

Fourier,Laplace and Z transform

Fourier transform:

In this lab, we learn how to compute the continuous-time Fourier transform (CTFT), normally referred to as Fourier transform, numerically and examine its properties.

Also, we explore noise cancellation and amplitude modulation as applications of Fourier transform. The continuous-time Fourier transform (CTFT) (commonly known as Fourier transform) of an aperiodic signal $x(t)$ is given by this expression:

$$X(\omega) = \int_{-\infty}^{\infty} x(t) * e^{-j\omega t} dt$$

The signal $x(t)$ can be recovered from $x(\omega)$ via this inverse Fourier transform (IFT) equation:

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) * e^{-j\omega t} d\omega$$

Some of the properties of CTFT are listed in Table 8.1.

Properties	Time domain	Frequency domain
Linearity	$a_1 x_1(t) + a_2 x_2(t)$	$a_1 X_1(\omega) + a_2 X_2(\omega)$
Time scaling	$x(at)$	$\frac{1}{ a } X\left(\frac{\omega}{a}\right)$
Time shifting	$x(t - t_o)$	$X(\omega) * e^{-j\omega t_o}$
Time convolution	$x(t) * h(t)$	$X(\omega) H(\omega)$
Frequency convolution	$x(t) h(t)$	$X(\omega) * H(\omega)$

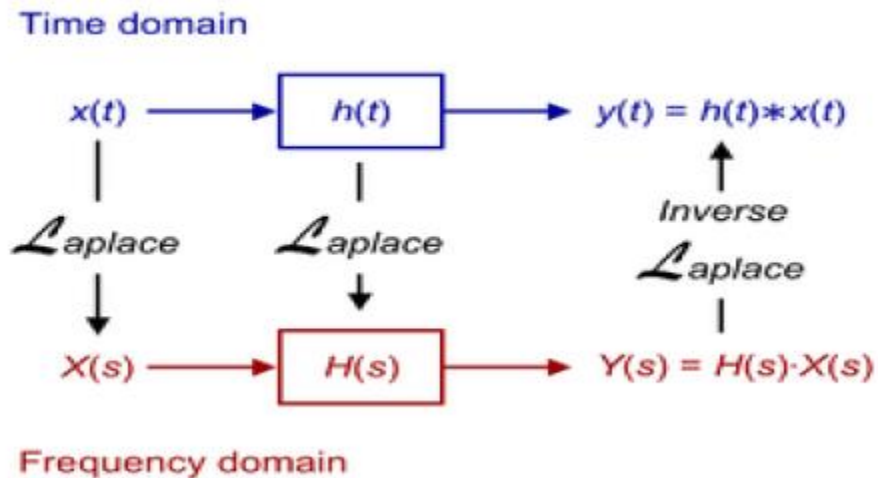
Fourier Transform and Inverse Fourier in Matlab

$f(t) = e^{-t^2}$	<pre>syms t w x=exp(-t^2); fourier(x)</pre>
$X(w) = \frac{1}{1+jw}$	<pre>syms n X=1/(1+j*w); ifourier(X,n)</pre>
$f(t) = e^{-t}u(t)$	<pre>syms t w x=exp(-t)*heaviside(t); X=fourier(x,w)</pre>
$f(t) = te^{- t }$	<pre>syms x u; f = x*exp(-abs(x)); fourier(f,u)</pre>

Laplace Transform:

Laplace transform is used for solving differential and integral equations. In physics and engineering, it is used for analysis of linear time-invariant systems such as electrical circuits, harmonic oscillators, optical devices, and mechanical systems. In this analysis, the Laplace transform is often interpreted as a transformation from the *time-domain*, in which inputs and outputs are functions of time, to the *frequency-domain*, where the same inputs and outputs are functions of complex angular frequency, in radians per unit time.

The Laplace transform has the useful property that many relationships and operations over the originals $f(t)$ correspond to simpler relationships and operations over the images $F(s)$.



Formal Definition

The Laplace transform of a function $x(t)$, is the function $X(s)$, defined by:

$$\mathcal{L}[x(t)] = X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt$$

Laplace in Matlab

Suppose we wished to find the output to a filter whose input is

$$x(t) = e^{-2t} u(t)$$

and whose transfer function is

$$H(s) = \frac{1}{s+3}$$

we could take the symbolic Laplace transform ,multiply it by $1/(s+3)$, and then take the inverse Laplace transform as follows:

```
syms x t s;
x = exp(-2*t);
X= laplace(x);
H = 1/(s+3);
Y = X*H;
y = ilaplace(Y);
```

Method of partial fraction expansion:

Consider a linear time-invariant system with transfer function $H(s)$

The impulse response is simply the inverse Laplace transform of this transfer function:

$$h(t) = \mathcal{L}^{-1}\{H(s)\}.$$

♦ Use the residue command to find the inverse Laplace transform of the following functions:

$$X(s) = (2s^2 + 5)/(s^2 + 3s + 2).$$

```
Num=[2 0 5];
Den=[1 3 2];
[r, p, k]=residue(Num, Den)
```

Z Transform

In mathematics and signal processing, the **Z-transform** converts a discrete time-domain signal, which is a sequence of real or complex numbers, into a complex frequency-domain representation.

It can be considered as a discrete equivalent of the Laplace transform.

$$X(z) = \mathcal{Z}\{x[n]\} = \sum_{n=0}^{\infty} x[n]z^{-n}.$$


```

y =
z/(z-1)^2
////////////////////////////////
syms n
y=ztrans(n^2)
y =
z*(z+1)/(z-1)^3
////////////////////////////////
syms n a T
y=ztrans(exp(-a*n*T))
y =
z/exp(-a*T)/(z/exp(-a*T)-1)
////////////////////////////////
syms n b
y=ztrans(cos(b*n))
y =
(z-cos(b))*z/(z^2-2*z*cos(b)+1)

```

2. Find the inverse z-transform of:

- $\frac{z}{z-a}$
- $\frac{z(2z-1)}{(z-1)(z+0.5)}$

