

Time domain analysis of C.T systems.

* If you want to study and analysis any system, you need to perform these steps:—

- [1] make modeling for a system like RLC circuit
- [2] develop mathematical equation.
- [3] Analysis
- [4] design the new system.

* Modeling is its a translator that represent a physical device by its equivalent equation which depend on your analysis type.

⇒ In our course we have two types for analysis a linear time invariant (LTI) system which are:—

- | | |
|---|--|
| <p>① Time domain</p> <ul style="list-style-type: none">- Differential equations (Derivatives)- Convolution | <p>② frequency domain</p> <ul style="list-style-type: none">- Transfer function (Laplace transform)- frequency Response |
|---|--|

⇒ In this chapter we will discuss the type one for analysis for continuous time system.

There is standard form for differential equation to LTI system :-

$$a_n \frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \dots + a_0 y(t) = b_m \frac{d^m x(t)}{dt^m} + \dots + b_0 x(t)$$

$a, b \Rightarrow$ are constants

* To solve the differential equation, you need to find $y(t)$ by substitute of $x(t)$ (given)

* Response of linear system can be expressed as the sum of two components :-

[1] Zero input response (natural response)

[2] Zero state response (forced response)

* The two component are independent of each other and each one can be computed independently of the other

Total response = Zero input response + Zero state response

- When $x(t) = 0$

- result from internal system condition only

- independent of $x(t)$

- response to non zero $x(t)$.

- all initial conditions are zero

- dependent on $x(t)$

این آخر // } Zero input resp
 output که جای قاعده
 Initial condition نه
 input نه
 $x(t) = 0$

Zero state response
 output که جای قاعده نه

initial condition = zero , input

[2]

$$y(t) = \underbrace{y(t)}_{ZI} + \underbrace{y(t)}_{ZS}$$



Example // $y(t) = \underbrace{y(t_0) e^{-(1/Rc)(t-t_0)}}_{\text{due to initial condition}} + \underbrace{\int_{t_0}^t \frac{1}{C} e^{-(1/Rc)(t-\tau)} d\tau}_{\text{due to input (Zero state response)}}$

* ملاحظة // الجزء الذي يتلوه عند $t = \infty$ هو الذي نعتبه على initial condition

وهذا الجزء يسمى natural response

* الجزء الذي يتلوه ← يكون نعتبه على Zero state

وهذا الجزء يسمى forced لأنه يتلوه ولا يتلوه، نعتبه كذلك $x(t)$ أي input

← جزء الـ Example //

In this system, the input starts at $t=t_0$ ($x(t)=0, t < t_0$)
 $y(t_0) \Rightarrow$ output voltage when the input is first applied
 (initial condition)

~ واجعل لي لسان صوفي في الذخيرة واجعلني من ورثة جنة النعيم ~

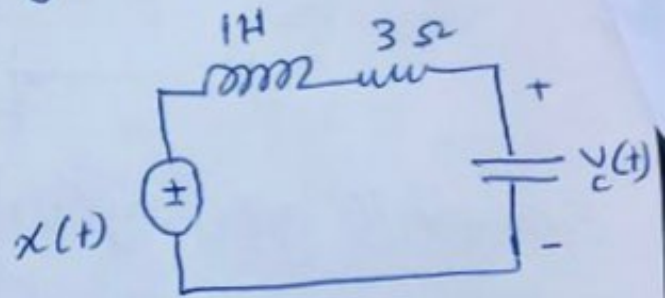


Example 1 Find $y_0(t)$ the zero input response for a LTI system described by the following differential equation //

$$(D^2 + 3D + 2)y(t) = Dx(t)$$

When initial conditions are

$$y_0(0) = 0, y_0'(0) = -5$$



Solution //

for Zero input response

طلب مني في السؤال أكتب Zero input response، وبما أن $x(t) = 0$ (input = صفر) ←

(initial condition = output) initial condition معطى

□ Characteristic equation for the system :-

$$(D^2 + 3D + 2)y_0(t) = 0$$

□ Find the roots //

$$\lambda^2 + 3\lambda + 2 = 0$$

$$(\lambda + 1)(\lambda + 2) = 0 \Rightarrow \boxed{\lambda = -1}, \boxed{\lambda = -2} \Rightarrow \text{roots}$$

□ Zero input response of $y(t)$ is given by:

$$y_0(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$$

$$y_0(t) = C_1 e^{-t} + C_2 e^{-2t}$$

□ Find the constants C_1, C_2 , using initial conditions

$$y_0(0) = 0, y_0'(0) = -5$$

$$\begin{aligned} 0 &= C_1 + C_2 \Rightarrow y_0(t) = C_1 e^{-t} + C_2 e^{-2t} \\ -5 &= -C_1 - 2C_2 \Rightarrow y_0'(t) = -C_1 e^{-t} - 2C_2 e^{-2t} \end{aligned}$$

□ 4

$$\Rightarrow \boxed{C_1 = -5} , \boxed{C_2 = 5}$$

then, Zero input response of $y(t)$

$$y_{ZI}(t) = -5e^{-t} + 5e^{-2t}$$

In this course, we will focus on finding the Zero state response (I.C = Zero)

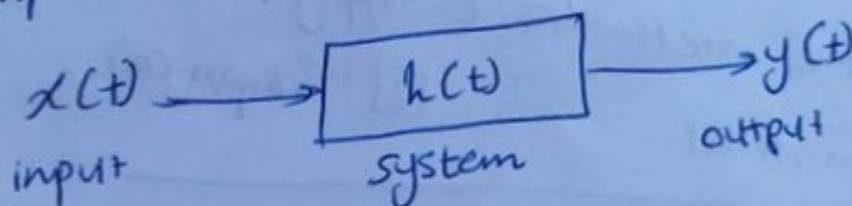
$$y_{zs}(t) = \int_{t_0}^t h(t-\lambda) x(\lambda) d\lambda \Rightarrow \text{General form for Zero-state response}$$

↳ This is called "Convolution"

😊 "واعبد ربك حتى تأتيك الساعة بغتة وهم لا يشعرون"

* Convolution

* It's an operation give the output of a system $h(t)$ if we applied the input $x(t)$.

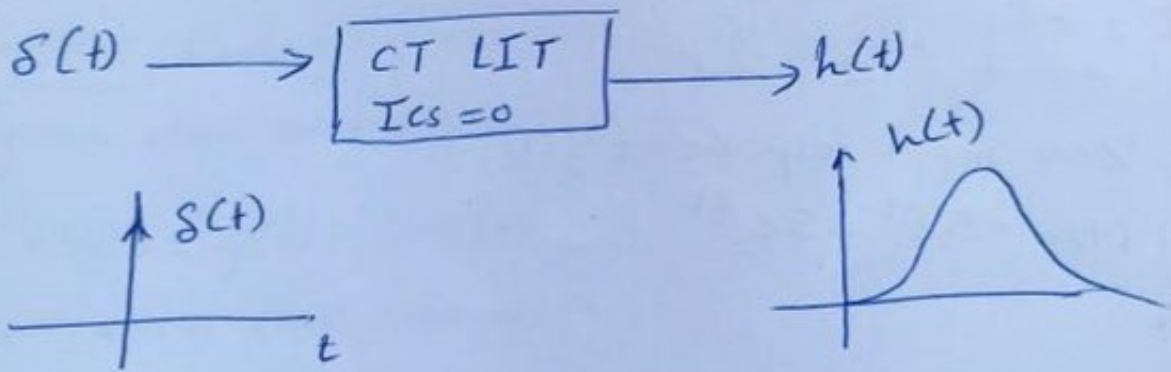


الحل آخر ← ال Convolution ~ فلابا جوت على output

لا system التي اسعو ~ $h(t)$ ~ ال يكون ال input $x(t)$

$h(t)$: impulse response of a system

↳ the response of the system when the input is unit impulse.



$$x(t) * s(t) = x(t)$$

↓
Convolution.

// ملاحظة

* Equation of Convolution :-

علاقة بين

$$y(t) = x(t) * h(t) = h(t) * x(t)$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau) d\tau$$

$$= x(t) * h(t)$$

$$\text{OR } y(t) = \int_{-\infty}^{\infty} x(t-\tau) h(\tau) d\tau$$

* There are two methods to apply Convolution :-

[1] Mathematical

[2] Graphical

* Mathematical Method :-

* Some properties of the Convolution //

[1] Commutative :-

$$x(t) * h(t) = h(t) * x(t)$$

[2] Associative : (series connection)

$$x(t) * h_1(t) * h_2(t) = [x(t) * h_1(t)] * h_2(t)$$

$$= [x(t) * h_2(t)] * h_1(t)$$

[6]

3] Distribution: (parallel connection)

$$x(t) * [h_1(t) + h_2(t)] = [x(t) * h_1(t)] + [x(t) * h_2(t)]$$

4] Convolution with an impulse:-

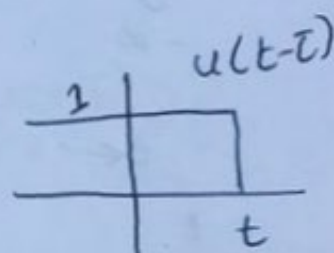
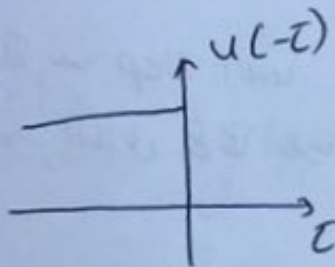
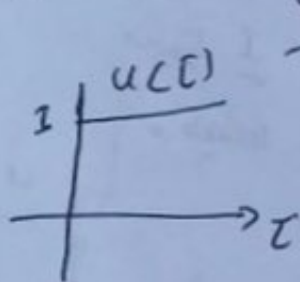
$$x(t) * \delta(t) = x(t)$$

5] Convolution with unit step

$$x(t) * u(t) = \int_{-\infty}^{\infty} x(\tau) u(t-\tau) d\tau = \int_{-\infty}^t x(\tau) d\tau$$

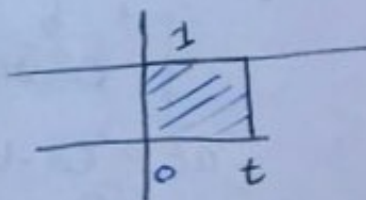
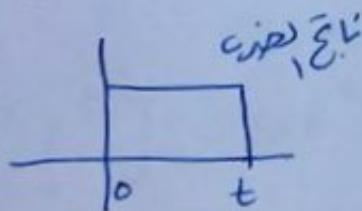
Example# Find the Convolution for $u(t) * u(t)$

$$u(t) * u(t) = \int_{-\infty}^{\infty} u(\tau) u(t-\tau) d\tau$$



$$= \int_{-\infty}^{\infty} u(\tau) u(t-\tau) d\tau = \int_0^t 1 d\tau = \tau \Big|_0^t = t$$

نقطة تقاطع المنحنى مع المحاور هي 0 و t



منطقة المساحة هي 0 ← t

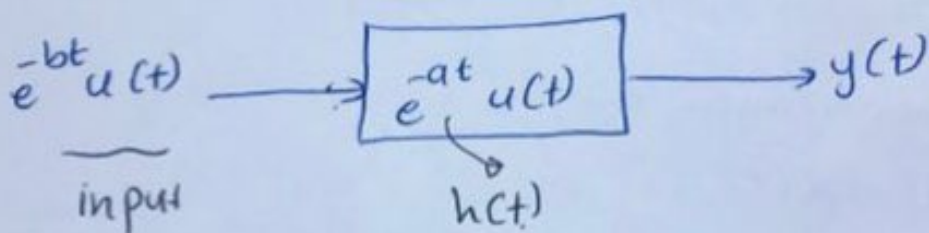
OR by properties //

$$u(t) * u(t) = \int_{-\infty}^{\infty} u(\tau) u(t-\tau) d\tau = \int_{-\infty}^t 1 d\tau = \tau \Big|_{-\infty}^t = t$$

[7]

Example #1 Find $y(t)$:

$y(t)$



$$a > b \\ b \neq a$$

$$y(t) = x(t) * h(t)$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

$$y(t) = \int_{-\infty}^{\infty} e^{-b\tau} u(\tau) \cdot e^{-a(t-\tau)} u(t-\tau) d\tau$$

$$= \int_{-\infty}^{\infty} e^{-b\tau} \cdot e^{-a(t-\tau)} \underbrace{u(t-\tau) \cdot u(\tau)} d\tau$$

ملاحظة 1 لأننا نعلم أن unit step

وظيفته 1 عندما $t \geq 0$ و 0 عندما $t < 0$

$$= \int_0^t e^{-b\tau} \cdot e^{-a(t-\tau)} d\tau$$

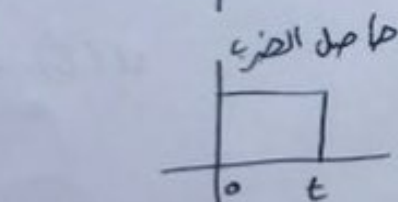
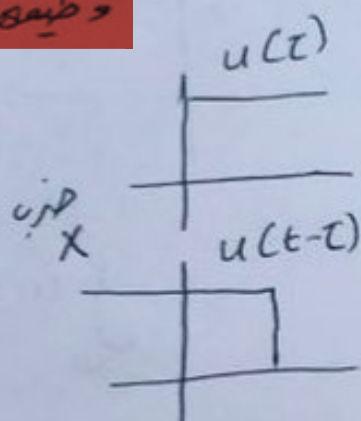
$$= \int_0^t e^{-b\tau} \cdot e^{-at} \cdot e^{a\tau} d\tau$$

$\downarrow$
 Constant

$$= e^{-at} \int_0^t e^{(a-b)\tau} d\tau$$

$$= e^{-at} \left[\frac{e^{(a-b)\tau}}{a-b} \right]_0^t$$

$$y(t) = e^{-at} \left[\frac{e^{(a-b)t}}{a-b} - \frac{1}{a-b} \right]$$



ملاحظة 2 عند التداخل $t \geq 0$

$$y(t) = \left(\frac{e^{-bt}}{a-b} - \frac{e^{-at}}{a-b} \right) u(t)$$

Example #1 if $x(t) = 10e^{-3t}u(t)$, $h(t) = [2e^{-2t} - e^{-t}]u(t)$

Find $y(t)$??

$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} 10e^{-3\tau}u(\tau) [2e^{-2(t-\tau)} - e^{-(t-\tau)}]u(t-\tau)d\tau$$

$$= \int_{-\infty}^{\infty} 10e^{-3\tau} \cdot 2e^{-2(t-\tau)} u(\tau)u(t-\tau)d\tau - \int_{-\infty}^{\infty} 10e^{-3\tau} e^{-(t-\tau)} u(\tau)u(t-\tau)d\tau$$

$$= \int_0^t 10e^{-3\tau} \cdot 2e^{-2(t-\tau)} d\tau - \int_0^t 10e^{-3\tau} \cdot e^{-(t-\tau)} d\tau$$

$$= 20 \int_0^t e^{-3\tau} \cdot e^{-2t+2\tau} d\tau - 10 \int_0^t e^{-3\tau} \cdot e^{-t+\tau} d\tau$$

$$= 20e^{-2t} \int_0^t e^{-\tau} d\tau - 10e^{-t} \int_0^t e^{-2\tau} d\tau$$

$$= 20e^{-2t} \cdot [-e^{-\tau}]_0^t - 10e^{-t} \left[-\frac{1}{2}e^{-2\tau} \right]_0^t$$

$$= 20e^{-2t} [-e^{-t} + 1] - 10e^{-t} \left[-\frac{1}{2}e^{-2t} + \frac{1}{2} \right]$$

$$= -20e^{-3t} + 20e^{-2t} + 5e^{-3t} - 5e^{-t}$$

$$= [-15e^{-3t} + 20e^{-2t} - 5e^{-t}]u(t)$$

#

[9]

Ex #1, $x(t) = t e^{-t} u(t)$, $h(t) = u(t)$, find $y(t)$

$y_2(t)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau) u(t-\tau) d\tau$$

$$= \int_{-\infty}^{\infty} \tau e^{-\tau} \underline{u(\tau)} \cdot \underline{u(t-\tau)} d\tau$$

$$= \int_0^t \tau e^{-\tau} d\tau$$

$$= -\tau e^{-\tau} - e^{-\tau} \Big|_0^t$$

$$= (-t e^{-t} - e^{-t}) - (0 - e^0)$$

$$= -t e^{-t} - e^{-t} + 1$$

$$\boxed{y(t) = [e^{-t}(-t-1) + 1] u(t)} \quad \#$$

If $x(t) = u(t)$, $h(t) = e^{-2t} u(t) + \delta(t)$

Find $y_1(t)$, $y_2(t)$, $y_T(t)$

$$y_1(t) = \int_{-\infty}^t u(t-\tau) e^{-2\tau} \underline{u(\tau)} d\tau$$

$$= \int_0^t e^{-2\tau} d\tau \Rightarrow -\frac{1}{2} e^{-2\tau} \Big|_0^t \Rightarrow -\frac{1}{2} [e^{-2t} - 1]$$

$$y_1(t) = -\frac{1}{2} [e^{-2t} - 1]$$

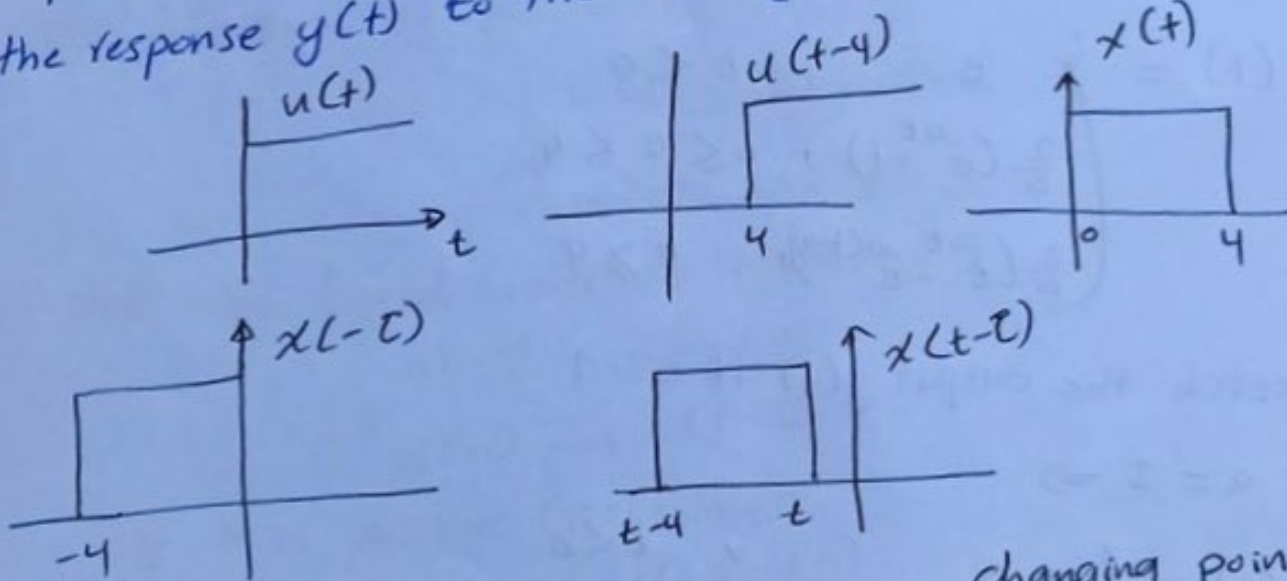
$$\boxed{y_1(t) = \left[\frac{-e^{-2t}}{2} + \frac{1}{2} \right] u(t)}$$

10

$$y(t) = \int_{-\infty}^{\infty} u(t-\tau) \delta(\tau) d\tau = u(t) \quad \text{shifting prop}$$

$$y_T = y_1 + y_2 = \left[\frac{e^{-2t}}{-2} + \frac{1}{2} \right] u(t) + u(t)$$

Example # | A linear time invariant has the following impulse response $h(t) = 2e^{-at} u(t)$, use convolution to find the response $y(t)$ to the following input $x(t) = u(t) - u(t-4)$



$$t=0 \Rightarrow t=4$$

$$\begin{aligned} t &< 0 \\ 0 &\leq t < 4 \\ t &\geq 4 \end{aligned}$$

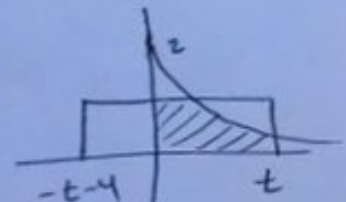
$$\text{for } t < 0 \Rightarrow y(t) = 0$$

لا يوجد تقاطع بين الدالتين

$$\begin{aligned} \text{for } 0 &\leq t < 4 \\ -4 &\leq t-4 < 0 \end{aligned}$$

changing point

$$\begin{aligned} t &\} \text{ from } x(t-\tau) \\ t-4 &\} \\ 0 &\} \text{ from } h(t) \end{aligned}$$



$$\begin{aligned} y(t) &= \int_0^t 2e^{-a\tau} d\tau = \left. \frac{2e^{-a\tau}}{-a} \right|_0^t \\ &= -\frac{2}{a} [e^{-at} - 1] \end{aligned}$$

$$t \geq 4$$

$$t-4 \geq 0$$

$$y(t) = \int_{t-4}^t 2e^{-a\tau} d\tau$$

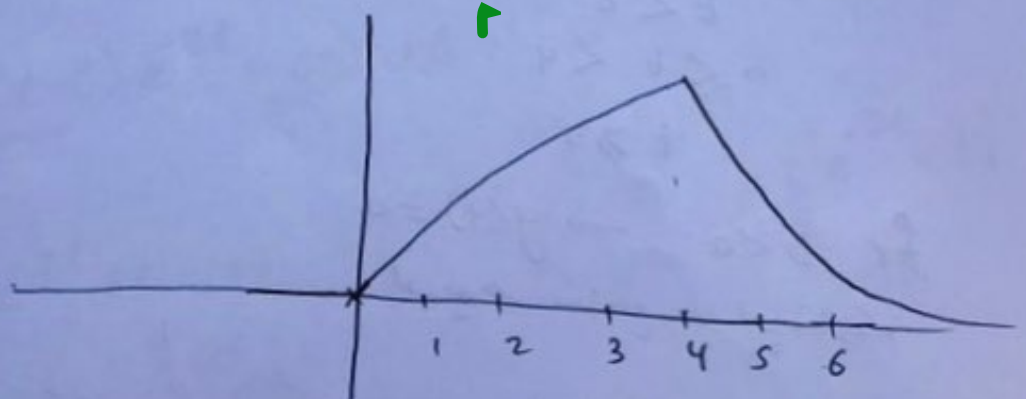
$$= -\frac{2}{a} [e^{-a\tau}] \Big|_{t-4}^t$$

$$= -\frac{2}{a} [e^{-at} - e^{-a(t-4)}]$$

$$y(t) = \begin{cases} 0, & t < 0 \\ -\frac{2}{a}(e^{-at} - 1), & 0 \leq t < 4 \\ -\frac{2}{a}(e^{-at} - e^{-a(t-4)}), & t \geq 4 \end{cases}$$

(b) sketch the output $y(t)$ if $a=1$
if $a=1 \Rightarrow$

$$y(t) = \begin{cases} 0, & t < 0 \\ -2(e^{-t} - 1), & 0 \leq t < 4 \\ -2(e^{-t} - e^{4-t}), & t \geq 4 \end{cases}$$



2] Graphical Convolution operation // many signals have no exact mathematical description so that it can be described only graphically.

* Steps to find the graphical convolution $x(t) * h(t)$??

[1] Convert the variable t by τ in $x(t) \rightarrow x(\tau)$
 $h(t) \rightarrow h(\tau)$.

[2] Keep one signal fixed like $x(\tau)$ or $h(\tau)$ then find the the inversion of the other one.

$$h(\tau) \rightarrow h(-\tau)$$

$$\text{or } x(\tau) \rightarrow x(-\tau)$$

[3] Shift the flipped (inverted) signal by t .

$$h(-\tau) \rightarrow h(t-\tau)$$

$$\text{or } x(-\tau) \rightarrow x(t-\tau)$$

If t is positive (right shift)

if t is negative (left shift)

[4] Determine the critical points or change points & divide the two signal into regions depend on this point

[5] The area under the product $x(\tau) \cdot h(t-\tau)$ is the value of convolution at t .

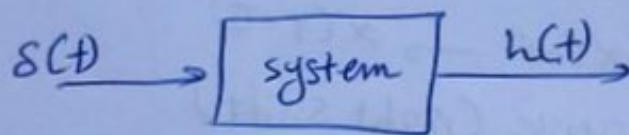
[6] Do the integral $y(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau) d\tau$.

Convolution جمع

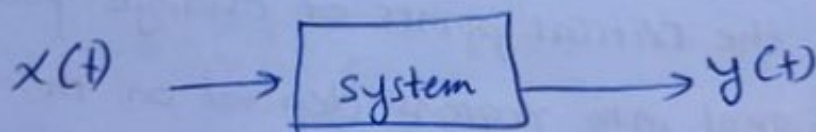
لو كان عندي LTI system ومعلوم response تابع ال impulse ($h(t)$)
فأنا بإمكانني الحصول على response تابع Input مع ذلك ال Convolution
كأنني //

نقسم ال Input ال مجموعة من impulses، بما أن ال system عندي
time invariant فكل impulse فزاحة سيألفنا انجاعة في output
(impulse response) وبما أن ال system عندي linear
فأنا صغرت ال input ال مجموعة من ال impulses، إذن ال output
عندي هو جمع response تابع كل واحدة.

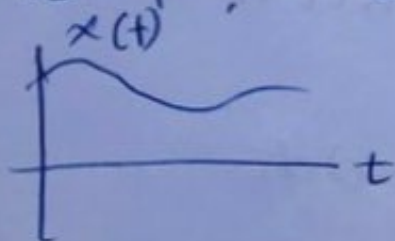
* ملاحظة // Convolution هو طريقة لإيجاد ال response تابع
ال system بدلاً من استخدام المعادلات التفاضلية.

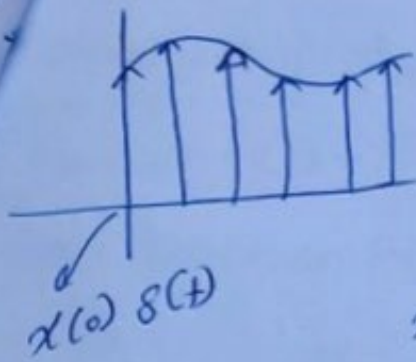


الآن نفرض أن لدينا input هو $x(t)$ ، وال output هو $y(t)$



نقسم ال input ال مجموعة من النبضات باستخدام طريقة Sifting





$$x(0)\delta(t) \rightarrow \boxed{\text{system}} \rightarrow x(0)h(t)$$

$$x(1)\delta(t-1) \rightarrow \boxed{\text{system}} \rightarrow x(1)h(t-1)$$

$$x(2)\delta(t-2) \rightarrow \boxed{\text{system}} \rightarrow x(2)h(t-2)$$

* $\delta(t)$ کا آٹھ ال system linear ← خاصہ خطہ اور input
 خاصہ خطہ $x(t)$ اور output خاصہ خطہ $x(t)$

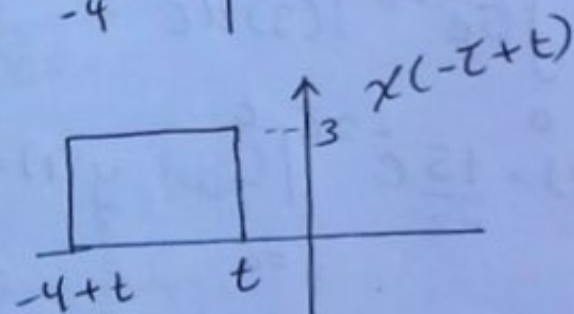
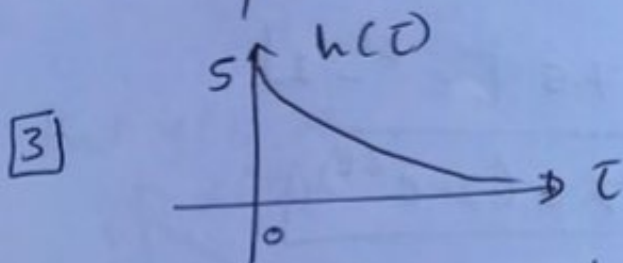
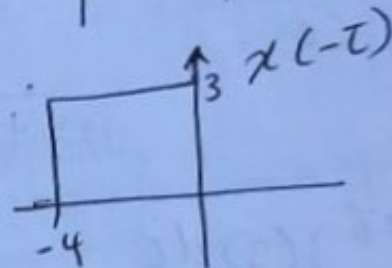
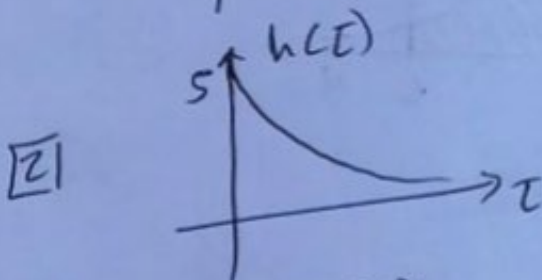
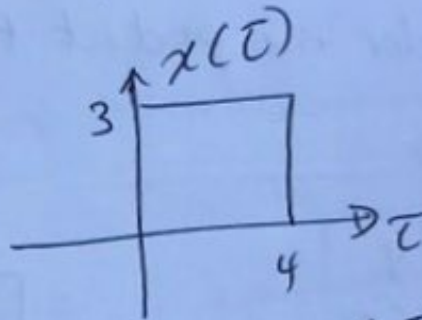
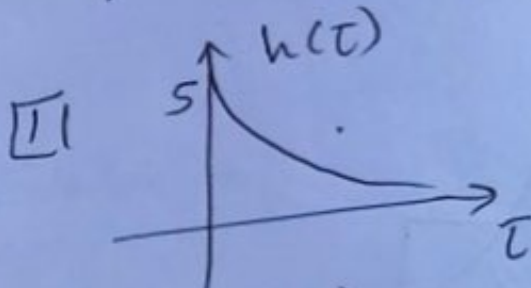
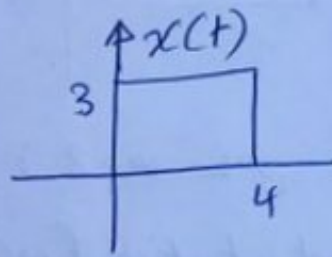
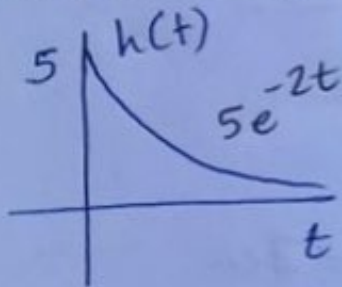
* $\delta(t)$ کا آٹھ system time invariant ہے
 یعنی آٹھ shift اور input خاصہ خطہ shift اور output
 وہ آٹھ خاصہ خطہ اور system

* $x(t)$ کے مجموعہ کے لئے pulses

وہ آٹھ اور system خاصہ خطہ linear خاصہ خطہ output
 مجموعہ اور response خاصہ خطہ impulses اور مجموعہ کے لئے

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

Example #1 | Given the impulse response of an LTI system is $h(t) = 5e^{-2t}u(t)$, Compute the output of this system due to an input which is a 4 second pulse of height 3.



changing point
 $t=0$

changing point
 t
 $-4+t$

نقطة التغير

Changing point نقطة التغير

$$t=0 \Rightarrow \boxed{t=0}$$

$$-4+t=0 \Rightarrow \boxed{t=4}$$

$$t < 0$$

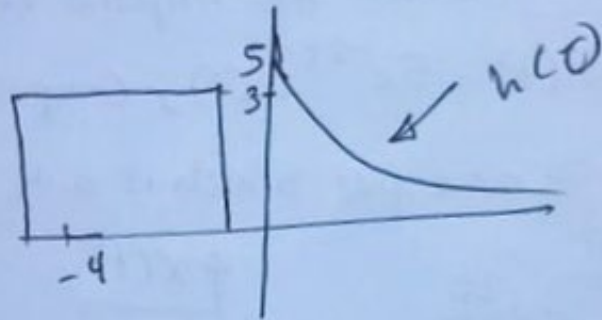
$$0 \leq t < 4$$

$$t \geq 4$$

$$t < 0$$

$$t-4 < -4$$

$h(t-\tau)$



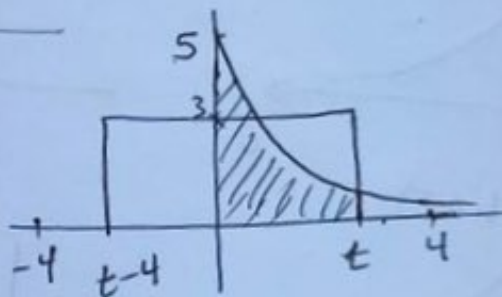
No overlap (لا يوجد تقاطع)

Area under the product two function = Zero

$$y(t) = 0$$

$$0 \leq t < 4$$

$$-4 \leq t-4 < 0$$



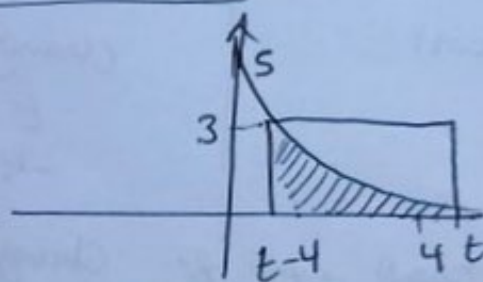
$$y(t) = \int_0^t (5e^{-2\tau})(3) d\tau$$

$$y(t) = \frac{15}{-2} e^{-2\tau} \Big|_{\tau=0}^t \Rightarrow y(t) = -7.5 [e^{-2t} - 1]$$

$$= 7.5 (1 - e^{-2t})$$

$$t \geq 4$$

$$t-4 \geq 0$$



$$y(t) = \int_{t-4}^t (3)(5e^{-2\tau}) d\tau$$

$$\tau = t-4$$

$$= \frac{15}{-2} e^{-2\tau} \Big|_{\tau=t-4}^t \Rightarrow -7.5 [e^{-2t} - e^{-2(t-4)}]$$

$$= -7.5 [e^{-2t} - e^{-2t} \cdot e^8]$$

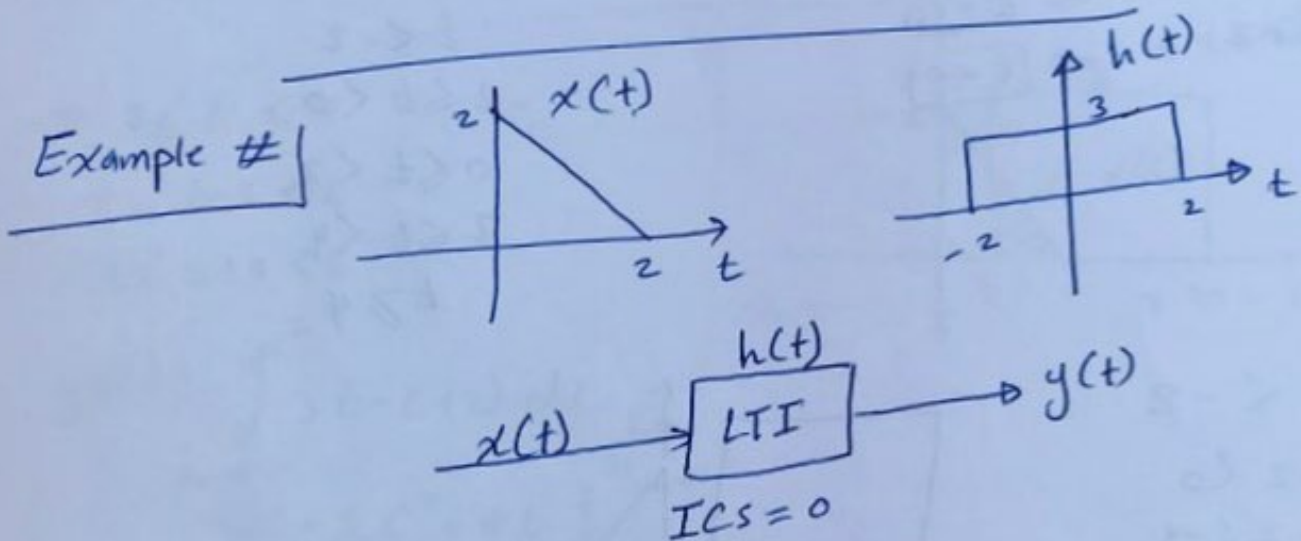
$$= -7.5 [e^{-2t} (1 - e^8)]$$

[17]

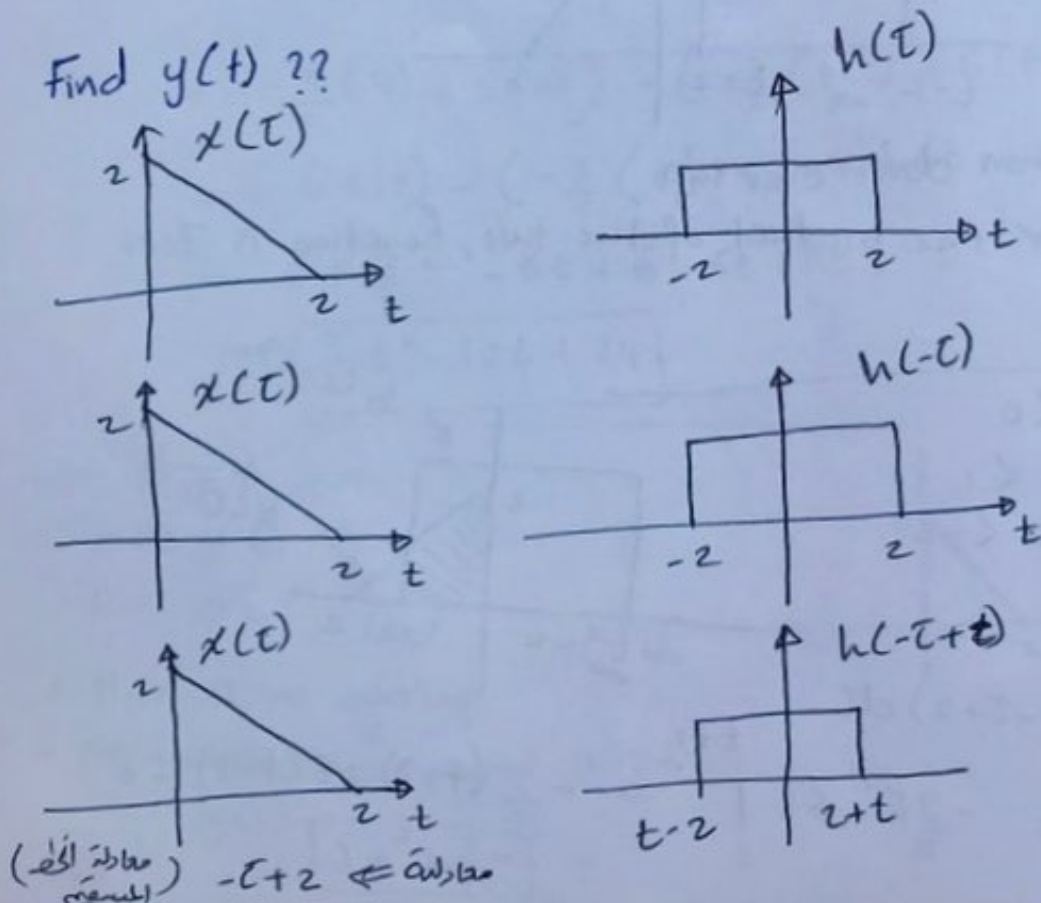
$$= 7.5 e^{-2t} (e^8 - 1)$$

$$y(t) = \begin{cases} 0 & , t < 0 \\ 7.5(1 - e^{-2t}) & , 0 \leq t < 4 \\ 7.5 e^{-2t} (e^8 - 1) & , t \geq 4 \end{cases}$$

ملاحظة // إذا قمنا بعمل flip و shift لـ $h(t)$ من أجل البقاء
فإننا نحصل على نفس الجواب



Find $y(t)$??



Changing point نقطة التغير

⇒ From $x(t)$: changing point $(0, 2)$

⇒ From $h(t-2)$: changing point $(t-2, t+2)$

نقطة التغير مع بعضهما البعض

$$t-2=0 \Rightarrow \boxed{t=2}$$

$$t-2=2 \Rightarrow \boxed{t=4}$$

$$t+2=0 \Rightarrow \boxed{t=-2}$$

$$t+2=2 \Rightarrow \boxed{t=0}$$

نقطة التغير

$$t < -2$$

$$-2 \leq t < 0$$

$$0 \leq t < 2$$

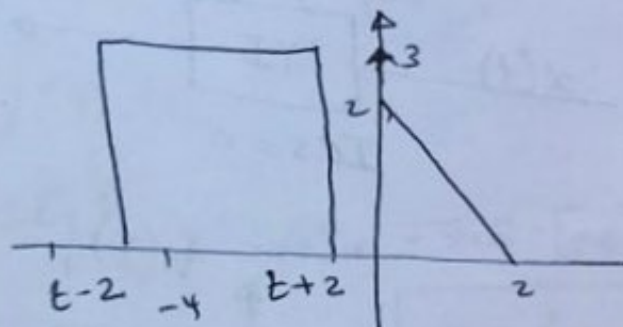
$$2 \leq t < 4$$

$$t \geq 4$$

$$\Rightarrow t < -2$$

$$t+2 < 0$$

$$t-2 < -4$$



* the function don't overlap

* area under the product of the two function is Zero

$$y(t) = 0$$

$$\Rightarrow -2 \leq t < 0$$

$$0 \leq t+2 < 2$$

$$-4 < t-2 < -2$$

$$t+2$$

$$y(t) = \int_0^{t+2} (3)(-t+2) dt$$

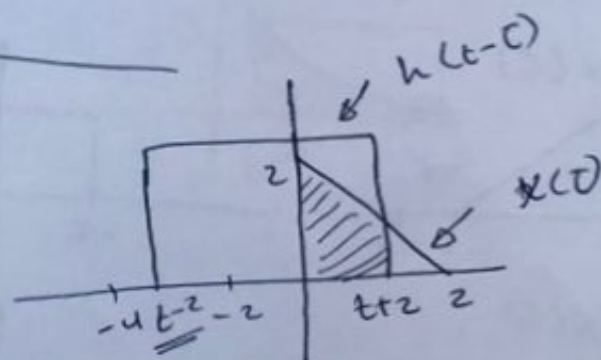
$$= \left. -\frac{3t^2}{2} + 6t \right|_0^{t+2}$$

$$t+2$$

$$\Rightarrow -\frac{3}{2}(t+2)^2 + 6(t+2) - 0$$

$$= \left[-\frac{3}{2}t^2 + 6 \right]$$

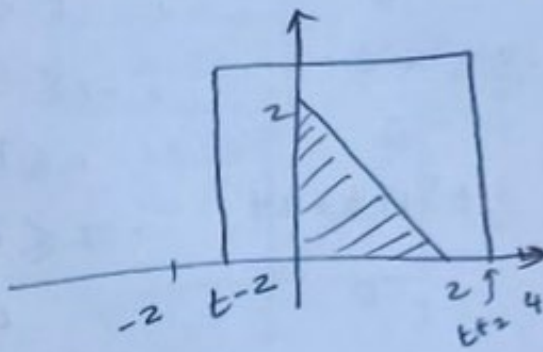
[9]



$$\Rightarrow 0 \leq t < 2$$

$$-2 \leq t-2 < 0$$

$$2 \leq t+2 < 4$$



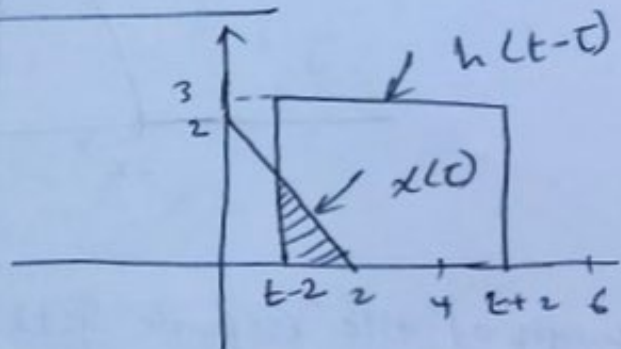
$$y(t) = \int_0^2 3(-\tau+2) d\tau$$

$$= \left. -\frac{3}{2}\tau^2 + 6\tau \right|_0^2 \Rightarrow \left(-\frac{3}{2}(4) + 12 \right) - (0) = \boxed{6}$$

$$\Rightarrow 2 \leq t < 4$$

$$0 \leq t-2 < 2$$

$$4 \leq t+2 < 6$$



$$y(t) = \int_{t-2}^2 3(-\tau+2) d\tau$$

$$= \left. -\frac{3}{2}\tau^2 + 6\tau \right|_{t-2}^2$$

$$= \left(-\frac{3}{2}(4) + 6(2) \right) - \left(-\frac{3}{2}(t-2)^2 + 6(t-2) \right)$$

$$= (-6 + 12) - \left(-\frac{3}{2}(t^2 - 4t + 4) + 6t - 12 \right)$$

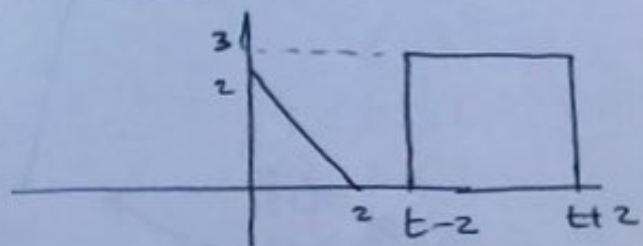
$$= 6 + \frac{3}{2}t^2 - 6t + 6 - 6t + 12$$

$$= \boxed{\frac{3}{2}t^2 - 12t + 24}$$

$$\Rightarrow t \geq 4$$

$$t+2 \geq 6$$

$$t-2 \geq 2$$



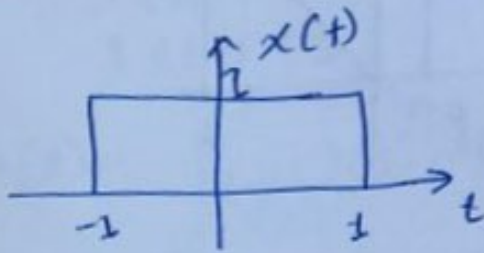
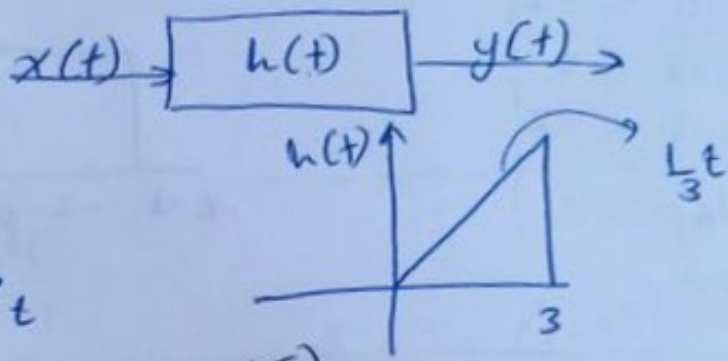
* There is no overlap

* Area under their period is zero

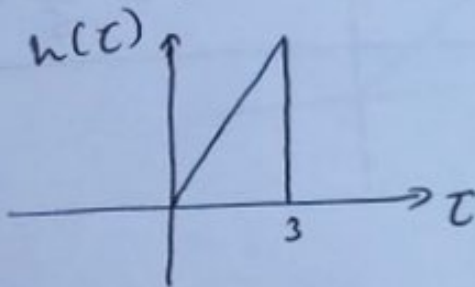
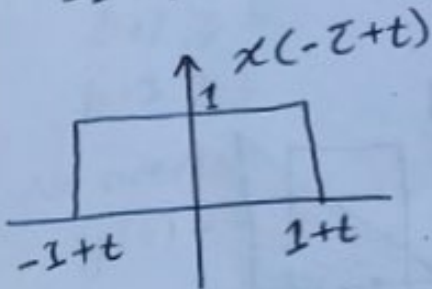
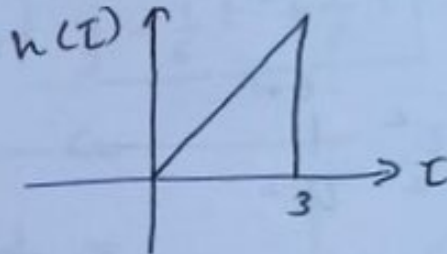
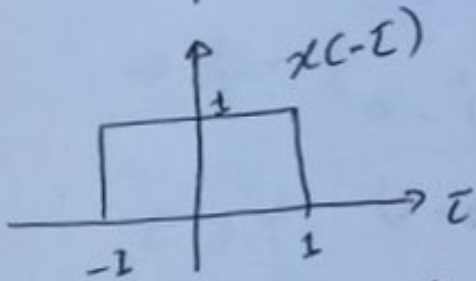
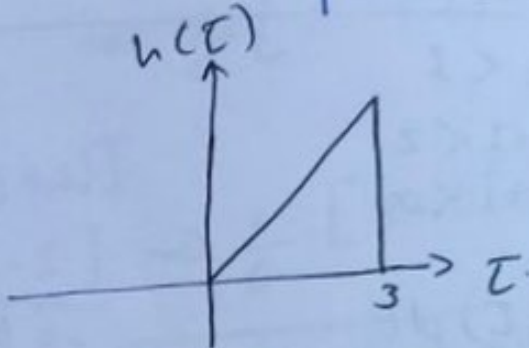
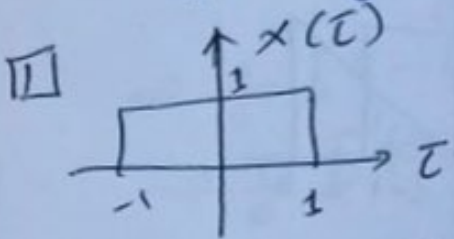
$$y(t) = 0$$

$\boxed{20}$

Example



Sketch $y(t)$.



changing point
 $-1+t, 1+t$

changing point
 $0, 3$

changing point نقاط كسابة

$$\begin{aligned} -1+t=0 &\Rightarrow \boxed{t=1} \\ -1+t=3 &\Rightarrow \boxed{t=4} \\ 1+t=0 &\Rightarrow \boxed{t=-1} \\ 1+t=3 &\Rightarrow \boxed{t=2} \end{aligned}$$

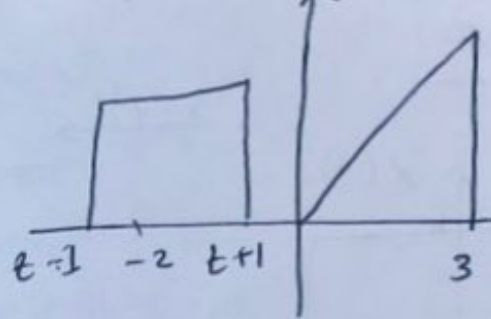
$$\begin{aligned} t &< -1 \\ -1 &\leq t < 1 \\ 1 &\leq t < 2 \\ 2 &\leq t < 4 \\ t &\geq 4 \end{aligned}$$

$$\Rightarrow \text{at } t < -1$$

$$t+1 < 0$$

$$t-1 < -2$$

No overlap
 $y(t) = 0$



$$\Rightarrow \text{at } -1 \leq t < 1$$

$$0 \leq t+1 < 2$$

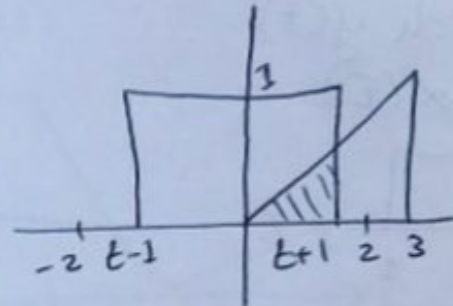
$$-2 \leq t-1 < 0$$

$$y(t) = \int_{\tau=0}^{t+1} (1) \left(\frac{1}{3}\tau\right) d\tau$$

$$y(t) = \frac{1}{3} \frac{\tau^2}{2} \Big|_{\tau=0}^{t+1}$$

$$\Rightarrow \frac{1}{6} (t+1)^2 - 0$$

$$= \frac{1}{6} (t^2 + 2t + 1)$$



$$1 \leq t < 2$$

$$2 \leq t+1 < 3$$

$$0 \leq t-1 < 1$$

$$y(t) = \int_{\tau=t-1}^{t+1} \left(\frac{1}{3}\tau\right) (1) d\tau$$

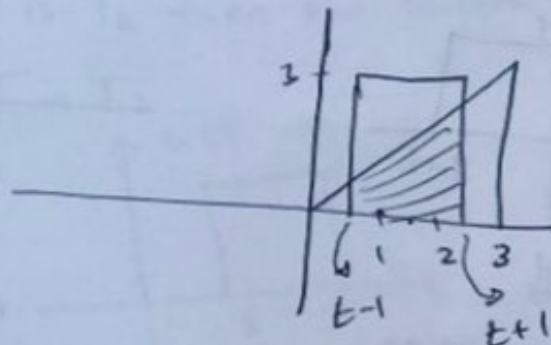
$$= \frac{1}{6} \tau^2 \Big|_{\tau=t-1}^{t+1}$$

$$\Rightarrow \frac{1}{6} [(t+1)^2 - (t-1)^2]$$

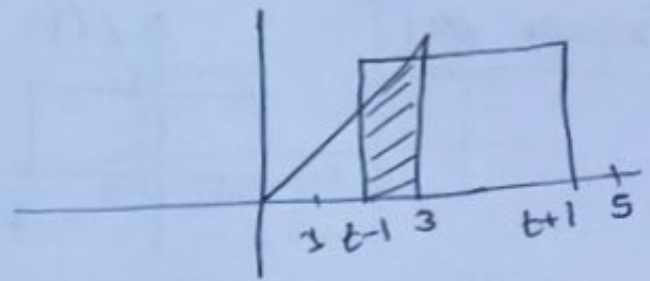
$$\frac{1}{6} [t^2 + 2t + 1 - (t^2 - 2t + 1)]$$

$$y(t) = \frac{1}{6} [t^2 + 2t + 1 - t^2 + 2t - 1]$$

$$y(t) = \frac{4}{6}t \Rightarrow \boxed{y(t) = \frac{2}{3}t}$$



$$\Rightarrow \begin{aligned} 2 < t < 4 \\ 3 < t+1 < 5 \\ 1 < t-1 < 3 \end{aligned}$$



$$y(t) = \int_{t-1}^3 \left(\frac{1}{3}\tau\right)(2) d\tau$$

$$\frac{1}{3} \frac{\tau^2}{2} \Big|_{t-1}^3 \Rightarrow \frac{1}{6} [3^2 - (t-1)^2]$$

$$\Rightarrow \frac{1}{6} [9 - (t^2 - 2t + 1)]$$

$$\Rightarrow \frac{1}{6} [9 - t^2 + 2t - 1] \Rightarrow \frac{1}{6} [8 - t^2 + 2t]$$

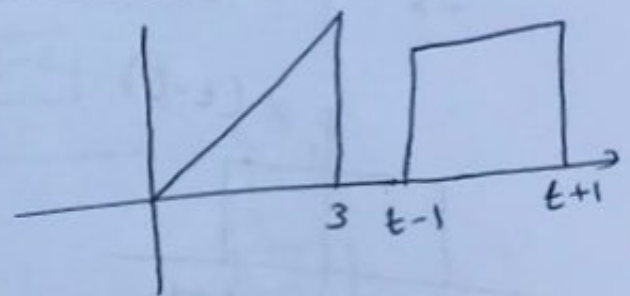
$$\Rightarrow \frac{8}{6} - \frac{1}{6}t^2 + \frac{2t}{6} \Rightarrow \boxed{\frac{4}{3} - \frac{1}{6}t^2 + \frac{1}{3}t}$$

$$t \geq 4$$

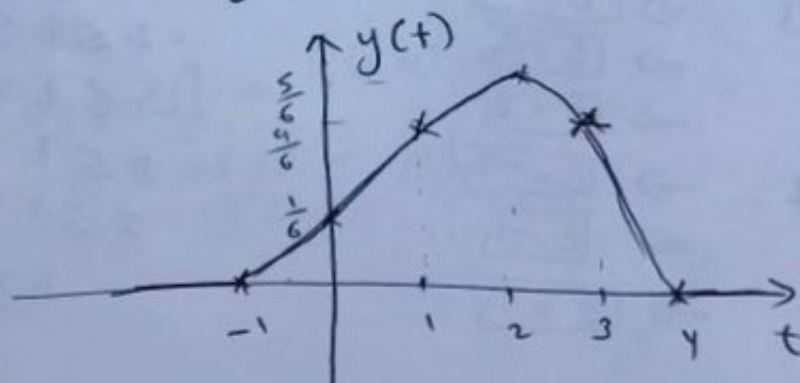
$$t+1 \geq 5$$

$$t-1 \geq 3$$

No overlap
 $y(t) = 0$

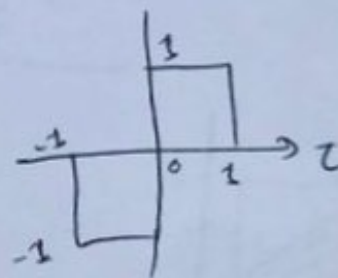
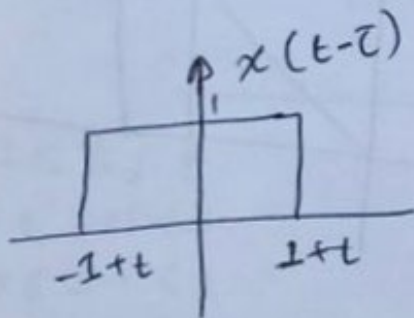
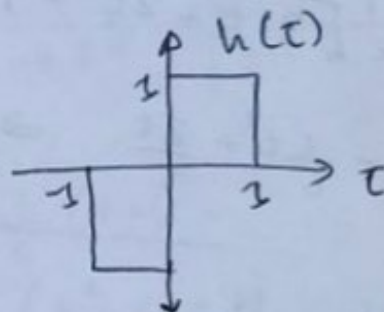
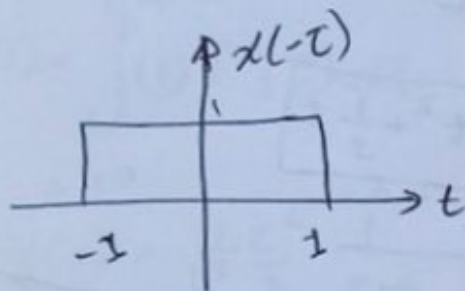
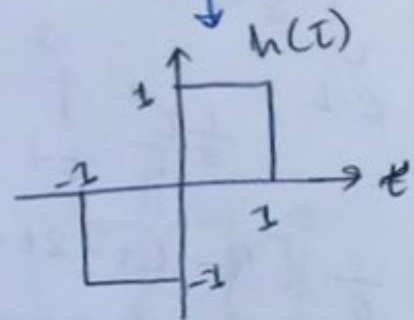
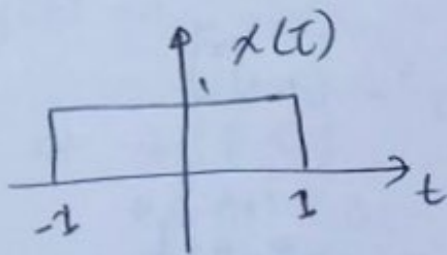
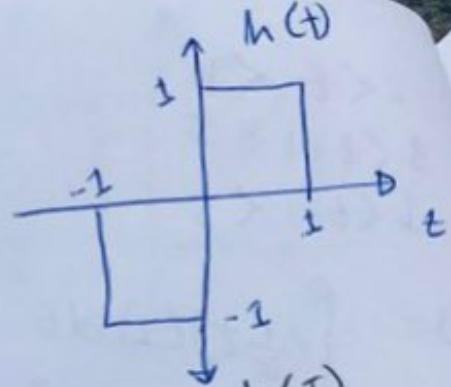
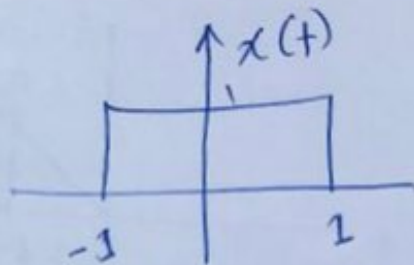


$$y(t) = \begin{cases} 0 & , t < -1 \\ \frac{1}{6}(t^2 + 2t + 1) & , -1 \leq t < 1 \\ \frac{2t}{3} & , 1 \leq t < 2 \\ \frac{4}{3} - \frac{1}{6}t^2 + \frac{1}{3}t & , 2 \leq t < 4 \\ 0 & , t \geq 4 \end{cases}$$



[24]

Example #1



Changing point
 $-1+t, 1+t$

Changing point
 $-1, 0, 1$

نقاط التغير ← Changing point مع تغير الإشارة

$$\begin{aligned} -1+t &= -1 &\Rightarrow t &= 0 \\ -1+t &= 0 &\Rightarrow t &= 1 \\ -1+t &= 1 &\Rightarrow t &= 2 \\ 1+t &= -1 &\Rightarrow t &= -2 \\ 1+t &= 0 &\Rightarrow t &= -1 \\ 1+t &= 1 &\Rightarrow t &= 0 \end{aligned}$$

$$\begin{aligned} t &< -2 \\ -2 &\leq t < -1 \\ -1 &\leq t < 0 \\ 0 &\leq t < 1 \\ 1 &\leq t < 2 \\ t &\geq 2 \end{aligned}$$

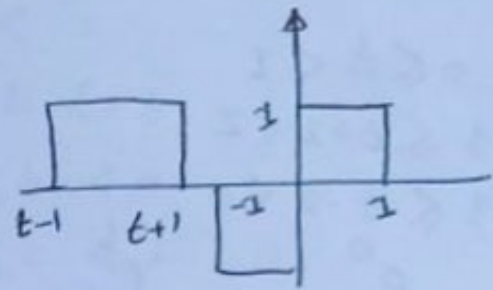
$\Rightarrow$ at $t < -2$

$$t+1 < -1$$

$$t-1 < -3$$

there is no overlap

$$y(t) = 0$$



$\Rightarrow$ at $-2 \leq t < -1$

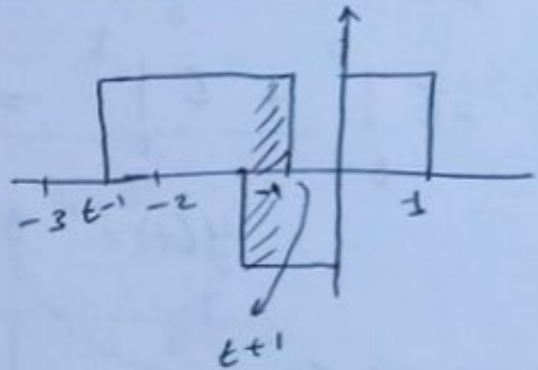
$$-1 \leq t+1 < 0$$

$$-3 \leq t-1 < -2$$

$$y(t) = \int_{-1}^{t+1} (1)(-1) d\tau$$

$$\Rightarrow -\tau \Big|_{-1}^{t+1} \Rightarrow -[(t+1) - (-1)]$$

$$y(t) = -t-1-1 = \boxed{-t-2}$$



$\Rightarrow$ at $-1 \leq t < 0$

$$0 \leq t+1 < 1$$

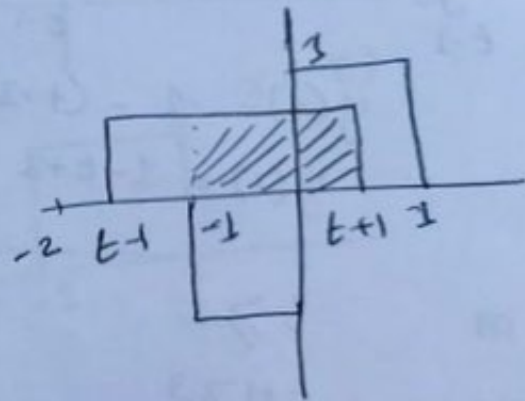
$$-2 \leq t-1 < -1$$

$$y(t) = \int_{-1}^0 (1)(-1) d\tau + \int_0^{t+1} (1)(1) d\tau$$

$$= -\tau \Big|_{-1}^0 + \tau \Big|_0^{t+1}$$

$$-[0 - (-1)] + ((t+1) - 0)$$

$$-1 + t+1 = \boxed{t}$$



$$\Rightarrow 0 \leq t < 1$$

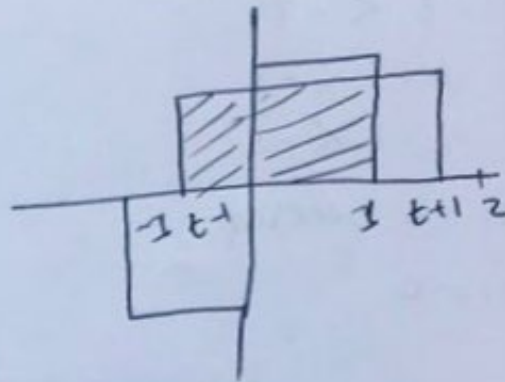
$$1 \leq t+1 < 2$$

$$-1 \leq t-1 < 0$$

$$y(t) = \int_{t-1}^0 (1)(-1) d\tau + \int_0^1 (1)(1) d\tau$$

$$-\tau \Big|_{t-1}^0 + \tau \Big|_0^1 \Rightarrow -(0 - (t-1)) + (1 - 0)$$

$$y(t) = t - 1 + 1 \Rightarrow \boxed{y(t) = t}$$



$$\Rightarrow 1 \leq t < 2$$

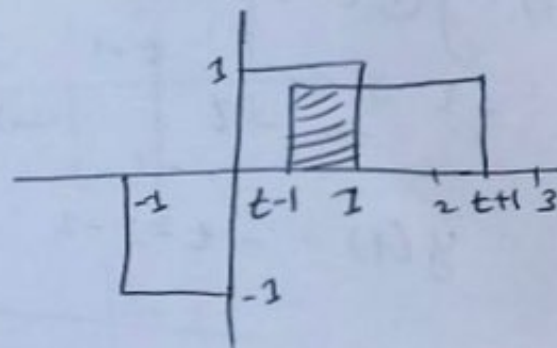
$$2 \leq t+1 < 3$$

$$0 \leq t-1 < 1$$

$$y(t) = \int_{t-1}^1 (1)(1) d\tau = \tau \Big|_{t-1}^1$$

$$y(t) = 1 - (t-1)$$

$$y(t) = 1 - t + 1 \Rightarrow \boxed{y(t) = -t + 2}$$



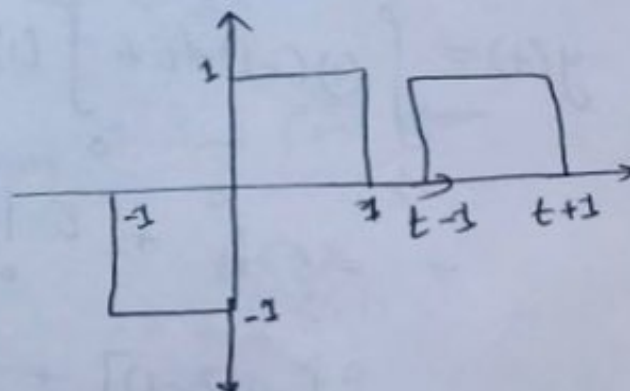
$$\Rightarrow \text{at } t \geq 2$$

$$t+1 \geq 3$$

$$t-1 \geq 1$$

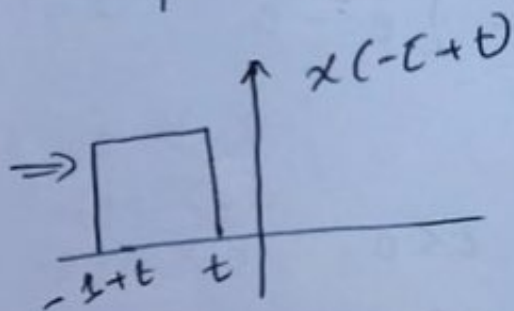
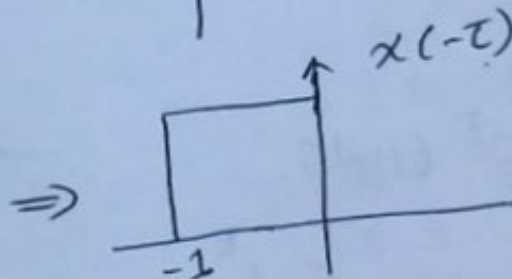
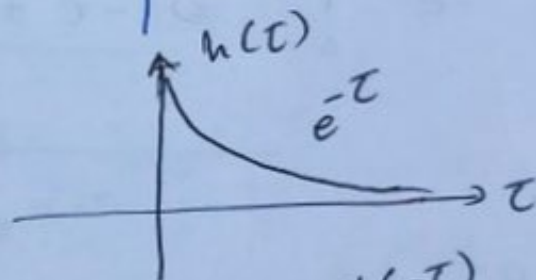
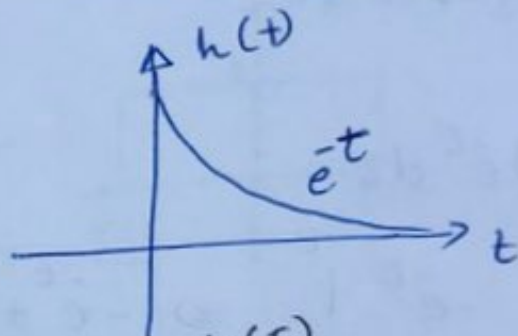
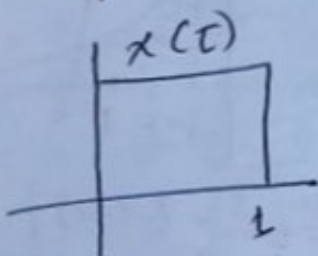
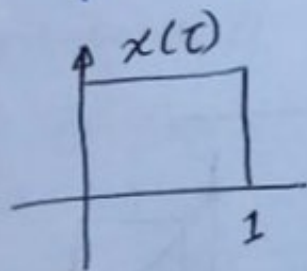
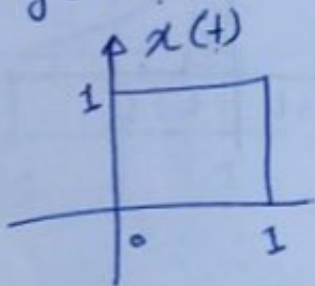
there is no overlap

$$y(t) = 0$$



$$y(t) = \begin{cases} 0 & , t < -2 \\ -t-2 & , -2 \leq t < -1 \\ t & , -1 \leq t < 0 \\ t & , 0 \leq t < 1 \\ -t+2 & , 1 \leq t < 2 \\ 0 & , t \geq 2 \end{cases}$$

find $y(t)$?



changing point

from $h(\tau) \Rightarrow 0$

from $x(t-\tau) \Rightarrow -1+t, t$

تغير إشارة changing point مع بعض البقي $t=1$ $\Rightarrow -1+t=0$
 $t=0$

$$\begin{aligned} t &< 0 \\ 0 &\leq t < 1 \\ t &\geq 1 \end{aligned}$$

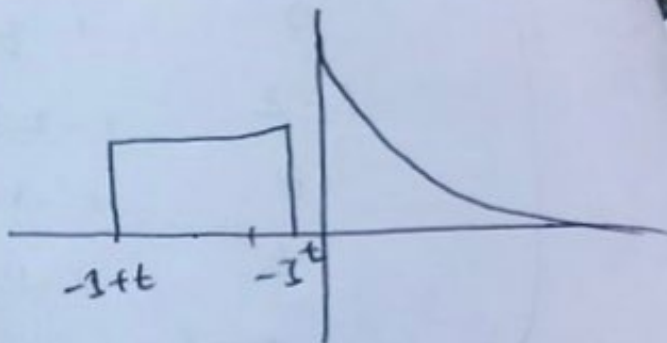
(28)

$$\Rightarrow \text{at } t < 0$$

$$-1+t < -1$$

there is no overlap

$$y(t) = 0$$

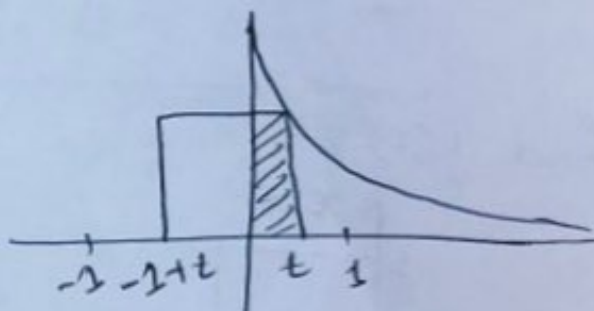


$$\Rightarrow \text{at } 0 \leq t < 1$$

$$-1 \leq -1+t < 0$$

$$y(t) = \int_0^t (1) e^{-\tau} d\tau$$

$$= -e^{-\tau} \Big|_0^t \Rightarrow \boxed{-e^{-t} + 1}$$



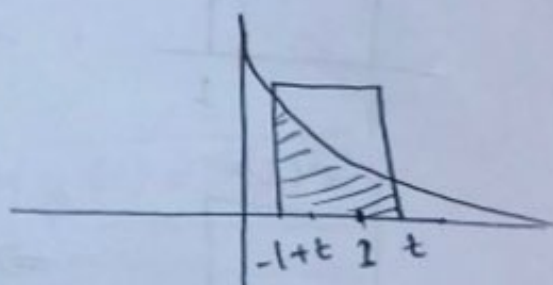
$$t \geq 1$$

$$-1+t \geq 0$$

$$y(t) = \int_{-1+t}^t e^{-\tau} (1) d\tau$$

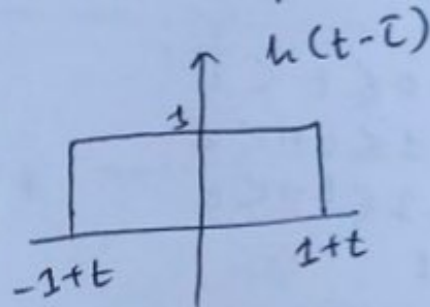
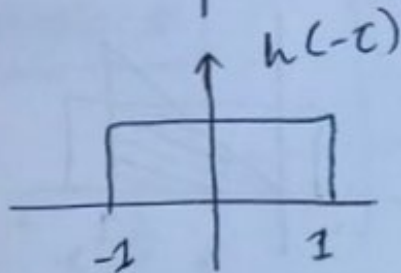
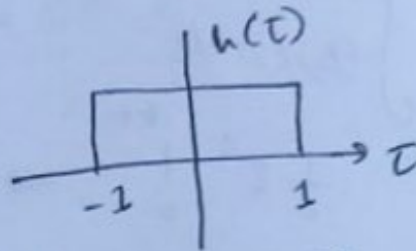
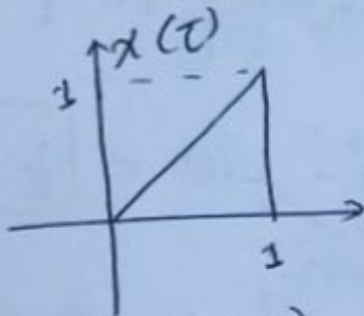
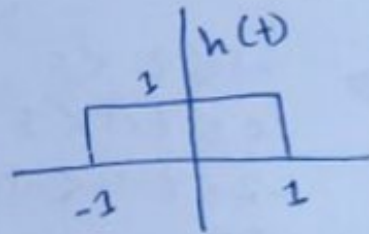
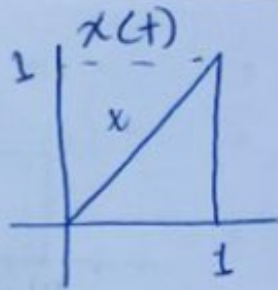
$$\Rightarrow -e^{-\tau} \Big|_{-1+t}^t \Rightarrow -[e^{-t} - e^{-(-1+t)}]$$

$$\boxed{y(t) = -e^{-t} + e^{-t+1}}$$



$$y(t) = \begin{cases} 0 & , t < 0 \\ -e^{-t} + 1 & , 0 \leq t < 1 \\ -e^{-t} + e^{-t+1} & , t \geq 1 \end{cases}$$

Example



changing point //

from $x(\tau) \Rightarrow 0, 1$

from $h(t-\tau) \Rightarrow -1+t, 1+t$

$$-1+t=0 \Rightarrow \boxed{t=1}$$

$$-1+t=1 \Rightarrow \boxed{t=2}$$

$$1+t=0 \Rightarrow \boxed{t=-1}$$

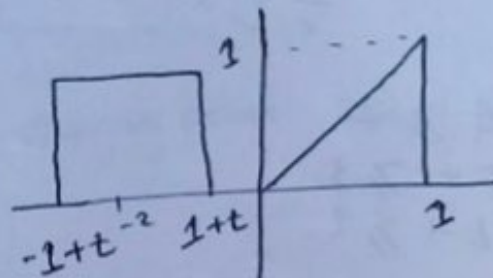
$$1+t=1 \Rightarrow \boxed{t=0}$$

$$\begin{aligned} t &< -1 \\ -1 &< t < 0 \\ 0 &\leq t < 1 \\ 1 &\leq t < 2 \\ t &\geq 2 \end{aligned}$$

$$\begin{aligned} \Rightarrow \text{at } t &< -1 \\ -1+t &< -2 \\ 1+t &< 0 \end{aligned}$$

there is no overlap

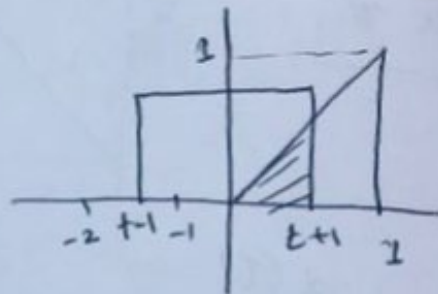
$$\Rightarrow y(t) = 0$$



$$\text{at } \Rightarrow \begin{aligned} -1 &\leq t < 0 \\ 0 &\leq t+1 < 1 \\ -2 &\leq t-1 < -1 \end{aligned}$$

$$y(t) = \int_0^{t+1} (\tau)(1) d\tau$$

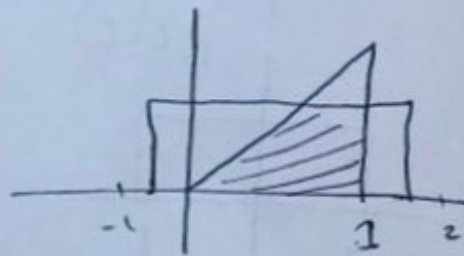
$$\frac{1}{2} \tau^2 \Big|_0^{t+1} \Rightarrow \frac{1}{2} (t+1)^2 - \frac{1}{2} (0) = \boxed{\frac{1}{2} (t+1)^2}$$



$$\text{at } \Rightarrow \begin{aligned} 0 &\leq t < 1 \\ 1 &\leq t+1 < 2 \\ -1 &\leq t-1 < 0 \end{aligned}$$

$$y(t) = \int_0^1 \tau \cdot d\tau$$

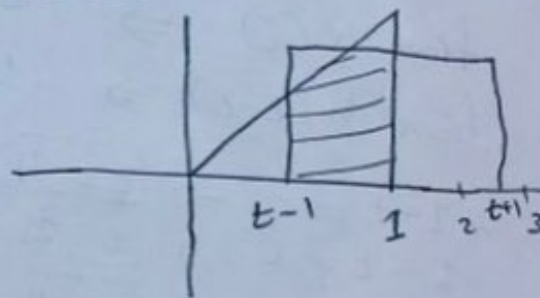
$$\frac{1}{2} \tau^2 \Big|_0^1 \Rightarrow \frac{1}{2} (1) = \boxed{\frac{1}{2}}$$



$$\text{at } \begin{aligned} 1 &\leq t < 2 \\ 2 &\leq t+1 < 3 \\ 0 &\leq t-1 < 1 \end{aligned}$$

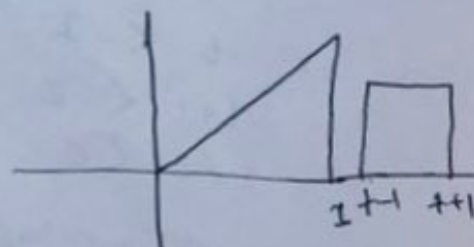
$$y(t) = \int_{t-1}^1 \tau \cdot (1) d\tau = \frac{1}{2} \tau^2 \Big|_{t-1}^1$$

$$\frac{1}{2} [1 - (t-1)^2] \Rightarrow \boxed{\frac{1}{2} [1 - (t-1)^2]}$$



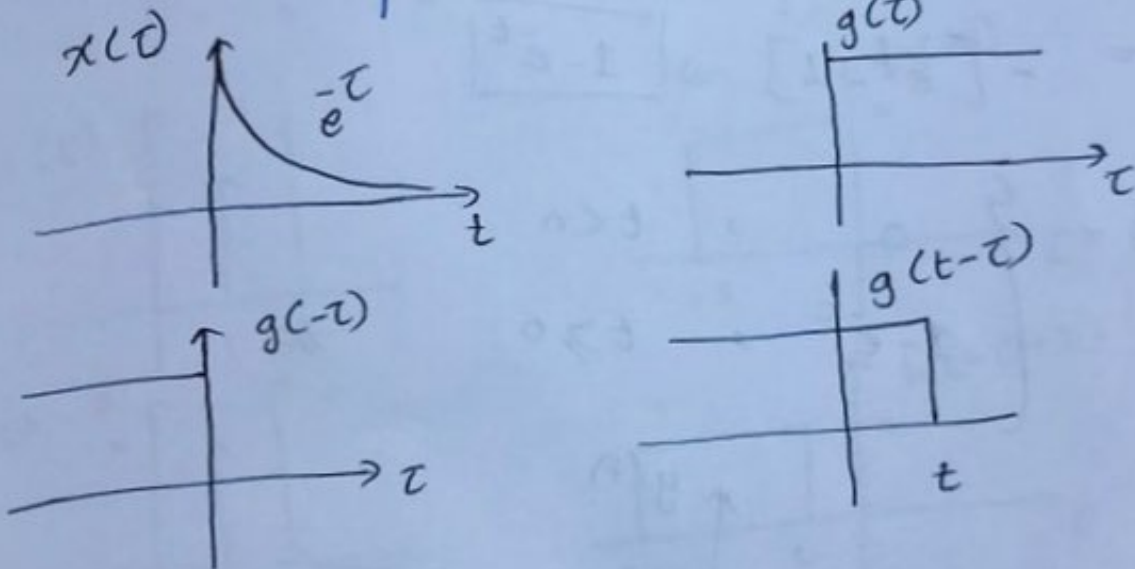
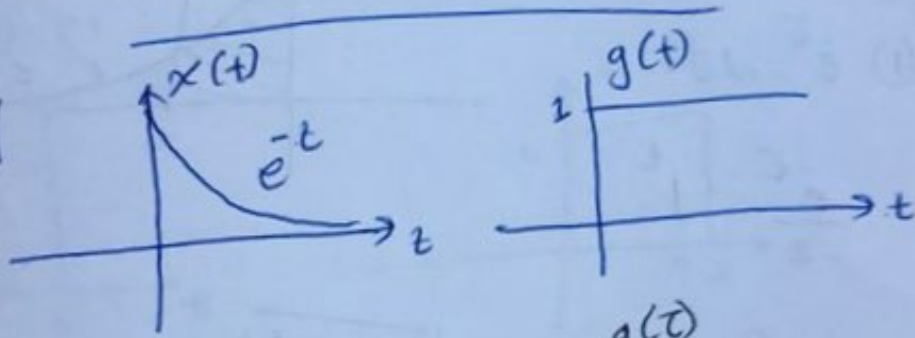
$$\text{at } \begin{aligned} t &\geq 2 \\ t+1 &\geq 3 \\ t-1 &\geq 1 \end{aligned}$$

there is no overlap
 $\Rightarrow y(t) = 0$



$$y(t) = \begin{cases} 0 & , t < -1 \\ \frac{1}{2}(t+1)^2 & , -1 \leq t < 0 \\ \frac{1}{2} & , 0 \leq t < 1 \\ \frac{1}{2}(1-(t-1)^2) & , 1 \leq t < 2 \\ 0 & , t \geq 2 \end{cases}$$

Example #1



changing point //

from $x(\tau) \Rightarrow 0$

from $g(t-\tau) \Rightarrow t$

$t=0$

← نقطة التحويل ← changing point

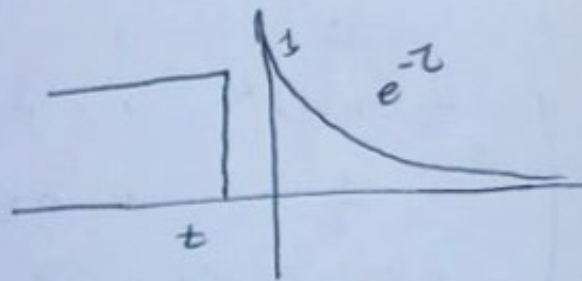
$t < 0$

$t \geq 0$

for $t < 0$

there is no overlap

$$\Rightarrow y(t) = 0$$

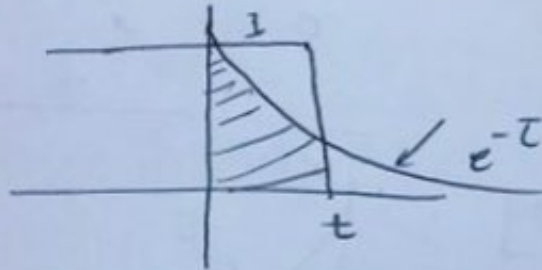


for $t \geq 0$

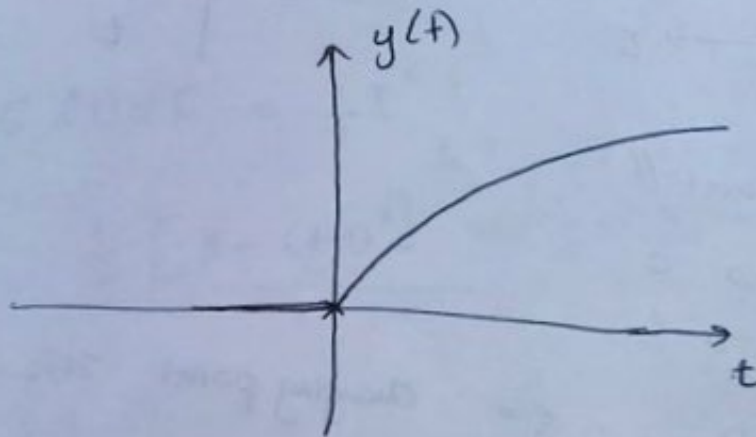
$$y(t) = \int_0^t (1) e^{-\tau} d\tau$$

$$= \left. \frac{e^{-\tau}}{-1} \right|_0^t$$

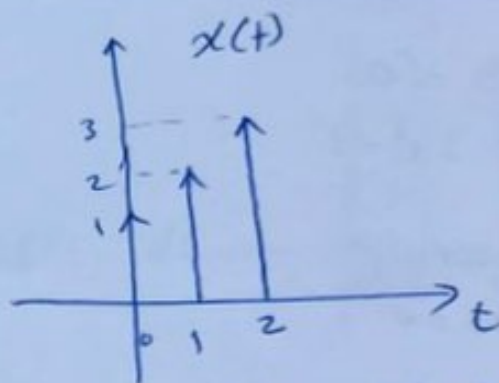
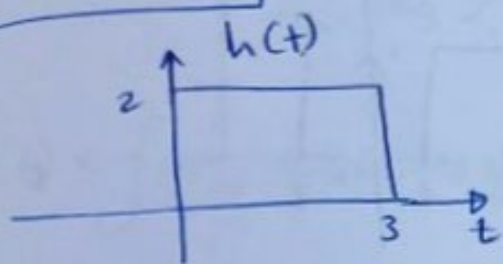
$$= -[e^{-t} - 1] \Rightarrow \boxed{1 - e^{-t}}$$



$$y(t) = \begin{cases} 0 & , t < 0 \\ 1 - e^{-t} & , t \geq 0 \end{cases} \quad \#$$

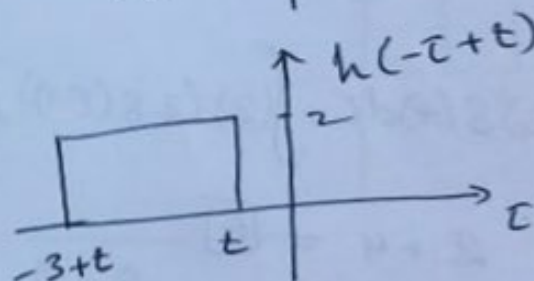
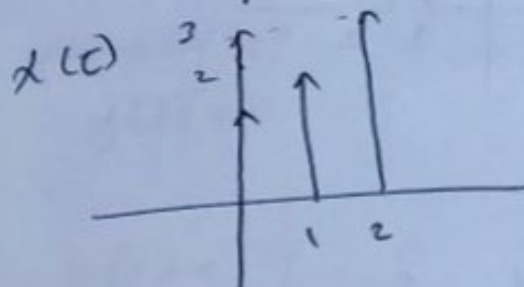
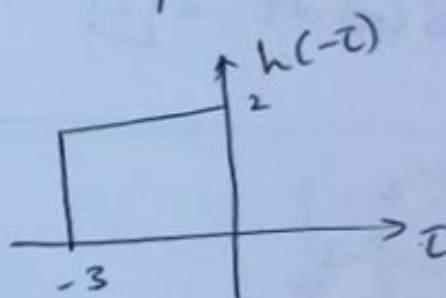
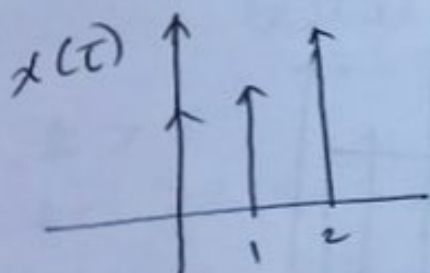
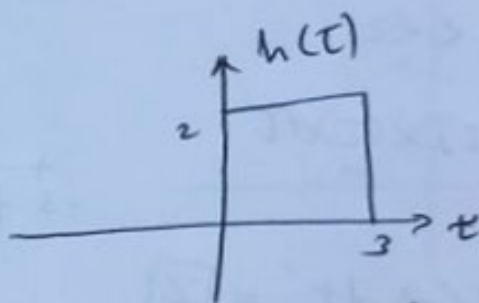
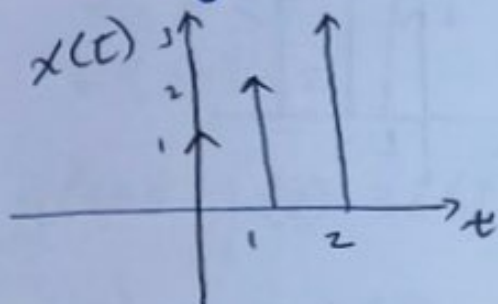


Example #1



$$x(t) = \delta(t) + 2\delta(t-1) + 3\delta(t-2)$$

Find $y(t)$??



changing point

0, 1, 2



changing point

$-3+t, t$

$$t=0$$

$$t=1$$

$$t=2$$

$$t-3=0 \Rightarrow \boxed{t=3}$$

$$t-3=1 \Rightarrow \boxed{t=4}$$

$$t-3=2 \Rightarrow \boxed{t=5}$$

$$t < 0$$

$$0 \leq t < 1$$

$$1 \leq t < 2$$

$$2 \leq t < 3$$

$$3 \leq t < 4$$

$$4 \leq t < 5$$

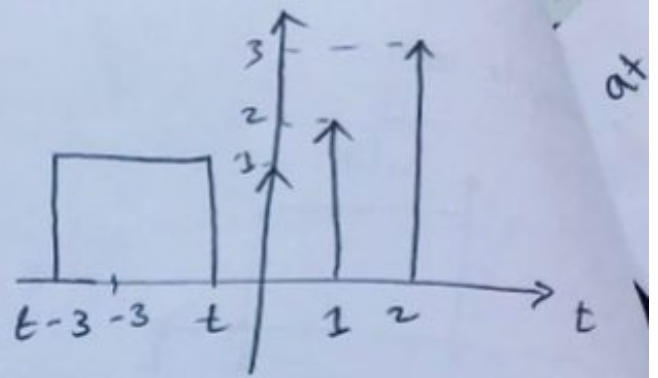
$$t \geq 5$$

34

$$\Rightarrow \text{at } t < 0$$

$$t-3 < -3$$

No overlap
 $y(t) = 0$

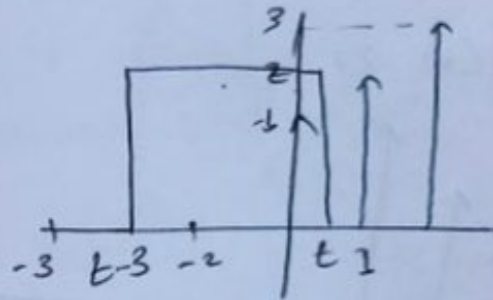


$$0 < t < 1$$

$$-3 < t-3 < -2$$

$$y(t) = \int h(t-\tau) x(\tau) d\tau$$

$$y(t) = \int (2) \delta(\tau) d\tau = \boxed{2}$$

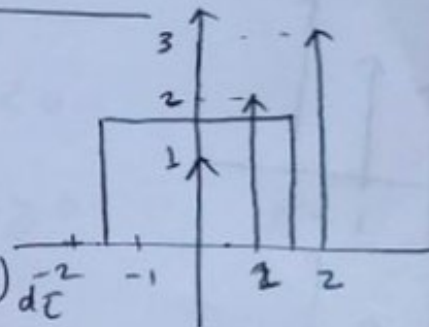


$$1 < t < 2$$

$$-2 < t-3 < -1$$

$$y(t) = \int (2) \delta(\tau) d\tau + \int (2) (2 \delta(\tau-1)) d\tau$$

$$2 + 4 = \boxed{6}$$

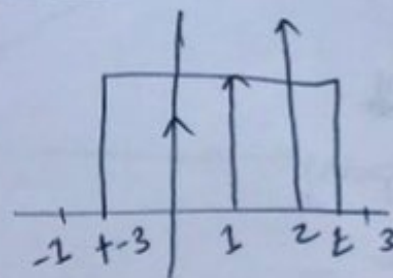


$$\Rightarrow 2 < t < 3$$

$$-1 < t-3 < 0$$

$$y(t) = \int_{-1}^2 h(t-\tau) \delta(\tau) d\tau + \int_{-1}^2 h(t-\tau) \cdot 2 \delta(\tau-1) d\tau + \int_{-1}^2 h(t-\tau) \cdot 3 \delta(\tau-2) d\tau$$

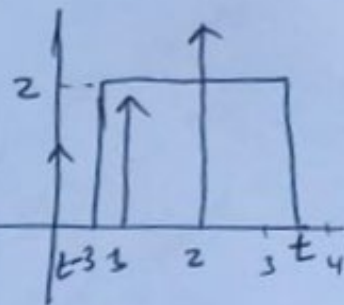
$$\Rightarrow 2 + 4 + 6 = \boxed{12}$$



35

$$at \Rightarrow \begin{aligned} 3 < t < 4 \\ 0 < t-3 < 1 \end{aligned}$$

$$y(t) = \int (2) \cdot 2\delta(\tau-1)d\tau + \int (2)(3)\delta(\tau-2)d\tau$$

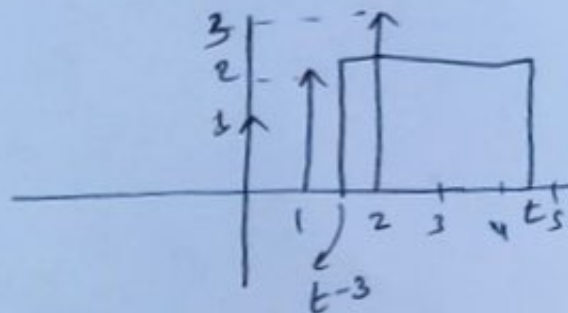


$$(2)(2) + (2)(3) = \boxed{10}$$

$$\begin{aligned} 4 &\leq t < 5 \\ 1 &\leq t-3 < 2 \end{aligned}$$

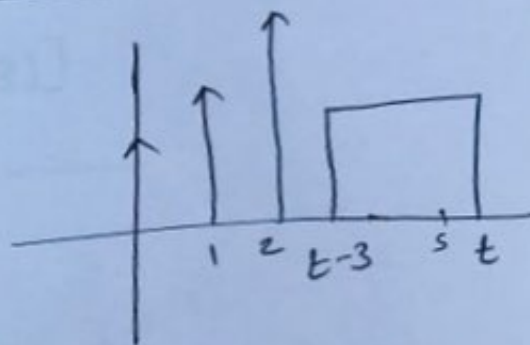
$$y(t) = \int (2) \cdot 3\delta(\tau-2)d\tau$$

$$(2)(3) = \boxed{6}$$



$$\begin{aligned} t &> 5 \\ t-3 &> 2 \end{aligned}$$

No overlap
 $y(t) = 0$



$$y(t) = \begin{cases} 0 & t < 0 \\ 2 & 0 \leq t < 1 \\ 6 & 1 \leq t < 2 \\ 12 & 2 \leq t < 3 \\ 10 & 3 \leq t < 4 \\ 6 & 4 \leq t < 5 \\ 0 & t \geq 5 \end{cases}$$

* rectangular function ||

$$x(t) = A \text{ rect} \frac{(t-n)}{m}$$

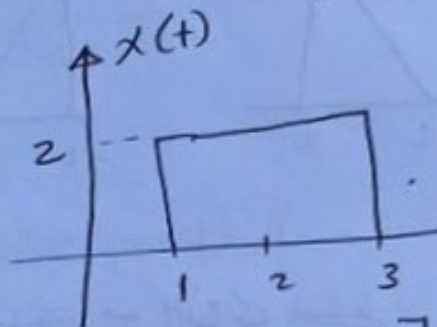
$A \Rightarrow$ amplitude

$n \Rightarrow$ center of rect

$m \Rightarrow$ length of rect

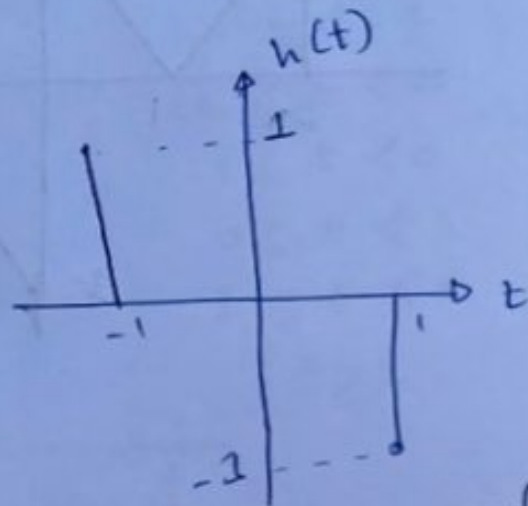
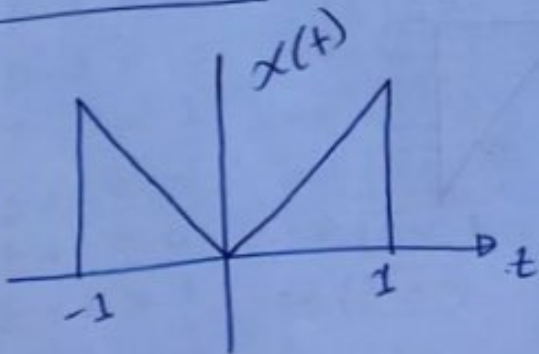
"وصف الليل ففجد به ناطلة لالك
عسى أنه يمشي ريلاً مقاماً محمداً"
- صورة لإبراهيم -
😊

Example # | $x(t) = 2 \text{ rect} \frac{(t-2)}{2}$



$$x(t) = 2 [u(t-1) - u(t-3)]$$

Example # | Find $y(t)$ if



$$h(t) = \delta(t+1) - \delta(t-1)$$

$$y(t) = x(t) * h(t)$$

$$= x(t) * [\delta(t+1) - \delta(t-1)]$$

$$= [x(t+1) - x(t-1)]$$

مع كضائف

الطريقة الأولى
بالحضائف

$$x(t) * \delta(t) = x(t)$$

بالمضاعف مرة //

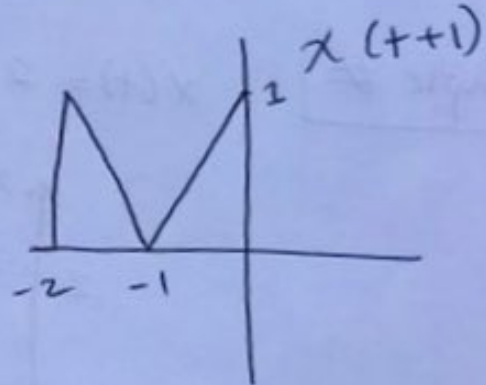
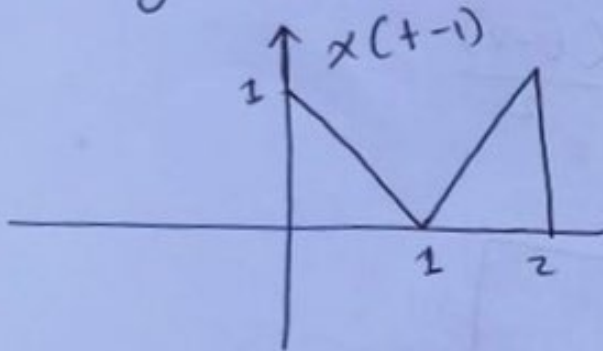
$$x(t) * \delta(t+1) = x(t+1)$$

$$x(t) * \delta(t-1) = x(t-1)$$

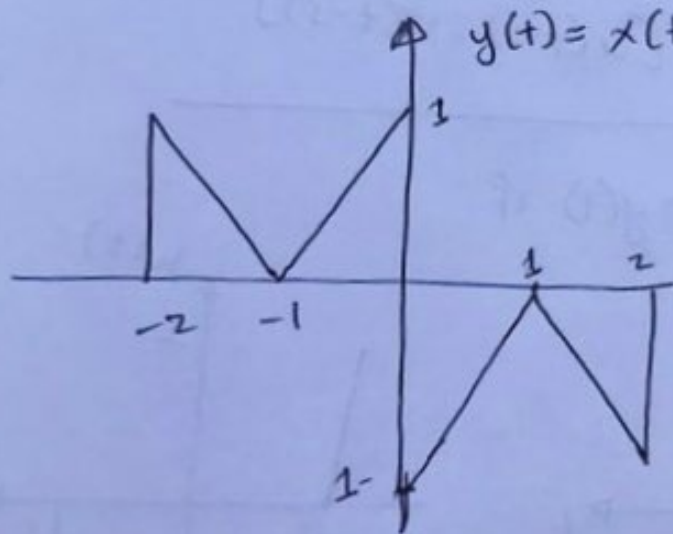
⋮

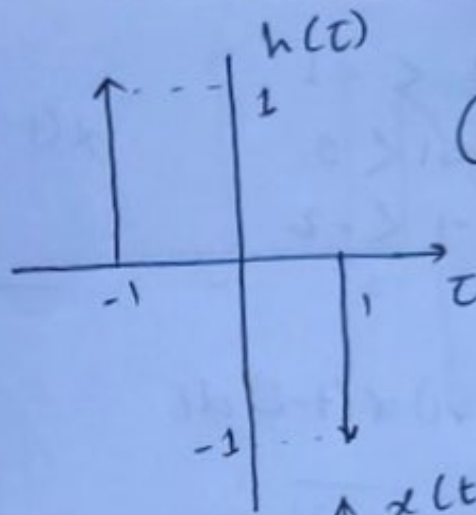
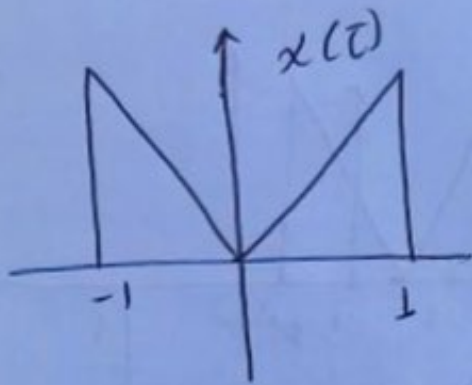
$$y(t) = x(t+1) - x(t-1)$$

تابع حل السؤال //

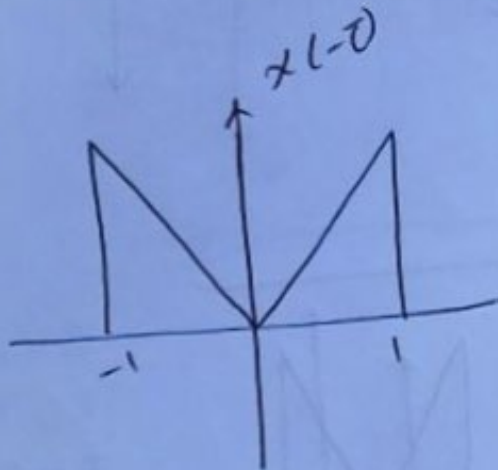


ناتج طرح الرسمة الثانية مع الأولى ←

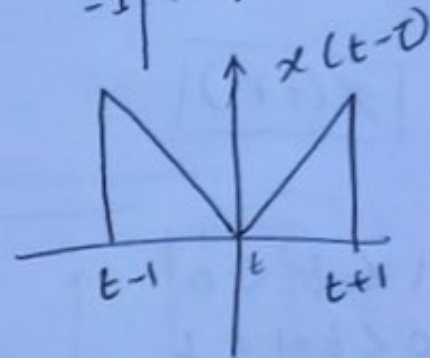




الطريقة الثانية



⇒



changing point (edge point) //

from $x(t-t) \Rightarrow t-1, 0, t+1$

from $x(t) \Rightarrow -1, 1$

نقطة التحوط

$$t-1 = -1 \Rightarrow \boxed{t=0}$$

$$t-1 = 0 \Rightarrow \boxed{t=1}$$

$$t-1 = 1 \Rightarrow \boxed{t=2}$$

$$t+1 = -1 \Rightarrow \boxed{t=-2}$$

$$t+1 = 0 \Rightarrow \boxed{t=-1}$$

$$t+1 = 1 \Rightarrow \boxed{t=0}$$

$$t < -2$$

$$-2 \leq t < -1$$

$$-1 \leq t < 0$$

$$0 \leq t < 1$$

$$1 \leq t < 2$$

$$t > 2$$

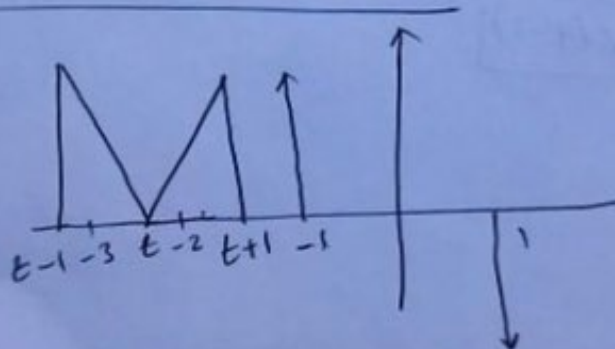
$$\text{at } t < -2$$

$$t+1 < -1$$

$$t-1 < -3$$

No overlap

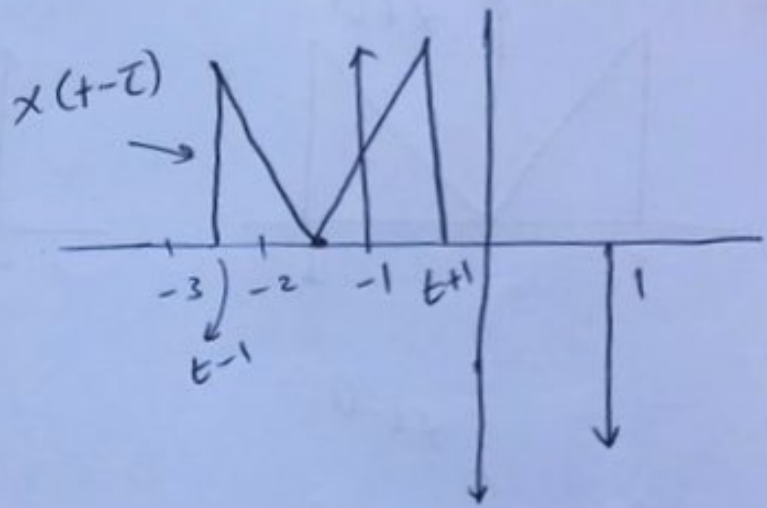
$$y(t) = 0$$



39

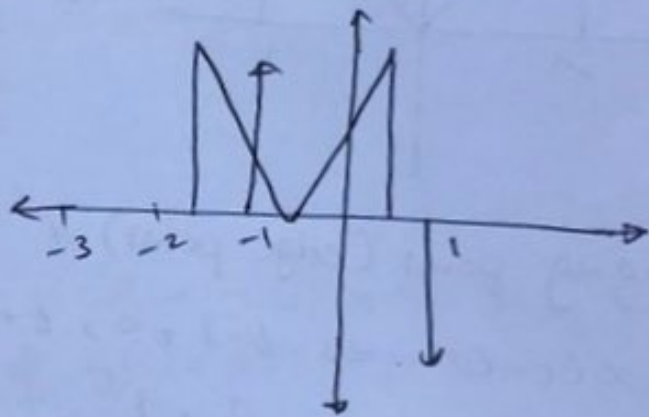
$$\begin{aligned} -2 < t < -1 \\ -1 < t+1 < 0 \\ -3 < t-1 < -2 \end{aligned}$$

$$\int \delta(t+1) x(t-\tau) d\tau = \boxed{x(t+1)}$$



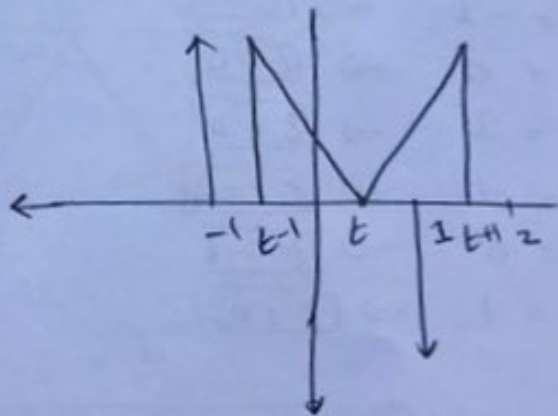
$$\begin{aligned} -1 < t < 0 \\ 0 < t+1 < 1 \\ -2 < t-1 < -1 \end{aligned}$$

$$\int \delta(t+1) x(t-\tau) d\tau = \boxed{x(t+1)}$$



$$\begin{aligned} 0 < t < 1 \\ 1 < t+1 < 2 \\ -1 < t-1 < 0 \end{aligned}$$

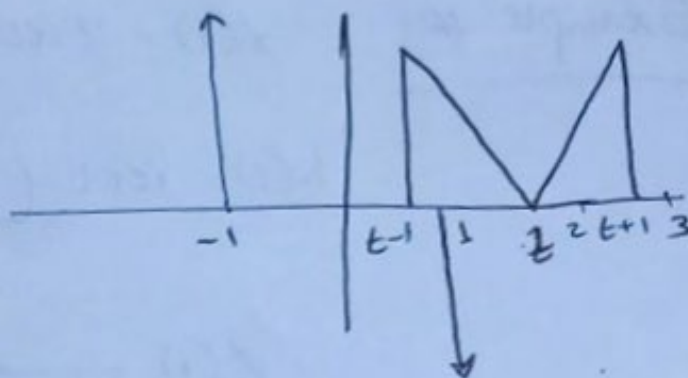
$$\int -\delta(t-1) x(t-\tau) d\tau = \boxed{-x(t-1)}$$



$$1 \leq t < 2$$

$$2 \leq t+1 < 3$$

$$0 \leq t-1 < 1$$



$$\int -x(t-\tau) \delta(t-1)$$

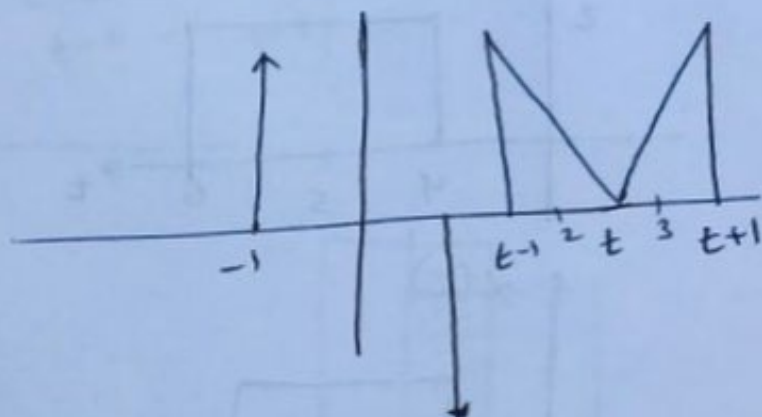
$$= \boxed{-x(t-1)}$$

$$t > 2$$

$$t+1 > 3$$

$$t-1 > 1$$

No overlap

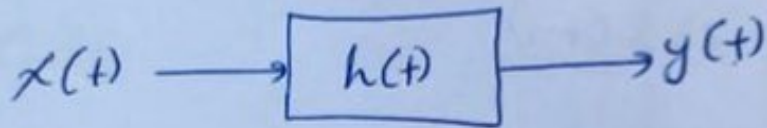


$$y(t) = \begin{cases} 0 & , t < -2 \\ x(t+1) & , -2 \leq t < -1 \\ x(t+1) & , -1 \leq t < 0 \\ -x(t-1) & , 0 \leq t < 1 \\ -x(t-1) & , 1 \leq t < 2 \\ 0 & , t > 2 \end{cases}$$

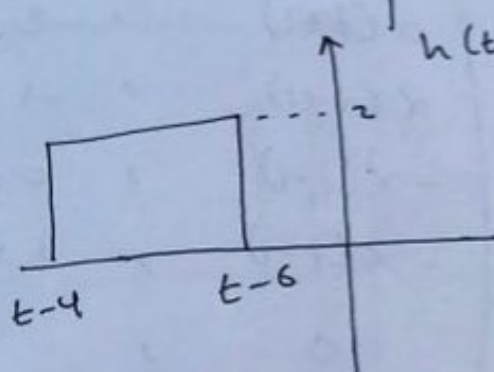
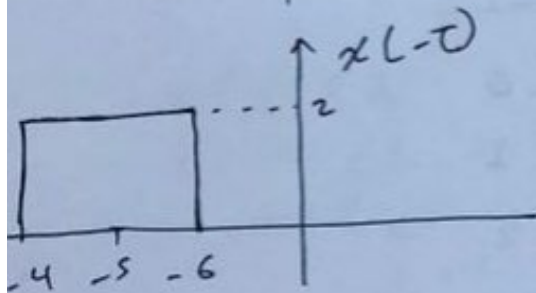
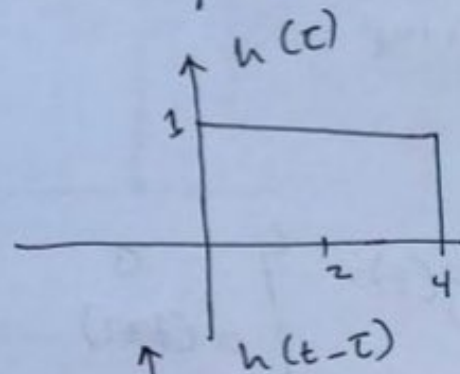
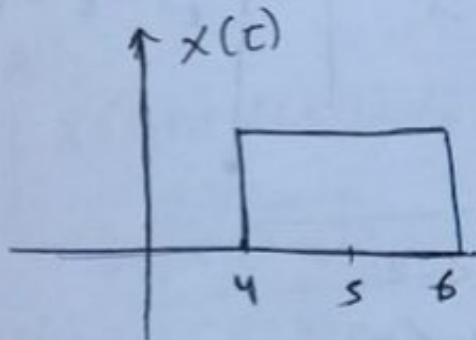
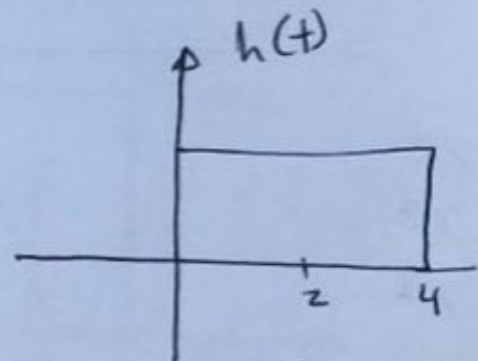
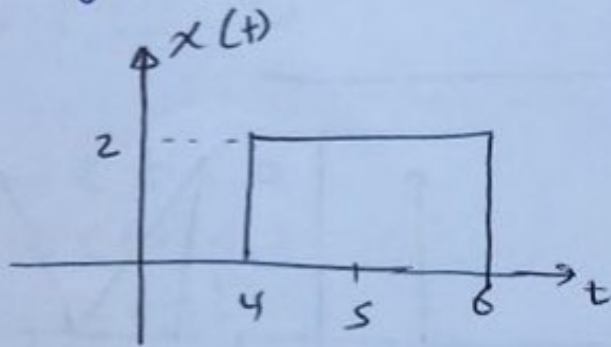
Example #

$$x(t) = 2 \operatorname{rect}\left(\frac{t-5}{2}\right)$$

$$h(t) = \operatorname{rect}\left(\frac{t-2}{4}\right)$$



Sketch $y(t)$??



changing point //

$$\begin{aligned} t-4 \\ t-6 \\ t=4 \\ t=0 \end{aligned}$$

$$\begin{aligned} t-4 &= 0 &\Rightarrow & \boxed{t=4} \\ t-4 &= 4 &\Rightarrow & \boxed{t=8} \\ t-6 &= 0 &\Rightarrow & \boxed{t=6} \\ t-6 &= 4 &\Rightarrow & \boxed{t=10} \end{aligned}$$

$$t < 4$$

$$4 \leq t < 6$$

$$6 \leq t < 8$$

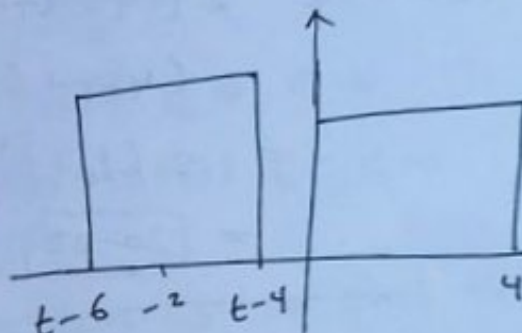
$$8 \leq t < 10$$

$$t \geq 10$$

for $t < 4$

No overlap

$$y(t) = 0$$



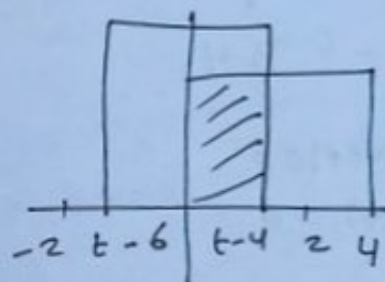
$$4 \leq t \leq 6$$

$$0 \leq t-4 < 2$$

$$-2 \leq t-6 < 0$$

$$\int_0^{t-4} (2)(1) d\tau \Rightarrow 2\tau \Big|_0^{t-4}$$

$$\Rightarrow 2(t-4) = \boxed{2t-8}$$



$$6 \leq t < 8$$

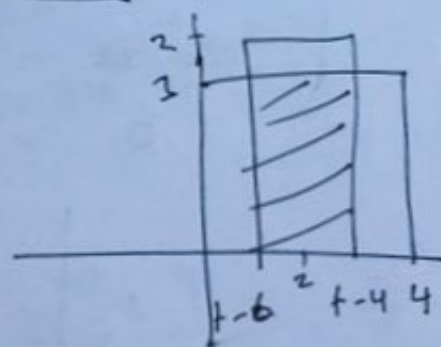
$$2 \leq t-4 < 4$$

$$0 \leq t-6 < 2$$

$$\int_{t-6}^{t-4} (2)(1) d\tau \Rightarrow 2\tau \Big|_{t-6}^{t-4}$$

$$\Rightarrow 2(t-4 - (t-6))$$

$$2(t-4 - t + 6) = \boxed{4}$$



$$8 \leq t < 10$$

$$4 \leq t-4 < 6$$

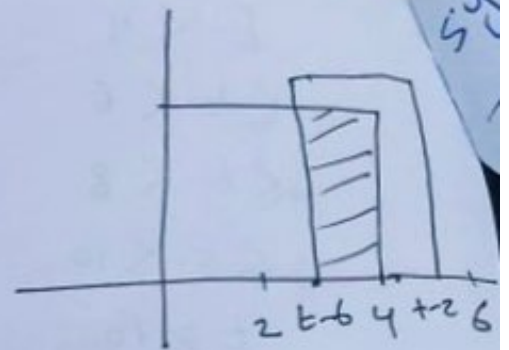
$$2 \leq t-6 < 4$$

$$\int_{t-6}^4 (2)(1) d\tau = 2[4 - (t-6)]$$

$$= 2(4 - t + 6)$$

$$= 2(10 - t)$$

$$= \boxed{20 - 2t}$$



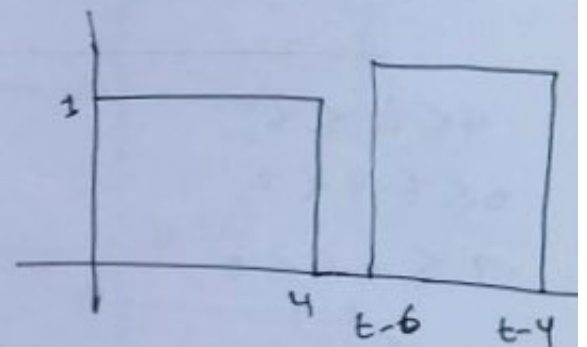
for $t \geq 10$

$$t-4 \geq 6$$

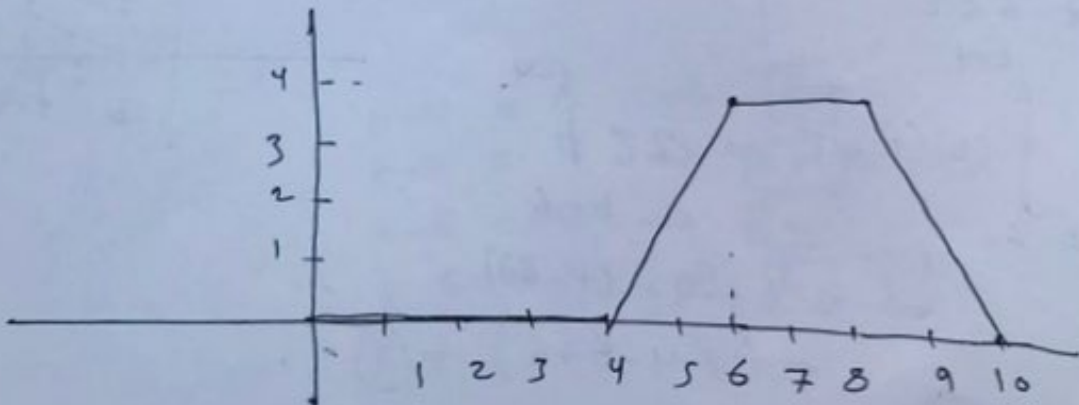
$$t-6 \geq 4$$

No overlap

$$y(t) = 0$$



$$y(t) = \begin{cases} 0 & , t < 4 \\ 2t-8 & , 4 \leq t < 6 \\ 4 & , 6 \leq t < 8 \\ 20-2t & , 8 \leq t < 10 \\ 0 & , t \geq 10 \end{cases}$$



system stability :

* The system is stable if it results in a bounded output for any bounded input (BIBO)

* A system is said to be BIBO stable if

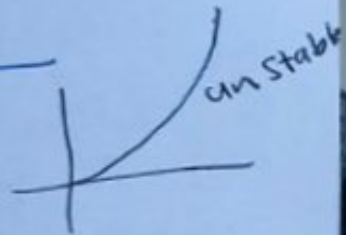
[1] for any bounded input $|x(t)| \leq c_1 < \infty$

[2] The output remains bounded $|y(t)| \leq c_2 < \infty$

* If we have C.T. L.T.I system $\Rightarrow$ we said that this system is stable if

$$\int_{-\infty}^{\infty} h(\tau) d\tau < \infty$$

"there is no final value"



Example:-

[1] $h(t) = -3e^{2t} u(t)$

$$\int_{-\infty}^{\infty} h(\tau) d\tau < \infty \Rightarrow$$

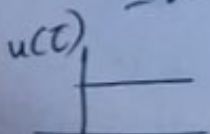
نظام غير مستقر

$$\int_{-\infty}^{\infty} -3e^{2\tau} u(\tau) d\tau \Rightarrow \int_0^{\infty} -3e^{2\tau} d\tau$$

$$\Rightarrow -\frac{3}{2} e^{2\tau} \Big|_0^{\infty} \Rightarrow -\frac{3}{2} [e^{\infty} - 1] \Rightarrow -\frac{3}{2} (\infty) = \boxed{\infty}$$

$\int_{-\infty}^{\infty} h(\tau) d\tau = \infty \Rightarrow$ so system is unstable #

نظام غير مستقر $u(t)$ // $u(t)$ غير محدود
 $\infty \leftarrow 0$ $u(t)$ غير محدود



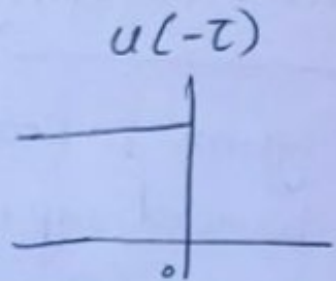
$$[2] h(t) = e^{4t} u(-t)$$

$$\int_{-\infty}^{\infty} e^{4\tau} u(-\tau) d\tau$$

$$\int_{-\infty}^0 e^{4\tau} d\tau$$

$$= \frac{e^{4\tau}}{4} \Big|_{-\infty}^0 \Rightarrow \frac{1}{4} \left[e^0 - \frac{0}{4}(\infty) \right]$$

$$= \frac{1}{4} (1) = \frac{1}{4} < \infty$$



$-\infty \rightarrow 0$ مبرور الأسان

$\Rightarrow$ so system is stable #

$e^{-\infty} \parallel$ ملاحظة

$$[3] h(t) = 5\delta(t+5)$$

$$\int_{-\infty}^{\infty} 5\delta(t+5) d\tau = 5 < \infty$$

sifting property

so system is stable #

$$[4] y(t) = e^{x(t)} u(t)$$

$$\text{If } x(t) = u(t)$$

$$y(t) = e^1 u(t) = \text{Constant} < \infty$$

$\Rightarrow$ system stable

ملاحظة // هنا مخرجنا $y(t)$ ليس $h(t)$ بل $x(t)$ متناهي
bounded، ولأنه $u(t)$ محدود في $u(t)$ متناهي 1، ومخرجنا
في الصفر لا يذهب إلى ∞