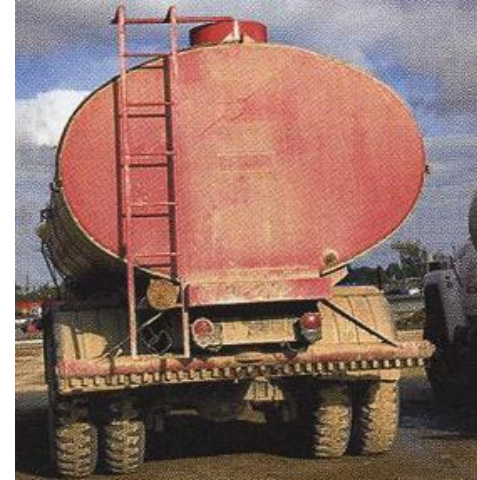


COMPOSITE BODIES

Today's Objective:

Students will be able to determine:

- a) The location of the center of gravity.
- b) The location of the center of mass.
- c) The location of the centroid using the method of composite bodies.

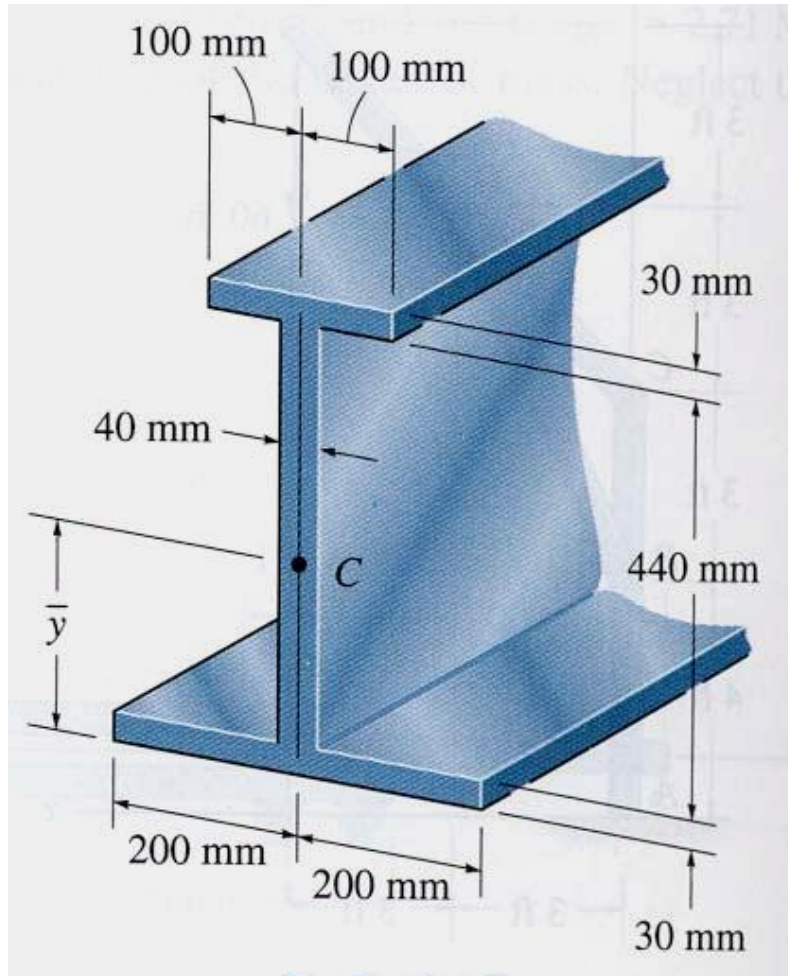


READING QUIZ

1. A composite body in this section refers to a body made of _____.
 - A) carbon fibers and an epoxy matrix
 - B) steel and concrete
 - C) a collection of “simple” shaped parts or holes
 - D) a collection of “complex” shaped parts or holes
2. The composite method for determining the location of the center of gravity of a composite body requires _____.
 - A) integration
 - B) differentiation
 - C) simple arithmetic
 - D) All of the above.



9.3 COMPOSITE BODIES



APPLICATIONS

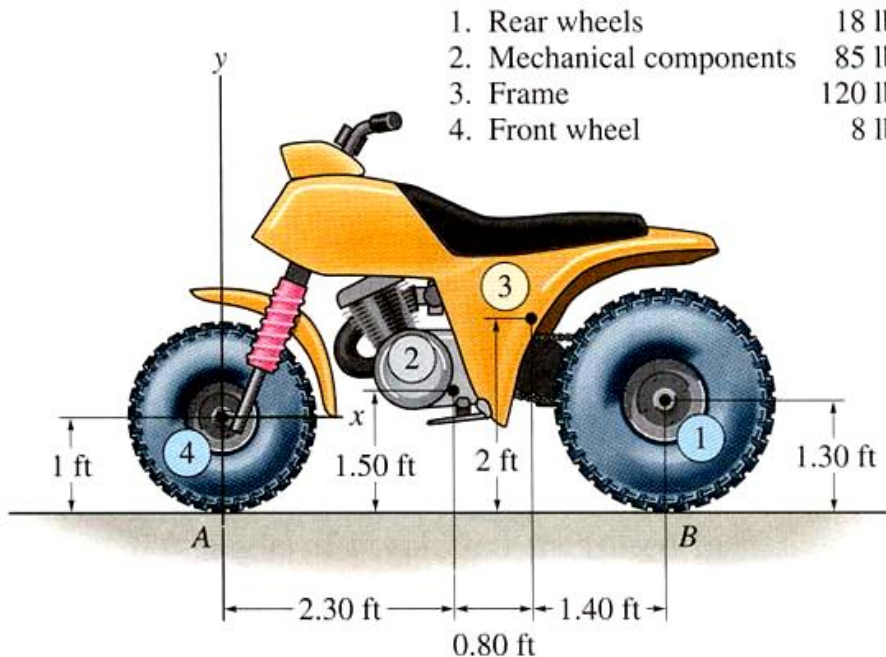
The I-beam is commonly used in building structures.

When doing a stress analysis on an I - beam, the location of the centroid is very important.

How can we easily determine the location of the centroid for a given beam shape?

APPLICATIONS

(continued)

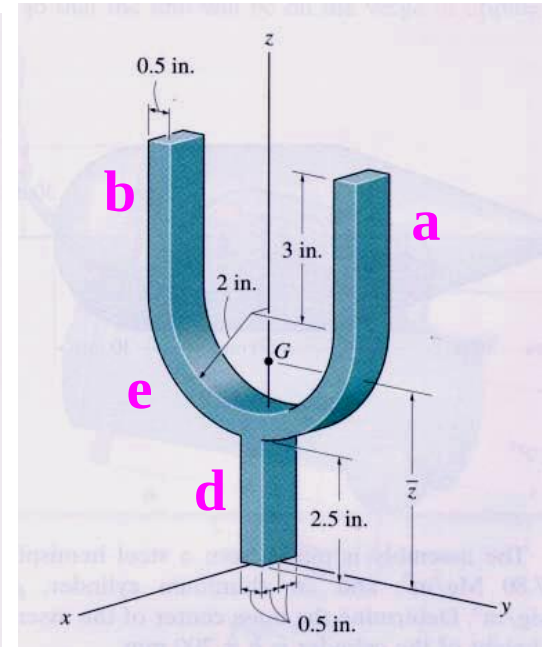
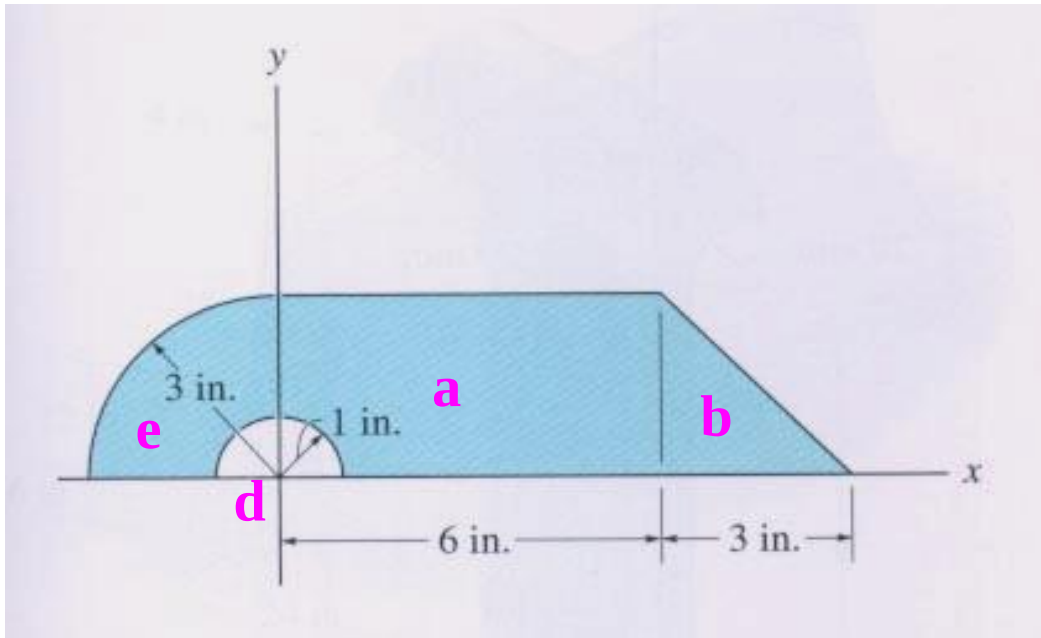


Cars, trucks, bikes, etc., are assembled using many individual components.

When designing for stability on the road, it is important to know the location of the bikes' center of gravity (CG).

If we know the weight and CG of individual components, how can we determine the location of the CG of the assembled unit?

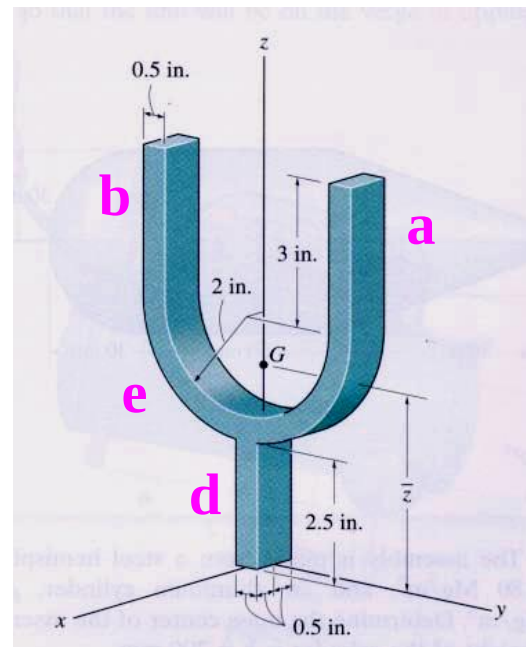
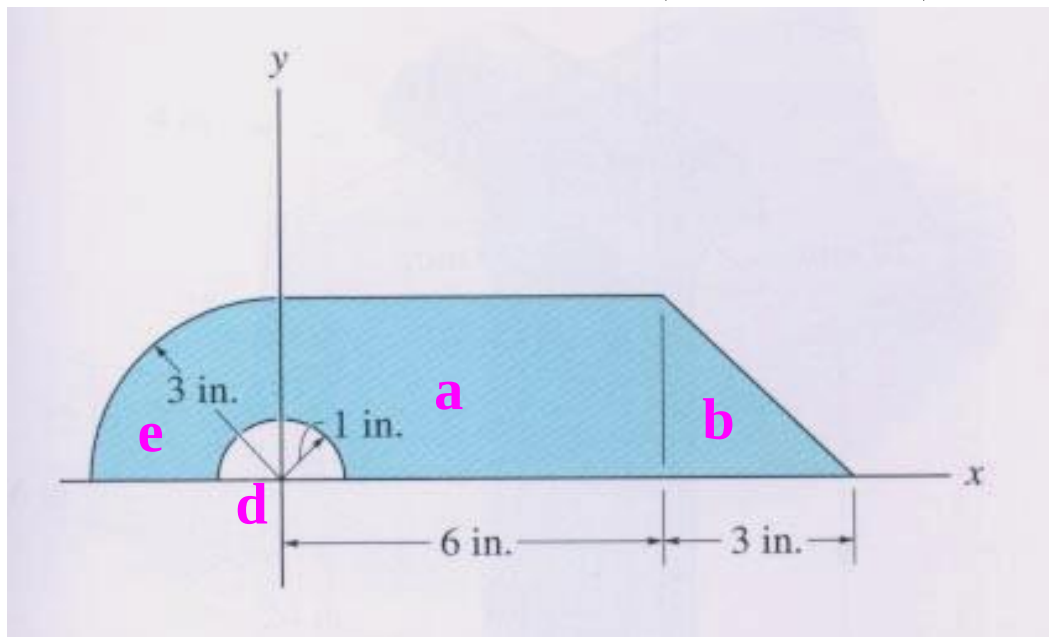
CONCEPT OF A COMPOSITE BODY



Many industrial objects can be considered as composite bodies made up of a series of connected “simpler” shaped parts or holes, like a rectangle, triangle, and semicircle.

Knowing the location of the centroid, C , or center of gravity, G , of the simpler shaped parts, we can easily determine the location of the C or G for the more complex composite body.

CONCEPT OF A COMPOSITE BODY (continued)



This can be done by considering each part as a “particle” and following the procedure as described in Section 9.1.

This is a simple, effective, and practical method of determining the location of the centroid or center of gravity.

STEPS FOR ANALYSIS

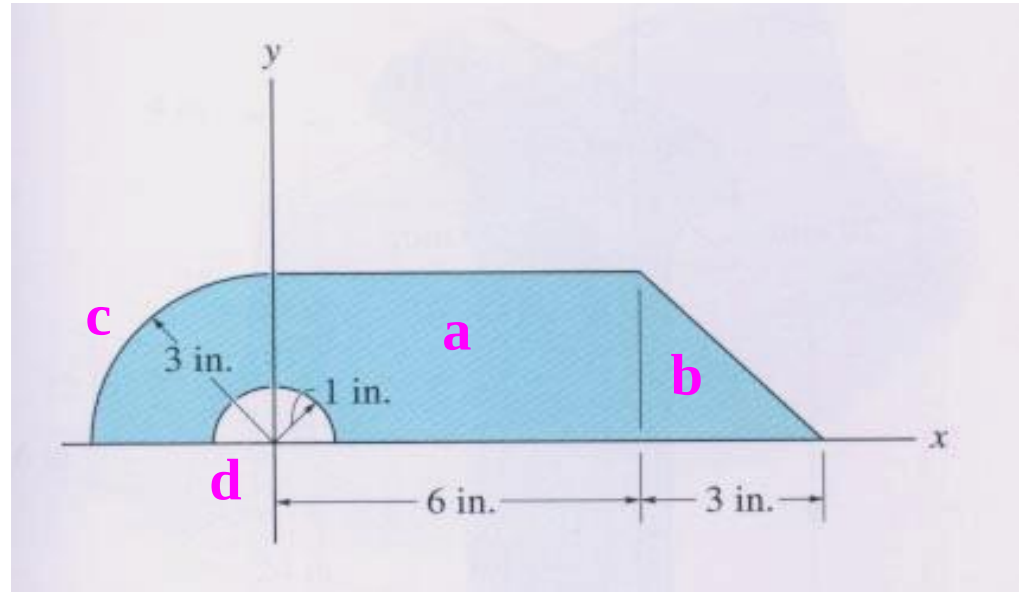
1. Divide the body into pieces that are known shapes.
Holes are considered as pieces with negative weight or size.
2. Make a table with the first column for segment number, the second column for weight, mass, or size (depending on the problem), the next set of columns for the moment arms, and, finally, several columns for recording results of simple intermediate calculations.
3. Fix the coordinate axes, determine the coordinates of the center of gravity of centroid of each piece, and then fill-in the table.
4. Sum the columns to get $\bar{x}$, $\bar{y}$, and $\bar{z}$. Use formulas like
$$\bar{x} = (\sum \tilde{x}_i A_i) / (\sum A_i) \text{ or } \bar{x} = (\sum \tilde{x}_i W_i) / (\sum W_i)$$
This approach will become clear by doing examples!

EXAMPLE

Given: The part shown.

Find: The centroid of the part.

Plan: Follow the steps for analysis.

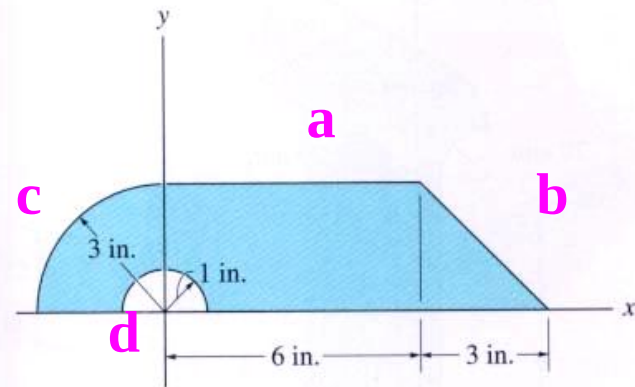


Solution:

1. This body can be divided into the following pieces:
rectangle (a) + triangle (b) + quarter circular (c) –
semicircular area (d)

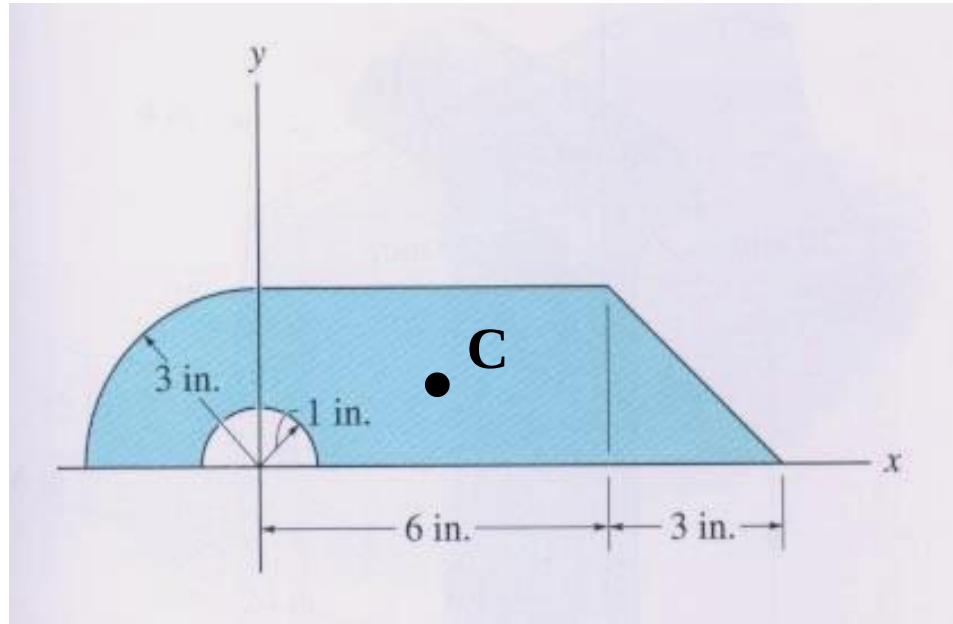
EXAMPLE (continued)

Steps 2 & 3: Make up and fill the table using parts a, b, c, and d.



Segment	Area A (in ²)	$\tilde{x}$ (in)	$\tilde{y}$ (in)	$A \tilde{x}$ (in ³)	$A \tilde{y}$ (in ³)
Rectangle	18	3	1.5	54	27
Triangle	4.5	7	1	31.5	4.5
Q. Circle	$9\pi/4$	$-4(3)/(3\pi)$	$4(3)/(3\pi)$	-9	9
Semi-Circle	$-\pi/2$	0	$4(1)/(3\pi)$	0	-2/3
Σ	28.0			76.5	39.83

EXAMPLE (continued)



4. Now use the table data and these formulas to find the coordinates of the centroid.

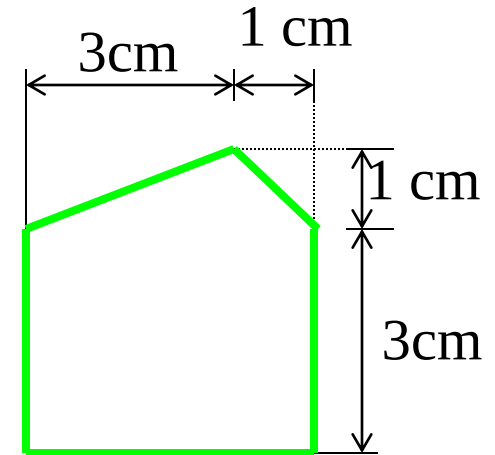
$$\bar{x} = (\Sigma \tilde{x} A) / (\Sigma A) = 76.5 \text{ in}^3 / 28.0 \text{ in}^2 = 2.73 \text{ in}$$

$$\bar{y} = (\Sigma \tilde{y} A) / (\Sigma A) = 39.83 \text{ in}^3 / 28.0 \text{ in}^2 = 1.42 \text{ in}$$

CONCEPT QUIZ

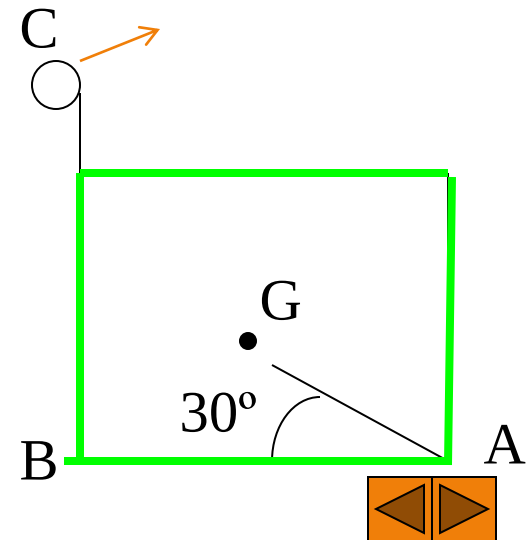
1. Based on the typical centroid information available in handbooks, what are the minimum number of segments you will have to consider for determining the centroid of the given area?

1, 2, 3, or 4



2. A storage box is tilted up to clean the rug underneath the box. It is tilted up by pulling the handle C, with edge A remaining on the ground. What is the maximum angle of tilt (measured between bottom AB and the ground) possible before the box tips over?

A) 30° B) 45° C) 60° D) 90°



GROUP PROBLEM SOLVING

Given: Two blocks of different materials are assembled as shown.

The densities of the materials are:

$$\rho_A = 150 \text{ lb / ft}^3 \text{ and}$$

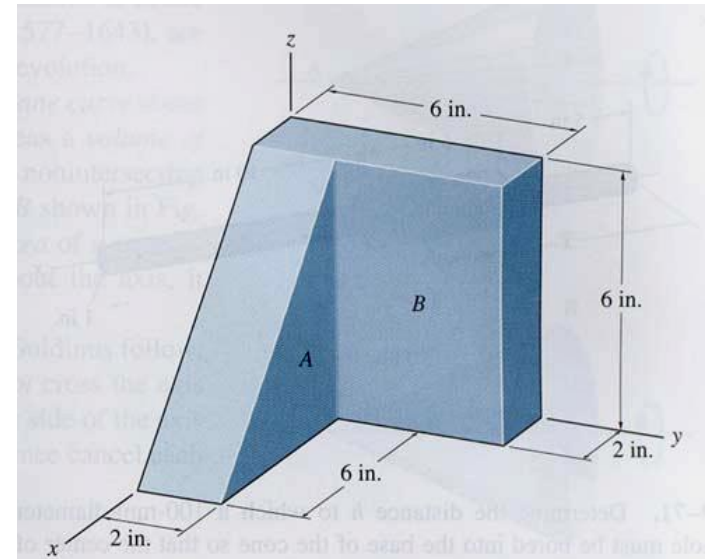
$$\rho_B = 400 \text{ lb / ft}^3.$$

Find: The center of gravity of this assembly.

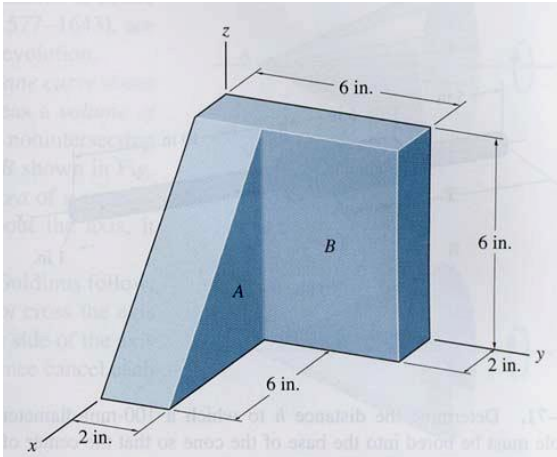
Plan: Follow the steps for analysis.

Solution

1. In this problem, the blocks A and B can be considered as two segments.



GROUP PROBLEM SOLVING (continued)



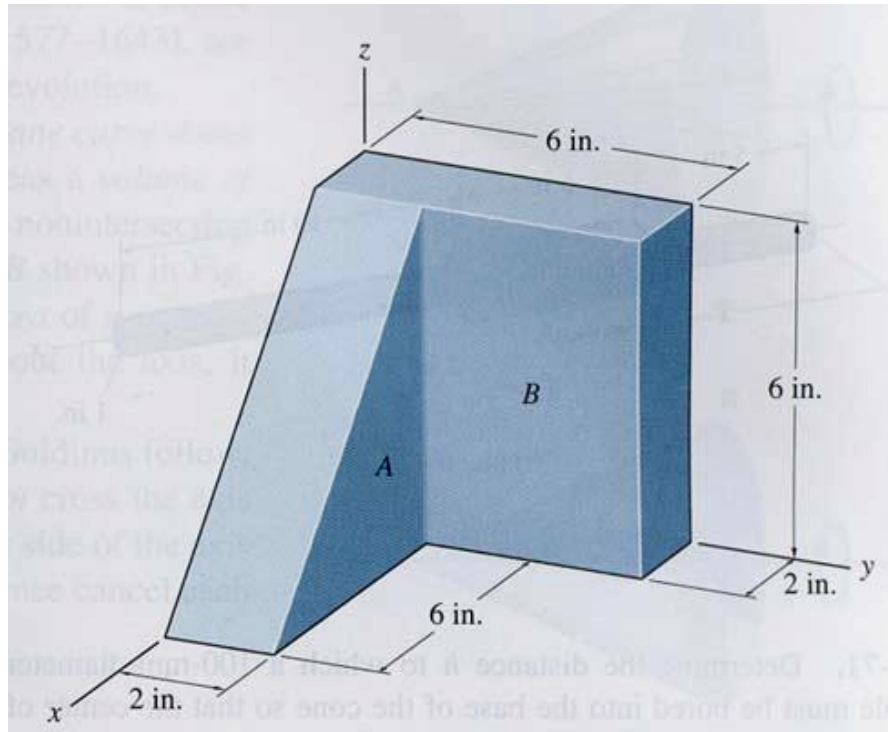
$$\text{Weight} = w = \rho (\text{Volume in ft}^3)$$

$$w_A = 150 (0.5) (6) (6) (2) / (12)^3 = 3.125 \text{ lb}$$

$$w_B = 400 (6) (6) (2) / (12)^3 = 16.67 \text{ lb}$$

Segment	w (lb)	$\tilde{x}$ (in)	$\tilde{y}$ (in)	$\tilde{z}$ (in)	w $\tilde{x}$ (lb·in)	w $\tilde{y}$ (lb·in)	w $\tilde{z}$ (lb·in)
A	3.125	4	1	2	12.5	3.125	6.25
B	16.67	1	3	3	16.67	50.00	50.00
Σ	19.79				29.17	53.12	56.25

GROUP PROBLEM SOLVING (continued)



$$\bar{x} = (\Sigma \tilde{x} w) / (\Sigma w) = 29.17/19.79 = 1.47 \text{ in}$$

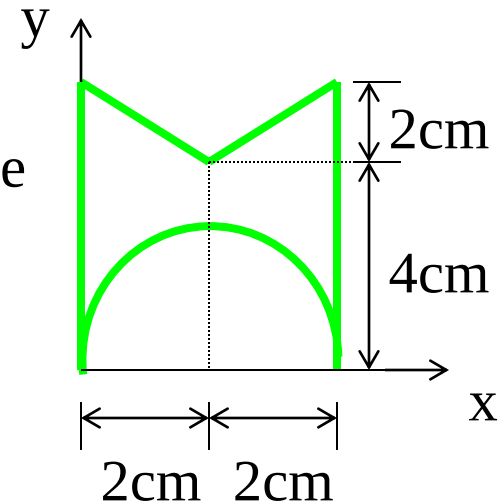
$$\bar{y} = (\Sigma \tilde{y} w) / (\Sigma w) = 53.12/19.79 = 2.68 \text{ in}$$

$$\bar{z} = (\Sigma \tilde{z} w) / (\Sigma w) = 56.25 / 19.79 = 2.84 \text{ in}$$

ATTENTION QUIZ

1. A rectangular area has semicircular and triangular cuts as shown. For determining the centroid, what is the minimum number of pieces that you can use?

A) Two B) Three
C) Four D) Five



2. For determining the centroid of the area, two square segments are considered; square ABCD and square DEFG. What are the coordinates $(\bar{x}, \bar{y})$ of the centroid of square DEFG?

A) (1, 1) m B) (1.25, 1.25) m
C) (0.5, 0.5) m D) (1.5, 1.5) m

