



Final exam ordinary differential equations

- موعد الامتحان النهائي هو الخميس 2021/6/24 الساعة 9:00 صباحا
- الامتحان النهائي يتكون من 20 سؤال اختيار من متعدد و مدة الامتحان 90 دقيقة
- الاسئلة موزعة على 10 ورقات، كل ورقة تحتوي سؤالين (2)، وغير مسموح التنقل بين الاوراق
- توزيع الدرجات على الاسئلة غير متساوي (درجة السؤال تظهر على جانب السؤال)
- اذا كان هناك اكثر من اجابة صحيحة فاختار واحدة منهم وسيعتبر حلك صحيح

2021-2022

Question 1

Not yet answered

Marked out of 2

Flag question

If $y(x) = \sum_{n=0}^{\infty} a_n x^n$ is the series solution of the initial value problem $(x^2 - 1)y'' + 3xy' + xy = 0$, $y(0) = 1$, $y'(0) = 2$, then

Select one:

- ☐ a_0 and a_1 are arbitrary
- ☐ $a_0 = 1$, $a_1 = 2$
- ☐ none of the given answers is true
- ☐ $a_0 = a_1 = 0$
- ☐ $a_0 = 2$, $a_1 = 1$

Question 2

Not yet answered

Marked out of 4

Flag question

If $y(x) = \sum_{n=0}^{\infty} a_n x^n$ is the series solution of the initial value problem

$$(1+x)y'' - xy' + 2y = 0, \quad y(0) = -1, \quad y'(0) = -1,$$

then find a_4 .

Select one:

- ☐ $a_4 = \frac{1}{12}$
- ☐ $a_4 = \frac{1}{6}$
- ☐ $a_4 = \frac{-1}{12}$
- ☐ none of the given answers is true
- ☐ $a_4 = \frac{-1}{6}$

Question 3

Not yet answered

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Flag question

In order to solve the Bernoulli equation $y' + \sin(t)y = ty^{9/4}$ we use the transformation

Select one:

- ☐ none of the given answers is true
- ☐ $v = y^{5/4}$
- ☐ $v = y^{-5/4}$
- ☐ $v = y^{9/4}$
- ☐ $v = y^{-9/4}$

Question 4

Not yet answered

Marked out of 2

Flag question

If $F(s) = \frac{1}{s^2 + s - 5}$ is Laplace transform of a function $f(t)$ with $f(0) = -1$, then find $\mathcal{L}\{f'(t)\}$.

Select one:

- ☐ $\frac{-s^2 + 5}{s^2 + s - 5}$
- ☐ $\frac{s^2 + 2s + 5}{s^2 + s - 5}$
- ☐ $\frac{s^2 - 5}{s^2 + s - 5}$
- ☐ none of the given answers is true
- ☐ $\frac{-s^2 + 2s - 5}{s^2 + s - 5}$

Question 5

Not yet answered

Marked out of 3

Flag question

Write $f(t) = \begin{cases} t+1, & 0 \leq x < 1, \\ 3, & x \geq 1, \end{cases}$ in terms of unit step function.

Select one:

- ☐ none of the given answers is true
- ☐ $f(t) = t + 1 - (t - 2)u_1(t)$
- ☐ $f(t) = t - 1 - (t + 2)u_1(t)$
- ☐ $f(t) = t + 1 - (t + 4)u_1(t)$
- ☐ $f(t) = t - 1 - (t - 4)u_1(t)$

Question 6

Not yet answered

Marked out of 3

Flag question

Find a suitable form for particular solution of the differential equation

$$y^{(4)} - 5y^{(3)} + 4y'' - 20y' = t + t^2 e^{-5t} + \sin(t).$$

Select one:

- ☐ $y_p = t(A_1 t + A_0) + e^{-5t}(B_2 t^2 + B_1 t + B_0) + C_0 \cos(t) + D_0 \sin(t)$
- ☐ none of the given answers is true
- ☐ $y_p = t(A_1 t + A_0) + t e^{-5t}(B_2 t^2 + B_1 t + B_0) + C_0 \cos(t) + D_0 \sin(t)$
- ☐ $y_p = t(A_1 t + A_0) + e^{-5t}(B_2 t^2 + B_1 t + B_0) + t[C_0 \cos(t) + D_0 \sin(t)]$
- ☐ $y_p = t(A_1 t + A_0) + t e^{-5t}(B_2 t^2 + B_1 t + B_0) + t[C_0 \cos(t) + D_0 \sin(t)]$

Question 7

Not yet answered

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Flag question

Use the method of reduction of order to find the general of the differential equation

$$(t^2 - 5)y'' - 2ty' + 2y = 12(t + 1)(t^2 - 5)^2$$

where $y_1(t) = t$ is a solution of

$$(t^2 - 5)y'' - 2ty' + 2y = 0.$$

Select one:

- ☐ none of the given answers is true
- ☐ $y = c_1 t + c_2(t^2 + 5) + t^5 + 2t^4 + 10t^3 + 30t^2$
- ☐ $y = c_1 t + c_2(t^2 - 5) + t^5 + 2t^4 + 10t^3 + 30t^2$
- ☐ $y = c_1 t + c_2(t^2 - 5) + t^5 + 2t^4 - 10t^3 - 30t^2$
- ☐ $y = c_1 t + c_2(t^2 + 5) + t^5 + 2t^4 - 10t^3 - 30t^2$

Question 8

Not yet answered

Marked out of 2

Flag question

The integrating factor of the first order linear equation $e^{5t}y' + e^{t-1}y = \sin(t)$ is

Select one:

- ☐ none of the given answers is true
- ☐ $\mu(t) = \exp\left[\int e^{t-1} dt\right]$
- ☐ $\mu(t) = \exp\left[\int e^{-4t+1} dt\right]$
- ☐ $\mu(t) = \exp\left[\int e^{t+1} dt\right]$
- ☒ $\mu(t) = \exp\left[\int e^{-4t-1} dt\right]$

Question 9

Not yet answered

Marked out of 3

Flag question

Find an integrating factor for the non-exact equation $(1 + 5ye^{5x-5y})dx + (e^{5x-5y} + 5x)dy = 0$.

Select one:

- ☐ $\mu(x) = e^{-5x}$
☐ $\mu(x) = e^{5x}$
☐ $\mu(y) = e^{5y}$
☐ none of the given answers is true
☐ $\mu(y) = e^{-5y}$

Question 10

Not yet answered

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If $y = \sum_{n=0}^{\infty} a_n x^n$, then $y' + (5x - 2)y =$

Select one:

- ☐ none of the given answers is true
☐ $-2a_0 + a_1 + \sum_{n=1}^{\infty} [(n+1)a_{n+1} - 2a_n + 5a_{n-1}]x^n$
☐ $2a_0 + a_1 + \sum_{n=1}^{\infty} [(n+1)a_{n+1} + 2a_n + 5a_{n-1}]x^n$
☐ $\sum_{n=0}^{\infty} [(n+1)a_{n+1} + 2a_n + 5a_{n-1}]x^n$
☐ $\sum_{n=0}^{\infty} [(n+1)a_{n+1} - 2a_n + 5a_{n-1}]x^n$

Question 11

Not yet answered

Marked out of 3

Flag question

Find the general solution of the differential equation $8y'' + 4y' + 5y = 0$.

Select one:

- ☐ none of the given answers is true
☐ $y = e^{t/4}[c_1 \cos(3t/4) + c_2 \sin(3t/4)]$
☐ $y = e^t[c_1 \cos(t/4) + c_2 \sin(t/4)]$
☐ $y = e^{-t/4}[c_1 \cos(3t/4) + c_2 \sin(3t/4)]$
☐ $y = e^{-t}[c_1 \cos(t/4) + c_2 \sin(t/4)]$

Question 12

Not yet answered

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Flag question

Find the inverse Laplace transform $f(t)$ of the function $F(s) = \frac{(2s^2 - 3s + 6)e^{-2s}}{(s-1)(s^2+4)}$.

Select one:

- ☐ $f(t) = u_2(t)[e^{-t+2} + \cos(2t-4) - \sin(2t-4)]$
☐ none of the given answers is true
☐ $f(t) = u_2(t)[e^{t-2} + \cos(2t-4) - \sin(2t-4)]$
☐ $f(t) = u_2(t)[e^{-t-2} + \cos(2t-4) - \sin(2t-4)]$
☐ $f(t) = u_2(t)[e^t + \cos(2t) - \sin(2t)]$

Question 13

Not yet answered

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Flag question

Find the solution of the initial value problem

$$25x^2y'' + 15xy' + y = 1, \quad y(1) = -5, \quad y'(1) = -1.$$

Select one:

- ☐ $y(x) = 2 - 7x^{1/5} + \frac{1}{5}x^{1/5} \ln x$
- ☐ $y(x) = 2 - 7x^{1/5} - \frac{1}{5}x^{1/5} \ln x$
- ☐ $y(x) = 1 - 6x^{1/5} - \frac{1}{5}x^{1/5} \ln x$
- ☐ none of the given answers is true
- ☐ $y(x) = 1 - 6x^{1/5} + \frac{1}{5}x^{1/5} \ln x$

Question 14

Not yet answered

Marked out of 4

Flag question

Find the Laplace transform $Y(s) = \mathcal{L}\{y(t)\}$ of the solution of the initial value problem

$$y''' + y'' + y' + y = 3 \cos(2t), \quad y(0) = y'(0) = 0, \quad y''(0) = -2.$$

Select one:

- ☐ $Y(s) = \frac{2s^2 + 3s - 8}{(s^2 + 4)(s^3 + s^2 + s + 1)}$
- ☐ none of the given answers is true
- ☐ $Y(s) = \frac{-2s^2 + 3s - 8}{(s^2 + 4)(s^3 + s^2 + s + 1)}$
- ☐ $Y(s) = \frac{2s^2 + 3s + 8}{(s^2 + 4)(s^3 + s^2 + s + 1)}$
- ☐ $Y(s) = \frac{-2s^2 + 3s + 8}{(s^2 + 4)(s^3 + s^2 + s + 1)}$

Question 15

Not yet answered

Marked out of 3

Flag question

Find the first order equation for v if $y' = v(y)$ and y satisfies the differential equation $y^3y'' + 5y^2(y')^2 = 1$.

Select one:

- ☐ $\frac{dv}{dy} - \frac{5}{y}v = \frac{1}{y^3}v^{-1}$
- ☐ $\frac{dv}{dy} + \frac{5}{y}v = \frac{1}{y^3}v^{-1}$
- ☐ none of the given answers is true
- ☐ $\frac{dv}{dy} + \frac{1}{y}v = \frac{1}{y^3}v^{-1}$
- ☐ $\frac{dv}{dy} - \frac{1}{y}v = \frac{1}{y^3}v^{-1}$

Question 16

Not yet answered

Marked out of 3

Flag question

Which of the following is a solution of the equation $y''' - 2y'' - 16y' + 32y = 0$.

Select one:

- ☐ $y = e^{2t} - e^{-4t}$
- ☐ none of the given equations
- ☐ $y = e^{-2t} - e^{4t}$
- ☐ $y = e^{-2t} - e^{-4t}$
- ☐ $y = e^{2t} - e^{-2t}$

Question 17

Not yet answered

Marked out of 2

Flag question

Which of the following first order differential equations is homogeneous?

Select one:

- ☐ $y' = 2 \ln(t + y) + \ln(t - y)$
- ☐ $y' = \ln(t + y) + \ln(t - y)$
- ☐ none of the given answers is true
- ☒ $y' = \ln(t + y) - \ln(t - y)$
- ☐ $y' = 2 \ln(t + y) - \ln(t - y)$

Question 18

Not yet answered

Marked out of 3

Flag question

Determine a lower bound for the radius of convergence of the series solutions of

$$(x^2 + 5)y'' + (x + 5)y' + (x^2 - 5)y = 0$$

about $x = 5$.

Select one:

- ☒ $\sqrt{5}$
- ☐ $\sqrt{30}$
- ☐ 5
- ☐ none of the given answers is true
- ☐ $\sqrt{15}$

Question 19

Not yet answered

Marked out of 4

Flag question

If $y_p = u_1(t) + t^{-1}u_2(t) + t^{-3}u_3(t)$ is a particular solution of the differential equation $y''' + p_1(t)y'' + p_2(t)y' + p_3(t)y = t^{-4}$, then $u_2(t) =$

Select one:

- ☐ none of the given answers is true
- ☐ $\frac{1}{2} \ln |t|$
- ☐ $\frac{1}{2} t$
- ☐ $\frac{-1}{2} t$
- ☐ $\frac{-1}{2} \ln |t|$

Question 20

Not yet answered

Marked out of 2

Flag question

The partial fraction decomposition of $\frac{s+9}{(s-1)^2(s^2+9)}$ has the form

Select one:

- ☐ $\frac{A}{(s-1)^2} + \frac{Bs+C}{(s^2+9)}$
- ☐ $\frac{A}{(s-1)} + \frac{B}{(s-1)^2} + \frac{C}{(s^2+9)}$
- ☐ $\frac{A}{(s-1)} + \frac{B}{(s-1)^2} + \frac{Cs+D}{(s^2+9)}$
- ☐ none of the given answers is true
- ☐ $\frac{A}{(s-1)} + \frac{Bs+C}{(s^2+9)}$