

Name : _____

Student's number: _____

Teacher's name: Prof. Dr. Raid Salha, Dr. Faten Abu-Shoga

Question:	1	2	3	4	5	6	7	Total
Points:	10	16	12	20	12	20	10	100
Score:								

1. (10 points) Solve the DE $t^3y' + 3t^2y = 2ty^3$, $t > 0$

2. (a) (8 points) Solve the initial value problem

$$y' + (\cos t)y^2 = 0, \quad y(0) = 2.$$

- (b) (8 points) If the wronskian of the functions f and g is $3e^{6t}$ and if $f(t) = e^{3t}$, find $g(t)$.

3. (12 points) Find the first five non-zero terms of the series solution of the DE

$$x^2y'' + y' - 6y = 0$$

4. (a) (12 points) Transform the differential equation $x^2y'' + 7xy' + 5y = x$, $x > 0$ into a differential equation with constant coefficients and solve it.

- (b) (8 points) Solve the DE $y^{(5)} - y' = 0$.

5. (12 points) Use Laplace transform methods to solve the following initial value problem:

$$y'' + 4y' + 4y = 3e^{-2t}, \quad y(0) = 1, y'(0) = -2$$

6. (a) (10 points) Find all singular points and determine whether each one is regular and irregular for the DE

$$(1 - x^2)^3 y'' + (1 - x^2)^2 y' + 2(1 + x)y = 0$$

- (b) (10 points) Find by using **the definition** the Laplace transform of the function: $f(t) = e^{-t} \sin(2t)$, $t > 0$

7. (10 points) Choose the correct answer for the following questions:
- (a) Suppose $y'' + y = 0$. Which of the following is a possibility for $y = f(x)$.
 A. $\tan x$. B. $\sin x$. C. $\csc x$. D. $\sec x$.
- (b) The particular solution of the D.E $y'' + 9y = 6 \sin(3x)$ has the form
 A. $a \cos 3x + b \sin 3x$. B. $ax \cos 3x + b \sin 3x$.
 C. $a \cos 3x + bx \sin 3x$. D. $ax \cos 3x + bx \sin 3x$.
- (c) The D.E. $(1 - y)y'' - 2y' = \cos 2t$ is
 A. non-linear, homogeneous second order differential equation.
 B. non-linear, non-homogeneous second order differential equation.
 C. linear, homogeneous second order differential equation.
 D. linear, non-homogeneous second order differential equation.
- (d) Consider the D.E. $ty' + y = \sin t$, $y(\frac{\pi}{2}) = 1$. The largest interval in which the solution exists
 A. $(0, \infty)$. B. $(-\infty, 0)$. C. $(-\infty, \infty)$. D. $(0, \frac{\pi}{2})$.
- (e) The general solution of the D.E $y'' + 4y' + 4y = 0$ is
 A. $y = C_1 e^{2t} + C_2 e^{-2t}$. B. $y = C_1 e^{2t} \cos 2t + C_2 e^{2t} \sin 2t$.
 C. $y = C_1 e^{2t} + C_2 t e^{-2t}$. D. non of the above.



Islamic University of Gaza
Department of Math.
Second Semester 19/5/2016

Differential Equations
Final Exam
Time: Two hours

	الاسم	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Total
	الرقم	10	14	14	14	16	18	14	100
	المدرس								

Solve the following questions:

Q1) [10 Marks] If $W(f, g)(t) = te^{-t} - \ln t$ and if $u = f + 3g$ and $v = f - g$, find $W(u, v)(t)$.

Q2) (a) [8Marks] Find the homogeneous differential equation which has the following general solution

$$y = c_1 e^t + c_2 e^{-t} + c_3 \cos t + c_4 \sin t .$$

(b) [6 Marks] Determine the interval in which the solution of the given initial value problem exists

$$(4 - x^2)y' + xy = -6x^2, \quad y(1) = 4.$$

Q3) [14 Marks] Suppose that $y_1(t) = e^t$ is one solution of the differential equation

$$ty'' - (t+1)y' + y = t^2e^t, \quad t > 0.$$

Use the method of reduction of order to solve the differential equation.

Q4 [14 Marks] Transform the following Euler equation into a differential equation with constant coefficients and then solve it.

$$x^2 y'' + 2xy' - 3y = \sin(\ln x), \quad x > 0.$$

Q5 a) [10 Marks] Find and classify the singular points of the following differential equation as regular or irregular

$$(x^2 - x^3)y'' - (1 + x)y' + 2xy = 0.$$

(b)[6 Marks] Determine a lower bound for the radius of convergence of the series solution for the given differential equation about $x_0 = 1$

$$(x^2 - 2x - 3)y'' - 2xy' + 4y = 0.$$

Q6 a) [8Marks] Use the definition to find the Laplace transformation of the function

$$f(t) = t e^t, \quad t \geq 0.$$

(b) [10 Marks] Use the Laplace transform to solve the initial value problem

$$y'' + 9y = 6, \quad y(0) = 1, y'(0) = 2.$$

Q7)[14 Marks] Find the first five terms in the series solution of the IVP
 $y'' + xy' + 2y = 0, y(0) = 4, y'(0) = -1.$

Good Luck

The Islamic University Of Gaza
 Department of Mathematics
 Ordinary differential equations (Math 2302)

Final Exam
 Date: 28/5/2015
 Time: 2 Hour

Name:	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Total
Student number:	10	16	16	16	16	10	16	100
Instructor:								

[Q1] If $y_1 = x$ is one solution of DE $x^2y'' - x(x+2)y' + (x+2)y = 0$
 find a second linearly independent solution of DE.

[Q2] Find the general solution of the DE

a) Solve the DE $y' + \frac{3}{x}y = x^2y^2, \quad x > 0.$

b) $y^{(4)} + 81y = 0$

[Q3] a) Find the radius of convergence of solutions of the DE
 $2x(x^2 - 2x + 2)y'' + (x - 1)y' + xy = 0$ about $x_0 = 2$

b) Classify all singular points as regular or irregular of DE
 $x^2(1 - x)y'' - 4y' = 3xy$

[Q4] Find the first three terms in each power series solution of the DE
 $y'' - xy = 0$, about $x_0 = -2$.

[Q5] a) Find the general solution of the Euler equation

$$(x+6)^2 y'' - (x+6)y' - 3y = \ln(x+6), \quad x > -6.$$

b) Use definition to find Laplace transform of $f(t) = t - 5$

[Q6] Use Laplace transform to solve the IVP
 $y'' + 4y' + 5y = e^t, \quad y(0) = 1, y'(0) = 2$

[Q7] Choose the correct answer for each of the following items:

- (1) The DE $(\sin x)u_{xx} + \tan(x + y)u_{yyy} = e^{x+y}$ is
(a) Ordinary DE and Linear (b) Partial DE and Linear.
(c) Ordinary DE and nonlinear (d) Partial DE and nonlinear.
- (2) The DE $(1 + t^2)y' - 2ty = \sqrt{(1 + t^2)}y$ is
(a) homogenous (b) Linear.
(c) Bernoulli (d) exact.
- (3) All singular points for the DE $(\tan x)y'' + 2xy' + 3x^2y = 0$ are
(a) $n\pi, n \in \mathbb{Z}$. (b) $(2n + 1)\frac{\pi}{2}, n \in \mathbb{Z}$.
(c) 0. (d) all of the above.
- (4) One solution of the DE $y'' + y = 0$ is
(a) e^{-t} (b) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$
(c) $\sum_{n=0}^{\infty} \frac{x^n}{(n)!}$ (d) non of the above.
- (5) The Wronskian of the two solutions of the DE $2ty'' - 4y' + y = 0$ equals
(a) t^{-2} . (b) t^2 .
(c) $c t^2, c$ is constant. (d) $c t^{-2}, c$ is constant.
- (6) The DE $ty' + \ln ty = e^t, y(-1) = 2$ has a solution on
(a) $(0, \infty)$. (b) $(-\infty, 0)$.
(c) $(-\infty, \infty)$. (d) $(1, \infty)$.
- (7) If the DE $(x^2 + \sin y)dx + (x^n \cos y + 3)dy = 0$ is exact then
(a) $n = -1$. (b) $n = 1$.
(c) $n = 2$. (d) $n = -2$.
- (8) The functions $f(x) = \sin^2 x, g(x) = 1 - \cos 2x$ are
(a) Linearly independent. (b) Linearly dependent.
(c) Form a fundamental set of solutions. (d) both a, c.



Islamic University of Gaza
Department of Math.
Second semester 22/5/2014

Differential Equations
Final exam
Time: Two hours

	الاسم	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Total
	الرقم	8	8	8	8	8	8	12	60
	شعبة								

Solve the following questions:

Q1) (a) [4 marks] Find the general solution of the differential equation

$$yy'' + (y')^2 = 0.$$



(b) [4 marks] Find a function $f(x)$ such that the differential equation

$f(0) = 0$ exact and an is $y \cos x + (\cos x) f(x) y' = 0$



Q2) (a)[4 marks] Solve the initial value problems

$$y + \sqrt{x^2 - y^2} - xy' = 0, \quad y(e^3) = 0.$$

(b) [4 marks] Solve the differential equation

$$y' + x(4y - 8y^{-3}) = 0.$$

Q3) Consider the differential equation

$$y''' - 5y'' - 2y' + 24y = e^{-2x} + xe^{3x} \cos 2x$$

(a)[5 marks] Find the complementary solution, y_c .

**(b) [3 marks] Find a formula for the particular solution, y_p .
(Don't evaluate the coefficients).**

Q4) [8 marks] Find a series solution in powers of $(x - 2)$ for the following differential equation (Include at least three nonzero terms in each series)

$$y'' + xy' + y = 0.$$

Q5(a)[4 marks] Solve the following initial value problem

$$x^2 y'' + 2x y' - 6y = 0, \quad y(1) = 0, y'(1) = 2.$$



(b)[4 marks] Classify the singular points of the following differential equation as regular or irregular

$$(x^4 - x^3)y'' + (x + 1)y' + y = 0.$$



Q6)[8 marks] Use the Laplace transformation to solve the initial value problem

$$y'' - 2y' + y = te^t, \quad y(0) = 1, \quad y'(0) = 2.$$

Q7)(a)[4 marks] Is the following initial value problem has a unique solution. Explain your answer.

$$y' = \frac{3x^2 + 2}{y + 1}, \quad y(1) = -1.$$

(b)[4 marks] Are the two functions $y_1 = x^3$, $y_2 = x^4$ form a fundamental set of solutions of the differential equation Explain your answer.

$$x^2 y'' - 6xy' + 12y = 0, \quad x > 0.$$

(c)[4marks] Suppose f, g and h are differential functions such that $f(2) = 3$ and $W(f, g, f, h)(2) = 4$. Find $W(g, h)(2)$.

Good Luck



Islamic University of Gaza
Department of Math.
Summer semester 27/8/2013

Differential Equations
Final exam
Time: Two hours

	الاسم	Q1	Q2	Q3	Q4	Q5	Q6	Total
	الرقم	16	18	16	16	16	18	100
	المدرس							

Solve the following questions:

Q1) (a) [6 Marks] Determine the interval in which the solution of the given initial value problem exists

$$y' + (\tan t)y = \sin t, \quad y(\pi) = 0.$$

b) [12 Marks] Suppose that $y_1(x) = x^2$ is one solution of the differential equation

$$x^2 y'' - 2y = 0.$$

Find a second linearly independent solution of the differential equation.

Q2) (a) [10 Marks] Find the general solution of the differential equation

$$y'' + 4y' + 4y = t^{-2}e^{-2t}, t > 0.$$

(b) [6 Marks] Solve the following differential equation

$$\frac{dy}{dx} = \frac{3y^2 - x^2}{2xy}.$$

Q3) a) [10 Marks] Consider the differential equation

$$y^{(4)} + y''' = 3t + 4e^{-t}.$$

(i) [5 Marks] Find the solution of the corresponding homogeneous equation.

(ii) [5 Marks] Use the method of undetermined coefficients to find a suitable form of the particular solution without evaluating the constants.

(b) [6 Marks] Solve the differential equation

$$x^2 y'' - 4xy' + 4y = 0.$$

Q4) [16 Marks] Find two linearly independent power series solutions at $x_0 = 0$ to the differential equation (include at least three nonzero terms in each series)

$$y'' + xy' + 2y = 0 .$$

Q5) (a) [6 Marks] Determine a lower bound for the radius of convergence of the series solution for the given differential equation about $x_0 = 0$

$$(1-x^2)y'' + 3xy' + y = 0.$$

(b) [10 Marks] Determine the singular points of the differential equation

$$2x(x-2)^2y'' + 3xy' + (x-2)y = 0,$$

and classify them as regular or irregular .

Q6) (a) [6 Marks] Use the definition to find the Laplace transformation of the function

$$f(t) = t^2 e^t.$$

(b) [12 Marks] Use the Laplace transform to solve the initial value problem

$$y'' - 2y' + 2y = \cos t, \quad y(0) = 1, y'(0) = 0.$$

Good Luck



Islamic University of Gaza
Department of Math.
Summer semester 14/8/2012

Differential Equations
Final exam
Time: Two hours

	اسم الطالب	Q1	Q2	Q3	Q4	Q5	Q6	Total
	الرقم الجامعي	20	20	20	20	20	20	120
	المدرس							

Solve the following questions:

Q1) [20 Marks] Mark each of the following: True (T) or False (F)

1. The differential equation $y' = \frac{xy^2 + x^3 + 2x^2y}{x^2y}$ is not homogeneous ().
2. For the differential equation $(\ln x)y'' + xy' + y = 0$, the point $x = 0$ is a singular point ().
3. The differential equation $(x - 2)^2y'' - 3(x - 2)y' - 4y = 0$ is an Euler equation ().
4. The solution of the IVP $(x - 3)y'' + xy' + (\ln |x|)y = 0, y(1) = 0, y'(1) = 1$ is valid in the interval $(0, 3)$.
5. The Wronskian of two Linearly independent solutions of the Differential equation $t^2y'' - t(t + 2)y' + (t + 2)y = 0$ is $W(t) = ct^2e^{-t}$ ().
6. If $\phi(t)$ is a solution of $y' + p(t)y^2 = 0$ then $c\phi(t)$ is also a solution for any value of c ().
7. The Laplace transform of the function $1 - \cos(10t)$ is $F(S) = \frac{100}{s(s^2 + 100)}$.
8. If $p(t)$ and $q(t)$ are continuous functions, then the differential equation $y''' + p(t)y'' + q(t)y' = g(t), g(t) \neq 0$ is ordinary, linear and nonhomogeneous ().
9. The differential equation $y'' - 9y = e^{3t}$ has a particular solution of the form $Y(t) = Ae^{3t}$ ().
10. If y_1 and y_2 are any two solutions of the differential equation $y'' + p(t)y' + q(t)y = 0$ then the general solution is $y = c_1y_1 + c_2y_2$, c_1 and c_2 are constants ().

Q2) (a) [Marks] Solve the differential equation

$$y^{(4)} + 8y'' + 16y = 0.$$

|

(b) [Marks] Find a function $f(x)$ such that the differential equation

$$xydx + e^x f(x)dy = 0 \text{ is exact and } f(0) = \frac{1}{2}.$$

|

Q3) (a) [Marks] Find the singular points of the differential equation $(x + 2)^2(x - 1)y'' + 3(x - 1)y' - 2(x + 2)y = 0$, and determine each one regular or irregular .

(b) [Marks] Use the substitution $x = e^z$ to transform the differential equation $x^2y'' + xy' + 4y = \sin(\ln x)$, $x > 0$ into a differential equation with constant coefficients, then solve the new equation.

Q4) [Marks] Find the general solution of the differential equation

$$x^2 y'' - 2xy' + 2y = x^4 e^x.$$



Q5) [Marks] Find only the recurrence relation of the series solution of the differential

$$y'' - xy' + y = 0$$

about the point $x_0 = 0$.

Q6) [Marks] Use laplace transform to solve the IVP

$$y'' - 2y' + 2y = \cos t, \quad y(0) = 1, y'(0) = 1.$$

Good Luck



Islamic University of Gaza
Department of Math.
Summer semester 1/9/2010

Differential Equations
Final exam
Time: Two hours

	الاسم	Q1	Q2	Q3	Q4	Q5	Total
	الرقم	15	12	11	10	12	60
	المدرس						

Solve the following questions:

Q1) (a) [6Marks] Solve the following differential equation

$$y'' + (y')^2 = 2e^{-y}.$$

(b) [5 Marks] Solve the following initial value problem

$$x \frac{dy}{dx} + 2y = \frac{\sin x}{x}, \quad y(2) = 1.$$

(c) [4 Marks] Determine the interval in which the solution of the given initial value problem exists

$$x(x-4)y'' + 3xy' + 4y = \ln x, \quad y(3) = 1, \quad y'(3) = -1.$$

Q2) Find the general solution of the following differential equations

(a) [6 Marks]

$$y^{(4)} + y''' = \sin 2t .$$

(b) [6 Marks]

$$y'' + 4y' + 4y = t^{-2}e^{-2t}, \quad t > 0.$$

Q3)(a) [6 Marks] Solve the following differential equation

$$3(x-1)^2 y'' - 6(x-1)y' + 6y = 6\ln(x-1).$$

(b) [5Marks] Classify the singular points of the following differential equation as regular or irregular

$$(x^2 - 2x - 3)y'' + xy' + 4y = 0.$$

Q4)[10 Marks] Find two linearly independent power series solutions at $x_0 = 1$ to the differential equation (include at least three nonzero terms in each series)

$$y'' - xy' - y = 0.$$

Q5) (a) [5 Marks] Use the definition to find of the Laplace transformation of the function

$$f(t) = \cos 2t, \quad t \geq 0.$$

(b) [7 Marks] Use the Laplace transform to solve the initial value problem

$$y'' - 2y' + 2y = \cos t, \quad y(0) = 1, y'(0) = 0.$$

Good Luck

Islamic University of Gaza
Department of Math.
Summer semester 1/9/2009



Differential Equations
Final exam
Time: Two hours

	الاسم	Q1	Q2	Q3	Q4	Q5	Q6	Total
	الرقم	15	10	6	10	7	12	60
	المدرس							

Solve the following questions:

Q1) (a) [4 Marks] Determine the interval in which the solution of the given initial value problem exists

$$(t - 4)y' + (\ln t)y = t, \quad y(2) = 1.$$

(b) [5 Marks] Solve the following differential equation

$$ydx + (2xy - e^{-2y})dy = 0.$$

(c) [6Marks] Solve the following differential equation

$$\frac{dy}{dx} = \frac{4y - 3x}{2x - y}.$$

Q2) (a) [6 Marks] Suppose that $y_1(x) = x$ is one solution of the differential equation $x^2 y'' - x(x+2)y' + (x+2)y = 0, \quad x > 0$. Find a second linearly independent solution of the differential equation.

(b) [4 Marks] If $W(f, g) = 4$, and if $u = f - g, v = f + g$. Find $W(u, v)$.

Q3) Consider the differential equation

$$y^{(4)} + 2y'' + y = 2\sin t + 3e^{-t}.$$

(a) [3 Marks] Find the solution of the corresponding homogeneous equation.

(b) [3Marks] Use the method of undetermined coefficients to find a suitable form of the particular solution without evaluating the constants.

Q4) Find two linearly independent power series solutions at $x_0 = 0$ to the differential equation (include at least three nonzero terms in each series)

$$y'' - x^2 y' + xy = 0.$$

Q5) (a) [4 Marks] Solve the differential equation

$$(x + 1)^2 y'' - 4(x + 1)y' + 4y = 0.$$

(b)[3 Marks] Determine a lower bound for the radius of convergence of the series solution for the given differential equation about $x_0 = 0$

$$(x^2 - 2x - 3)y'' + 3xy' + y = 0.$$

Q6) (a) [4 Marks] Use the definition to find of the Laplace transformation of the function

$$f(t) = te^{-t}.$$

(b) [8 Marks] Use the Laplace transform to solve the initial value problem

$$y'' - 2y' + 2y = e^{-t}, \quad y(0) = 0, y'(0) = 1.$$

Good Luck



Islamic University of Gaza
Department of Math.
Summer semester 12/8/2008

Differential Equations
Final exam
Time: Two hours

	الاسم	Q1	Q2	Q3	Q4	Q5	Q6	Total
	الرقم	16	26	16	14	18	10	100
	المدرس							

Q1) Solve the initial value problems

(a) $y' + 2y = te^{-2t}$, $y(1) = 0$.

(b) $y'' - 3y^2 = 0$, $y(0) = 2$, $y'(0) = 4$.

Q2) Solve the following differential equations:

a) $y''' + y = 0.$

b) $(\cos 2y - \sin x) dx - 2 \tan x \sin 2y dy = 0.$

c) $4y'' + y = 2\sec(t/2), \quad -\pi < t < \pi$

Q3) Find a series solution in powers of x for the differential equation
$$(3-x^2)y'' - 3xy' - y = 0.$$

Q4) Solve the Euler equation

$$x^2 y'' - 3x y' + 4y = x^2, \quad x > 0.$$

Q5)(a) Use the Laplace transformation to solve the initial value problem

$$y'' + 4y' + 5y = 0, \quad y(0) = 1, \quad y'(0) = 0.$$

(b) Use the definition to find the Laplace transformation for $f(t) = e^t \cos t$.

Q6) Consider the differential equation

$$(1-x^2)y'' + x(1-x)y' + (1+x)y = 0.$$

(a) Determine a lower bound for the radius of convergence of series solution about $x = 0$ for this differential equation.

(b) Determine the singular points of this differential equation and classify them as regular or irregular.

Islamic University of Gaza

Department of Math.

Second semester 22/5/2014



Differential Equations

Final exam

Time: Two hours

	الاسم	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Total
	الرقم	8	8	8	8	8	8	12	60

Solve the following questions:

Q1) (a) [4 marks] Find the general solution of the differential equation

$$yy'' + (y')^2 = 0.$$

(b) [4 marks] Find a function $f(x)$ such that the differential equation

$$y \cos x + (\cos x) f(x) y' = 0 \text{ is an exact and } f(0) = 0.$$

Q2) (a) [4 marks] Solve the initial value problems

$$y + \sqrt{x^2 - y^2} - xy' = 0, \quad y(e^3) = 0.$$

(b) [4 marks] Solve the differential equation

$$y' + x(4y - 8y^{-3}) = 0.$$

Q3) Consider the differential equation

$$y''' - 5y'' - 2y' + 24y = e^{-2x} + xe^{3x} \cos 2x$$

(a) [5 marks] Find the complementary solution, y_c .

(b) [3 marks] Find a formula for the particular solution, y_p .

(Don't evaluate the coefficients).

Q4) [8 marks] Find a series solution in powers of $(x - 2)$ for the following differential equation (Include at least three nonzero terms in each series)
 $y'' + xy' + y = 0.$

Q5)(a)[4 marks] Solve the following initial value problem

$$x^2 y'' + 2x y' - 6y = 0, \quad y(1) = 0, \quad y'(1) = 2.$$

(b)[4 marks] Classify the singular points of the following differential equation as regular or irregular

$$(x^4 - x^3)y'' + (x + 1)y' + y = 0.$$

Q6)[8 marks] Use the Laplace transformation to solve the initial value problem

$$y'' - 2y' + y = te^t, \quad y(0) = 1, \quad y'(0) = 2.$$

Q7)(a)[4 marks] Is the following initial value problem has a unique solution. Explain your answer.

$$y' = \frac{3x^2 + 2}{y + 1}, \quad y(1) = -1.$$

(b)[4 marks] Are the two functions $y_1 = x^3$, $y_2 = x^4$ form a fundamental set of solutions of the differential equation Explain your answer.

$$x^2 y'' - 6xy' + 12y = 0, \quad x > 0.$$

(c)[4marks] Suppose f, g and h are differential functions such that $f(2) = 3$ and $W(g, h)(2) = 4$. Find $W(fg, fh)(2)$.

Islamic University of Gaza

Department of Math.

Summer semester 27/8/2013



Differential Equations

Final exam

Time: Two hours

	الاسم	Q1	Q2	Q3	Q4	Q5	Q6	Total
	الرقم	16	18	16	16	16	18	100

Solve the following questions:

Q1) (a) [6 Marks] Determine the interval in which the solution of the given initial value problem exists $y' + (\tan t)y = \sin t, y(\pi) = 0$.

b) [12 Marks] Suppose that $y_1(x) = x^2$ is one solution of the differential equation

$$x^2 y'' - 2y = 0.$$

Find a second linearly independent solution of the differential equation.

Q2) (a) [10 Marks] Find the general solution of the differential equation

$$y'' + 4y' + 4y = t^{-2}e^{-2t}, t > 0.$$

(b) [6 Marks] Solve the following differential equation

$$\frac{dy}{dx} = \frac{3y^2 - x^2}{2xy}.$$

Q3) a) [10 Marks] Consider the differential equation

$$y^{(4)} + y''' = 3t + 4e^{-t}.$$

(i) [5 Marks] Find the solution of the corresponding homogeneous equation.

(ii) [5 Marks] Use the method of undetermined coefficients to find a suitable form of the particular solution without evaluating the constants.

(b) [6 Marks] Solve the differential equation

$$x^2 y'' - 4xy' + 4y = 0.$$

Q4) [16 Marks] Find two linearly independent power series solutions at $x_0 = 0$ to the differential equation (include at least three nonzero terms in each series)

$$y'' + xy' + 2y = 0.$$

Q5) (a) [6 Marks] Determine a lower bound for the radius of convergence of the series solution for the given differential equation about $x_0 = 0$

$$(1 - x^2)y'' + 3xy' + y = 0.$$

(b) [10 Marks] Determine the singular points of the differential equation

$$2x(x - 2)^2 y'' + 3xy' + (x - 2)y = 0,$$

and classify them as regular or irregular .

Q6) (a) [6 Marks] Use the definition to find the Laplace transformation of the function

$$f(t) = t^2 e^t.$$

(b) [12 Marks] Use the Laplace transform to solve the initial value problem

$$y'' - 2y' + 2y = \cos t, \quad y(0) = 1, y'(0) = 0$$

Islamic University of Gaza

Department of Math.

Summer semester 1/9/2010



Differential Equations

Final exam

Time: Two hours

	الاسم	Q1	Q2	Q3	Q4	Q5	Total
	الرقم	15	12	11	10	12	60

Solve the following questions:

Q1) (a) [6Marks] Solve the following differential equation

$$y'' + (y')^2 = 2e^{-y}.$$

(b) [5 Marks] Solve the following initial value problem

$$x \frac{dy}{dx} + 2y = \frac{\sin x}{x}, \quad y(2) = 1.$$

(c) [4 Marks] Determine the interval in which the solution of the given initial value problem exists

$$x(x-4)y'' + 3xy' + 4y = \ln x, \quad y(3) = 1, \quad y'(3) = -1.$$

Q2) Find the general solution of the following differential equations

(a) [6 Marks] $y^{(4)} + y''' = \sin 2t.$

(b) [6 Marks] $y'' + 4y' + 4y = t^{-2}e^{-2t}, \quad t > 0.$

Q3)(a) [6 Marks] Solve the following differential equation

$$3(x-1)^2 y'' - 6(x-1)y' + 6y = 6\ln(x-1).$$

(b) [5Marks] Classify the singular points of the following differential equation as regular or irregular $(x^2 - 2x - 3)y'' + xy' + 4y = 0.$

Q4)[10 Marks] Find two linearly independent power series solutions at $x_0 = 1$ to the differential equation (include at least three nonzero terms in each series)

$$y'' - xy' - y = 0.$$

Q5) (a) [5 Marks] Use the definition to find of the Laplace transformation of the function

$$f(t) = \cos 2t, \quad t \geq 0.$$

(b) [7 Marks] Use the Laplace transform to solve the initial value problem

$$y'' - 2y' + 2y = \cos t, \quad y(0) = 1, y'(0) = 0.$$