

Experiment 7: Fourier Series

Theory

A Fourier series is an infinite sum of harmonic functions (sines and cosines) with every term in the series having a frequency which is an integral multiple of some “principal” frequency and an amplitude that varies inversely with its frequency. The usefulness of such series is that any periodic function f with period T can be written as a Fourier series.

- **Trigonometric fourier series**

$$f(t) = a_0 + \sum_{k=1}^{\infty} [a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t)]$$

where ω_0 is the fundamental frequency of the function; that is,

$$\omega_0 = \frac{2\pi}{T}$$

Where T is the fundamental period of the function.

The Fourier coefficients, the a 's and b 's in the equation, may be computed by the following set of integrals:

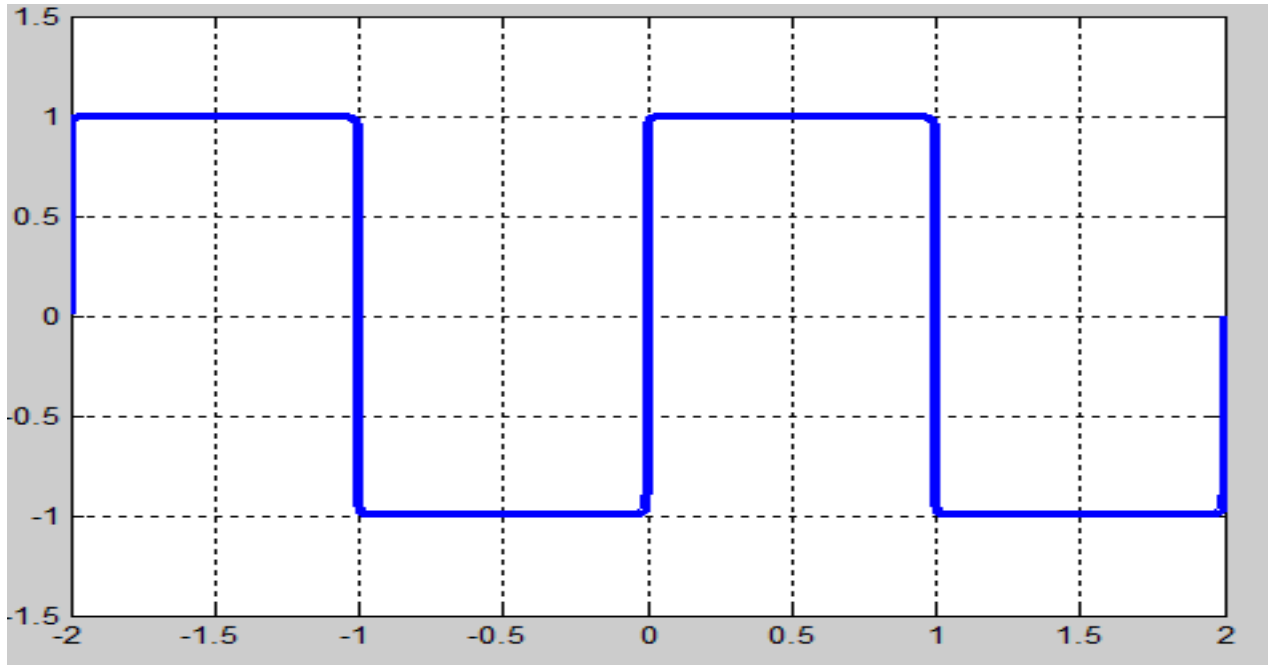
$$a_0 = \frac{1}{T} \int_0^T f(t) dt$$

$$a_k = \frac{2}{T} \int_0^T f(t) \cos(k\omega_0 t) dt$$

$$b_k = \frac{2}{T} \int_0^T f(t) \sin(k\omega_0 t) dt$$

Code in matlab:

Example:



```
T=2;  
w=2*pi/T;  
t=-T:T/1000:T;  
N=input('# harmonics = ');  
a0 = 0;  
xN=a0*ones(1,length(t));  
for n= 1 :N  
    an = 0;  
    bn=(2/(n*pi)) .* (1-cos(n*pi));  
    xN=xN+an*cos(n*w*t)+bn*sin(n*w*t);  
end  
plot(t,xN,'linewidth',3)
```

```
axis([-T T -1.5 1.5])
title(['N= ', num2str(N)])
grid on
```

- **Exponential Fourier Series**

The periodic function can be expressed using sine and cosine functions, such expressions, however, are not as convenient as the expression using complex exponentials.

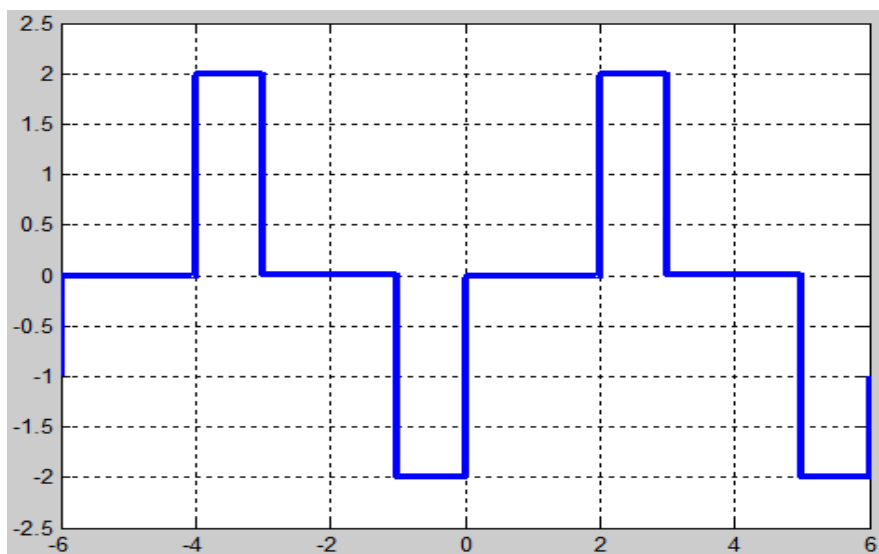
$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$\text{Where } C_n = \frac{1}{T} \int_{t_0}^{t_0+T} f(t) e^{-jn\omega_0 t} dt$$

$$a_0 = c_0$$

Code in matlab:

Example:



```

T=6;
w=2*pi/6;
t=-T:T/1000:T;
N=input('# harmonics = ');
c0= 0;
xN=c0*ones(1,length(t));
for n= 1 :N   cn=(-
4*j./(n*pi))*sin(n*pi/6)*sin(n*pi
i/2)*exp(-j*n*pi/3);
c_n = conj(cn);
xN=xN+cn*exp(j*n*w*t)+c_n*exp(-
j*n*w*t);
end
plot(t,xN,'linewidth',3)
title(['N= ',num2str(N)])
grid on

```

- **Line Spectra: (Amplitude Spectrum & Phase Spectrum)**

The complex exponential Fourier series of a signal consists of a summation of phasor.

The periodic signal to be characterized graphically in the frequency domain is, by making 2 plots.

The first, showing amplitude versus frequency is known as amplitude spectrum of the signal.

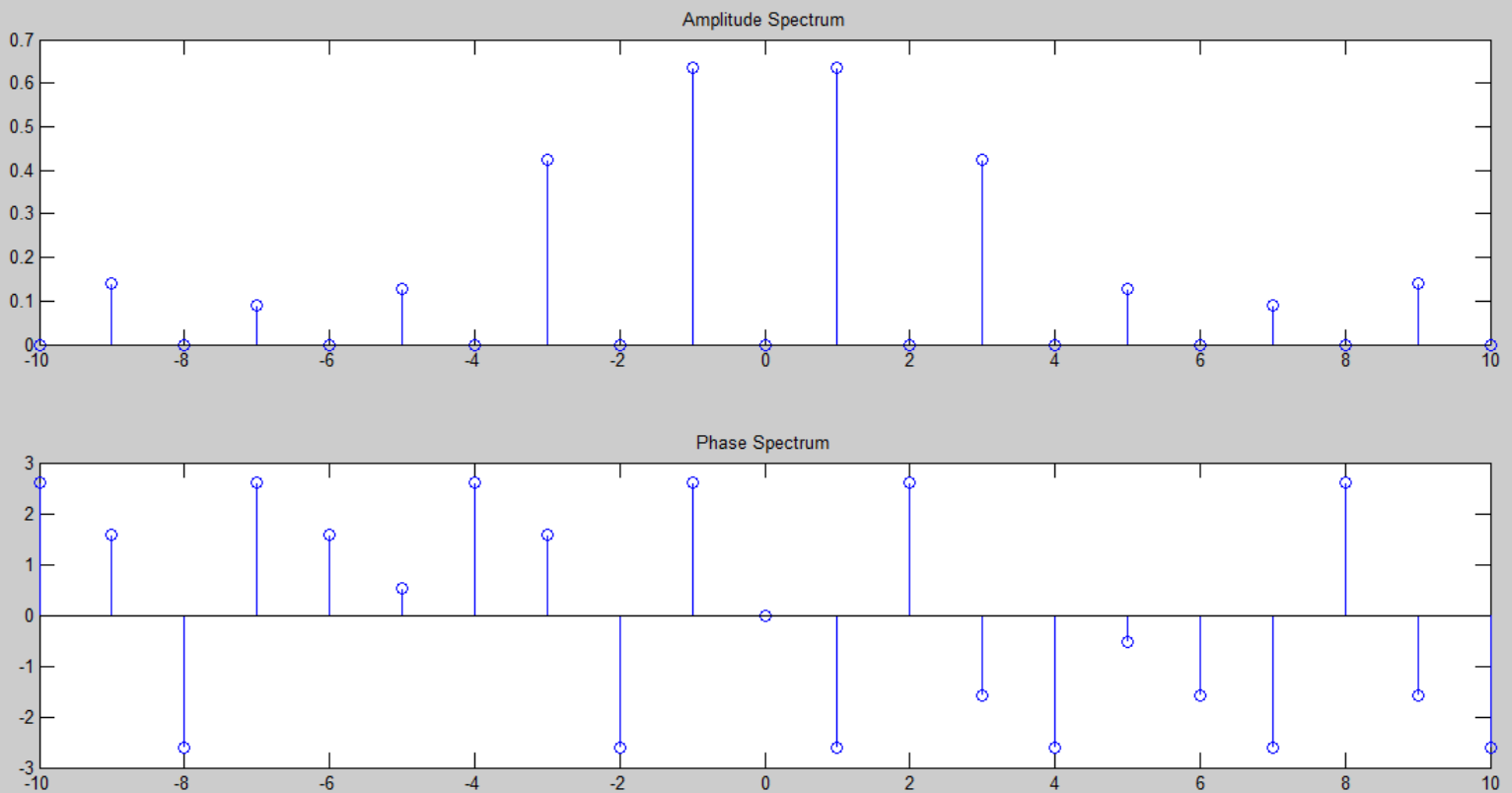
$$C_n = |C_n|e^{j\theta_n} = |C_n|C_n^0 \quad \text{Polar Form}$$

The amplitude spectrum is the plot of $|C_n|$ versus $\omega = n\omega_0$

The second, showing the phase of each component versus frequency is called the phase spectrum of the signal.

Code in matlab:

Example: Amplitude Spectrum & Phase Spectrum to above example



```
n = 1 : 10;  
cn=(-  
4*j./(n*pi)).*sin(n*pi/6).*sin(n  
*pi/2).*exp(-j*n*pi/3);  
n = -10 : -1;
```

```
c_n = (-  
4*j./(n*pi)).*sin(n*pi/6).*sin(n  
*pi/2).*exp(-j*n*pi/3);  
n = -10 : 10;  
c0 = 0;  
cn = [c_n c0 cn];  
subplot 211  
stem(n,abs(cn))  
title('Amplitude Spectrum')  
subplot 212  
stem(n,angle(cn))  
title('Phase Spectrum')
```