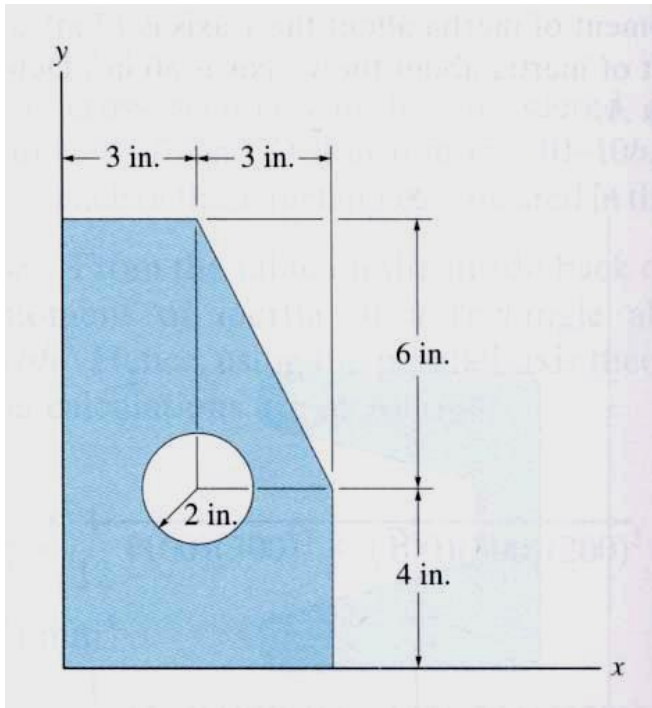


10.5 MOMENT OF INERTIA FOR A COMPOSITE AREA



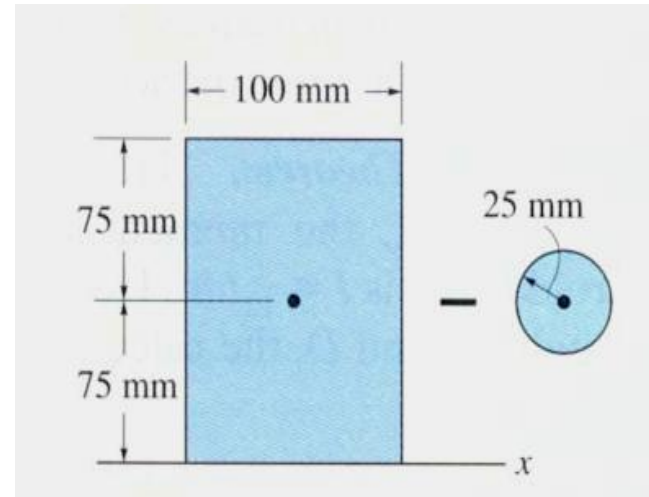
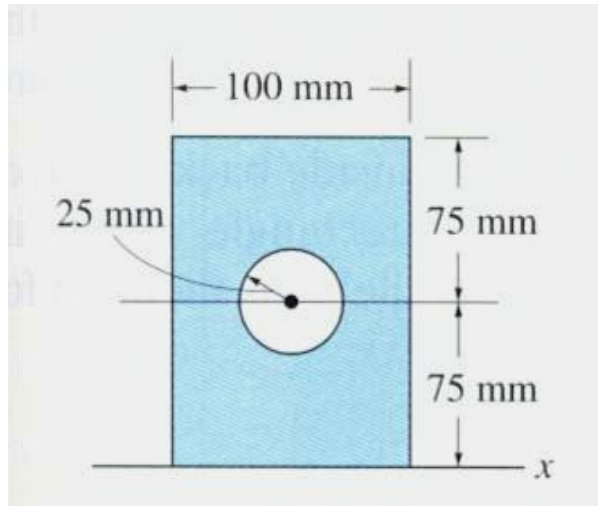
A composite area is made by adding or subtracting a series of “simple” shaped areas like rectangles, triangles, and circles.

For example, the area on the left can be made from a rectangle minus a triangle and circle.

The MoI of these “simpler” shaped areas about their centroidal axes are found in most engineering handbooks as well as the inside back cover of the textbook.

Using these data and the parallel-axis theorem, the MoI for a composite area can easily be calculated.

Steps for calculating MoI for composite sections



1. [Divide the given area](#) into its simpler shaped parts.
2. [Locate the centroid of each part and indicate the perpendicular distance](#) from each centroid to the desired reference axis.
3. Determine the MoI of each “simpler” shaped part about the desired reference axis using the parallel-axis theorem $(I_X = I_{X'} + A (d_y)^2)$.
4. MoI of entire area about the reference axis is determined by performing an [algebraic summation](#) of the individual MoIs obtained in Step 3.

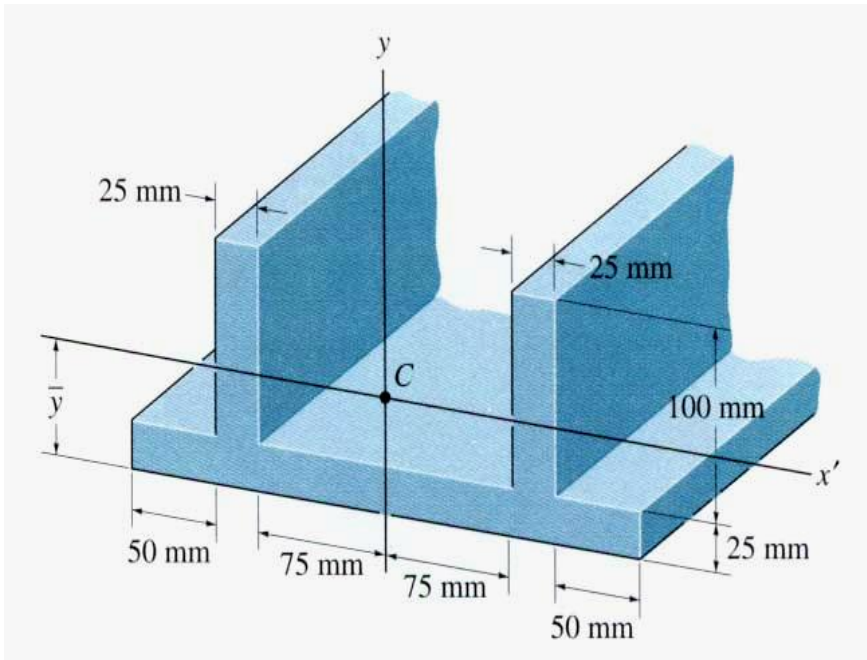
A 3D perspective view of a Z-section. The section has a total height of 100 mm and a total width of 150 mm. The flanges are 25 mm thick. The web thickness is 25 mm. The distance from the outer edge of the top flange to the center of the web is 75 mm. The distance from the outer edge of the bottom flange to the center of the web is 75 mm. The distance from the outer edge of the left flange to the center of the web is 50 mm. The distance from the outer edge of the right flange to the center of the web is 50 mm. The centroid is labeled C . The y -axis is vertical, and the x' -axis is horizontal, passing through C . The $\bar{y}$ axis is also shown, representing the distance from the top edge to the centroid.

Find: The moment of inertia of the area about the y-axis and the radius of gyration k_y .

A diagram of a 1D lattice with three sites. The sites are represented by blue vertical bars on a horizontal base. The central site is labeled [1], the site to its left is labeled [2], and the site to its right is labeled [3].

1. The cross-section can be divided into 3 rectangles ([1], [2], [3]) as shown.
2. The centroids of these three rectangles are in their center. The distances from these centers to the y-axis are 0 mm, 87.5 mm, and 87.5 mm, respectively.

EXAMPLE 10.3 (continued)



3. From the inside back cover of the book, the MoI of a rectangle about its centroidal axis is $(1/12) b h^3$.

$$\begin{aligned} I_{y[1]} &= (1/12) (25\text{mm})(300\text{mm})^3 \\ &= 56.25 (10^6) \text{ mm}^4 \end{aligned}$$

Using the parallel-axis theorem,

$$\begin{aligned} I_{Y[2]} &= I_{Y[3]} = I_{Y'} + A (d_X)^2 \\ &= (1/12) (100) (25)^3 + (25) (100) (87.5)^2 \\ &= 19.27 (10^6) \text{ mm}^4 \end{aligned}$$

EXAMPLE 10.3 (continued)

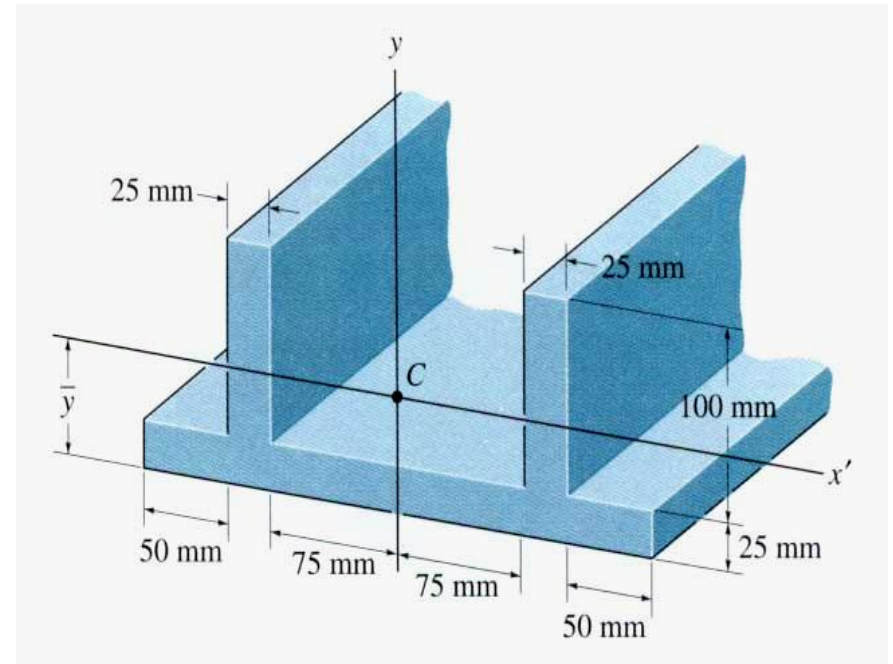
$$\begin{aligned} 4. \quad I_y &= I_{y1} + I_{y2} + I_{y3} \\ &= 94.8 (10^6) \text{ mm}^4 \end{aligned}$$

5. Radius of gyration

$$k_y = \sqrt{(I_y / A)}$$

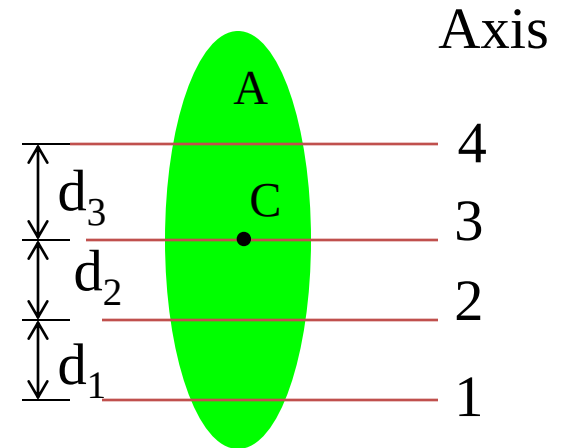
$$A = 300(25) + 25(100) + 25(100) = 12,500 \text{ mm}^2$$

$$k_y = \sqrt{(94.79)(10^6) / (12500)} = 87.1 \text{ mm}$$



CONCEPT QUIZ

1. For the area A , we know the centroid's (C) location, area, distances between the four parallel axes, and the MoI about axis 1. We can determine the [MoI about axis 2](#) by applying the parallel axis theorem ____ .

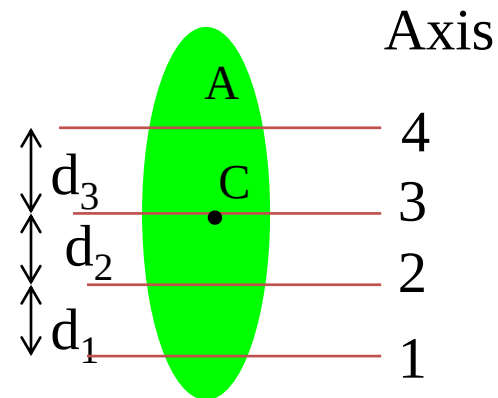


- A) directly between the axes 1 and 2.
- B) between axes 1 and 3 and then between the axes 3 and 2.
- C) between axes 1 and 4 and then axes 4 and 2.
- D) None of the above.

CONCEPT QUIZ (continued)

2. For the same case, consider the MoI about each of the four axes. About which axis will the MoI be the smallest number?

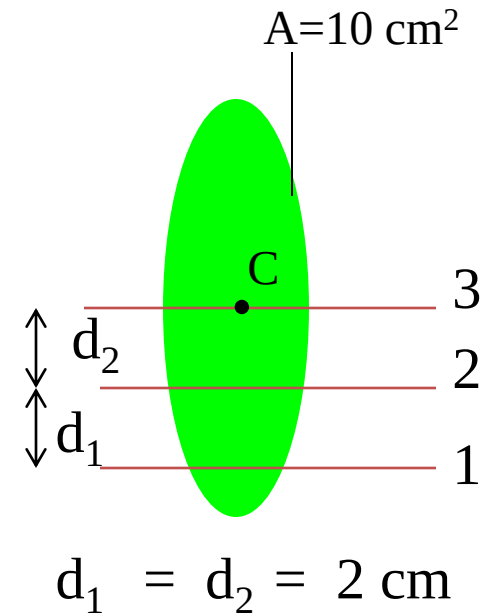
- A) Axis 1
- B) Axis 2
- C) Axis 3
- D) Axis 4
- E) Can not tell.



ATTENTION QUIZ

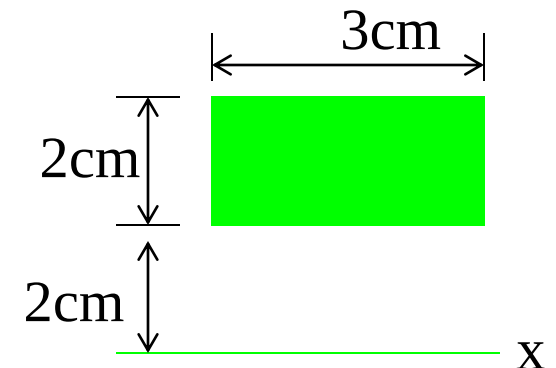
1. For the given area, the moment of inertia about axis 1 is 200 cm^4 . What is the MoI about axis 3 (the centroidal axis)?

- A) 90 cm^4 B) 110 cm^4
C) 60 cm^4 D) 40 cm^4



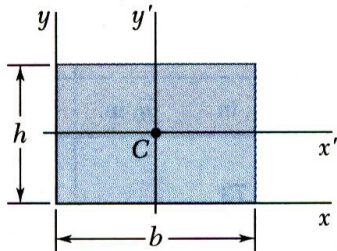
2. The moment of inertia of the rectangle about the x-axis equals

- A) 8 cm^4 . B) 56 cm^4 .
C) 24 cm^4 . D) 26 cm^4 .



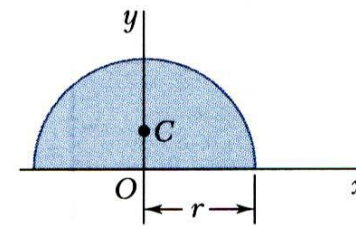
Moments of Inertia of simple shapes

Rectangle



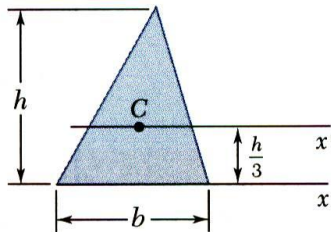
$$\begin{aligned}\bar{I}_{x'} &= \frac{1}{12}bh^3 \\ \bar{I}_{y'} &= \frac{1}{12}b^3h \\ I_x &= \frac{1}{3}bh^3 \\ I_y &= \frac{1}{3}b^3h \\ J_C &= \frac{1}{12}bh(b^2 + h^2)\end{aligned}$$

Semicircle



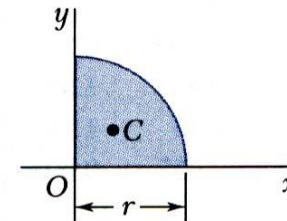
$$\begin{aligned}I_x &= I_y = \frac{1}{8}\pi r^4 \\ J_O &= \frac{1}{4}\pi r^4\end{aligned}$$

Triangle



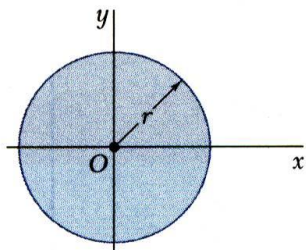
$$\begin{aligned}\bar{I}_{x'} &= \frac{1}{36}bh^3 \\ I_x &= \frac{1}{12}bh^3\end{aligned}$$

Quarter circle



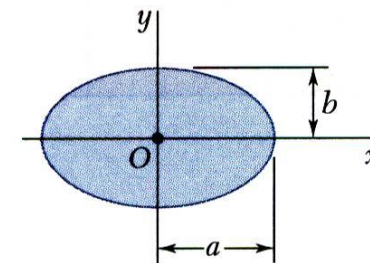
$$\begin{aligned}I_x &= I_y = \frac{1}{16}\pi r^4 \\ J_O &= \frac{1}{8}\pi r^4\end{aligned}$$

Circle



$$\begin{aligned}\bar{I}_x &= \bar{I}_y = \frac{1}{4}\pi r^4 \\ J_O &= \frac{1}{2}\pi r^4\end{aligned}$$

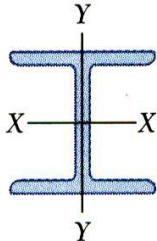
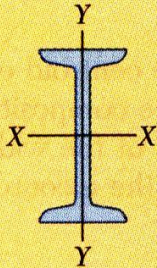
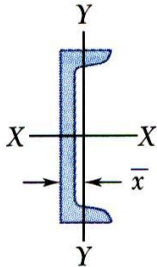
Ellipse



$$\begin{aligned}\bar{I}_x &= \frac{1}{4}\pi ab^3 \\ \bar{I}_y &= \frac{1}{4}\pi a^3b \\ J_O &= \frac{1}{4}\pi ab(a^2 + b^2)\end{aligned}$$

Moments of Inertia of standard sections

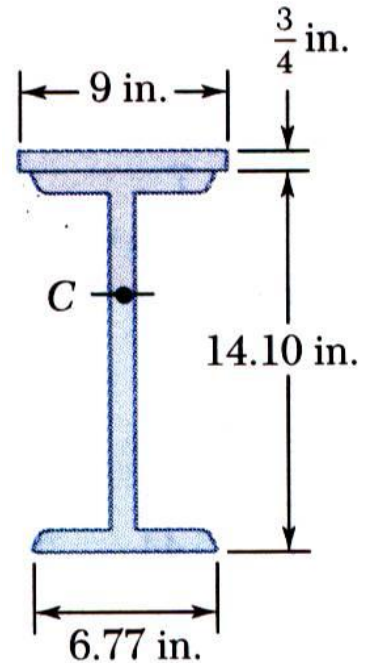
(see steel design handbook for details)

		Designation	Area mm ²	Depth mm	Width mm	Axis X-X			Axis Y-Y		
						$\bar{I}_x$ 10 ⁶ mm ⁴	$\bar{k}_x$ mm	$\bar{y}$ mm	$\bar{I}_y$ 10 ⁶ mm ⁴	$\bar{k}_y$ mm	$\bar{x}$ mm
W Shapes (Wide-Flange Shapes)		W460 × 113†	14400	463	280	554	196.3		63.3	66.3	
		W410 × 85	10800	417	181	316	170.7		17.94	40.6	
		W360 × 57	7230	358	172	160.2	149.4		11.11	39.4	
		W200 × 46.1	5890	203	203	45.8	88.1		15.44	51.3	
S Shapes (American Standard Shapes)		S460 × 81.4†	10390	457	152	335	179.6		8.66	29.0	
		S310 × 47.3	6032	305	127	90.7	122.7		3.90	25.4	
		S250 × 37.8	4806	254	118	51.6	103.4		2.83	24.2	
		S150 × 18.6	2362	152	84	9.2	62.2		0.758	17.91	
C Shapes (American Standard Channels)		C310 × 30.8†	3929	305	74	53.7	117.1		1.615	20.29	17.73
		C250 × 22.8	2897	254	65	28.1	98.3		0.949	18.11	16.10
		C200 × 17.1	2181	203	57	13.57	79.0		0.549	15.88	14.50
		C150 × 12.2	1548	152	48	5.45	59.4		0.288	13.64	13.00

Sample Problem 10.4

The strength of a W14x38 rolled steel beam is increased by attaching a plate to its upper flange.

Determine the moment of inertia and radius of gyration with respect to an axis which is parallel to the plate and passes through the centroid of the section.



SOLUTION:

- Determine location of the centroid of composite section with respect to a coordinate system with origin at the centroid of the beam section.
- Apply the parallel axis theorem to determine moments of inertia of beam section and plate with respect to composite section centroidal axis.
- Calculate the radius of gyration from the moment of inertia of the composite section.

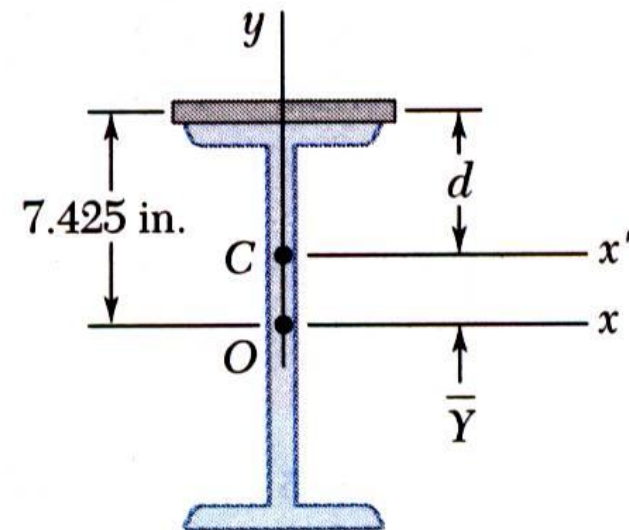
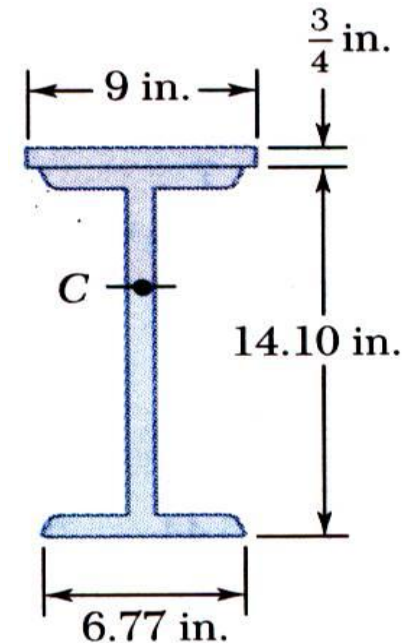
Sample Problem 10.4

SOLUTION:

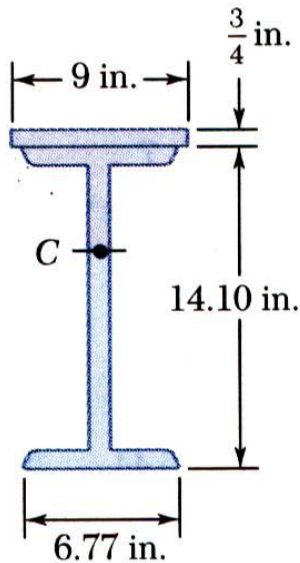
- Determine location of the centroid of composite section with respect to a coordinate system with origin at the centroid of the beam section.

Section	A, in^2	$\bar{y}, \text{in.}$	$\bar{y}A, \text{in}^3$
Plate	6.75	7.425	50.12
Beam Section	11.20	0	0
	$\sum A = 17.95$		$\sum \bar{y}A = 50.12$

$$\bar{Y} = \frac{\sum \bar{y}A}{\sum A} = \frac{50.12 \text{ in}^3}{17.95 \text{ in}^2} = 2.792 \text{ in.}$$



Sample Problem 10.4



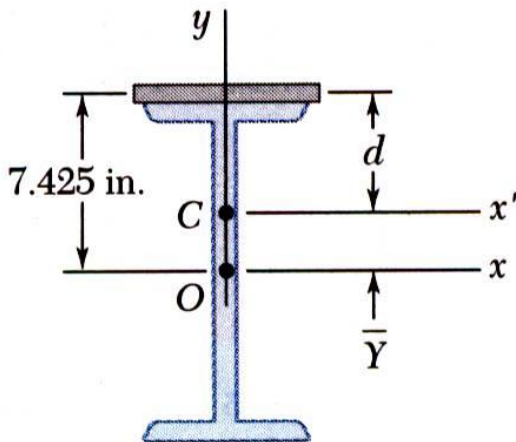
- Apply the parallel axis theorem to determine moments of inertia of beam section and plate with respect to composite section centroidal axis.

$$I_{x', \text{beam section}} = \bar{I}_x + A\bar{Y}^2 = 385 + (11.20)(2.792)^2 = 472.3 \text{ in}^4$$

$$I_{x', \text{plate}} = \bar{I}_x + Ad^2 = \frac{1}{12}(9)\left(\frac{3}{4}\right)^3 + (6.75)(7.425 - 2.792)^2 = 145.2 \text{ in}^4$$

$$I_{x'} = I_{x', \text{beam section}} + I_{x', \text{plate}} = 472.3 + 145.2$$

$$I_{x'} = 618 \text{ in}^4$$



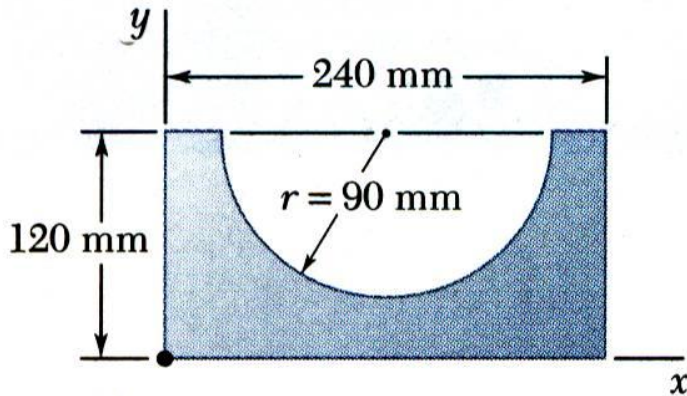
- Calculate the radius of gyration from the moment of inertia of the composite section.

$$k_{x'} = \sqrt{\frac{I_{x'}}{A}} = \frac{617.5 \text{ in}^4}{17.95 \text{ in}^2}$$

$$k_{x'} = 5.87 \text{ in.}$$

Sample Problem 10.5

SOLUTION:



Determine the moment of inertia of the shaded area with respect to the x axis.

- Compute the moments of inertia of the bounding rectangle and half-circle with respect to the x axis.
- The moment of inertia of the shaded area is obtained by subtracting the moment of inertia of the half-circle from the moment of inertia of the rectangle.

Sample Problem 10.5

SOLUTION:

- Compute the moments of inertia of the bounding rectangle and half-circle with respect to the x axis.

Rectangle:

$$I_x = \frac{1}{3}bh^3 = \frac{1}{3}(240)(120) = 138.2 \times 10^6 \text{ mm}^4$$

Half-circle:

moment of inertia with respect to AA' ,

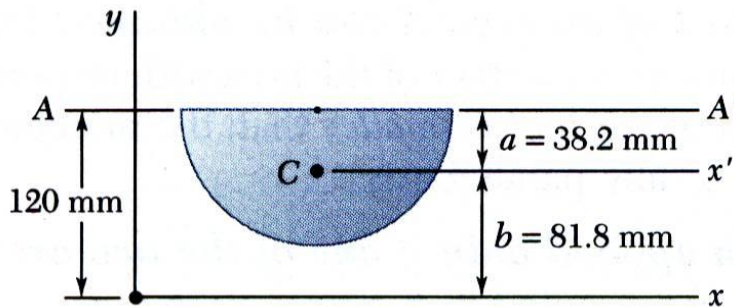
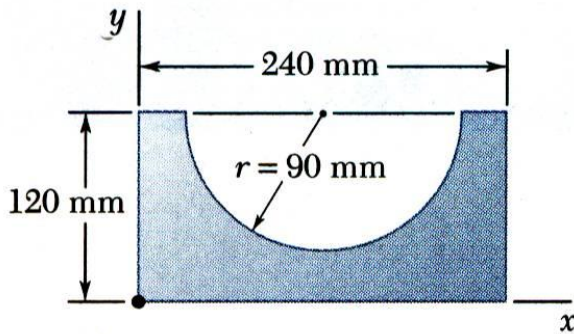
$$I_{AA'} = \frac{1}{8}\pi r^4 = \frac{1}{8}\pi(90)^4 = 25.76 \times 10^6 \text{ mm}^4$$

moment of inertia with respect to x' ,

$$\begin{aligned}\bar{I}_{x'} &= I_{AA'} - Aa^2 = (25.76 \times 10^6) - (12.72 \times 10^3) \\ &= 7.20 \times 10^6 \text{ mm}^4\end{aligned}$$

moment of inertia with respect to x ,

$$\begin{aligned}I_x &= \bar{I}_{x'} + Ab^2 = 7.20 \times 10^6 + (12.72 \times 10^3)(81.8)^2 \\ &= 92.3 \times 10^6 \text{ mm}^4\end{aligned}$$



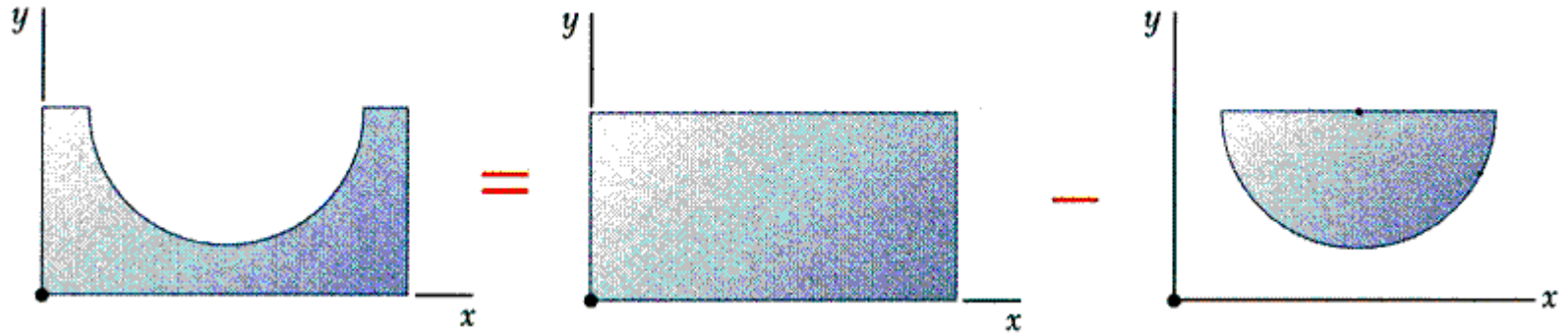
$$a = \frac{4r}{3\pi} = \frac{(4)(90)}{3\pi} = 38.2 \text{ mm}$$

$$b = 120 - a = 81.8 \text{ mm}$$

$$\begin{aligned}A &= \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(90)^2 \\ &= 12.72 \times 10^3 \text{ mm}^2\end{aligned}$$

Sample Problem 10.5

- The moment of inertia of the shaded area is obtained by subtracting the moment of inertia of the half-circle from the moment of inertia of the rectangle.



$$I_x = 138.2 \times 10^6 \text{ mm}^4 - 92.3 \times 10^6 \text{ mm}^4$$

$$I_x = 45.9 \times 10^6 \text{ mm}^4$$