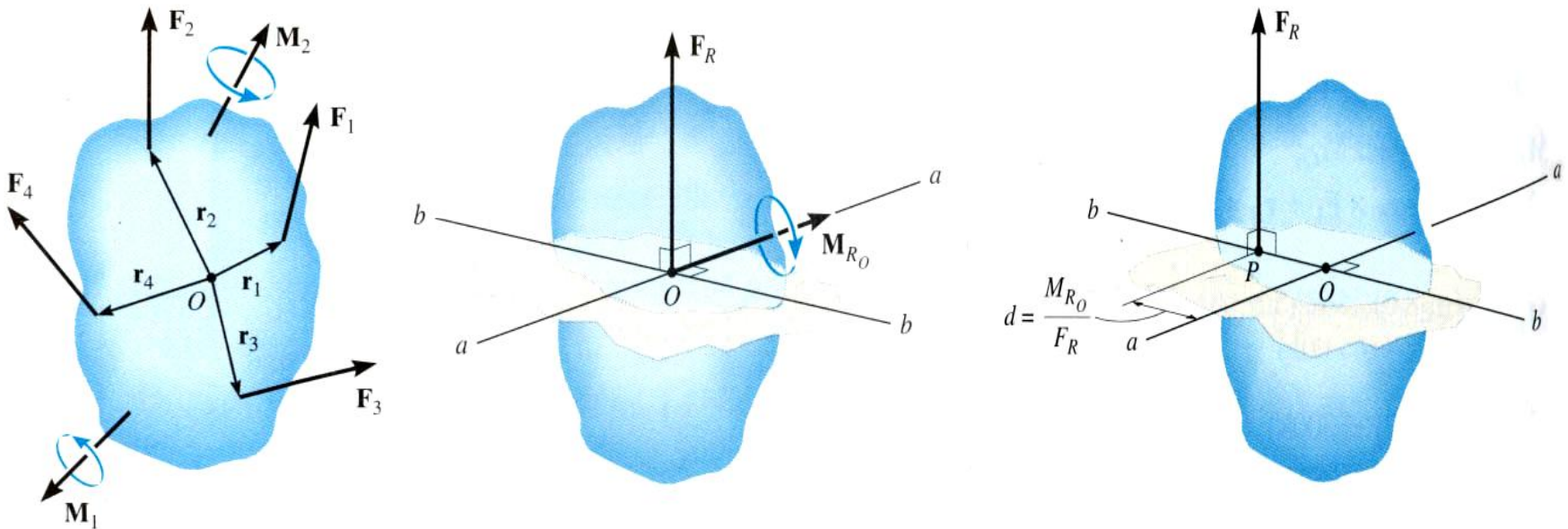


If the force system lies in the x-y plane (the 2-D case), then the reduced equivalent system can be obtained using the following three scalar equations.

$$\begin{aligned}F_{R_x} &= \Sigma F_x \\F_{R_y} &= \Sigma F_y \\M_{R_O} &= \Sigma M_c + \Sigma M_O\end{aligned}$$

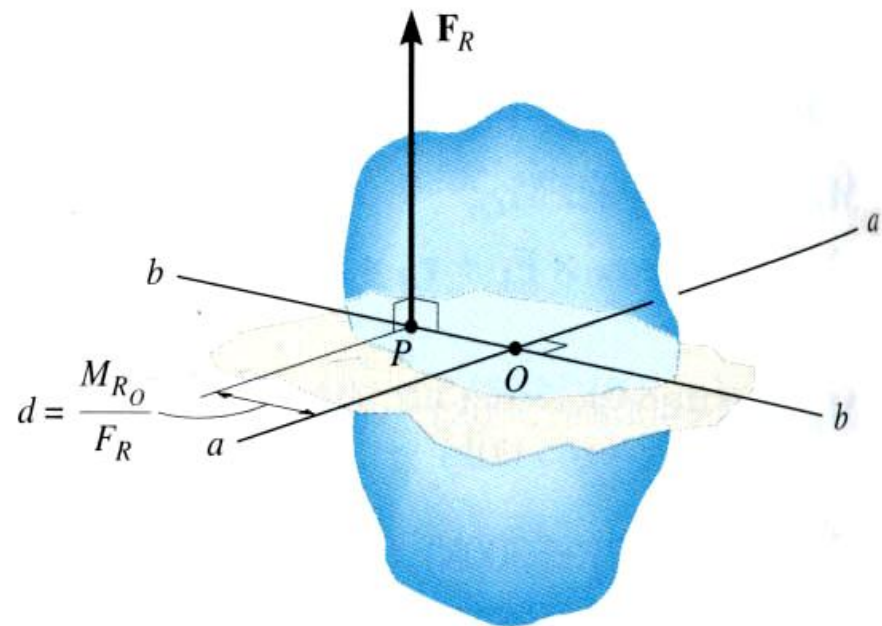
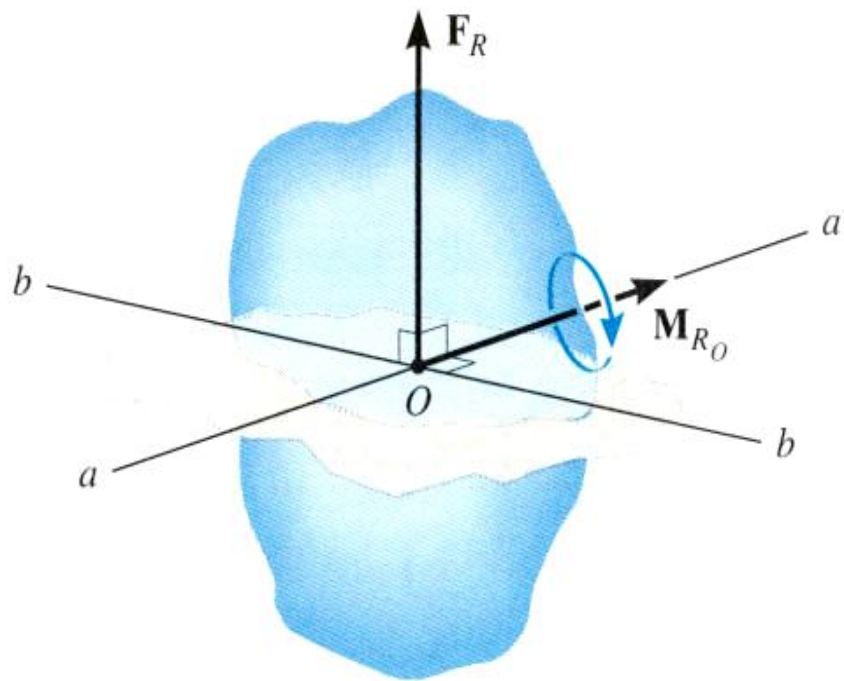
Section 4.9

FURTHER REDUCTION OF A FORCE AND COUPLE SYSTEM

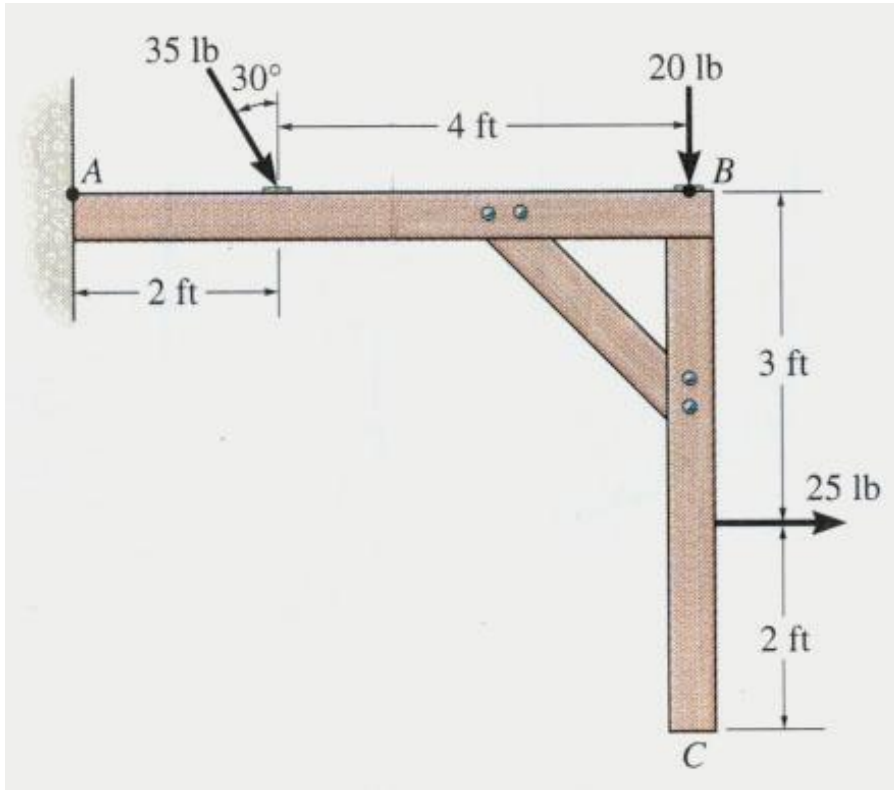


If F_R and M_{RO} are perpendicular to each other, then the system can be further reduced to a single force, F_R , by simply moving F_R from O to P .

In three special cases, concurrent, coplanar, and parallel systems of forces, the system can always be reduced to a single force.



EXAMPLE #1



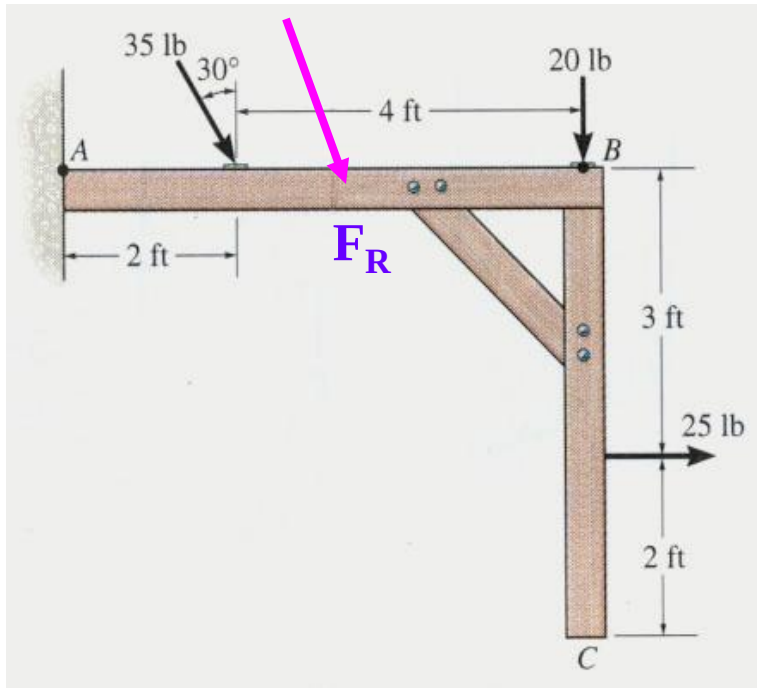
Given: A 2-D force and couple system as shown.

Find: The equivalent resultant force and couple moment acting at A and then the equivalent single force location along the beam AB.

Plan:

- 1) Sum all the x and y components of the forces to find F_{RA} .
- 2) Find and sum all the moments resulting from moving each force to A.
- 3) Shift the F_{RA} to a distance d such that $d = M_{RA}/F_{Ry}$

EXAMPLE #1 (continued)



$$+ \rightarrow \Sigma F_{Rx} = 25 + 35 \sin 30^\circ = 42.5 \text{ lb}$$

$$+ \downarrow \Sigma F_{Ry} = 20 + 35 \cos 30^\circ = 50.31 \text{ lb}$$

$$+ \curvearrowright M_{RA} = 35 \cos 30^\circ (2) + 20(6) - 25(3) = 105.6 \text{ lb}\cdot\text{ft}$$

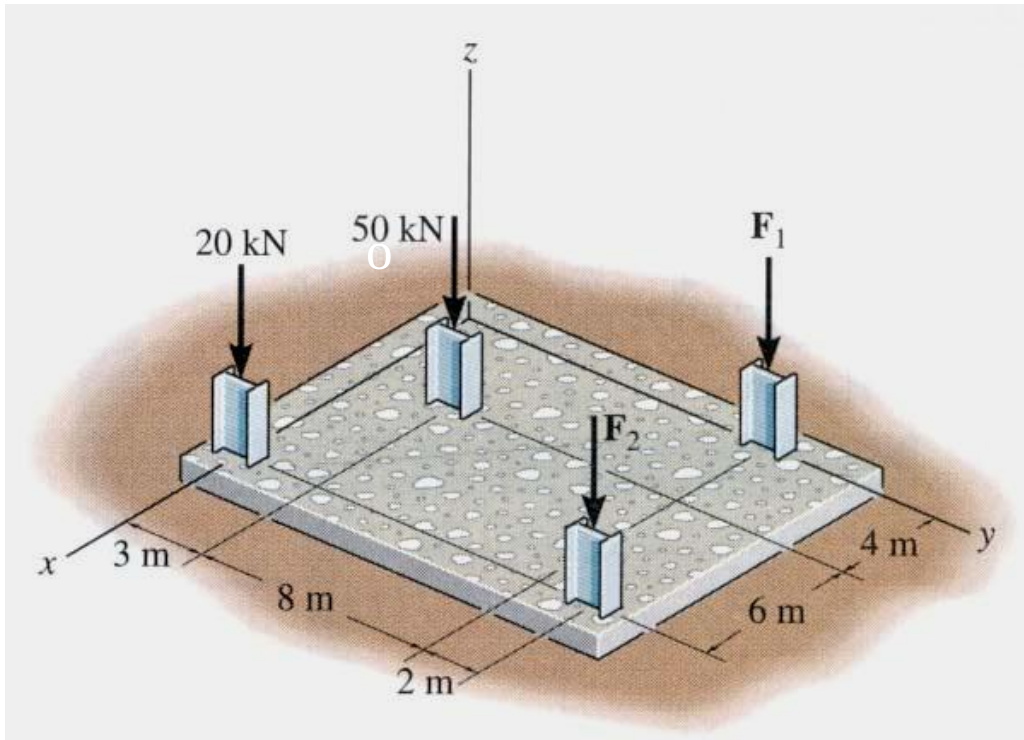
$$F_R = (42.5^2 + 50.31^2)^{1/2} = 65.9 \text{ lb}$$

$$\searrow \theta = \tan^{-1} (50.31/42.5) = 49.8^\circ$$

The equivalent single force F_R can be located on the beam AB at a distance d measured from A.

$$d = M_{RA}/F_{Ry} = 105.6/50.31 = 2.10 \text{ ft.}$$

EXAMPLE #2



Given: The building slab has four columns. F_1 and $F_2 = 0$.

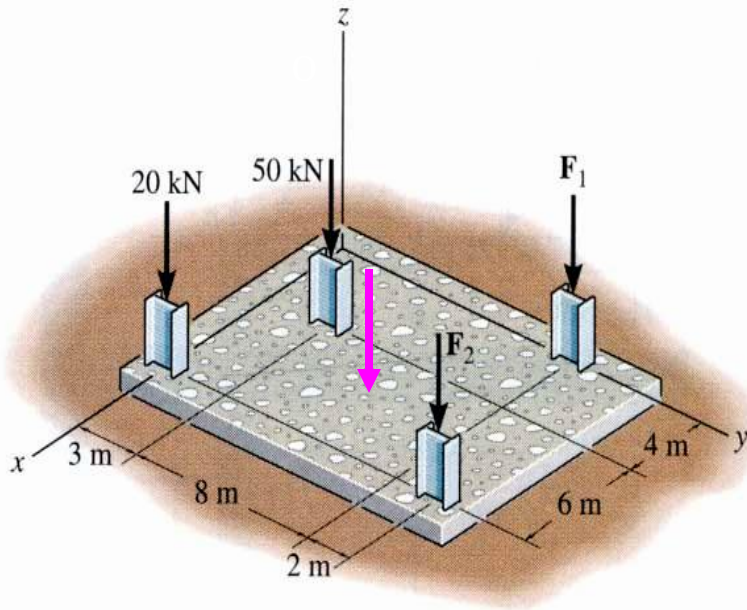
Find: The equivalent resultant force and couple moment at the origin O. Also find the location (x,y) of the single equivalent resultant force.

Plan:

- 1) Find $\mathbf{F}_{RO} = \sum \mathbf{F}_i = F_{RzO} \mathbf{k}$
- 2) Find $\mathbf{M}_{RO} = \sum (\mathbf{r}_i \times \mathbf{F}_i) = M_{RxO} \mathbf{i} + M_{RyO} \mathbf{j}$
- 3) The location of the single equivalent resultant force is given as;

$$x = -M_{RyO}/F_{RzO} \quad \text{and} \quad y = M_{RxO}/F_{RzO}$$

EXAMPLE #2 (continued)



$$F_{RO} = \{-50 \mathbf{k} - 20 \mathbf{k}\} = \{-70 \mathbf{k}\} \text{ kN}$$

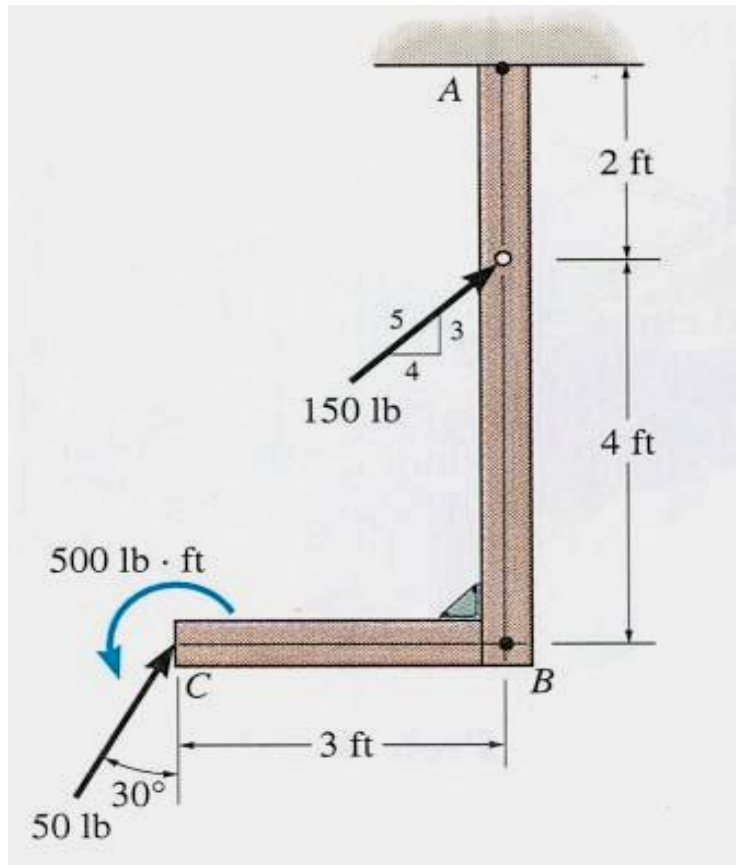
$$\begin{aligned} M_{RO} &= (10 \mathbf{i}) \times (-20 \mathbf{k}) + (4 \mathbf{i} + 3 \mathbf{j}) \times (-50 \mathbf{k}) \\ &= \{200 \mathbf{j} + 200 \mathbf{j} - 150 \mathbf{i}\} \text{ kN}\cdot\text{m} \\ &= \{-150 \mathbf{i} + 400 \mathbf{j}\} \text{ kN}\cdot\text{m} \end{aligned}$$

The location of the single equivalent resultant force is given as,

$$x = -M_{RyO}/F_{RzO} = -400/(-70) = 5.71 \text{ m}$$

$$y = M_{RxO}/F_{RzO} = (-150)/(-70) = 2.14 \text{ m}$$

GROUP PROBLEM SOLVING



Given: A 2-D force and couple system as shown.

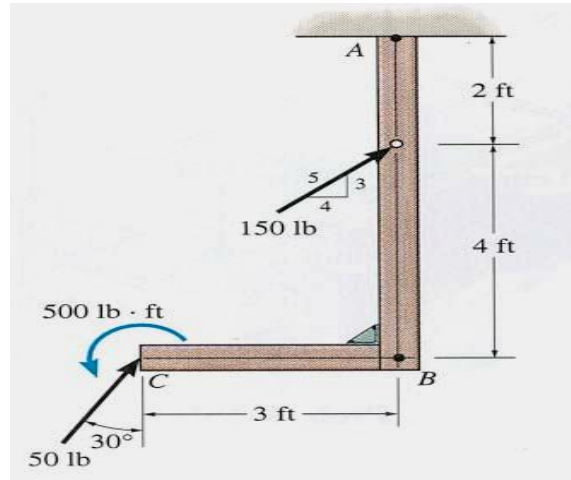
Find: The equivalent resultant force and couple moment acting at A.

Plan:

- 1) Sum all the x and y components of the forces to find F_{RA} .
- 2) Find and sum all the moments resulting from moving each force to A and add them to the 500 lb · ft free moment to find the resultant M_{RA} .

GROUP PROBLEM SOLVING (continued)

Summing the
force
components:



$$+ \rightarrow \Sigma F_x = (4/5) 150 \text{ lb} + 50 \text{ lb} \sin 30^\circ = 145 \text{ lb}$$

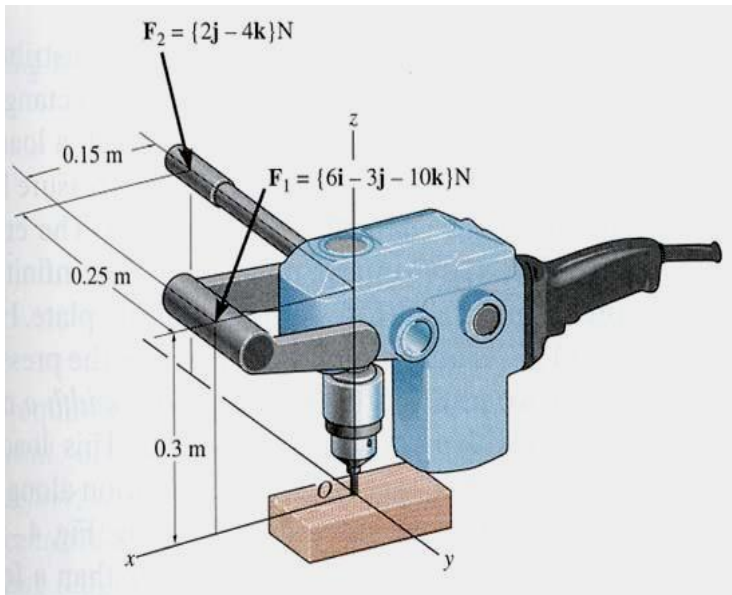
$$+ \uparrow \Sigma F_y = (3/5) 150 \text{ lb} + 50 \text{ lb} \cos 30^\circ = 133.3 \text{ lb}$$

Now find the magnitude and direction of the resultant.

$$F_{RA} = (145^2 + 133.3^2)^{1/2} = 197 \text{ lb} \quad \text{and} \quad \theta = \tan^{-1} (133.3/145) = 42.6^\circ \angle$$

$$+ \curvearrowleft M_{RA} = \{ (4/5)(150)(2) - 50 \cos 30^\circ (3) + 50 \sin 30^\circ (6) + 500 \} = 760 \text{ lb}\cdot\text{ft}$$

GROUP PROBLEM SOLVING (continued)



Given: Handle forces F_1 and F_2 are applied to the electric drill.

Find: An equivalent resultant force and couple moment at point O.

Plan:

a) Find $F_{RO} = \Sigma F_i$

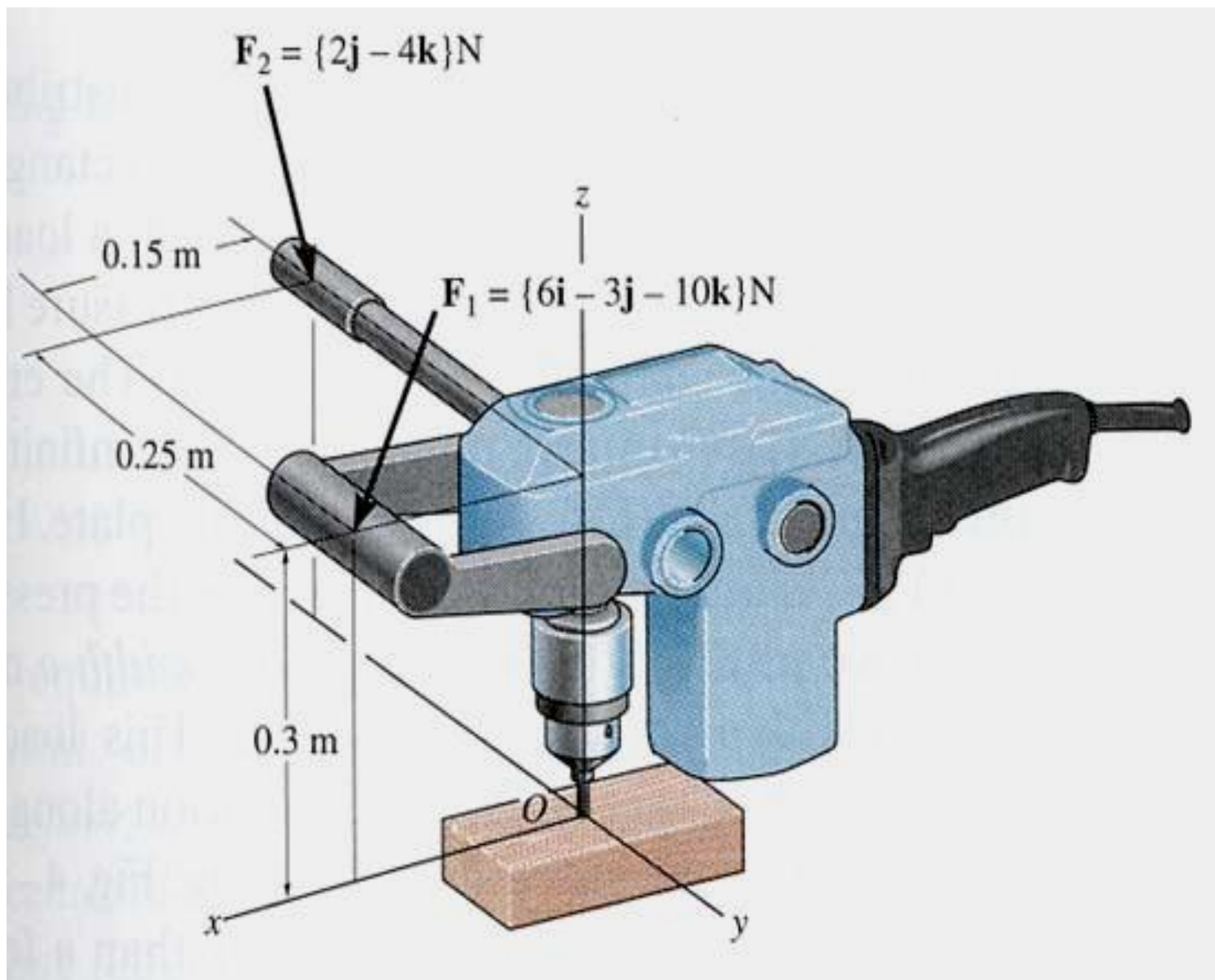
b) Find $M_{RO} = \Sigma M_C + \Sigma (r_i \times F_i)$

Where,

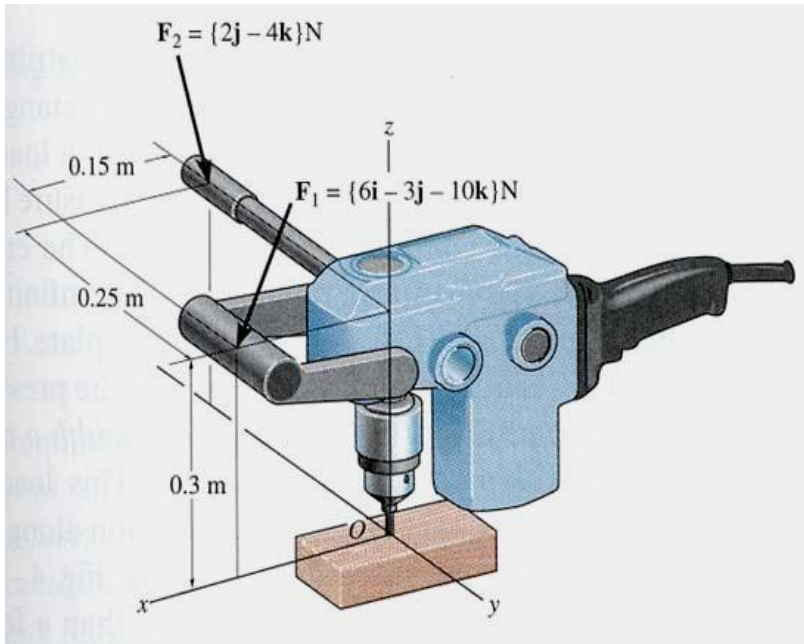
F_i are the individual forces in Cartesian vector notation (CVN).

M_C are any free couple moments in CVN (none in this example).

r_i are the position vectors from the point O to any point on the line of action of F_i .



SOLUTION



$$F_1 = \{6\mathbf{i} - 3\mathbf{j} - 10\mathbf{k}\} \text{ N}$$

$$F_2 = \{0\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}\} \text{ N}$$

$$F_{RO} = \{6\mathbf{i} - 1\mathbf{j} - 14\mathbf{k}\} \text{ N}$$

$$\mathbf{r}_1 = \{0.15\mathbf{i} + 0.3\mathbf{k}\} \text{ m}$$

$$\mathbf{r}_2 = \{-0.25\mathbf{j} + 0.3\mathbf{k}\} \text{ m}$$

$$\mathbf{M}_{RO} = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2$$

$$\mathbf{M}_{RO} = \left\{ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.15 & 0.0 & 0.3 \\ 6 & -3 & -10 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.0 & -0.25 & 0.3 \\ 0 & 2 & -4 \end{vmatrix} \right\} \text{ N}\cdot\text{m}$$

$$= \{0.9\mathbf{i} + 3.3\mathbf{j} - 0.45\mathbf{k} + 0.4\mathbf{i} + 0\mathbf{j} + 0\mathbf{k}\} \text{ N}\cdot\text{m}$$

$$= \{1.3\mathbf{i} + 3.3\mathbf{j} - 0.45\mathbf{k}\} \text{ N}\cdot\text{m}$$