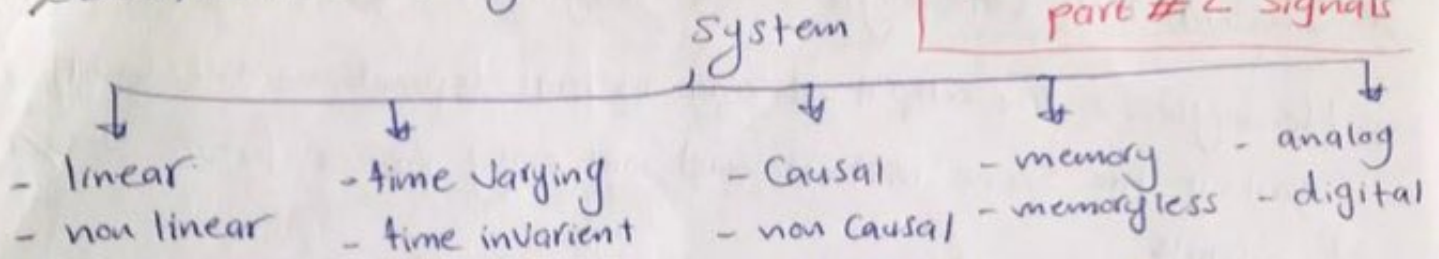


Classification of systems :-



* linear and non linear system //

To test if the system linear or non linear we make linearly test (superposition).

$$x_1(t) \rightarrow y_1(t)$$

$$x_2(t) \rightarrow y_2(t)$$

$$x_3(t) = ax_1(t) + bx_2(t) \rightarrow y_3(t)$$

If $ay_1(t) + by_2(t) = y_3(t)$ then system is linear

* Time Varying & time invariant //

- If the characteristic of a system don't change with time, so this system called time invariant.

$$\text{If } x(t) \rightarrow y(t)$$

$$x(t-t_0) \rightarrow y(t-t_0)$$

إذا كان النظام لا يتغير مع الزمن
system لا يتغير مع الزمن
time invariant

output لا يتغير مع الزمن ← input لا يتغير مع الزمن
time invariant ← system لا يتغير مع الزمن

* Causal and non Causal //

Causal system is one for which the output at any instant "to" depends only on the value of the input $x(t)$ for $t \leq t_0$.

* The value of the present depends on the present and or on the past not the future.

present	$y(t) = x(t)$	Causal
past	$y(t) = x(t-1)$	Causal
future	$y(t) = x(t+1)$	non Causal

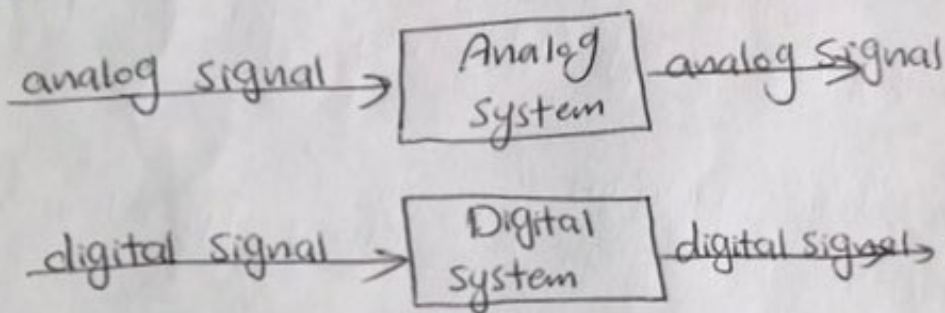
* Instantaneous (memoryless) & Dynamic (memory) systems

→ Memoryless: The output at any instant depends on the input signal at the same instant and not only past or future values of signals

→ Dynamic (memory): system whose response at t is completely determined by the input signal over the past T second

Notes: If the present value of output depends only on the present value of input, A system is said memoryless

Analog & Digital system //



Examples //

System 1 & 2

$$\boxed{1} \quad y(t) = \frac{dx}{dt} + x(t)$$

$$x_1(t) \rightarrow y_1(t) = \frac{dx_1(t)}{dt} + x_1(t)$$

$$x_2(t) \rightarrow y_2(t) = \frac{dx_2(t)}{dt} + x_2(t)$$

$$x_3(t) = ax_1(t) + bx_2(t) \rightarrow y_3(t)$$

$$\text{If } ay_1(t) + by_2(t) = y_3(t) \rightarrow \text{system is linear.}$$

// بعبارة أخرى //

$$y_3(t) = \frac{dx_3(t)}{dt} + x_3(t)$$

$$= \frac{d}{dt} (ax_1(t) + bx_2(t)) + (ax_1(t) + bx_2(t))$$

$$= \left[a \frac{dx_1(t)}{dt} + b \frac{dx_2(t)}{dt} + ax_1(t) + bx_2(t) \right] \dots \textcircled{1}$$

input

$$y_1(t) + by_2(t) = \left[a \frac{dx_1(t)}{dt} + ax_1(t) + b \frac{dx_2(t)}{dt} + bx_2(t) \right] \dots (2)$$

$$\Rightarrow ay(t) + by_2(t) = y_3(t)$$

$\Rightarrow$ then the system is linear #

time variant $\leftrightarrow$ system $\leftrightarrow$ time invariant

$$x_1(t) \rightarrow y_1(t)$$

$$x_2(t) = x_1(t-t_0) \rightarrow y_2(t)$$

$$\text{If } y_1(t-t_0) = y_2(t)$$

"time invariant" system

$$y_1(t-t_0) = \frac{d}{dt} x_1(t-t_0) + x_1(t-t_0) \dots (*)$$

$$y_2(t) = \frac{d}{dt} x_2(t) + x_2(t)$$

$$= \frac{d}{dt} x_1(t-t_0) + x_1(t-t_0)$$

$$\Rightarrow y_1(t-t_0) = y_2$$

..... (*)

$\Rightarrow$ system is time invariant

memory

memoryless

system

$\Rightarrow$ system is memory (لا تخزن على مئة)
initial condition

Causal

non causal

system

$\Rightarrow$ system is Causal

input لا يحد مع مئة output يحد مع مئة الكالية لا input في مستقبل

[3]

ملاحظات حول علية كل عنصر النظر

1) إذا كان في مجموع في input يكون الـ system $\Leftarrow$ non linear

2) إذا كان في ثابت مجموع الـ input و output يكون الـ system $\Leftarrow$ non linear

3) إذا كان في t متغيرة بـ input يكون الـ system $\Leftarrow$ time varying

4) إذا كانت t داخل $x(t)$ متغيرة بأي معامل مثل $x(at)$

بكون الـ system $\Leftarrow$ time varying

5) إذا كان $x(t+a)$ بمعنى مجموع رقم موجب و t يكون الـ system $\Leftarrow$ non causal

6) إذا كان في متغيرة أو كمال السؤال يكون Memory

7) إذا كان $x(t \pm a)$ يكون Memory

8) إذا كان $x(at)$ يكون non causal بحيث ينظره صفة $a =$

- present $y(t) = x(t) \Rightarrow$ Causal , Memoryless
- past $y(t) = x(t-1) \Rightarrow$ Causal , Memory
- future $y(t) = x(t+1) \Rightarrow$ non causal , Memory

$$y(t) = 2x(t) + 3$$

$$x_1(t) \rightarrow y_1(t) = 2x_1(t) + 3$$

$$x_2(t) \rightarrow y_2(t) = 2x_2(t) + 3$$

$$x_3(t) = ax_1(t) + bx_2(t) \rightarrow y_3(t)$$

If $ay_1(t) + by_2(t) = y_3(t) \Rightarrow$ system linear
 خاصية نتيجة التراكب

$$y_3(t) = 2x_3(t) + 3 = 2(ax_1(t) + bx_2(t)) + 3 = \boxed{2ax_1(t) + 2bx_2(t) + 3} \dots (*)$$

$$ay_1(t) + by_2(t) \Rightarrow \boxed{2ax_1(t) + 3a + 2bx_2(t) + 3b} = \boxed{2ax_1(t) + 2bx_2(t) + 3(a+b)} \dots (**)$$

$ay_1(t) + by_2(t) \neq y_3(t) \Rightarrow$ system is non linear

$$x_1(t) \rightarrow y_1(t)$$

$$x_2(t) = x_1(t-t_0) \rightarrow y_2(t)$$

$$\text{If } y_1(t-t_0) = y_2(t)$$

$\Rightarrow$ system is time invariant

$$\boxed{y_1(t-t_0) = 2x_1(t-t_0) + 3} \dots (*)$$

$$\boxed{y_2(t) = 2x_2(t) + 3 = 2x_1(t-t_0) + 3} \dots (**)$$

$$y_2(t) = y_1(t-t_0)$$

$\Rightarrow$ system is time invariant

time varying

$\Rightarrow$ system is memoryless (present value of the output depends only on the present value of input)

$\Rightarrow$ system is causal (output doesn't depend on future)

3] $y(t) = 2x^2(t) + 3x(t)$

$x_1(t) \rightarrow y_1(t) = 2x_1^2(t) + 3x_1(t)$

$x_2(t) \rightarrow y_2(t) = 2x_2^2(t) + 3x_2(t)$

$x_3(t) = ax_1(t) + bx_2(t) \rightarrow y_3(t)$

If $ay_1(t) + by_2(t) = y_3(t) \rightarrow$ system linear

$\Rightarrow ay_1(t) + by_2(t) \Rightarrow 2ax_1^2(t) + 3ax_1(t) + 2bx_2^2(t) + 3bx_2(t)$

$\Rightarrow y_3(t) = 2x_3^2(t) + 3x_3(t)$
 $= 2(ax_1(t) + bx_2(t))^2 + 3(ax_1(t) + bx_2(t))$

$\Rightarrow ay_1(t) + by_2(t) \neq y_3(t)$

$\Rightarrow$ system is non linear

$x_1(t) \rightarrow y_1(t)$

$x_2(t) = x_1(t-t_0) \rightarrow y_2(t)$

If $y_1(t-t_0) = y_2(t) \Rightarrow$ system is time invariant

$y_1(t-t_0) = 2x_1^2(t-t_0) + 3x_1(t-t_0) \dots (*)$

$y_2(t) = 2x_2^2(t) + 3x_2(t)$
 $= 2x_1^2(t-t_0) + 3(x_1(t-t_0)) \dots (**)$

$y_1(t-t_0) = y_2(t) \Rightarrow$ system is time invariant

$\rightarrow$ system is memory less

$\rightarrow$ system is causal

linearity

time varying

$$y(t) = Atx(t)$$

$$x_1(t) \rightarrow y_1(t) = Atx_1(t)$$

$$x_2(t) \rightarrow y_2(t) = Atx_2(t)$$

$$x_3(t) = ax_1(t) + bx_2(t) \rightarrow y_3(t)$$

$$ay_1(t) + by_2(t) \Rightarrow [aAtx_1(t) + bAtx_2(t)] \dots (*)$$

$$y_3(t) = Atx_3(t)$$

$$= At(ax_1(t) + bx_2(t)) \Rightarrow [aAtx_1(t) + bAtx_2(t)] \dots (**)$$

$$ay_1(t) + by_2(t) = y_3(t) \Rightarrow \text{system is linear}$$

$$x_1(t) \rightarrow y_1(t)$$

$$x_2(t) = x_1(t-t_0) \rightarrow y_2(t)$$

$$[y_1(t-t_0) = A(t-t_0)x_1(t-t_0)] \dots (*)$$

$$y_2(t) = Atx_2(t)$$

$$[y_2(t) = Atx_1(t-t_0)] \dots (**)$$

$$y_1(t-t_0) \neq y_2(t) \text{ so system is time varying}$$

$$\Rightarrow \text{system is memoryless}$$

$$\Rightarrow \text{system is Causal}$$

$$[5] y(t) = x(t) \cdot x(t-2)$$

$$x_1(t) \rightarrow y_1(t) = x_1(t) \cdot x_1(t-2)$$

$$x_2(t) \rightarrow y_2(t) = x_2(t) \cdot x_2(t-2)$$

$$x_3(t) = ax_1(t) + bx_2(t) \rightarrow y_3(t)$$

$$ay_1(t) + by_2(t) \Rightarrow [ax_1(t)x_1(t-2) + bx_2(t)x_2(t-2)] \dots (*)$$

$$y_3(t) = x_3(t) \cdot x_3(t-2)$$

$$= [ax_1(t) + bx_2(t)] \cdot [ax_1(t-2) + bx_2(t-2)] \dots (**)$$

$$ay_1(t) + by_2(t) \neq y_3(t) \therefore \text{non linear}$$

Linearity test

$$x_1(t) \rightarrow y_1(t)$$

$$x_2(t) = x_1(t-t_0) \rightarrow y_2(t)$$

$$y_1(t-t_0) = x_1(t-t_0) \cdot x_1(t-t_0-2)$$

$$y_2(t) = x_2(t) \cdot x_2(t-2) \\ = x_1(t-t_0) x_1(t-t_0-2)$$

$y_1(t-t_0) = y_2(t)$, so system is time invariant

→ system is memory

→ system is causal

$$[6] y(t) = \log x(t+2) + 5$$

$$x_1(t) \rightarrow y_1(t) = \log x_1(t+2) + 5$$

$$x_2(t) \rightarrow y_2(t) = \log x_2(t+2) + 5$$

$$x_3(t) = ax_1(t) + bx_2(t) \rightarrow y_3(t)$$

$$ay_1(t) + by_2(t) = \log ax_1(t+2) + 5a + \log bx_2(t+2) + 5b \\ = \log ax_1(t+2) + \log bx_2(t+2) + 5(a+b)$$

$$y_3(t) = \log x_3(t+2) + 5$$

$$y_3(t) = \log (ax_1(t+2) + bx_2(t+2)) + 5$$

$$y_3(t) = \log ax_1(t+2) + \log bx_2(t+2) + 5$$

$$x_1(t) \rightarrow y_1(t)$$

$$x_2(t) = x_1(t-t_0) \rightarrow y_2(t)$$

$$y_1(t-t_0) = \log x_1(t-t_0+2) + 5$$

$$y_2(t) = \log x_2(t+2) + 5$$

$$= \log x_1(t-t_0+2) + 5$$

$y_1(t-t_0) = y_2(t) \Rightarrow$ time invariant system

$\Rightarrow$ memory system / $\Rightarrow$ non causal system

up
time varying

$$y(t) = tx(t+10)$$

$$x_1(t) \rightarrow y_1(t) = tx_1(t+10)$$

$$x_2(t) \rightarrow y_2(t) = tx_2(t+10)$$

$$x_3(t) = ax_1(t) + bx_2(t) \rightarrow y_3(t)$$

$$ay_1(t) + by_2(t) \Rightarrow [atx_1(t+10) + btx_2(t+10)]$$

$$y_3(t) = tx_3(t+10)$$

$$= t[ax_1(t+10) + bx_2(t+10)]$$

$$= [atx_1(t+10) + btx_2(t+10)]$$

$$ay_1(t) + by_2(t) = y_3(t) \Rightarrow \text{Linear system}$$

$$x_1(t) \rightarrow y_1(t)$$

$$x_2(t) = x_1(t-t_0) \rightarrow y_2(t)$$

$$[y_1(t-t_0) = tx_1(t-t_0+10)] \rightarrow (t-t_0)x_1(t-t_0+10) \dots (*)$$

$$[y_2(t) = tx_2(t+10) \Rightarrow tx_1(t-t_0+10)] \dots (**)$$

System is time variant because ~~ay₁ + by₂~~

$$y_1(t-t_0) \neq y_2(t)$$

$\Rightarrow$ non causal system

$\Rightarrow$ memory system

$$[8] y(t) = x(at)$$

$$x_1(t) \rightarrow y_1(t) = x_1(at)$$

$$x_2(t) \rightarrow y_2(t) = x_2(at)$$

$$x_3(t) = ax_1(t) + bx_2(t) \rightarrow y_3(t)$$

$$ay_1(t) + by_2(t) = ax_1(at) + bx_2(at) \dots (*)$$

$$y_3(t) = x_3(at) = ax_1(at) + bx_2(at) \dots (**)$$

$$ay_1(t) + by_2(t) = y_3(t)$$

$\Rightarrow$ So system is linear

$$x_1(t) \rightarrow y_1(t)$$

$$x_2(t) = x_1(t - t_0) \rightarrow y_2(t)$$

$$y_1(t - t_0) = x_1(a(t - t_0))$$

$$y_2(t) = x_2(at) = x_1(at - t_0)$$

$$y_1(t - t_0) = y_2(t) \Rightarrow \text{system is time invariant}$$

$\Rightarrow$ system is memoryless

$\Rightarrow$ system is Causal

non.

($x(at)$)

بسیار آسان است

$$[9] \frac{dy}{dt} + 2y(t) = x^2(t)$$

$$x_1(t) \Rightarrow \frac{dy_1}{dt} + 2y_1(t) = x_1^2(t)$$

$$x_2(t) \Rightarrow \frac{dy_2}{dt} + 2y_2(t) = x_2^2(t)$$

$$x_3(t) = ax_1(t) + bx_2(t)$$

$$\frac{dy_3}{dt} + 2y_3 = (ax_1 + bx_2)^2 = a^2x_1^2 + b^2x_2^2 + 2abx_1x_2$$

$$\neq ay_1 + by_2$$

$\Rightarrow$ system is non linear

$\Rightarrow$ system is memory ($\hat{a} \hat{b} \hat{c} \hat{d} \hat{e} \hat{f} \hat{g} \hat{h} \hat{i} \hat{j} \hat{k} \hat{l} \hat{m} \hat{n} \hat{o} \hat{p} \hat{q} \hat{r} \hat{s} \hat{t} \hat{u} \hat{v} \hat{w} \hat{x} \hat{y} \hat{z}$)

$\Rightarrow$ system is Causal

$\Rightarrow$ system is time invariant.

20 $y' + 10y + 5 = x$

$$x_1(t) \rightarrow x_1 = y_1' + 10y_1 + 5$$

$$x_2(t) \rightarrow x_2 = y_2' + 10y_2 + 5$$

$$x_3(t) = ax_1(t) + bx_2(t) \Rightarrow$$

$$ax_1 + bx_2 = ay_1' + 10ay_1 + 5a + by_2' + 10by_2 + 5b$$

$$\Rightarrow ax_1 + bx_2 = ay_1' + by_2' + 10(ay_1 + by_2) + 5(a+b)$$

$$x_3(t) = y_3' + 10y_3 + 5[a+b]$$

المفروض نظر من المعطيات x_3
 $\rightarrow x_3(t) = y_3' + 10y_3 + 5$

$\Rightarrow$ system is non linear.

$\Rightarrow$ system is memory (لا يوجد الذاكرة)

$\Rightarrow$ system is causal

II $ty(t) = tu(t)$

$$u_1(t) = x_1(t) \rightarrow y_1(t) = tu_1(t)$$

$$u_2(t) = x_2(t) \rightarrow y_2(t) = tu_2(t)$$

$$u_3 = x_3(t) = ax_1(t) + bx_2(t) \rightarrow y_3(t)$$

If $ay_1(t) + by_2(t) = y_3(t) \rightarrow$ linear system

$$[atu_1(t) + btu_2(t)] \Rightarrow ay_1(t) + by_2(t) \quad \text{نفس الشرط}$$

$$y_3(t) = tu_3(t)$$

$$= t(ax_1(t) + bx_2(t))$$

$$= t(ax_1(t) + bx_2(t)) = [atu_1(t) + btu_2(t)]$$

$\Rightarrow$ system is linear #

$x(t)$ هي $u(t)$ المدخلة
 - input -

t is time varying ← t is time varying

$$y(t) = tu(t)$$

$$u(t) \rightarrow y_1(t)$$

$$u_1(t) = u(t-t_0) \rightarrow y_2(t)$$

$$\text{If } y_1(t-t_0) = y_2(t) \rightarrow \text{time invariant}$$

هل النظام ثابت زمني

$$y_1(t-t_0) = (t-t_0)u_1(t-t_0) \quad \dots (*)$$

$$y_2(t) = tu(t) = tu_1(t-t_0) \quad \dots (**)$$

$$y_1(t-t_0) \neq y_2(t) \quad \text{so system is time varying}$$

$\Rightarrow$ System with Memoryless.

$\Rightarrow$ System is Causal.