

Name :

Student's number:

Teacher's name: Prof. Dr. Raid Salha, Dr. Faten Abu-Shoga

Question:	1	2	3	4	5	6	7	Total
Points:	10	16	12	20	12	20	10	100
Score:								

1. (10 points) Solve the DE $t^3 y' + 3t^2 y = 2ty^3$, $t > 0$

Bernouli

$$\left(y' + \frac{3t^x}{t^3} y = \frac{2t^x y^3}{t^3} \right) \times y^{-3}$$

$$y^{-3} y' + \frac{3}{t} y^{-2} y = \frac{2}{t^2}$$

$$z = y^{-2}$$

$$z' = -2 y^{-3} y'$$

$$\times -2 \left(-\frac{z'}{2} + \frac{3}{t} z = \frac{2}{t^2} \right)$$

$$z' + \frac{6}{t} z = -\frac{4}{t^2}$$

By Factor method

$$\mu = e^{\int \frac{6}{t} dt} = e^{6 \ln t} = t^6$$

$$t^{-6} z' - 6t^{-7} z = -4t^{-8}$$

$$\int (t^{-6} z)' = \int -4t^{-8}$$

$$t^{-6} z = \frac{-4}{-7} t^{-7} + C_1 \Rightarrow z = \frac{4 \cdot 1}{7t} + t^6 C_1$$

$$\frac{1}{y^2} = \frac{4}{7} \frac{1}{t} + \frac{t^7}{t} C_1$$

$$y = \sqrt{\frac{t}{\frac{4}{7} + t^7 C_1}} = \sqrt{\frac{7t}{4 + t^7 C_1}}$$

2. (a) (8 points) Solve the initial value problem

$$y' + (\cos t)y^2 = 0, \quad y(0) = 2.$$

separable

$$y' = -\cos t y^2$$

$$\int \frac{y'}{y^2} = \int -\cos t$$

$$-1 \times \left(-\frac{1}{y} \right) = -\sin t + C$$

$$\frac{1}{y} = \sin t - C \Rightarrow \frac{1}{2} = \sin 0 - C$$

$$\boxed{C = -\frac{1}{2}}$$

$$y = \frac{1}{\sin t + \frac{1}{2}}$$

- (b) (8 points) If the wronskian of the functions f and g is $3e^{6t}$ and if $f(t) = e^{3t}$, find $g(t)$.

$$W(f, g) = \begin{vmatrix} f & g \\ f' & g' \end{vmatrix} = \begin{vmatrix} e^{3t} & g \\ 3e^{3t} & g' \end{vmatrix} = 3e^{6t}$$

$$g' e^{3t} - 3g e^{3t} = 3e^{6t}$$

$$g' - 3g = 3e^{3t}$$

By Factor method

$$\mu = e^{\int -3} = e^{-3t}$$

$$\int (e^{-3t} g)' = \int 3 \Rightarrow e^{-3t} g = 3t + C$$

$$\boxed{g = 3te^{3t} + Ce^{3t}}$$

3. (12 points) Find the first five non-zero terms of the series solution of the DE

$$x^2 y'' + y' - 6y = 0$$

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$x^2 \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} + \sum_{n=1}^{\infty} n a_n x^{n-1} - 6 \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=2}^{\infty} n(n-1) a_n x^n + \sum_{n=1}^{\infty} n a_n x^{n-1} - 6 \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\boxed{h = n+1}$$

$$\sum_{n=2}^{\infty} n(n-1) a_n x^n + \sum_{n=0}^{\infty} (n+1) a_{n+1} x^n - 6 \sum_{n=0}^{\infty} a_n x^n = 0$$

$$a_1 + 2a_2 x - 6a_0 - 6a_1 x + \sum_{n=2}^{\infty} [n(n-1)a_n + (n+1)a_{n+1} - 6a_n] x^n = 0$$

$$a_1 - 6a_0 = 0 \Rightarrow \boxed{a_1 = 6a_0}$$

$$(2a_2 - 6a_1)x = 0 \Rightarrow \boxed{a_2 = 3a_1 = 18a_0}$$

$$\boxed{n=2} \quad (2a_2 + 3a_3 - 6a_2)x^2 = 0 \Rightarrow \boxed{a_3 = \frac{4}{3}a_2 = 24a_0}$$

$$\boxed{n=3} \quad (6a_3 + 4a_4 - 6a_3)x^3 = 0 \Rightarrow \boxed{a_4 = 0}$$

$$\boxed{n=4} \quad (12a_4 + 5a_5 - 6a_4)x^4 = 0 \Rightarrow \boxed{a_5 = 0}$$

4. (a) (12 points) Transform the differential equation $x^2 y'' + 7xy' + 5y = \frac{e^z}{x}, x > 0$ into a differential equation with constant coefficients and solve it.

$$x = e^z \Rightarrow z = \ln x$$

$$y = e^{rz}$$

$$\frac{d^2 y}{dz^2} + (7-1) \frac{dy}{dz} + 5y = 0$$

$$r^2 + 6r + 5 = 0$$

$$(r+5)(r+1) = 0$$

$$r = -5, r = -1$$

$$y_c = c_1 e^{-5z} + c_2 e^{-z} \Rightarrow y_c = c_1 x^{-5} + c_2 x^{-1}$$

$$y_p = A e^z \Rightarrow y_p = Ax + B$$

$$y = y_c + y_p = c_1 x^{-5} + c_2 x^{-1} + Ax + B$$

- (b) (8 points) Solve the DE $y^{(5)} - y' = 0$.

$$y = e^{rt}$$

$$r^5 - r = 0$$

$$y' = r e^{rt}$$

$$r(r^4 - 1) = 0$$

$$r(r^2 - 1)(r^2 + 1) = 0$$

$$r(r-1)(r+1)(r^2+1) = 0$$

$$r = 0$$

$$r = 1$$

$$r = -1$$

$$r = \pm i$$

$$y = c_1 + c_2 e^t + c_3 e^{-t} + c_4 \cos t + c_5 \sin t$$

5. (12 points) Use Laplace transform methods to solve the following initial value problem:

$$y'' + 4y' + 4y = 3e^{-2t}, \quad y(0) = 1, y'(0) = -2$$

$$s^2 Y(s) - s y(0) - y'(0) + 4s Y(s) - 4y(0) + 4Y(s) = 3L\{e^{-2t}\}$$

$$= \frac{3}{s+2}$$

$$Y(s)[s^2 + 4s + 4] - s + 2 - 4 = \frac{3}{s+2}$$

$$Y(s) = \frac{s+2-2}{(s+2)^2} + \frac{2}{(s+2)^2} + \frac{3}{(s+2)^3}$$

$\hookrightarrow \boxed{n=1}$ $\hookrightarrow \boxed{n=2}$

$$Y(s) = \frac{1}{s+2} - \frac{2}{(s+2)^2} + \frac{2}{(s+2)^2} + \frac{3}{(s+2)^3}$$

$$= e^{-2t} - 2e^{-2t} + 2e^{-2t} + \frac{3}{2}t^2 e^{-2t}$$

$$Y(s) = e^{-2t} + \frac{3}{2}t^2 e^{-2t}$$

6. (a) (10 points) Find all singular points and determine whether each one is regular and irregular for the DE

$$(1-x^2)^3 y'' + (1-x^2)^2 y' + 2(1+x)y = 0$$

$$1-x^2=0 \rightarrow x=\pm 1$$

$$\boxed{x=1}$$

$$\lim_{x \rightarrow 1} (x-1) \cdot \frac{1}{(1-x)(1+x)} = \boxed{-\frac{1}{2}}$$

$$\lim_{x \rightarrow 1} (x-1)^2 \cdot \frac{2(1+x)}{(1-x^2)^3}$$

$$= \lim_{x \rightarrow 1} \frac{(x-1)^2 \cdot 2(1+x)}{(1-x)^3 (1+x)^3} = \infty \text{ d.n.e.}$$

irregular

$$\boxed{x=-1}$$

$$\lim_{x \rightarrow -1} (x+1) \cdot \frac{1}{(1-x)(1+x)} = \boxed{\frac{1}{2}}$$

$$\lim_{x \rightarrow -1} (x+1)^2 \cdot \frac{2(1+x)}{(1-x)^3 (1+x)^3} = \boxed{\frac{1}{4}}$$

regular

- (b) (10 points) Find by using the definition the Laplace transform of the function: $f(t) = e^{-t} \sin(2t)$, $t > 0$

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} \cdot e^{-t} \sin(2t) dt$$

$$= \lim_{b \rightarrow \infty} \int_0^b e^{(-s-1)t} \sin(2t) dt$$

$$= \lim_{b \rightarrow \infty} \left[\left(-\frac{1}{2} e^{(-s-1)t} \cos 2t \right) + \frac{(-s-1)}{4} e^{(-s-1)t} \sin 2t \right] \cdot \frac{4}{4+(s+1)^2} \Big|_0^b$$

$$= \lim_{b \rightarrow \infty} \left[\frac{4}{4+(s+1)^2} \left[-\frac{1}{2} \cos 0 \right] \right] = \frac{-2}{4+(s+1)^2} \checkmark$$

$$\begin{array}{l} e^{(-s-1)t} \sin 2t \\ (-s-1)e^{(-s-1)t} \cdot \frac{-\cos 2t}{2} \\ (-s-1)^2 e^{(-s-1)t} \cdot \frac{-\sin 2t}{4} \end{array}$$

$$\begin{array}{l} e^{-\infty} = 0 \\ \left[-\frac{1}{2} \cos 2t \right] \Big|_0^b \\ \left[\frac{(-s-1)}{4} \sin 2t \right] \Big|_0^b \end{array}$$

$$\int e^{(-s-1)t} \sin 2t dt = -\frac{1}{2} e^{(-s-1)t} \cos 2t + \frac{(-s-1)}{4} e^{(-s-1)t} \sin 2t - \int \frac{(-s-1)^2}{4} e^{(-s-1)t} \sin 2t dt$$

$$\left(\frac{4+(-s-1)^2}{4} \right) \int e^{(-s-1)t} \sin 2t dt = -\frac{1}{2} e^{(-s-1)t} \cos 2t + \frac{(-s-1)}{4} e^{(-s-1)t} \sin 2t$$

with $\frac{4}{4+(s+1)^2}$

$$y = \tan x$$

$$y = \sin x$$

$$y' = \cos x$$

$$y'' = -\sin x$$

$$-\sin x + \sin x = 0$$

7. (10 points) Choose the correct answer for the following questions:

(a) Suppose $y'' + y = 0$. Which of the following is a possibility for $y = f(x)$.

A. $\tan x$. B. $\sin x$. C. $\csc x$. D. $\sec x$.

(b) The particular solution of the D.E $y'' + 9y = 6 \sin(3x)$ has the form

A. $a \cos 3x + b \sin 3x$. B. $ax \cos 3x + b \sin 3x$. $r^2 + 9 = 0$

C. $a \cos 3x + bx \sin 3x$. D. $ax \cos 3x + bx \sin 3x$. $r = \pm 3i$

(c) The D.E. $(1 - y)y'' - 2y' = \cos 2t$ is $\frac{1}{2} \cos 2t$ non linear

A. non-linear, homogeneous second order differential equation.

B. non-linear, non-homogeneous second order differential equation.

C. linear, homogeneous second order differential equation.

D. linear, non-homogeneous second order differential equation.

$$\frac{1}{t} \quad R/203$$

(d) Consider the D.E. $ty' + y = \sin t$, $y\left(\frac{\pi}{2}\right) = 1$. The largest interval in which the solution exists

A. $(0, \infty)$. B. $(-\infty, 0)$. C. $(-\infty, \infty)$. D. $(0, \frac{\pi}{2})$. $(0, \infty)$

(e) The general solution of the D.E $y'' + 4y' + 4y = 0$ is

A. $y = C_1 e^{2t} + C_2 e^{-2t}$.

B. $y = C_1 e^{2t} \cos 2t + C_2 e^{2t} \sin 2t$.

C. $y = C_1 e^{2t} + C_2 t e^{-2t}$.

D. non of the above.

$$r^2 + 4r + 4 = 0$$

$$(r+2)^2 = 0$$

repeat

$$r = -2$$

$$y = C_1 e^{-2t} + C_2 t e^{-2t}$$

Name: <i>عليه السلام</i>	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Total
Student number:	10	16	16	16	16	10	16	100
Instructor:								

[Q1] If $y_1 = x$ is one solution of DE $x^2 y'' - x(x+2)y' + (x+2)y = 0$
 find a second linearly independent solution of DE.

$$y = xv$$

$$y' = v + xv'$$

$$y'' = v' + v' + xv'' = 2v' + xv''$$

$$x^2 [2v' + xv''] - (x^2 + 2x) [v + xv'] + (x+2)(xv) = 0$$

$$2x^2 v' + \boxed{x^3 v''} - x^2 v - \boxed{x^3 v'} - 2xv - 2x^2 v' + x^2 v + 2xv = 0$$

$$x^3 v'' = x^3 v'$$

$$\int \frac{v''}{v'} = \int \frac{1}{x} \quad \text{seperab.}$$

$$\ln v' = \ln x + C_1$$

$$\int v' = \int \frac{1}{x} e^x$$

$$(v = C_1 e^x + C_2) \times x$$

$$y = vx = C_1 e^x x + \underbrace{(C_2 x)}_{y_1}$$

$$y_2 = x e^x$$

[Q2] Find the general solution of the DE

a) Solve the DE $y' + \frac{3}{x}y = x^2y^2$, $x > 0$. $\leftarrow x(y^2)$

Bernoulli: $\bar{y}^2 y' + \frac{3}{x} \bar{y}^1 = x^2$

$z = y^{-1}$
 $(z' = -y^{-2} y') \times -1$

$-z' + \frac{3}{x} z = x^2$

$z' - \frac{3}{x} z = -x^2$

By Factor method

$\mu = e^{\int -\frac{3}{x} dx} = e^{-3 \ln x} = x^{-3}$

$x^{-3} z' - 3x^{-4} z = -\frac{1}{x}$

$\int (x^{-3} z)' = \int -\frac{1}{x} \Rightarrow x^{-3} z = -\ln x + C$

$z = -x^3 \ln x + x^3 C$

$\frac{1}{y} = -x^3 \ln x + x^3 C \Rightarrow y = \frac{1}{-x^3 \ln x + x^3 C}$

b) $y^{(4)} + 81y = 0$

$y = e^{rt}$

$r^4 + 81 = 0$

$r^4 = -81 \Rightarrow r = (-1)^{\frac{1}{4}} 3$

$w_{0,3} = \frac{3}{\sqrt{2}} \pm i \frac{3}{\sqrt{2}} \Rightarrow \lambda = \frac{3}{\sqrt{2}}, \mu = \frac{3}{\sqrt{2}}$

$w_{1,2} = -\frac{3}{\sqrt{2}} \mp i \frac{3}{\sqrt{2}} \Rightarrow \lambda = -\frac{3}{\sqrt{2}}, \mu = \frac{3}{\sqrt{2}}$

$y = C_1 e^{\frac{3}{\sqrt{2}}t} \cos\left(\frac{3}{\sqrt{2}}t\right) + C_2 e^{\frac{3}{\sqrt{2}}t} \sin\left(\frac{3}{\sqrt{2}}t\right)$

$+ C_3 e^{-\frac{3}{\sqrt{2}}t} \cos\left(\frac{3}{\sqrt{2}}t\right) + C_4 e^{-\frac{3}{\sqrt{2}}t} \sin\left(\frac{3}{\sqrt{2}}t\right)$

$(-1)^{\frac{1}{4}} \quad r = \sqrt{a^2 + b^2} = \sqrt{(-1)^2 + 0^2} = 1$

$\theta = \tan^{-1}\left(\frac{0}{-1}\right) = \pi$

$k = 0, 1, 2, 3$

$(-1)^{\frac{1}{4}} = r \left[\cos\left(\frac{\pi + 2\pi k}{4}\right) + i \sin\left(\frac{\pi + 2\pi k}{4}\right) \right]$

$w_0 = \cos\left(\frac{\pi}{4}\right) + i \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}}$

$w_1 = \cos\left(\frac{3\pi}{4}\right) + i \sin\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}}$

$w_2 = \cos\left(\frac{5\pi}{4}\right) + i \sin\left(\frac{5\pi}{4}\right) = -\frac{1}{\sqrt{2}} - i \frac{1}{\sqrt{2}}$

$w_3 = \cos\left(\frac{7\pi}{4}\right) + i \sin\left(\frac{7\pi}{4}\right) = \frac{1}{\sqrt{2}} - i \frac{1}{\sqrt{2}}$

ضرب 3

[Q3] a) Find the radius of convergence of solutions of the DE

$$2x(x^2 - 2x + 2)y'' + (x - 1)y' + xy = 0 \quad \text{about } \boxed{x_0 = 2}$$

$$y'' + \frac{(x-1)}{2x(x^2-2x+2)} y' + \frac{x}{2x(x^2-2x+2)} y = 0$$

$$P(x) = \frac{x-1}{2x(x^2-2x+2)}$$

cont. $\phi \setminus \{0, 1 \pm i\}$
 $\hookrightarrow q(x)$ $\frac{1}{x^2-2x+2}$

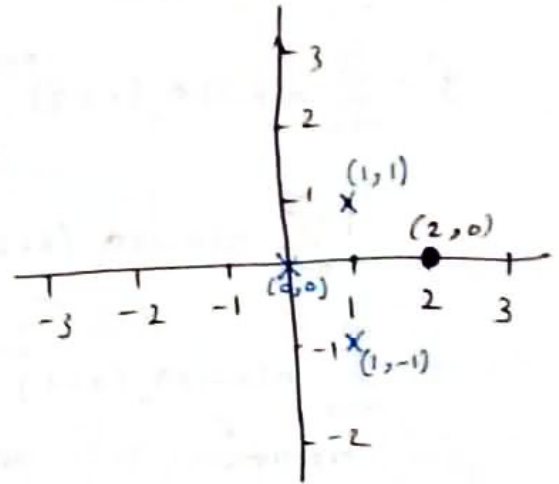
$$(1,1) \quad \boxed{x=2}$$

$$L = \sqrt{(1-0)^2 + (1-2)^2} = \sqrt{2}$$

$$L = \sqrt{(0-1)^2 + (2-1)^2} = \sqrt{2}$$

$$L = \sqrt{(0-0)^2 + (2-0)^2} = 2$$

$$\text{radius}_{\min} = \sqrt{2}$$



$$x^2 - 2x + 2 = 0$$

$$x = \frac{2 \pm \sqrt{4 - 4 \cdot 2}}{2} = 1 \pm i$$

b) Classify all singular points as regular or irregular of DE

$$x^2(1-x)y'' - 4y' = 3xy$$

$$x^2(1-x) = 0 \quad x=0, x=1$$

$$\boxed{x=0}$$

$$y'' - \frac{4}{x^2(1-x)} y' - \frac{3x}{x^2(1-x)} y = 0$$

$$\boxed{x=1}$$

$$\lim_{x \rightarrow 0} x \cdot \frac{-4}{x^2(1-x)} = \mp \infty \rightarrow \begin{cases} \lim_{x \rightarrow 0^+} = \infty \\ \lim_{x \rightarrow 0^-} = -\infty \end{cases}$$

d.n.e.

$$\lim_{x \rightarrow 1} (x-1) \cdot \frac{-4}{x^2(1-x)} = \boxed{4}$$

$$\lim_{x \rightarrow 0} x^2 \cdot \frac{-3x}{x^2(1-x)} = 0$$

irregular

$$\lim_{x \rightarrow 1} (x-1)^2 \cdot \frac{-3x}{x^2(1-x)} = \lim_{x \rightarrow 1} \frac{-3(1-x)}{x} = 0$$

regular

[Q4] Find the first three terms in each power series solution of the DE

$$y'' - xy = 0, \quad \text{about } \underline{x_0 = -2}.$$

$$y = \sum_{n=0}^{\infty} a_n (x+2)^n$$

$$y' = \sum_{n=1}^{\infty} n a_n (x+2)^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} n(n-1) a_n (x+2)^{n-2}$$

$$\sum_{n=2}^{\infty} n(n-1) a_n (x+2)^{n-2} - (x+2) \sum_{n=0}^{\infty} a_n (x+2)^n = 0$$

$$= \sum_{n=2}^{\infty} n(n-1) a_n (x+2)^{n-2} - \sum_{n=0}^{\infty} a_n (x+2)^{n+1} + 2 \sum_{n=0}^{\infty} a_n (x+2)^n = 0$$

$\downarrow$ $\downarrow$
 $n = n+2$ $n = n-1$

$$= \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} (x+2)^n - \sum_{n=1}^{\infty} a_{n-1} (x+2)^n + 2 \sum_{n=0}^{\infty} a_n (x+2)^n = 0$$

$$2a_2 + 2a_0 + \sum_{n=1}^{\infty} [(n+2)(n+1)a_{n+2} - a_{n-1} + 2a_n] (x+2)^n = 0$$

$$2a_2 + 2a_0 = 0 \Rightarrow \boxed{a_2 = -a_0}$$

$$\boxed{n=1} \rightarrow (3 \times 2 a_3 - a_0 + 2a_1) = 0 \Rightarrow a_3 = \frac{a_0}{6} - \frac{a_1}{3}$$

$$\boxed{n=2} \rightarrow (4 \times 3 a_4 - a_1 + 2a_2) = 0 \Rightarrow a_4 = \frac{a_1}{12} - \frac{a_2}{6}$$

$$= \frac{a_1}{12} + \frac{a_0}{6}$$

[Q5] a) Find the general solution of the Euler equation

$$(x+6)^2 y'' - (x+6)y' - 3y = \ln(x+6), \quad x > -6.$$

$$y = x^r$$

$$y' = r x^{r-1}$$

$$y'' = r(r-1)x^{r-2}$$

~~$$(x+6)^2 y'' - (x+6)y' - 3y = \ln(x+6)$$~~

$$x+6 = t$$

$$t^2 y'' - t y' - 3y = \ln(t)$$

$$\text{Ch. eq. } t^2 r(r-1) t^{r-2} - t r t^{r-1} - 3 t^r = 0$$

$$r(r-1) - r - 3 = 0$$

$$r^2 - 2r - 3 = 0$$

$$(r-3)(r+1) = 0$$

$$r = 3, r = -1$$

$$y = t^3 + t^{-1} = (x+6)^3 + (x+6)^{-1}$$

$$y_p = A \ln(x+6) + B$$

or

$$t = e^z, \quad z = \ln t$$

$$\frac{d^2 y}{dz^2} + (-1-1) \frac{dy}{dz} - 3y = z$$

$$r^2 - 2r - 3 = 0$$

$$r = 3, r = -1$$

$$y_c = C_1 e^{3z} + C_2 e^{-z} \Rightarrow y_c = C_1 (x+6)^3 + C_2 (x+6)^{-1}$$

$$y_p = A z + B \Rightarrow y_p = A \ln(x+6) + B$$

b) Use definition to find Laplace transform of $f(t) = t - 5$

$$L\{f(t)\} = \lim_{b \rightarrow \infty} \int_0^b (t-5) e^{-st} dt = \lim_{b \rightarrow \infty} \int_0^b (t e^{-st} - 5 e^{-st}) dt$$

$$L \int_0^\infty e^{-st} f(t) dt$$

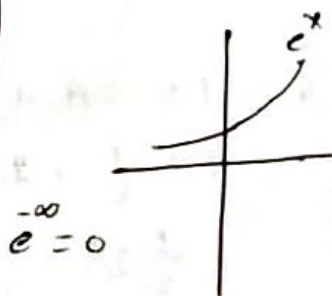
$$= \lim_{b \rightarrow \infty} \left[-\frac{t}{s} e^{-st} \Big|_0^b - \frac{e^{-st}}{s^2} \Big|_0^b + \frac{5 e^{-st}}{s} \Big|_0^b \right]$$

$$= \lim_{b \rightarrow \infty} \left[-\frac{t}{s} e^{-sb} - \frac{e^{-sb}}{s^2} + \frac{1}{s^2} + \frac{5 e^{-sb}}{s} - \frac{5}{s} \right]$$

$$= \frac{1}{s^2} - \frac{5}{s}$$

بسیار
محدود

$$\begin{array}{l} 1 \\ t \\ 1 \\ 0 \end{array} \begin{array}{l} \int \\ \downarrow \\ \downarrow \\ + \end{array} \begin{array}{l} e^{-st} \\ e^{-st} \\ -\frac{e^{-st}}{s} \\ \frac{e^{-st}}{s^2} \end{array}$$



[Q6] Use Laplace transform to solve the IVP

$$y'' + 4y' + 5y = e^t, \quad y(0) = 1, y'(0) = 2$$

$$L\{y''\} = s^2 Y(s) - s y(0) - y'(0)$$

$$L\{y'\} = s Y(s) - y(0)$$

$$s^2 Y(s) - s y(0) - y'(0) + 4s Y(s) - 4y(0) + 5Y(s) = L\{e^t\} = \frac{1}{s-1}$$

$$Y(s) [s^2 + 4s + 5] - s - 2 - 4 = \frac{1}{s-1}$$

$$Y(s) [s^2 + 4s + 5] = s + 6 + \frac{1}{s-1}$$

$$Y(s) = \frac{s+6}{s^2+4s+5-1+1} + \frac{1}{(s-1)(s^2+4s+5)}$$

$$= \frac{s+6}{(s+2)^2+1} + \frac{1}{10} \cdot \frac{1}{s-1} + \frac{\frac{13}{20}s + \frac{1}{2}}{(s+2)^2+1}$$

$$= \frac{s+2}{(s+2)^2+1} + \frac{4}{(s+2)^2+1} + \frac{1}{10} \cdot \frac{1}{s-1} + \frac{13}{20} \frac{s+2}{(s+2)^2+1} + -\frac{7}{10} \cdot \frac{1}{(s+2)^2+1}$$

$$= e^{-2t} \cos t + 4e^{-2t} \sin t + \frac{1}{10} e^t + \frac{13}{20} e^{-2t} \cos t - \frac{7}{10} e^{-2t} \sin t$$

$$\Rightarrow \frac{1}{(s-1)(s^2+4s+5)} = \frac{A}{s-1} + \frac{Bs+C}{s^2+4s+5}$$

$$1 = A(s^2+4s+5) + (Bs+C)(s-1)$$

$$\boxed{s=1} \quad 1 = 10A \Rightarrow \boxed{A = \frac{1}{10}}$$

$$\boxed{s=0} \quad 1 = \frac{1}{2}A \cdot 5 + -C$$

$$\boxed{C = \frac{1}{2}}$$

$$\boxed{s=-1} \quad 1 = 2A + 2B - C$$

$$1 = \frac{1}{5} + 2B - \frac{1}{2} \Rightarrow 1 = 2B - \frac{3}{10}$$

$$\frac{4}{5}2B = \frac{13}{10} \Rightarrow \boxed{B = \frac{13}{20}}$$

[Q7] Choose the correct answer for each of the following items:

- (1) The DE $(\sin x)u_{xx} + \tan(x+y)u_{yyy} = e^{x+y}$ is
 (a) Ordinary DE and Linear
 (b) Partial DE and Linear.
 (c) Ordinary DE and nonlinear
 (d) Partial DE and nonlinear.

- (2) The DE $(1+t^2)y' - 2ty = \sqrt{(1+t^2)}y$ is $\left(\frac{1}{2}\right)$
 (a) homogenous
 (b) Linear.
 (c) Bernoulli
 (d) exact.

- (3) All singular points for the DE $(\tan x)y'' + 2xy' + 3x^2y = 0$ are
 (a) $n\pi, n \in \mathbb{Z}$.
 (b) $(2n+1)\frac{\pi}{2}, n \in \mathbb{Z}$.
 (c) 0.
 (d) all of the above.
- $\tan x = \frac{\sin x}{\cos x} = 0$
 $0, \pi, 2\pi$

- (4) One solution of the DE $y'' + y = 0$ is
 (a) e^{-t}
 (b) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$
 (c) $\sum_{n=0}^{\infty} \frac{x^n}{(n)!} e^y$
 (d) non of the above.
- $y = \cos x$
 $y' = -\sin x$
 $y'' = -\cos x$

- (5) The Wronskian of the two solutions of the DE $2ty'' - 4y' + y = 0$ equals
 (a) t^{-2} .
 (b) t^2 .
 (c) $c t^2, c$ is constant.
 (d) $c t^{-2}, c$ is constant.
- $w = c e^{\int \frac{4}{2t} dt} = c t^2$

- (6) The DE $ty' + \ln ty = e^t, y(-1) = 2$ has a solution on
 (a) $(0, \infty)$.
 (b) $(-\infty, 0)$.
 (c) $(-\infty, \infty)$.
 (d) $(1, \infty)$.
- $p(t) = \frac{\ln t}{t} \rightarrow \text{cont. } (0, \infty)$
 $g(t) = \frac{e^t}{t} \rightarrow \text{cont. } \mathbb{R} \setminus \{0\}$

- (7) If the DE $(x^2 + \sin y)dx + (x^n \cos y + 3)dy = 0$ is exact then
 (a) $n = -1$.
 (b) $n = 1$.
 (c) $n = 2$.
 (d) $n = -2$.
- $N_x = n x^{n-1} \cos y$
 $M_y = \cos y$
 $n=1$

- (8) The functions $f(x) = \sin^2 x, g(x) = 1 - \cos 2x$ are
 (a) Linearly independent.
 (b) Linearly dependent.
 (c) Form a fundamental set of solutions.
 (d) both a, c.

$$2f(x) = 2\sin^2 x = 1 - \cos 2x$$

$$-g(x) = -1 + \cos 2x$$

$$2f(x) - g(x) = 0$$

$$c_1, c_2 \neq 0$$

$$c_1 = 2$$

$$c_2 = -1$$