

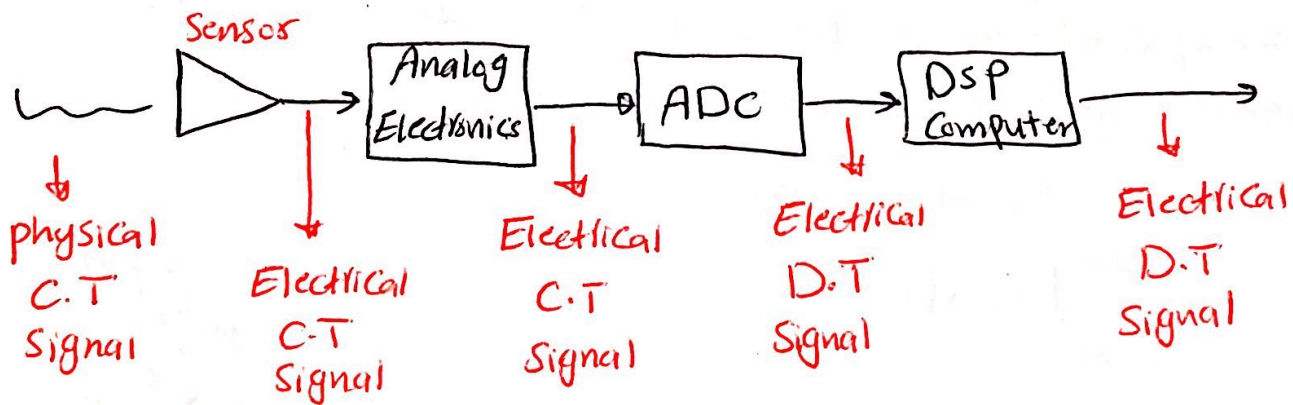
* Most of the processing of a signal in electronic systems is done using a computer, and computer can't directly process a C.T signal, but it needs a stream of numbers which is a D.T signal (Discrete time signal)

* So, we need to find an interface between external environment (C.T signal) and computer (D.T signal), which called

(AD & DA) Converter

↓
analog to digital Converter

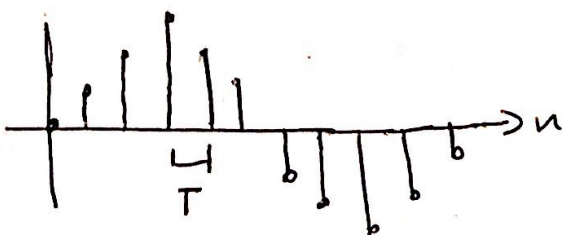
→ Digital to analog Converter



* What's a Discrete Time Signal (D.T)?

is a sequence of numbers indexed by integers

$n \Rightarrow$ integer number



$n = nT$
 $T \rightarrow$ sampling time

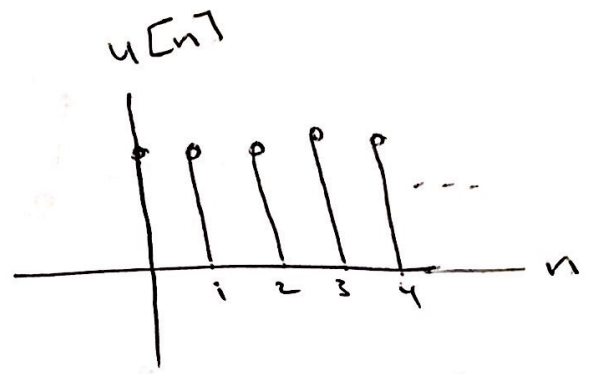
* If the input and outputs for a system are D.T, then the system is called (D.T system)

useful signal operation

(scaling, shifting, flipping), the same properties as we learned in C.T signals

* Step function (unit step) :

$$u[n] = \begin{cases} 0 & , n < 0 \\ 1 & , n \geq 0 \end{cases}$$



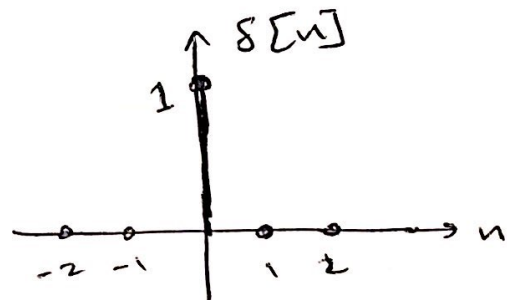
* Ramp function :

$$r[n] = \begin{cases} 0 & , n < 0 \\ n & , n \geq 0 \end{cases}$$

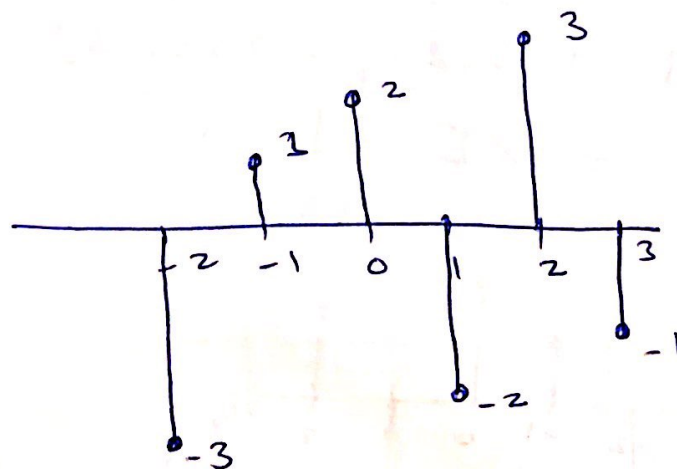


* unit impulse :

$$\delta[n] = \begin{cases} 1 & , n = 0 \\ 0 & , n \neq 0 \end{cases}$$

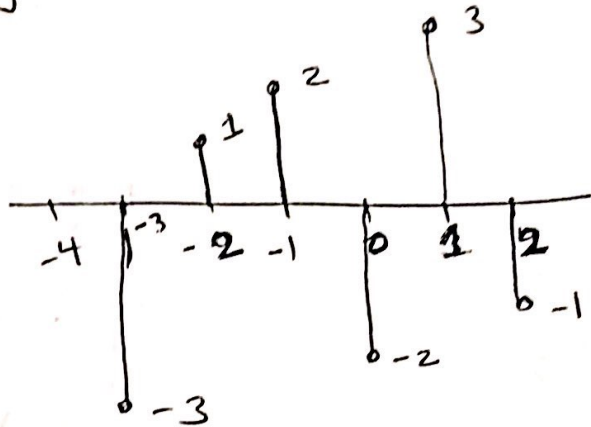


Example # | sketch $x[\frac{2}{3}n+1]$



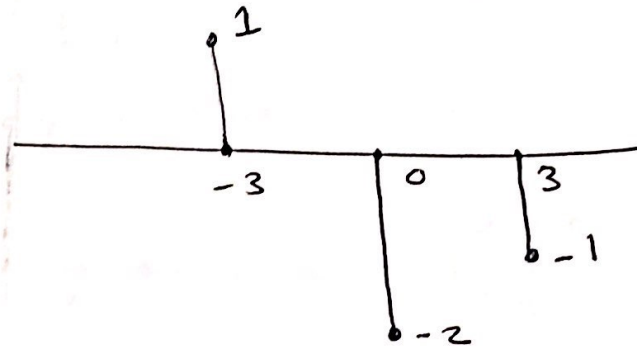
flip , scale , shift D.T. $x[n-4]$

$x[n+1]$



$x[\frac{2}{3}n+1]$

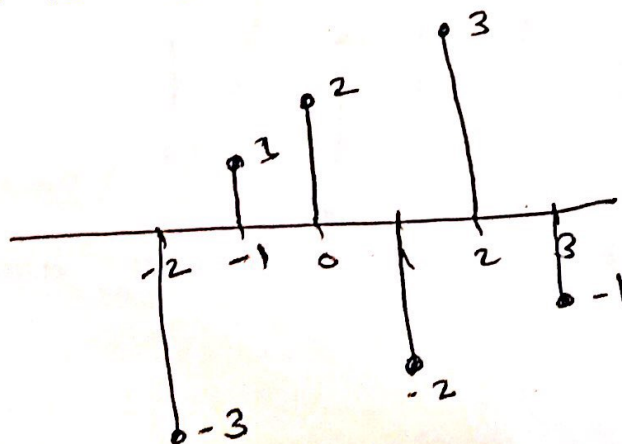
متغير n في $(\frac{3}{2})$



* بعد ما خدنا ان n في $(\frac{3}{2})$ الاعداد الكسرية ليس لها معنى لانه بنسأل discrete system و يجب ان يكون قيم n صحيحة فقط

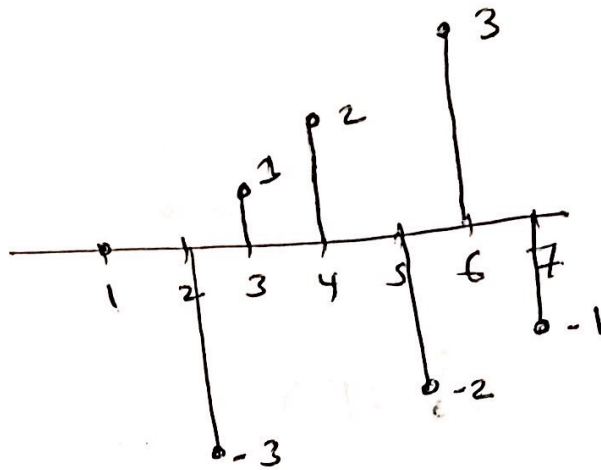
Example#] $x[n] = \{-3, 1, 2, -2, 3, -1\}$

Find $x[3n-4]$ و $x[2-n]$



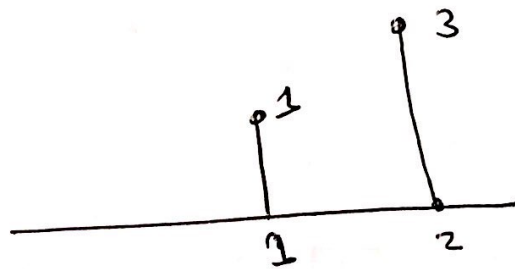
Q. $x[n-4]$

4 جانات الى اليمين shift



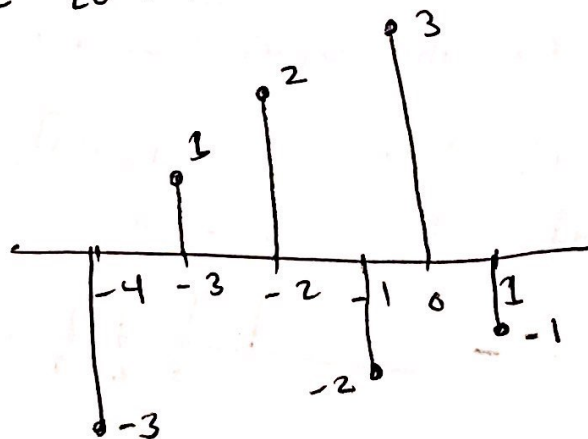
* Scale by 3

$[3n-4]$



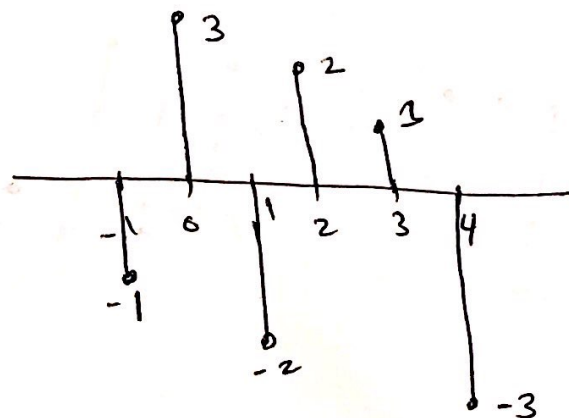
⑥ $x[2-n] \Rightarrow x[-n+2]$

$\Rightarrow$ shift 2 to left



$x[n+2]$

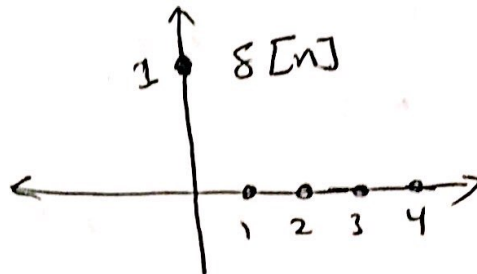
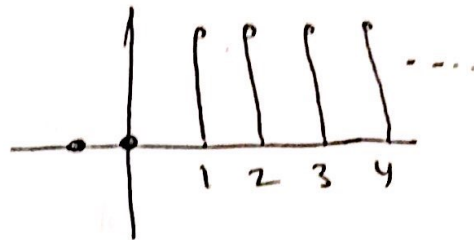
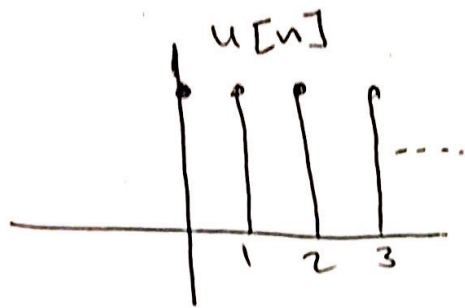
$\Rightarrow$ flip



$x[-n+2]$

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$$\delta[n] = u[n] - u[n-1]$$



$$* u[n] = \sum_{k=-\infty}^{\infty} \delta[k]$$

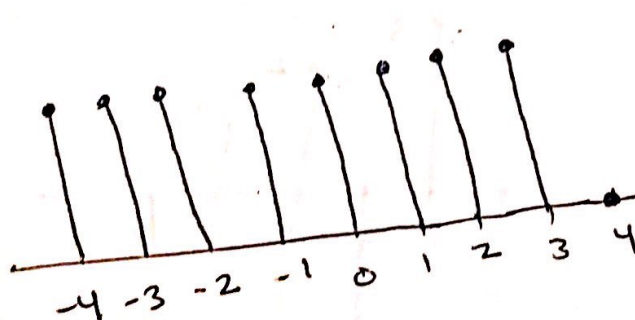
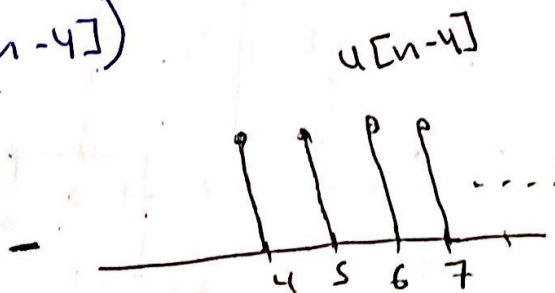
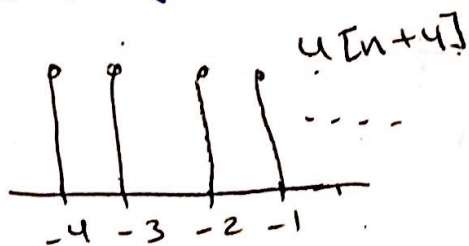
$$* x[n] \cdot \delta[n] = x[0] \delta[n]$$

$$* x[n] \cdot \delta[n-k] = x[k] \delta[n-k]$$

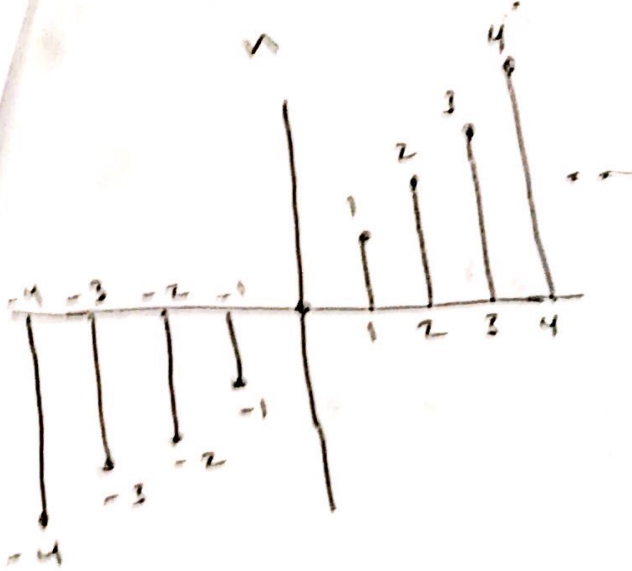
$$* x[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot \delta[n-k]$$

Example #1 sketch the following signals:-

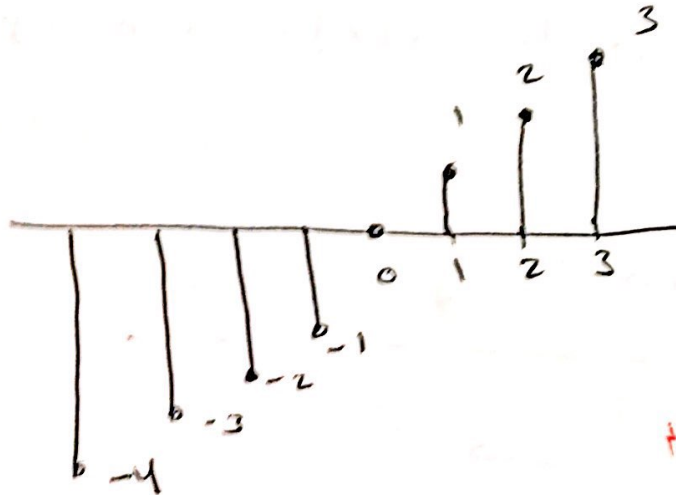
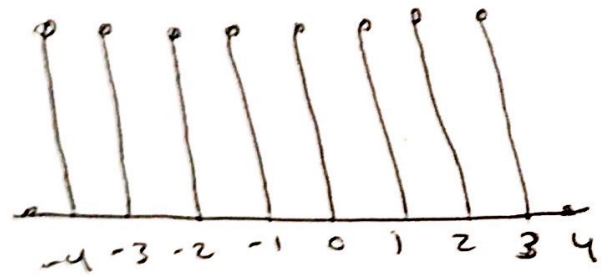
$$\text{II] } n (u[n+4] - u[n-4])$$



$$u[n+4] - u[n-4]$$

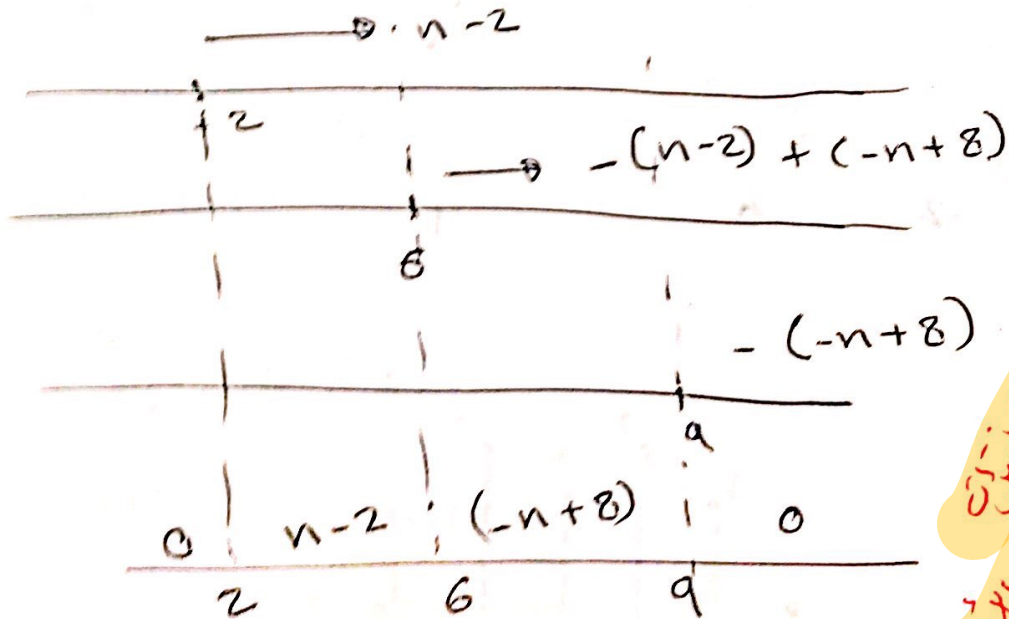


*



این ای لا آرنی الیة
سنة الفصلي
+

$$(n-2) [u[n-2] - u[n-6]] + (-n+8) [u[n-6] - u[n-9]]$$

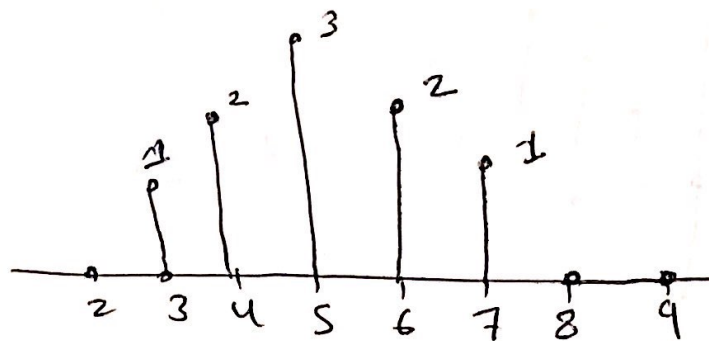


* الیة ختم بترتیب
طریقه سلة فاصلة
Continuous
discrete

[7] =>

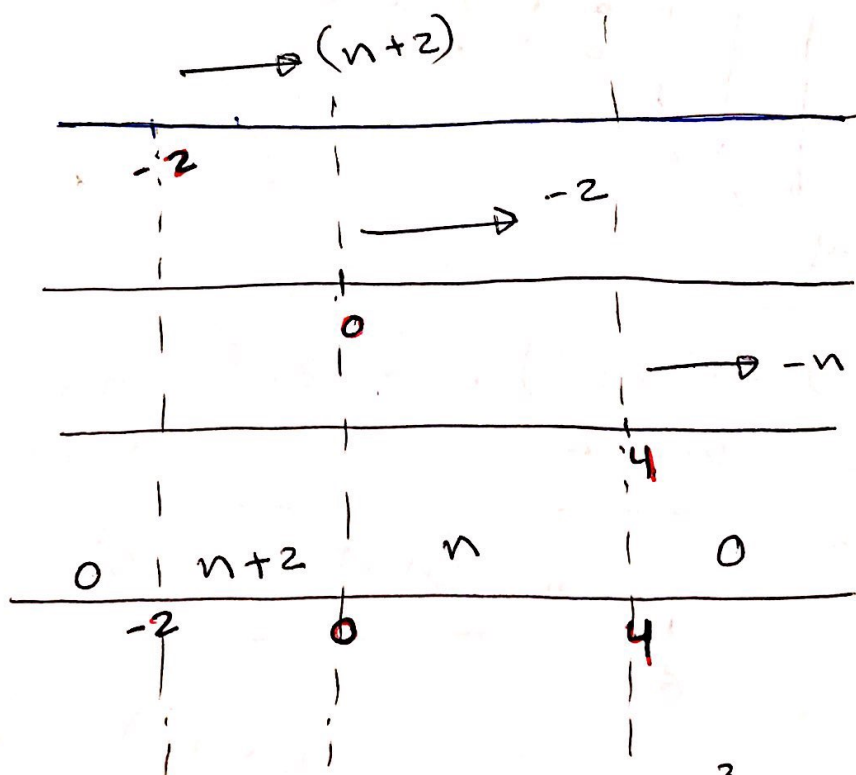
$$(n-2)(u[n-2] - u[n-6]) + (-n+8)(u[n-6] - u[n-8])$$

sample



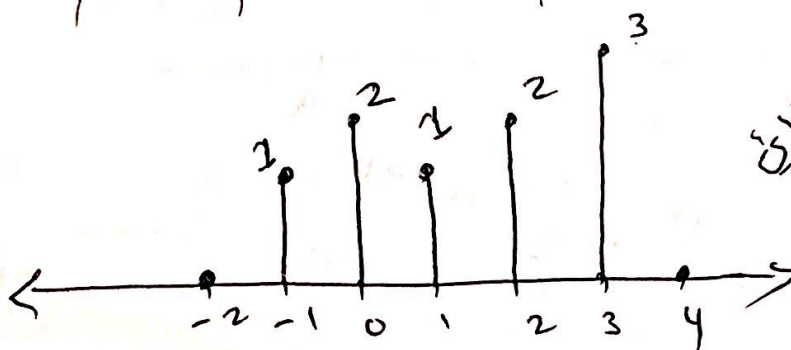
$$x[n] = (n+2)u[n+2] - 2u[n] - nu[n-4]$$

النقاط $-2, 0, 4$



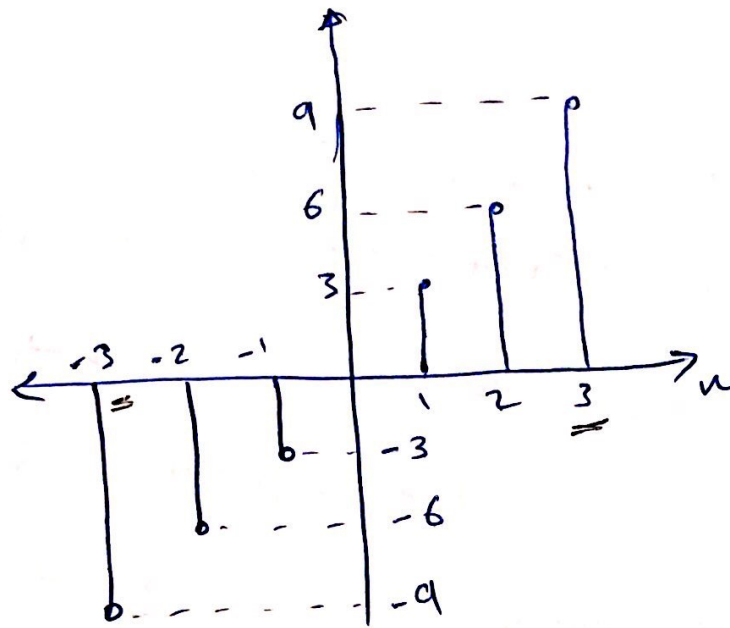
هذا هو الـ signal
الذي نريد
الذي نريد
الذي نريد

الـ signal



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Example #1 Describe the signal: —



هنا إشارة للناسه دهنه
موسميه للمصنعه
آل عمران

$$x[n] = 3n [u[n+3] - u[n-4]]$$

* في المناقشه راجع اوضح انكم كيف لحيثوا الاقترانه او الجارده من خلال النظر .

Classification of signals —

Linear , Time invariant, memory, Causal , stable
راجع اباريه الثاني من الساتر الاول الخاص بي ، نفس
الشرح تماماً ، حاطه على الـ Continuous ينطبق على discrete

Example 11

$$y[n] = n^2 x[n]$$

$$x_1[n] \rightarrow y_1[n] = n^2 x_1[n]$$

$$x_2[n] \rightarrow y_2[n] = n^2 x_2[n]$$

$$x_3[n] = a x_1[n] + b x_2[n]$$

$$a y_1[n] + b y_2[n] \stackrel{??}{=} y_3[n]$$

لتحقق الشرط

9

$$a n^2 x_1[n] + b n^2 x_2[n] = a y_1[n] + b y_2[n] \dots (1)$$

$$y_3[n] = n^2 x_3[n]$$

$$= n^2 [a x_1[n] + b x_2[n]]$$

$$y_3[n] = n^2 a x_1[n] + n^2 b x_2[n] \dots (2)$$

$$a y_1[n] + b y_2[n] = y_3[n] \leftarrow (2) \text{ also } (1) \text{ also}$$

linear system

* system is time varying " n^2 is time "

* system is causal (output depends on the input in present)

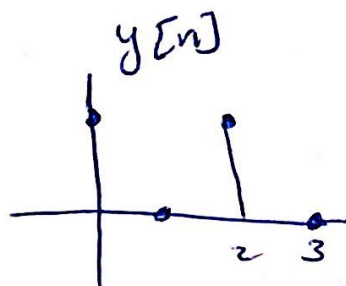
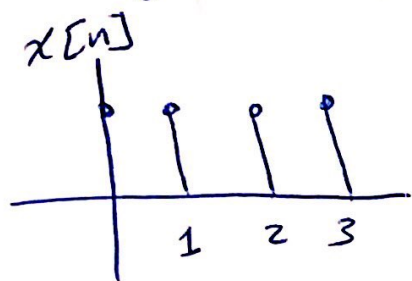
* system is memoryless.

1

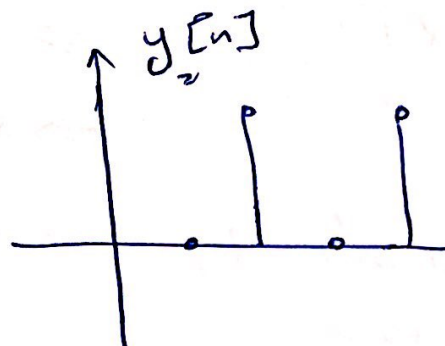
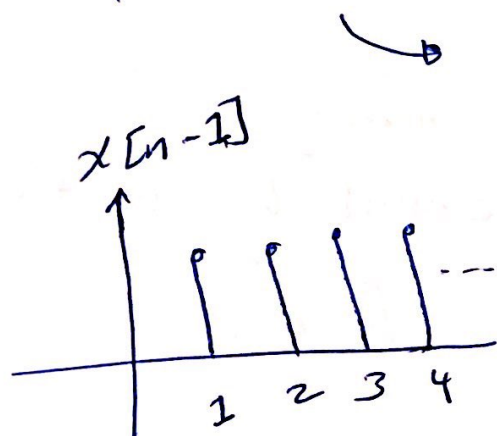
Example #

$$y[n] = \begin{cases} x[n], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$$

is the system time invariant??



// لک

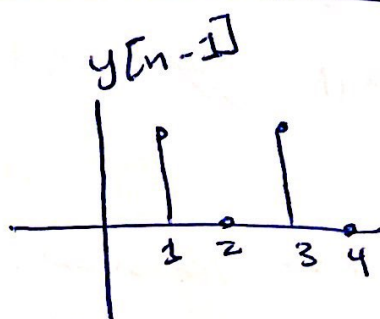
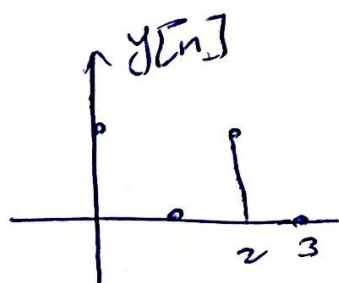


ملاحظة // قنا بفرمت $x[n]$

*

ملاحظة // shift الانصحت لا input في shift مع output
 تغيرت مميزات او system

$\Rightarrow$ system time varying



ملاحظة //

**

ملاحظة // shift الانصحت لا output في shift مع input
 تغيرت مميزات او system

غير متساوية. *

U

* Is the system $y[n-1] = x[n]$ Causal

$$\Rightarrow y[n] = x[n+1]$$

output, input $n-1$ ~~is~~ ^{is}

كل $y[n]$ $n-1$

$\Rightarrow$ system is not causal because the output depends on the input in future

$$* y[n+1] = \frac{x[n]}{x[n+1]}$$

$$y[n] = \frac{x[n-1]}{x[n]}$$

$\Rightarrow$ system is memory

$\Rightarrow$ system is causal because the

output doesn't depend on the input in future

$$* y[n] = e^{j2\pi n} x[n]$$

$$e^{j2\pi n} = \cos(2\pi n) + j\sin(2\pi n)$$

$$\Rightarrow \boxed{e^{j2\pi n} = 1}$$

$y[n] = x[n] \Rightarrow$ causal, memoryless, linear
time invariant. //

اَقُلْ اَللّٰهُمَّ مَا لَكَ بِمَلِكٍ تُوَيِّىْ بِمَلِكٍ مَدَنِيٍّ رَمَتْهُ مَلَكٌ مَدَنِيٍّ وَتَقَرَّ بِهِ
كَأَنَّ رَمَتْهُ مَدَنِيٍّ بِكَ اَكْبَرُ اَبْنِىْ عَلَى كُلِّ خَيْرٍ وَحَدِيٍّ
- آل عمران -

The general form for sinusoidal in D.T.

$$x[n] = A \sin(\Omega n + \theta)$$

$\Omega \Rightarrow$ in rad/sample , Ωn in radian

$$\Omega = 2\pi f$$

$$\Omega = \frac{2\pi}{N}$$

($f \Rightarrow$ frequency in Hz)

$$* y[n] = x[2n]$$

linear, non causal (لا فاعلية زمنية) , memory

$$x_1[n] \rightarrow y_1[n] = x_1[2n]$$

$$x_2[n] \rightarrow y_2[n] = x_2[2n]$$

$$x_3[n] = ax_1[n] + bx_2[n]$$

$$ay_1[n] + by_2[n] = ax_1[2n] + bx_2[2n]$$

$$y_3[n] = x_3[2n] = ax_1[2n] + bx_2[2n]$$

$\Rightarrow$ system is linear

$$x_1[n] \rightarrow y_1[n] = x_1[2n]$$

$$x_1[n-n_0] \Rightarrow x_2(n) \rightarrow y_2(n)$$

$$y_1(n-n_0) = x_1[2(n-n_0)]$$

$$y_2(n) = x_2[n] = x_1[2n-n_0]$$

$$y_1(n-n_0) \neq y_2[n] \Rightarrow \text{system is time variant.}$$

"ربنا ولا تحملنا طائفة لنا. ارحمنا وارحمنا أنت مولانا
فاغفرنا على إصغار الكافرين" - سورة البقرة -

$$* y[n] + 2y[n-1] = x[\underline{n+1}]$$

$\Rightarrow$ memory, non causal

$\Rightarrow$ linear (superposition is hold)

$\Rightarrow$ time invariant.

$$* y[n+1] + 4y[n] = 3x[n+1] - x[n]$$

$$y[n] + 4y[n-1] = 3x[\underline{n}] - x[\underline{n-1}]$$

* memory

* Causal (difference equation)

* time invariant

* linear (superposition is hold)

DT system
Differ
Differ

DT system Equations :-

Difference equation (discrete system)

Differential equation (continuous system)

DT system is modeled by difference equations

$$y[n] + a_1 y[n-1] + \dots + a_N y[n-N] = b_0 x[n] + b_1 x[n-1] + \dots + b_M x[n-M]$$

most advanced output sample $\rightarrow$ (pointing to $y[n]$)

least advanced output sample $\rightarrow$ (pointing to $x[n-M]$)

* The difference between these two index values is the order of the difference equation $\Rightarrow n - (n-N) = N$

* Some times difference equation presented in advanced form like

$$y[n+2] - y[n+1] + y[n] = x[n+2] - 2x[n+1] \dots (*)$$

$\Rightarrow$ Advanced form (2nd order)

Advanced form

* To present the equation in delay form :

$$y[n] - y[n-1] + y[n-2] = x[n] - 2x[n-1] \dots (**)$$

advance or delay $\rightarrow$ (pointing to $n-2$) $\leftarrow$ (pointing to n)

(*) $\rightarrow$ Advance form , (**) $\rightarrow$ delay form

Solving difference equation

يستخدم لحل المعادلات التفاضلية

We use recursive methode, but the disadvantage of using this method is that it doesn't provide closed form solution or you don't get an equation that describes the output. But you get a finite duration sequence of numbers that shows part of output.

مثال: اوجد اى جيبيل يلى بالجدول، المعطاة الحسنة، وحلهم بالى هي اى

Example #1

$$y[n+2] - 1.5y[n+1] + y[n] = 2x[n]$$

for $x[n] = u[n]$ and ICS of $\begin{cases} y[2] = 2 \\ y[-1] = 1 \end{cases}$

x تنظر اى initial condition اعطاة في السؤال ~ بناءً عليها
 advance ←
 delay ←

* عنا في المعادلة صا ~ درجة المعادلة هي 2 ~ المعادلة advance
 عنا في المجموع كله معطى initial condition بالباب ~ delay

عنا ~ استفيد منها مضطرة اقول ان system لصفحة delay
 ونجعل delay عقبار 2 لأنو درجة المعادلة 2

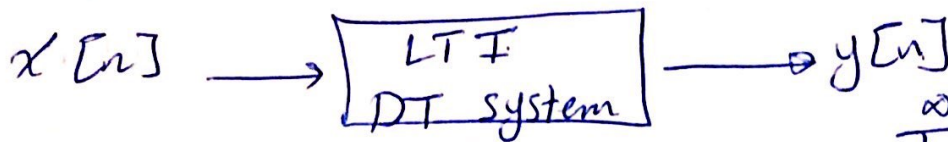
$$\Rightarrow y[n] - 1.5y[n-1] + y[n-2] = 2x[n-2]$$

$$y[n] = 1.5y[n-1] - y[n-2] + 2x[n-2]$$

ملاحظة // لو كان معطى $y[2]$ أو $y[1]$ قبل المعادلة
 advance الـ لصفحة

تابع

Convolution # used to find the Zero state solution



$$y[n] = x[n] * h[n] = h[n] * x[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

$h[n] \Rightarrow$ impulse response (output of the system when the input = $\delta[n]$)

Example # | $x[n] = \left(\frac{1}{4}\right)^n u[n-3]$, $h[n] = \left(\frac{1}{3}\right)^n u[n-5]$

Find $y[n]$??

Sol //

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

$$y[n] = \sum_{k=-\infty}^{\infty} \left(\frac{1}{4}\right)^k u[k-3] \cdot \left(\frac{1}{3}\right)^{n-k} u[n-k-5]$$

$u[k-3] \cdot u[n-k-5] \Rightarrow$ قيد 1 ومنه ههنا
 Summation نقوم بتغيير حدود الـ

$$k-3 \geq 0 \Rightarrow k \geq 3$$

$$n-k-5 \geq 0 \Rightarrow n-5 \geq k$$

$$y[n] = \sum_{k=3}^{n-5} \left(\frac{1}{4}\right)^k \left(\frac{1}{3}\right)^n \left(\frac{1}{3}\right)^{-k}$$

$3 \leq k \leq n-5$
 الحدود

$\left(\frac{1}{3}\right)^n \sum_{k=3}^{n-5} \left(\frac{3}{4}\right)^k$

$$\sum_{k=m}^n r^k = \frac{r^{n+1} - r^m}{r-1}$$

Summation نقوم بتغيير (بأي)

$$y[n] = \left(\frac{1}{3}\right)^n \frac{\left(\left(\frac{3}{4}\right)^{n-4} - \left(\frac{3}{4}\right)^3\right)}{\left(\frac{3}{4} - 1\right)} \Rightarrow y[n] = \left(\frac{1}{3}\right)^n \frac{\left(\frac{3}{4}\right)^n \cdot \frac{3}{4} - \left(\frac{3}{4}\right)^3}{-\frac{1}{4}}$$

$\Rightarrow$ 19

$$= -4 \left(\frac{1}{3}\right)^n \frac{\left[\left(\frac{3}{4}\right)^{n-4} - \left(\frac{3}{4}\right)^3\right]}{1}$$

لأنه $\left(\frac{3}{4}\right)^3$ ثابت

$$= -4 \left(\frac{1}{3}\right)^n \left(\frac{3}{4}\right)^3 \left[\frac{\left(\frac{3}{4}\right)^{n-4}}{\left(\frac{3}{4}\right)^3} - 1\right]$$

$$= \boxed{-4 \left(\frac{1}{3}\right)^n \left(\frac{3}{4}\right)^3 \left[\left(\frac{3}{4}\right)^{n-7} - 1\right] u[n-8]} \quad \#$$

n-5 > 8 example
n > 8
u[n-8] y[n]

Example # | $x[n] = e^{-n} u[n+1]$, $h[n] = (-2)^n u[n-1]$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k]$$

$$\Rightarrow \sum_{k=-\infty}^{\infty} -2^k \cdot u[k-1] \cdot e^{-(n-k)} u[n-k+1]$$

$$\begin{array}{ccc} u[k-1] \cdot u[n-k+1] & & \\ \downarrow & \downarrow & \\ k-1 \geq 0 & n-k+1 \geq 0 & \\ k \geq 1 & n+1 \geq k & \end{array}$$

$$\Rightarrow \boxed{1 \leq k \leq n+1}$$

$$y[n] = \sum_{k=1}^{n+1} -2^k \cdot e^{-(n-k)}$$

$$= \sum_{k=1}^{n+1} -2^k \cdot e^{-n} e^k \Rightarrow e^{-n} \sum_{k=1}^{n+1} (-2e)^k$$

$$y[n] = e^{-n} \left[\frac{(-2e)^{n+2} - (-2e)^1}{-2e - 1} \right] u[n]$$

$$\begin{array}{l} n+1 \geq 1 \\ n \geq 0 \\ u[n] \end{array}$$

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Example # $x[n] = \left(\frac{1}{2}\right)^n u[n]$, $h[n] = \left(\frac{1}{3}\right)^n u[n-1] - \left(\frac{1}{4}\right)^n u[n-3]$

$$y[n] = \left(\frac{1}{2}\right)^n u[n] * \left[\left(\frac{1}{3}\right)^n u[n-1] - \left(\frac{1}{4}\right)^n u[n-3] \right]$$

$$y[n] = \underbrace{\left(\frac{1}{2}\right)^n u[n] * \left(\frac{1}{3}\right)^n u[n-1]}_{y_1[n]} - \underbrace{\left(\frac{1}{2}\right)^n u[n] * \left(\frac{1}{4}\right)^n u[n-3]}_{y_2[n]}$$

$$y_1[n] = \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^k u[k] \cdot \left(\frac{1}{3}\right)^{n-k} u[n-k-1]$$

$$u[k] \cdot u[n-k-1]$$

قمتنا 1 وهو حلاكا بنجد
Summation ال حدود

$$k \geq 0 \Rightarrow k \geq 0$$

$$n-k-1 \geq 0 \Rightarrow n-1 \geq k$$

$$\Rightarrow \boxed{0 \leq k \leq n-1}$$

الحدود
المجموع

ملحوظة // $u[k]$ تكون معرفة لـ $k \geq 0$ (مجال للتوزيع)
 $u[k-1]$ تكون معرفة لـ $k-1 \geq 0$ (مجال للتوزيع)

$$y_1[n] = \sum_{k=0}^{n-1} \left(\frac{1}{2}\right)^k \cdot \left(\frac{1}{3}\right)^n \cdot \left(\frac{1}{3}\right)^{-k}$$

$$y_1[n] = \left(\frac{1}{3}\right)^n \sum_{k=0}^{n-1} \left(\frac{\frac{1}{2}}{\frac{1}{3}}\right)^k$$

$$y_1[n] = \left(\frac{1}{3}\right)^n \sum_{k=0}^{n-1} \left(\frac{3}{2}\right)^k = \left(\frac{1}{3}\right)^n \left[\frac{\left(\frac{3}{2}\right)^n - \left(\frac{3}{2}\right)^0}{\frac{3}{2} - 1} \right]$$

$$y_1[n] = \left(\frac{1}{3}\right)^n \left[\frac{\left(\frac{3}{2}\right)^n - 1}{\frac{1}{2}} \right] = 2 \left(\frac{1}{3}\right)^n \left(\left(\frac{3}{2}\right)^n - 1 \right) u[n-1]$$

$$n-1 \geq 0$$

$$n \geq 1$$

$$u[n-1] \leftarrow$$

Summation على الـ $y[n]$ معرف

(21)

$$y[n] = \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^k u[k] \cdot \left(\frac{1}{4}\right)^{n-k} u[n-k-3]$$

$$u[k] \cdot u[n-k-3]$$

$$k \geq 0 \Rightarrow \boxed{k \geq 0}$$

$$n-k-3 \geq 0 \Rightarrow \boxed{n-3 \geq k}$$

$$\Rightarrow \boxed{0 \leq k \leq n-3}$$

$$\sum_{k=0}^{n-3} \left(\frac{1}{2}\right)^k \left(\frac{1}{4}\right)^{n-k}$$

$$\Rightarrow \left(\frac{1}{4}\right)^n \sum_{k=0}^{n-3} \left(\frac{1}{2}\right)^k \cdot \left(\frac{1}{4}\right)^{-k}$$

$$\left(\frac{1}{4}\right)^n \sum_{k=0}^{n-3} \left(\frac{\frac{1}{2}}{\frac{1}{4}}\right)^k$$

$$\Rightarrow \boxed{\left(\frac{1}{4}\right)^n \sum_{k=0}^{n-3} (2)^k}$$

$$\boxed{\sum_{k=m}^n r^k = \frac{r^{n+1} - r^m}{r-1}}$$

عشان نطبقه بقية القافضه التالي

$$y[n] = \left(\frac{1}{4}\right)^n \left(\frac{2^{n-2} - 2^0}{2-1} \right)$$

عشان نطبقه بقية القافضه التالي

$$y[n] = \left(\frac{1}{4}\right)^n (2^{n-2} - 1) u[n-3]$$

$$n \geq 3$$

$$u[n-3]$$

$$y[n] = y_1[n] \ominus y_2[n]$$

عشان في السؤال حاصل طرح

$$= 2 \left(\frac{1}{3}\right)^n \left(\left(\frac{3}{2}\right)^n - 1\right) u[n-1] - \left(\frac{1}{4}\right)^n (2^{n-2} - 1) u[n-3]$$

#

* نَقَم كِبَاب $y[n]$ حِين بَدَأ مـ -1 وَنَقَم عِنْد $\underline{4}$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k]$$

* عَمِ لِكْمَة جَعِيد كُل n نَحِط بِكُل مَخَالِصَة $x[k]$ وَ $h[n-k]$ دَفْعَة دَفْعَة

$$y[-1] = 1 \times 1 = \boxed{1}$$

$$y[0] = 2 \times 1 + 0 \times 1 = \boxed{2}$$

$$y[1] = -1 \times 1 + 2 \times 0 + 1 \times -1 = \boxed{-2}$$

$$y[2] = -1 \times 1 + 0 \times 1 + 2 \times -1 = \boxed{-3}$$

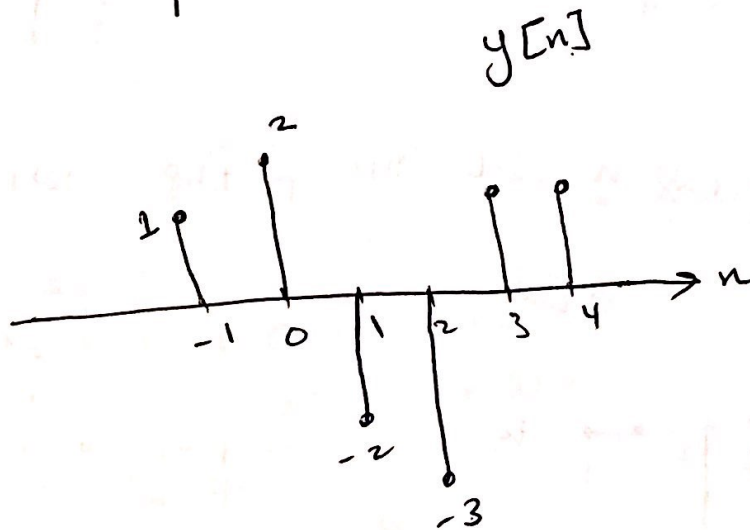
$$y[3] = -1 \times 0 + -1 \times -1 + 2 \times 0 = \boxed{1}$$

$$y[4] = -1 \times -1 + -1 \times 0 + 0 + 0 = \boxed{1}$$

for $n > 4$, $y[n] = 0$

for $n < -1$, $y[n] = 0$

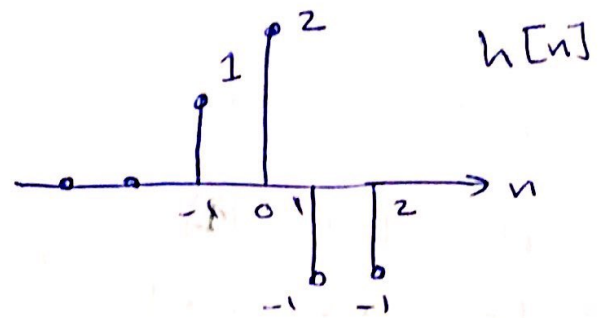
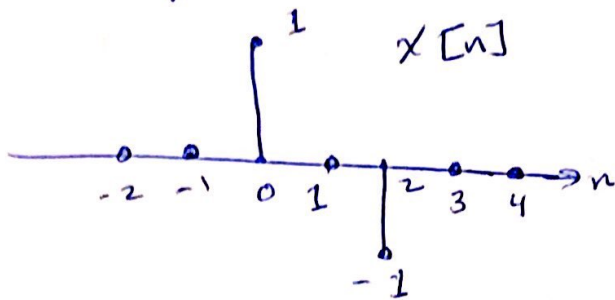
$$y[n] = \{ 1, 2, -2, -3, 1, 1 \}$$



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* Graphical Convolution

Example: Given $x[n]$ and $h[n]$, find $y[n]$

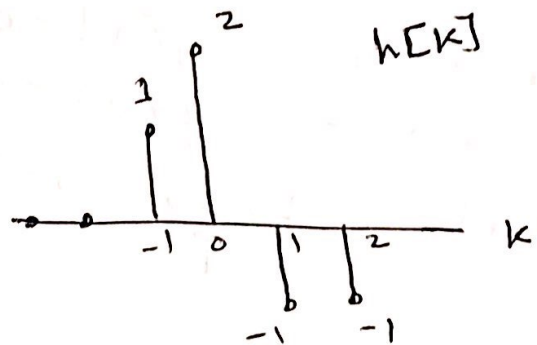
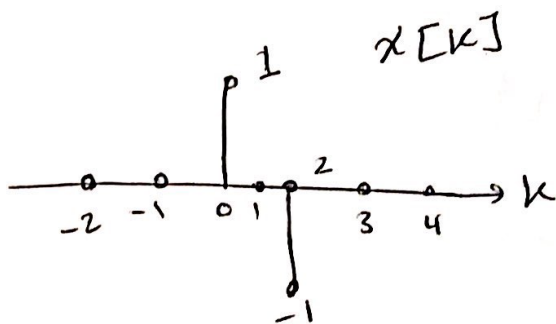


$$y[n] = x[n] * h[n]$$

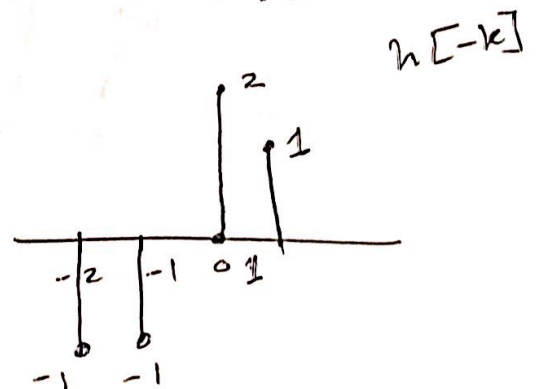
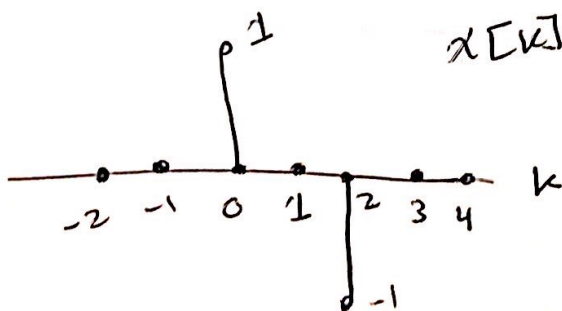
$y[n]$ start at $n = 0 + (-1) = \boxed{-1}$

$y[n]$ end at $n = 2 + 2 = \boxed{4}$

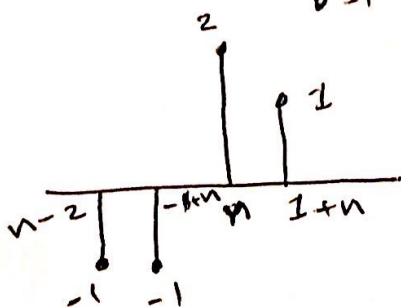
[1] $x[n] \rightarrow x[k]$ $h[n] \rightarrow h[k]$ نقطة n ك k $\leftarrow$



[2] $x[k] \rightarrow x[-k]$ نقطة k $\rightarrow$ flip $\rightarrow$ n $\rightarrow$ shift $\rightarrow$ n



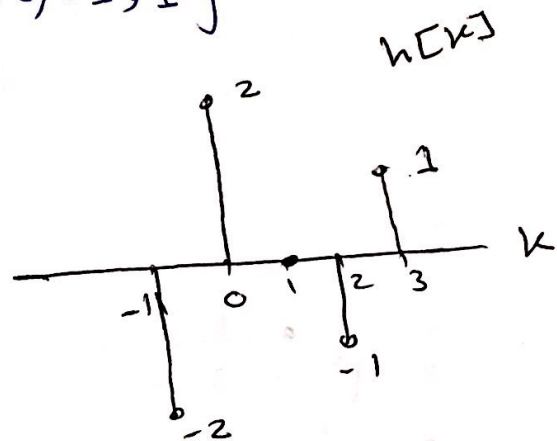
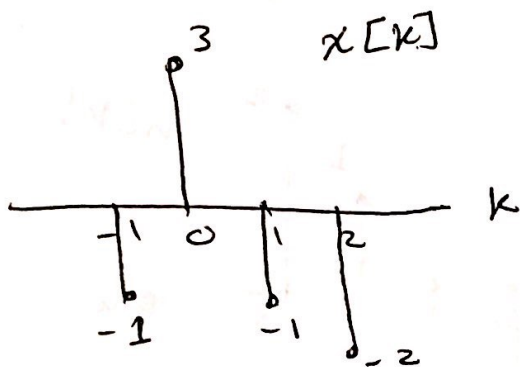
[3] $x[-k] \rightarrow y[n]$ shift $\rightarrow$ n $\rightarrow$ $h[-k]$ $\rightarrow$ shift $\rightarrow$ n



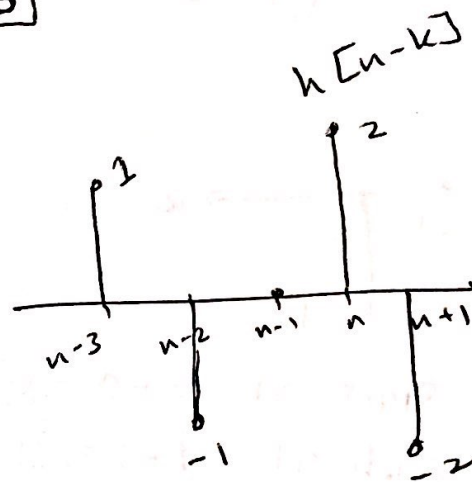
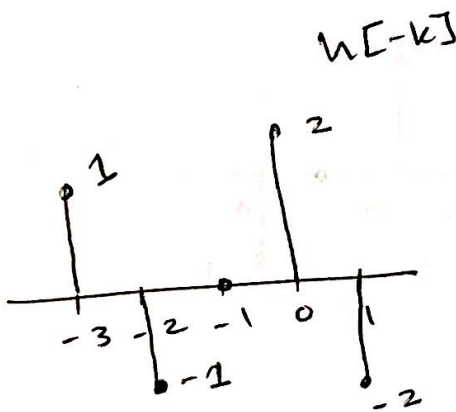
Example #

$$x[n] = \{-1, 3, -1, -2\}$$

$$h[n] = \{-2, 2, 0, -1, 1\}$$



$y[n]$: starts at $-1 + (-1) = \boxed{-2}$
ends at $2 + 3 = \boxed{5}$



$y[n] = 0$, for $n < -2$

$$y[-2] = (-2)(-1) = \boxed{2}$$

$$y[-1] = (2)(-1) + (3)(-2) = \boxed{-8}$$

$$y[0] = (-2)(-1) + (2)(3) = \boxed{8}$$

$$y[1] = (-1)(-1) + 0 \times 3 + (2)(-1) + (-2)(-2) = \boxed{3}$$

$$y[2] = (-1)(1) + (-1)(3) + (0)(-1) + (2)(-2) = \boxed{-8}$$

$$y[3] = (1)(3) + (-1)(-1) + (0)(-2) = \boxed{4}$$

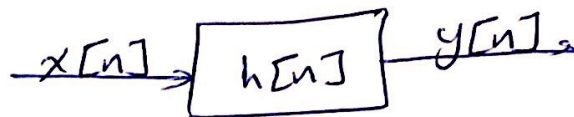
$$y[4] = (1)(-1) + (-1)(-2) + 0 = \boxed{1}$$

$$y[5] = (1)(-2) = \boxed{-2}$$

$$y[n] = \{2, -8, 8, 3, -8, 4, 1, -2\}$$

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Example #



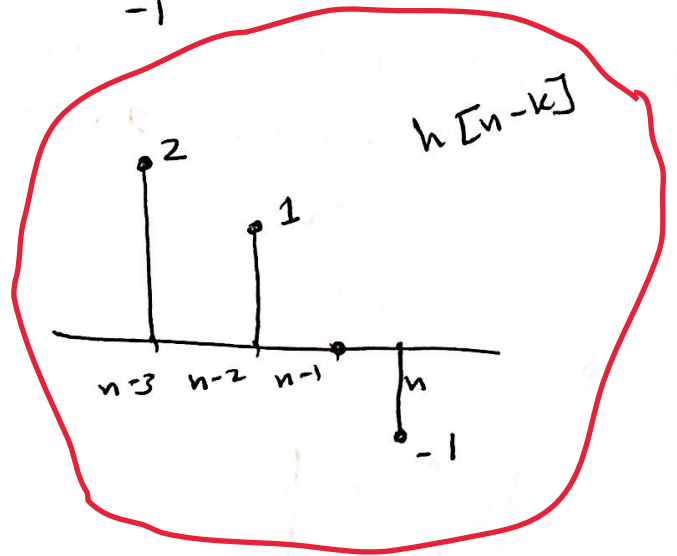
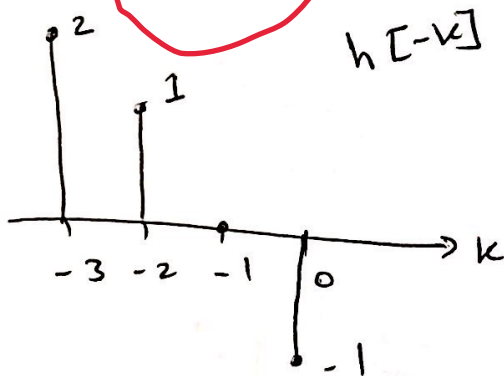
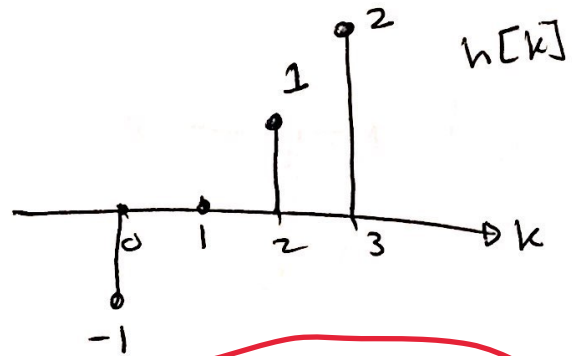
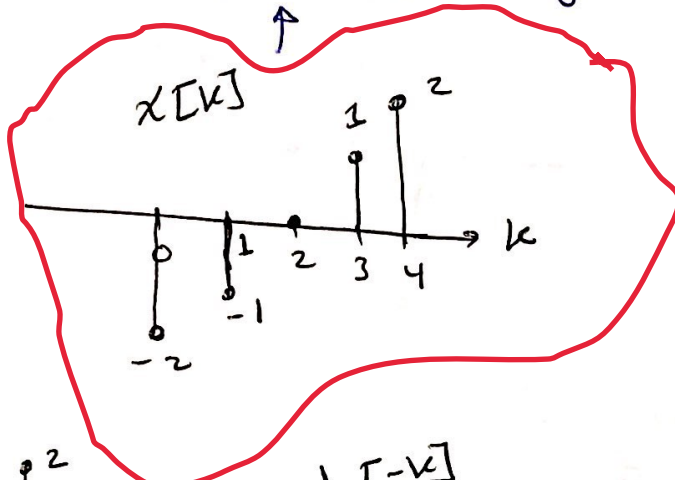
$$x[n] = \{-2, -1, 0, 1, 2\}$$

$$h[n] = \{-1, 0, 1, 2\}$$

$$N=5$$

$$M=4$$

عدد اقمار
~ عدد اقمار



$y[n]$: starts at $0+0=0$
ends at $4+3=7$

$$N+M-1$$

$$4+5-1=8$$

عدد اقمار $y[n]$

$$y[n] = 0 \text{ for } n < 0$$

$$y[0] = (-1)(-2) = 2$$

$$y[1] = (0)(-2) + (-1)(-1) = 1$$

$$y[2] = (1)(-2) + (0)(-1) + (-1)(0) = -2$$

$$y[3] = (2)(-2) + (1)(-1) + (0)(0) + (-1)(1) = -6$$

$$y[4] = (2)(-1) + (1)(0) + (0)(1) + (-1)(2) = -4$$

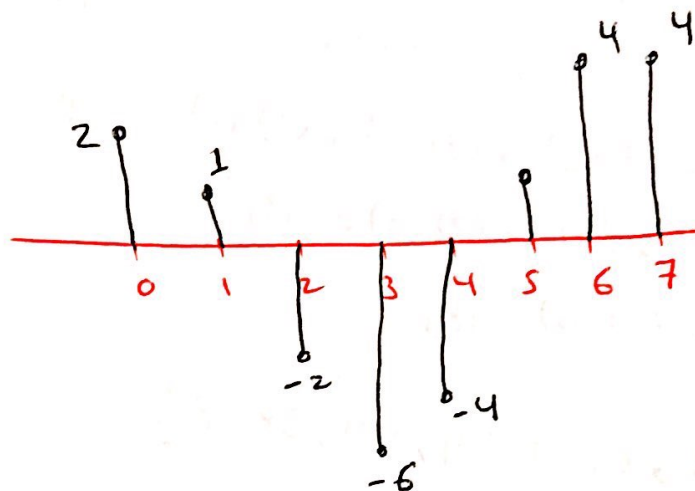
$$y[5] = (2)(0) + (1)(1) + (0)(2) + (-1)(0) = 1$$

$$y[6] = (2)(1) + (1)(2) = 4$$

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$$[7] = (2)(2) + (1)(0) = [4]$$

$$y[n] = \{ \underset{\uparrow}{2}, 1, -2, -6, -4, 1, 4, 4 \}$$

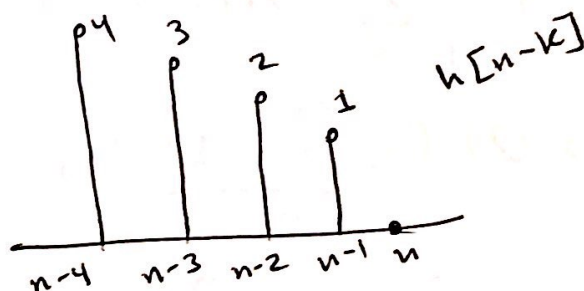
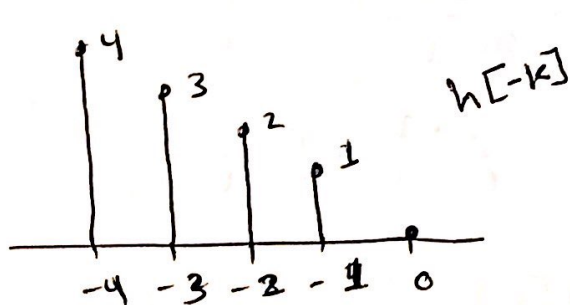
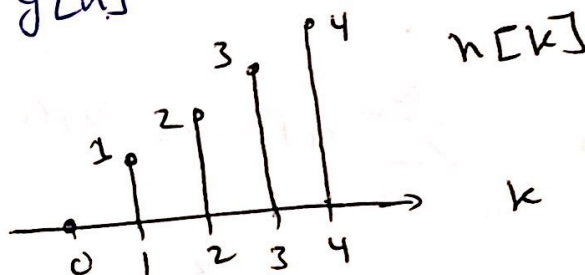
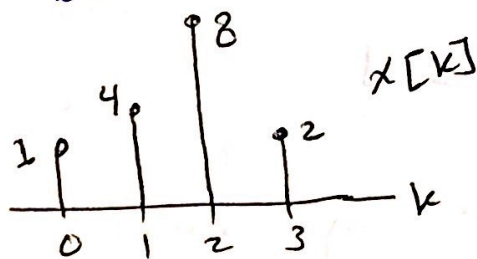


Example #1

$$x[n] = [1, 4, 8, 2]$$

$$h[n] = [0, 1, 2, 3, 4]$$

both start at $n=0$, Find $y[n]$



$y[n] \Rightarrow$ starts at $0+0 = [0]$
ends at $4+3 = [7]$

[27]

$$y[n] = 0, \text{ for } n < 0$$

$$y[0] = (0)(1) = \boxed{0}$$

$$y[1] = (1)(1) = \boxed{1}$$

$$y[2] = (2)(1) + (1)(4) = \boxed{6}$$

$$y[3] = (3)(1) + (2)(4) + (1)(8) = \boxed{19}$$

$$y[4] = (4)(1) + (3)(4) + (2)(8) + (1)(2) = \boxed{34}$$

$$y[5] = (4)(4) + (3)(8) + (2)(2) + (1)(0) = \boxed{44}$$

$$y[6] = (4)(8) + (3)(2) + (2)(0) = \boxed{38}$$

$$y[7] = (4)(2) + (3)(0) = \boxed{8}$$

$$y[n] = \{ \underset{\uparrow}{0}, 1, 6, 19, 34, 44, 38, 8 \}$$

$$x[n] = u[n], h[n] = 2(0.8)^n u[n]$$

Mathematical

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k]$$

$$y[n] = \sum_{k=-\infty}^{\infty} u[n-k] \cdot 2(0.8)^{n-k} u[n-k]$$

$$y[n] = \sum_{k=0}^n 2(0.8)^k = 2 \left(\frac{0.8^{n+1} - 0.8^0}{0.8 - 1} \right)$$

$$\Rightarrow 2 \left(\frac{0.8^{n+1} - 1}{-0.2} \right) \Rightarrow -10(0.8^{n+1} - 1) u[n]$$

$$\Rightarrow [(-10)(0.8)^n \cdot (0.8) + 10] u[n]$$

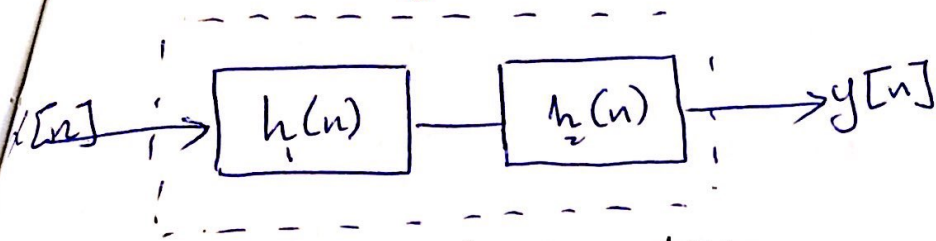
$$\Rightarrow [-8 \cdot (0.8)^n + 10] u[n]$$

#

$$\left\{ \begin{array}{l} n-k \geq 0 \\ n \geq k \\ \hline k \geq 0 \\ \hline 0 \leq k \leq n \\ \hline n \geq 0 \\ u[n] \end{array} \right.$$

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Interconnection system :-

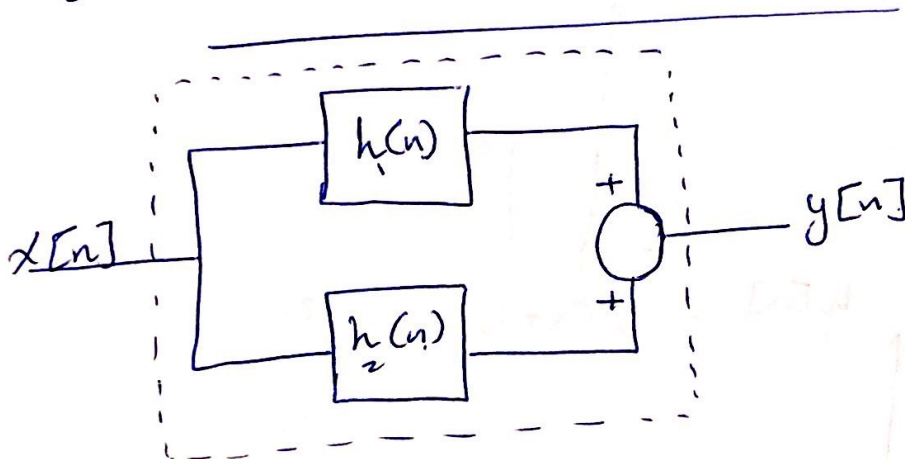


Cascade Connection

$$h_s[n] = h_1[n] * h_2[n]$$

$$y[n] = x[n] * h_1[n] * h_2[n]$$

#

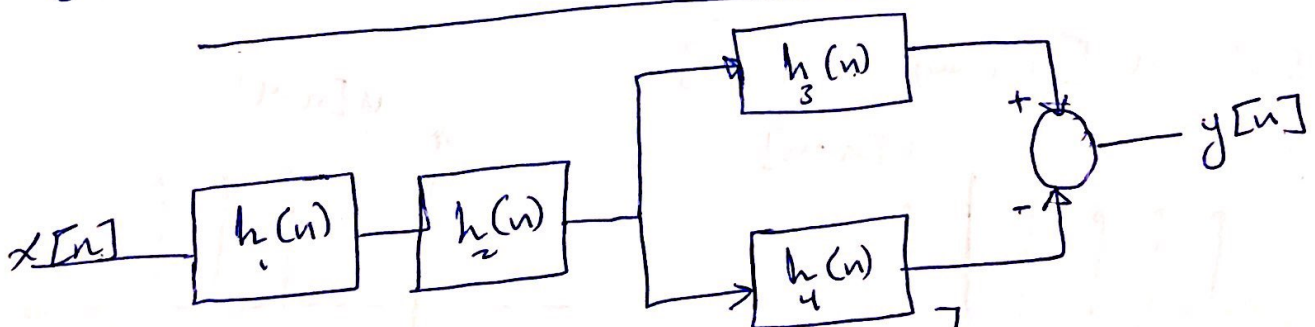


parallel connection

#

$$h_s[n] = h_1[n] + h_2[n]$$

$$y[n] = x[n] * [h_1[n] + h_2[n]]$$

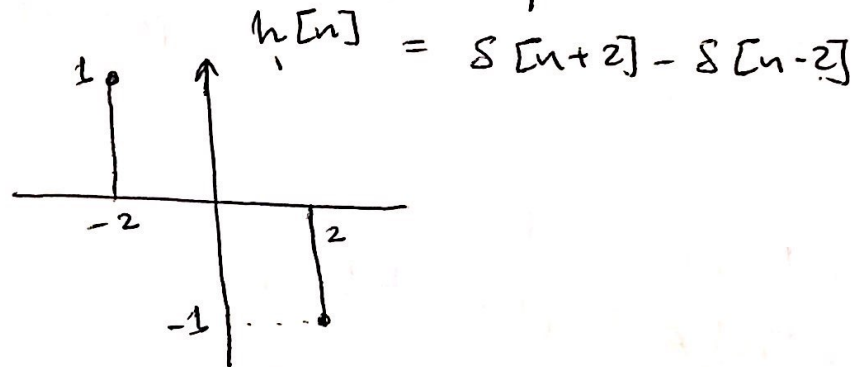
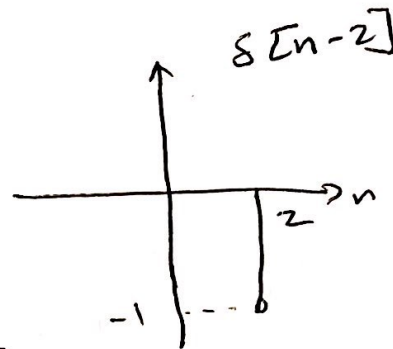
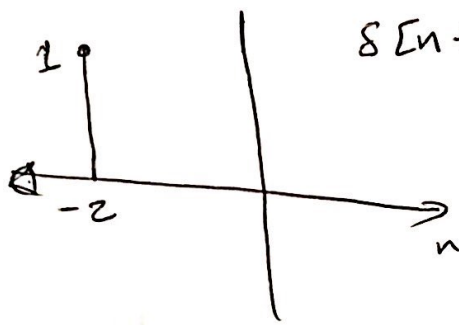
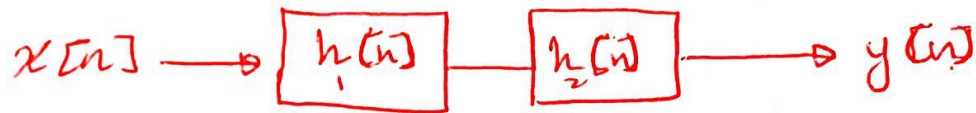


$$h_s[n] = h_1[n] * h_2[n] * [h_3[n] - h_4[n]]$$

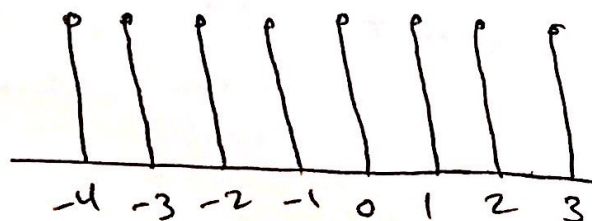
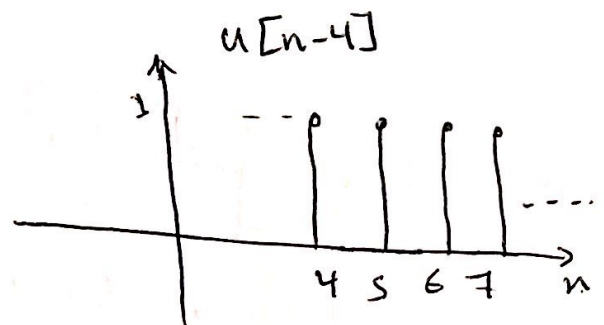
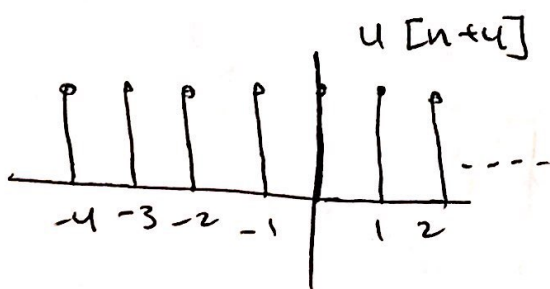
$$y[n] = x[n] * (h_1[n] * h_2[n] * [h_3[n] - h_4[n]])$$

#

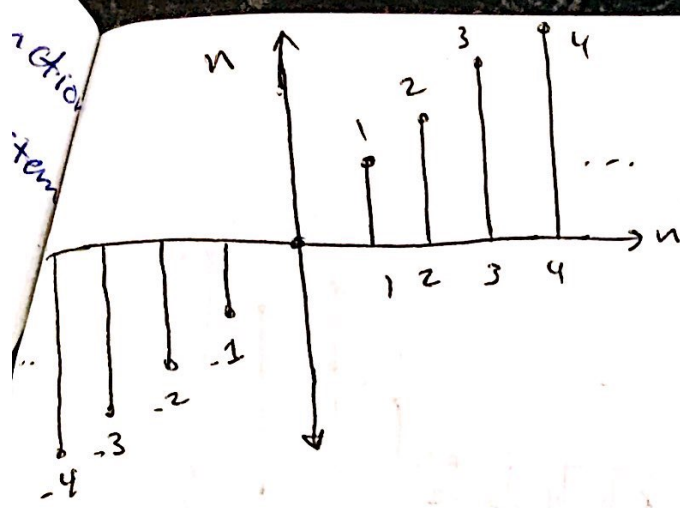
Ex: 1 An LTI system has an impulse response function given by $h_1[n] = \delta[n+2] - \delta[n-2]$, a second LTI system has an impulse function given by $h_2[n] = n(u[n+1] - u[n-4])$. Assume two systems are connected in cascade as shown find $y[n]$.



$$h_2[n] = n [u[n+4] - u[n-4]]$$

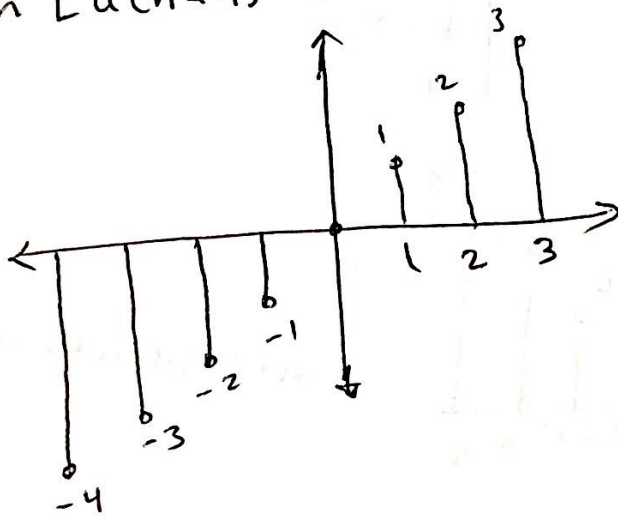


$$u[n+4] - u[n-4]$$



"دفع السماء زلزالكم وناوخلون"

$$\Rightarrow n [u(n+4) - u(n-4)]$$



$$\Rightarrow h_2[n]$$

* ملاحظة // من المفترض أنه تكون لدينا، الفترة مع الرسم مباشرة دور، للبعد
للخطوات التفصيلية اختصاراً للوقت (الرسم بالنظر)

$$\Rightarrow h_s[n] = h_1[n] * h_2[n]$$

graphical
أو باستخدام الخصائص

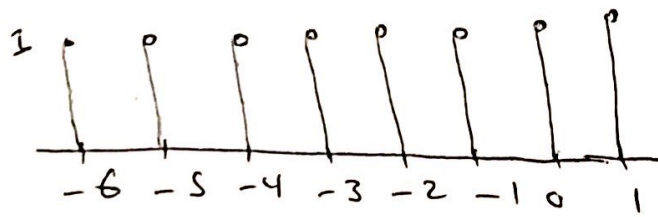
أولاً // باستخدام الخصائص :

$$\begin{aligned} h_s[n] &= h_1[n] * h_2[n] \\ &= (\delta[n+2] - \delta[n-2]) * (n(u(n+4) - u(n-4))) \\ &= (n+2)u(n+6) - (n+2)u(n-2) - (n-2)u(n+2) + (n-2)u(n-6) \\ &= (n+2)[u(n+6) - u(n-2)] - (n-2)[u(n+2) - u(n-6)] \end{aligned}$$

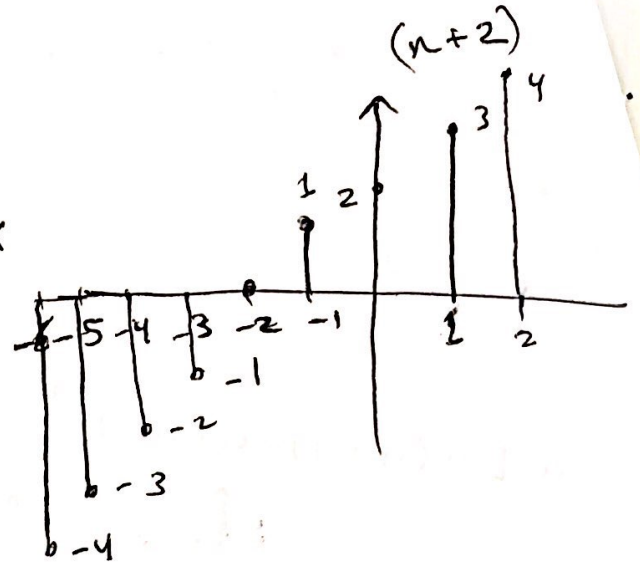
$$u(n+6) - u(n-2)$$

↓

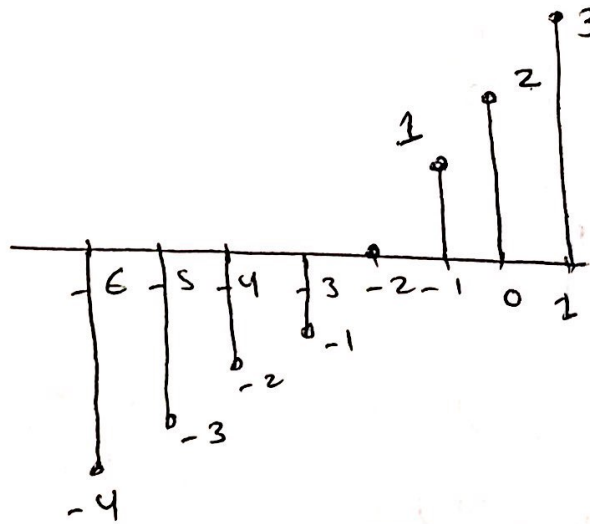
$$[u(n+6) - u(n-2)]$$



x

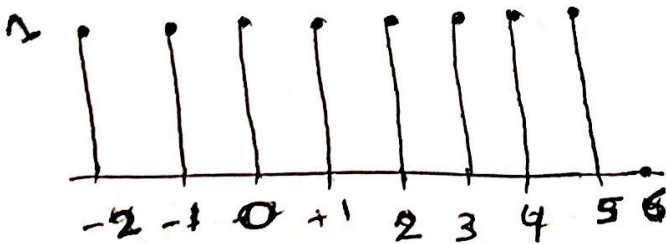


(*)

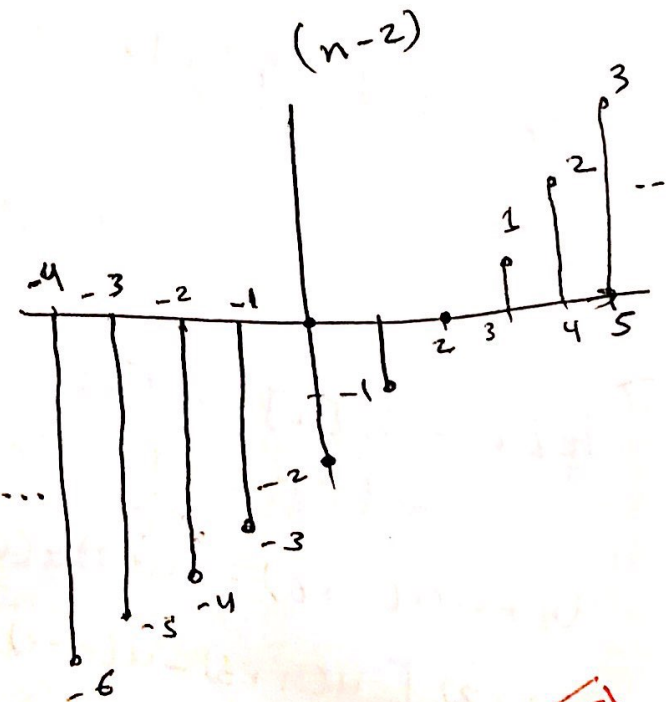


$$\Rightarrow (n+2)[u(n+6) - u(n-2)]$$

$$u(n+2) - u(n-6)$$

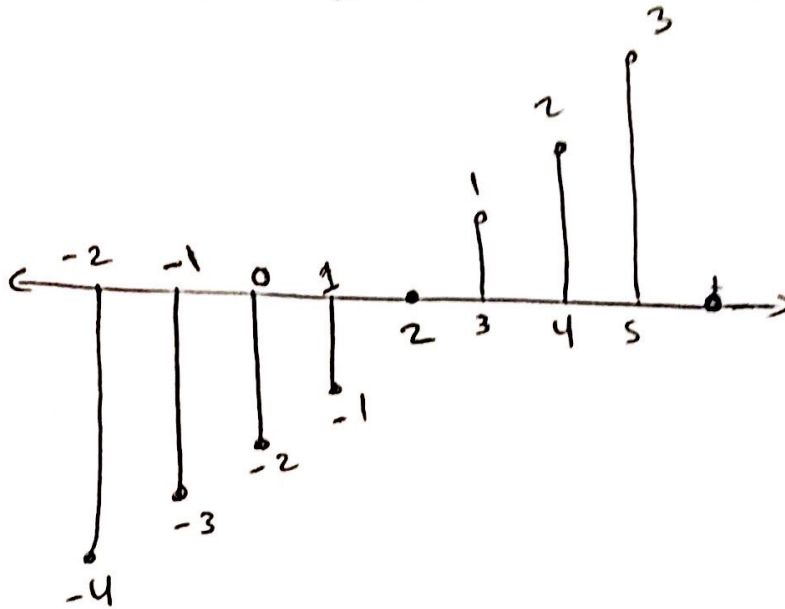


*



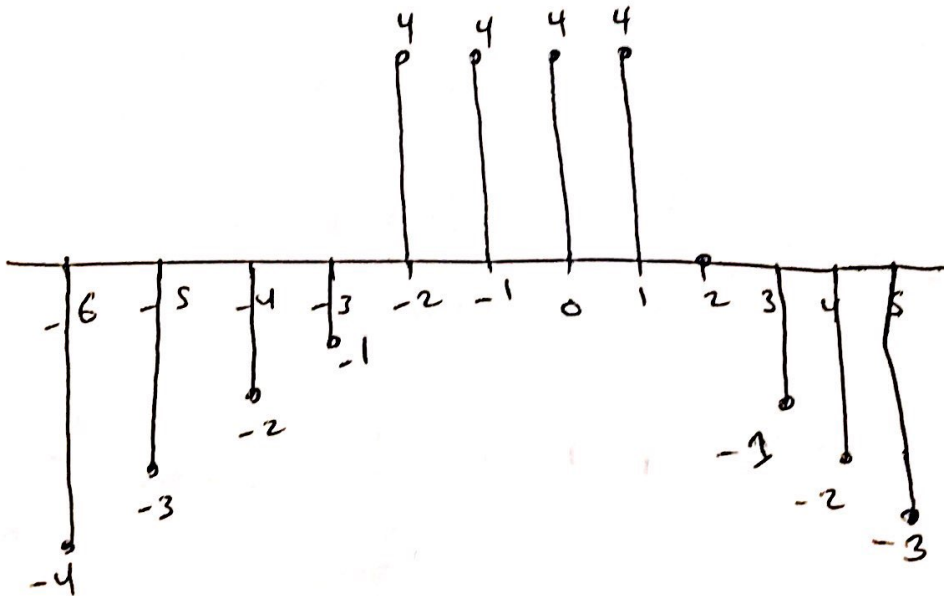
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$$(n-2) [u(n+2) - u(n-6)]$$



✱✱

$$h_5[n] = h_1[n] * h_2[n]$$



بالخطأ

ملاحظة مهمة على السؤال //

رسالة //

$$x(n) * \delta(n+1) \rightarrow x(n+1)$$

$$x(n+2) * \delta(n+3) \rightarrow x(n+5)$$

بعضنا قد نعتقد أن n موجودة في x بالبرهان لوجود داخل impulse

System Stability //

LTID with $h[n]$ is BIBO Stable if:

$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty \quad (\text{finite})$$

Example #1

$$h[n] = \begin{cases} \left(\frac{1}{2}\right)^n, & n \geq 0 \\ \left(\frac{1}{3}\right)^n, & n < 0 \end{cases}$$

$$\left(\frac{1}{2}\right)^n, n \geq 0 \Rightarrow \left(\frac{1}{2}\right)^n u[n]$$

$$\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^0 + \left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \dots$$

$$= 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$$

$$\sim \text{decrease} \Rightarrow \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n < \infty \quad (\text{finite})$$

$$\text{or } \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n = \frac{1}{1 - \frac{1}{2}} \quad (\text{ratio})$$

$$\Rightarrow < \infty \Rightarrow \text{finite}$$

$$\sum_{n=-\infty}^{-1} \left(\frac{1}{3}\right)^n, n < 0$$

assume $m = -n$

$$\sum_{-m=-\infty}^{-m=-1} \left(\frac{1}{3}\right)^{-m} \Rightarrow \sum_{m=\infty}^1 \left(\frac{1}{3}\right)^m \Rightarrow$$

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$$\sum_{m=1}^{\infty} \left(\frac{1}{3}\right)^{-m} \Rightarrow \left(\frac{1}{3}\right)^{-1} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{3}\right)^{-3} + \left(\frac{1}{3}\right)^{-4} + \dots$$

$$= (3)^1 + (3)^2 + (3)^3 + (3)^4 + \dots$$

increase (not finite)

$\Rightarrow$ system is not stable

system is BIUBO

Ex #1 A discrete time response has impulse response given by $h[n] = \delta[n] + \left(\frac{1}{3}\right)^n u(n-1)$ $1 \leq n < \infty$

* Is the system stable.

$$\sum_{n=-\infty}^{\infty} \delta[n] + \sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n$$

$\downarrow$

$$\sum_{n=-\infty}^{\infty} \delta[n] = 1$$

$$\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n = \left(\frac{1}{3}\right)^1 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)^4 + \dots \quad \text{decrease}$$

$\Rightarrow$ so the system is stable.

* Find $h[n]$, the unit impulse response of LTID system specified by the following equation:-

$$y[n] = 2x[n] - 2x[n-1]$$

$\Rightarrow$ We need $y[n] = h[n]$ at $x[n] = \delta[n]$

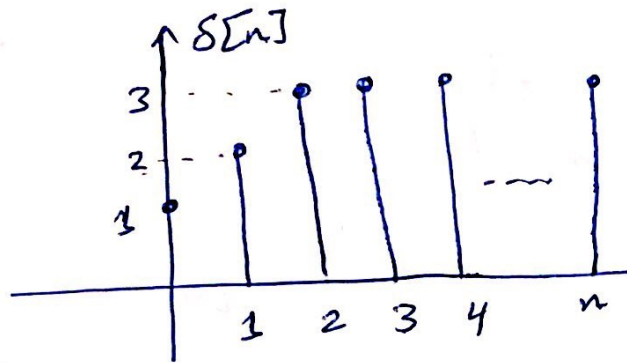
$$h[n] = 2\delta[n] - 2\delta[n-1]$$

$$x[n] * \delta[n] = x[n]$$

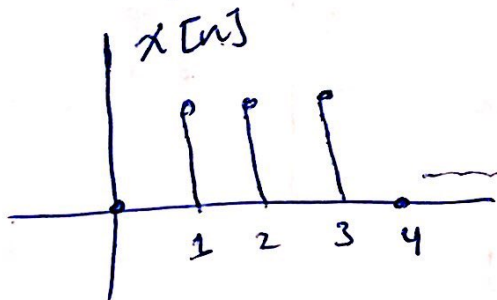
$$x[n] * \delta[n-1] = \underline{x[n-1]}$$

(26)

If the step response of discrete time LTI system $s[n]$ as shown in figure below



Sketch the system response to input $x[n]$ as shown in fig.



Solution //

بنا نتخرج معطيات السؤال //

نقطة LTI معطيات أي shift في input نقابل shift في output دون تغير خصائص الـ system.

Step response [2] ← الـ input للـ system عبارة عن unit step

[3] معطيات السؤال //



مطلوب (موجب) response تابع الـ system إلى الـ input تابع عبارة عن $x[n]$.

الـ input $x[n]$ ← بقدر أجر عنو بدلالة unit step

$$x[n] = u[n-1] - u[n-4]$$

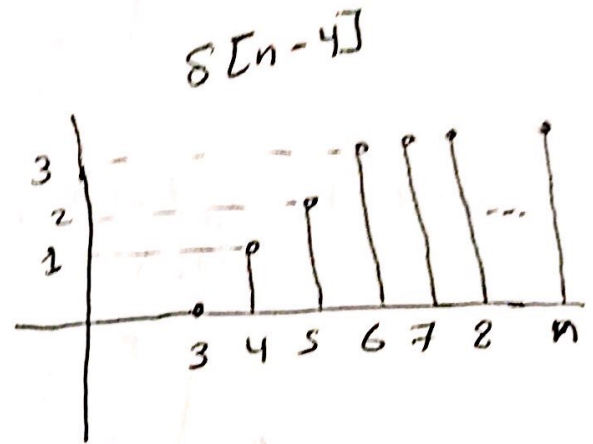
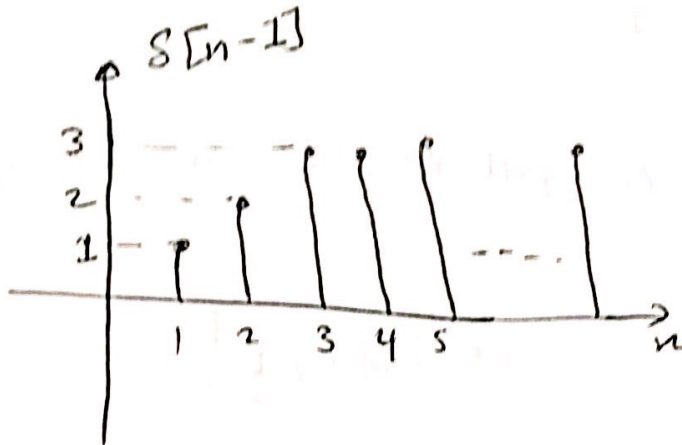
ونلاحظ أن $u[n]$ عبارة عن input للـ system الأول

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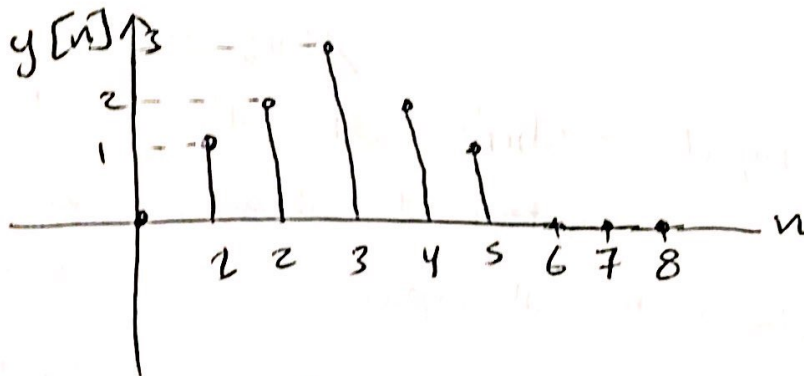
← LTI as a system it will
output is shift of input, is shift of

$$x[n] = u[n-1] - u[n-4]$$

$$y[n] = s[n-1] - s[n-4]$$



$$y[n] = s[n-1] - s[n-4]$$



ملحوظة // حاولت أبسط حل السؤال لأنني لم أكن أعرف فكرة نوعاً ما .
أي هو من راضع إلى كل شيء لأنني كنت قادراً أكثر من حينئذ (٢)
* بالإنجليزية راجع أوضاع كل شيء ستاد الله . (٣)

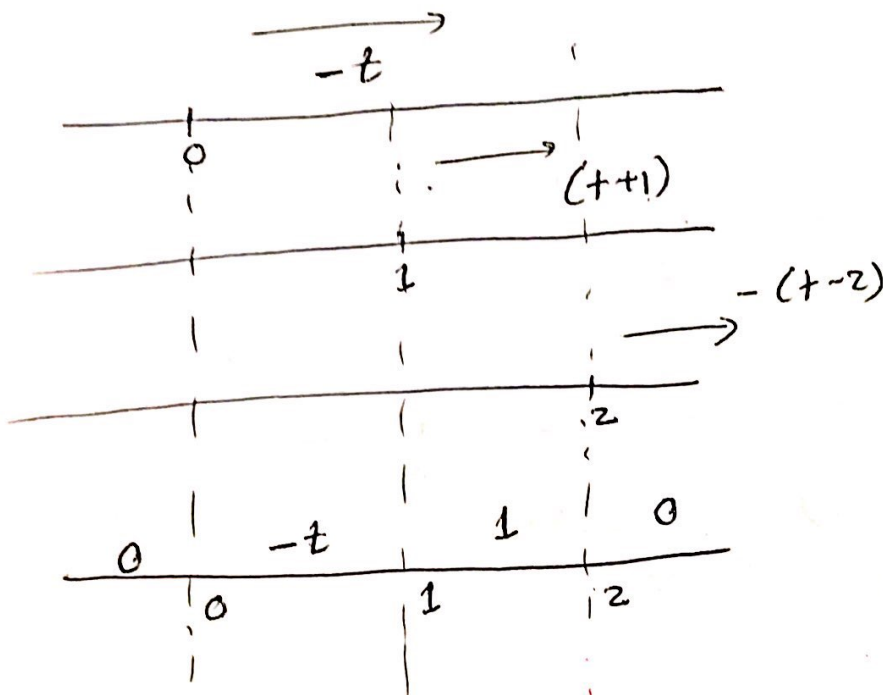
"ربما يجب لنا من أن نراجعها ودراسنا مرة أخرى (رجو لنا للمفصلة) (٤)
- صورة الفرقه -

discrete or Continuous

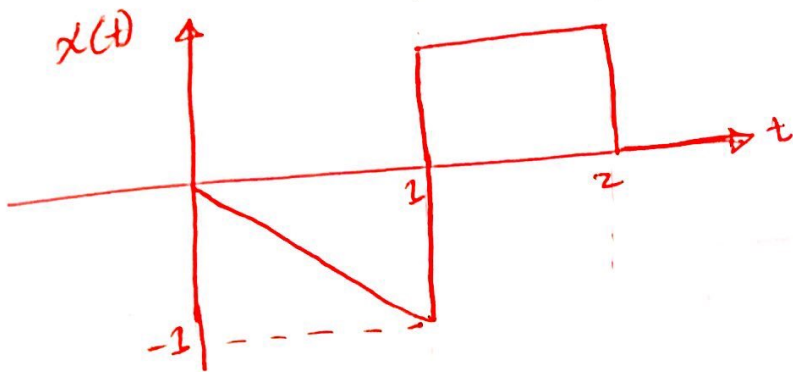
اسئلة هرة سواد على

Ex # | Sketch :

$$x(t) = (t+1)u(t-1) - tu(t) - u(t-2)$$



نقسم الى فترات
نخرج الفترات
ونرسم الامتزاز

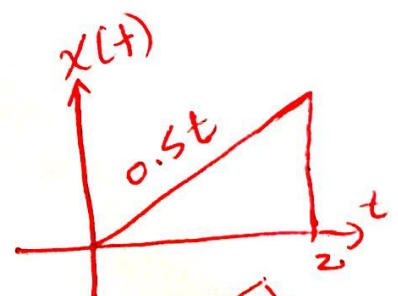


$$\begin{aligned} x(t) &= 0.5r(t)u(2-t) \\ &= 0.5tu(t) \cdot u(2-t) \end{aligned}$$

$$u(t) \rightarrow t \geq 0 \quad u(2-t) \Rightarrow \begin{aligned} 2-t &\geq 0 \\ 2 &\geq t \end{aligned}$$

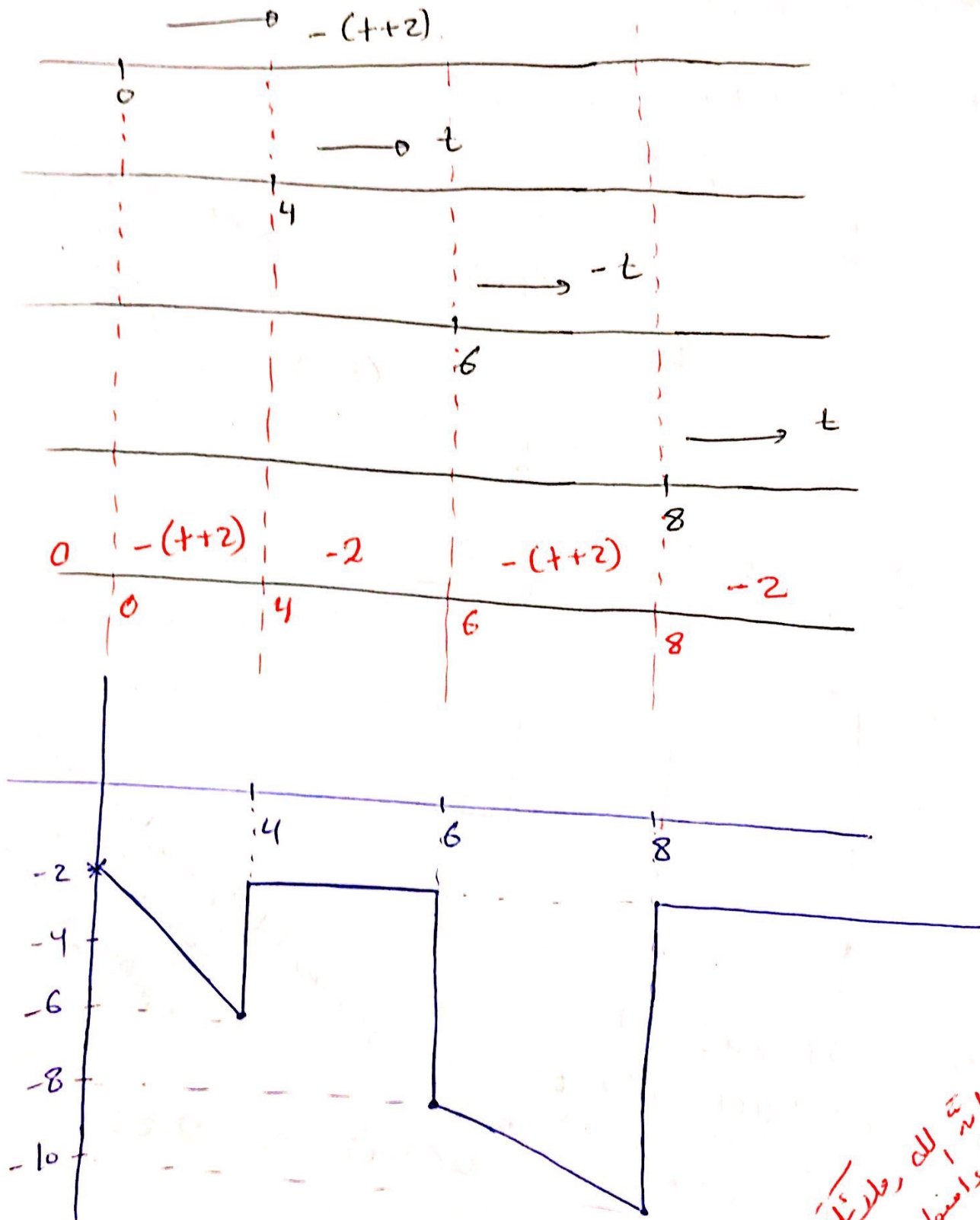
$$0 \leq t \leq 2$$

$$x(t) = 0.5t, \quad 0 \leq t \leq 2$$



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Ex #1 $x(t) = -(t+2)u(t) + t(u(t-4) - u(t-6) + u(t-8))$ 6

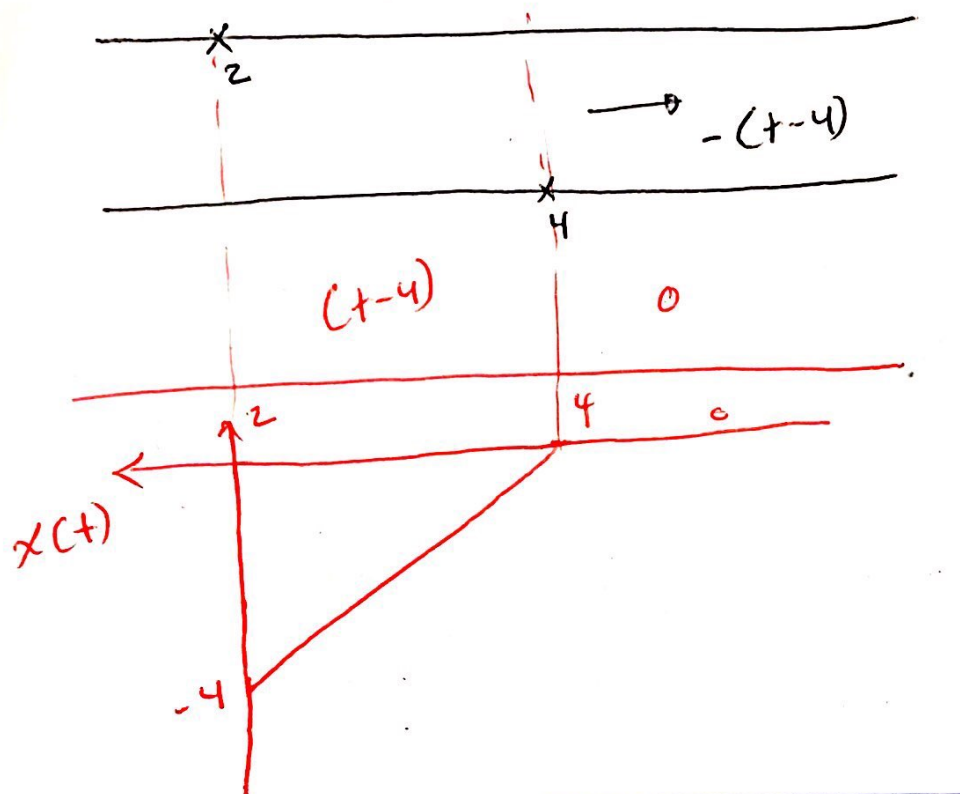


الله، فليزني رطلون على اليني يا زها، زني
 دامنوا مملوا عليه، لحو سلباً
 سونة الأخرين -
 (ن)

1) sketch # $(t-4) [u(t-2) - u(t-4)]$

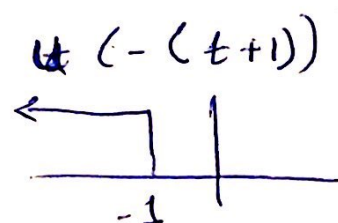
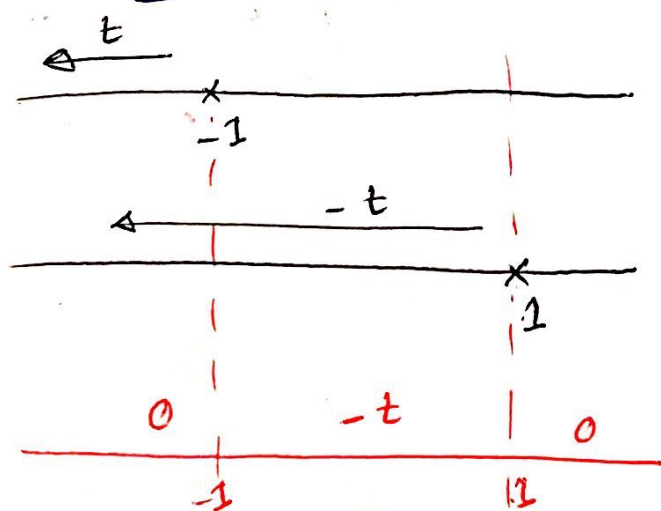
$\rightarrow (t-4)$

2003 (صن)

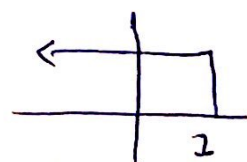


$$x(t) = t[u(-t-1) - u(-t+1)]$$

2007 (صن)

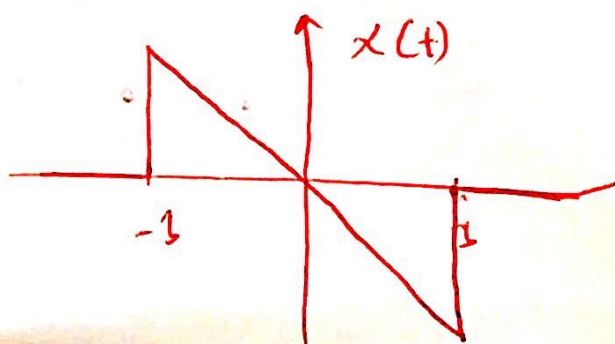


$$u(-(t+1))$$



$$(t) - \infty \rightarrow -1$$

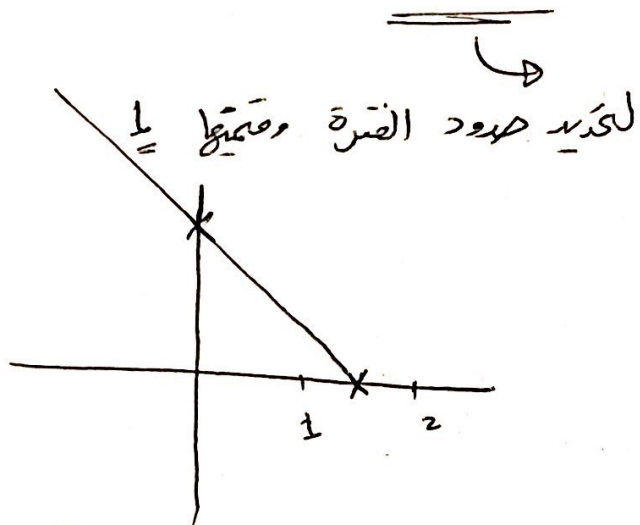
$$(-t) - \infty \rightarrow 1$$



$$x(t) = r(-2t+3)$$

2005 ن. 15

$$x(t) = (-2t+3)u(-2t+3)$$



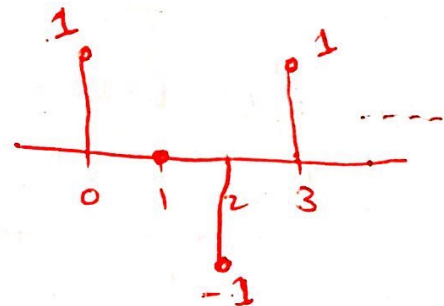
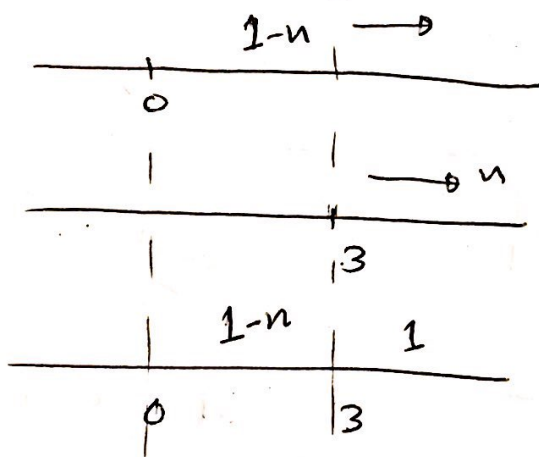
$$-2t+3 > 0$$

$$3 > 2t$$

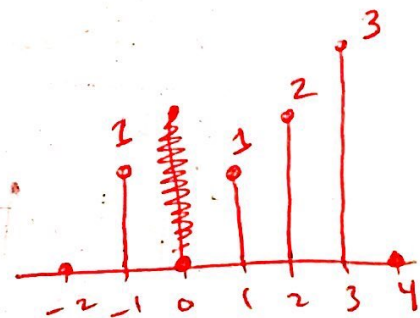
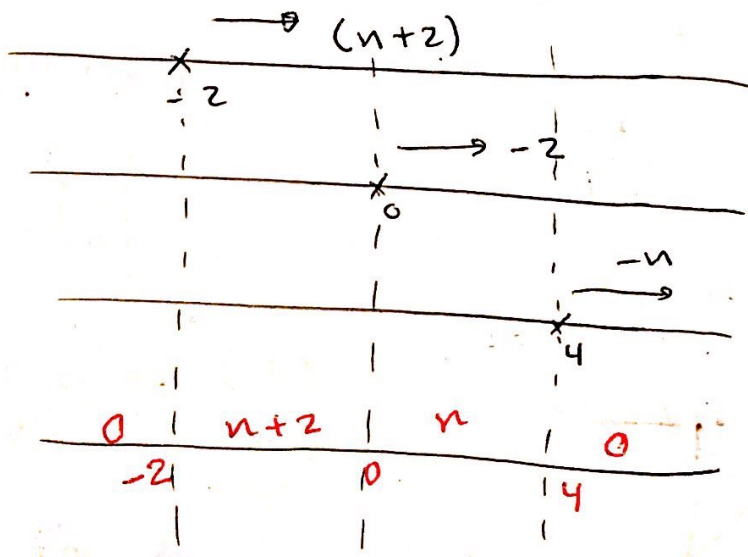
$$\frac{3}{2} > t$$

$$t < \frac{3}{2}$$

$$x[n] = (1-n)u[n] + nu[n-3]$$



$$x(n) = (n+2)u(n+2) - 2u(n) - nu[n-4]$$



at $n=0 \Rightarrow x[n]=0$

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