

Part #1
أول

في السابق تابعنا Fourier transform وحيث أنو Fourier Transform هو حالة خاصة من Laplace Transform أي أن Laplace هو حالة العامة
 هناك علاقة بين Laplace transform و Fourier transform حيث أن:

$$s = j\omega$$

حيث أنو Laplace transform يربط كل الإشارة ال Continuous بكونه شرط أما Fourier transform لا يربط بين بعض الشرط

$$\int_{-\infty}^{\infty} |x(t)| dt < \infty$$

لوحقق هذا الشرط يتوجب علينا Fourier transform

كلها تحويل للإشارة من time domain إلى frequency domain

The Laplace transform:

$$X(s) = \mathcal{L}\{x(t)\} = \int_{-\infty}^{\infty} x(t) e^{-st} dt$$

$$s = \sigma + j\omega$$

$x(s) \Rightarrow$ Laplace transform of $x(t)$

$$x(t) = \mathcal{L}^{-1}[X(s)] = \frac{1}{2\pi j} \int X(s) e^{st} dt$$

$x(t)$: inverse Laplace transform of $x(t)$

"ما أطلبك من حسنة فمن الله وما أطلبك من حسنة فمن نفسي" - سورة النساء -

- * Motivation for the Laplace Transform: —
- CT Fourier transform enables us to do a lot of things, e.g.
- * Analyze frequency response of LTI system.
 - * Sampling.
 - * Modulation.

* Why we need yet another transform?

→ one view of Laplace Transform is as an extension of the Fourier transform to allow analysis of broader class of signals and systems.

— In particular, Fourier transform cannot handle large (and important) classes of signals and unstable systems i.e. when

$$\int_{-\infty}^{\infty} |x(t)| dt < \infty$$

* Many of the signals don't have Fourier transform such as $e^{\alpha t} u(t)$, $t u(t)$, and other time signals that are not absolutely integrable, so this problem can be solved by using Laplace transform.

$$s = \sigma + j\omega$$

s	→ complex number
σ	→ real part
ω	→ imaginary part

* There is no conditions or limitations for using Laplace transform.

Ex #1 $x(t) = e^{\alpha t} u(t)$, find the frequency domain?

By using Fourier transform:

$$\int_{-\infty}^{\infty} e^{\alpha t} u(t) dt \Rightarrow \int_0^{\infty} e^{\alpha t} dt = \boxed{\infty}$$

⇒ there is no Fourier transform.

2

By Laplace transform: $\int_{-\infty}^{\infty} \underbrace{e^{\alpha t}}_{u(t)} dt = \int_0^{\infty} e^{\alpha t} dt = \infty$

but the L.T is exist (Laplace exists)

$$\int_0^{\infty} e^{\alpha t} \cdot e^{-st} dt \Rightarrow \int_0^{\infty} e^{(\alpha-s)t} dt \Rightarrow \frac{e^{(\alpha-s)t}}{(\alpha-s)} \Big|_0^{\infty}$$

سایه
قطب
ب
اذا
($\alpha-s$)
نقطه

$$\frac{e^{(\alpha-s)\infty}}{(\alpha-s)} - \frac{1}{(\alpha-s)} \Rightarrow \boxed{\frac{1}{s-\alpha}}$$

Roc $\Rightarrow \text{Re}\{s\} = \sigma > \alpha$

$s = \sigma + j \text{Im}$

Real part

$x(s) = \mathcal{L}\{x(t)\} = \int_{-\infty}^{\infty} x(t) \cdot e^{-st} dt$ (for causal and non causal)

Bilateral (two sided Laplace transform)

$x(s) = \mathcal{L}\{x(t)\} = \int_0^{\infty} x(t) \cdot e^{-st} dt$

(Just for causal signal)

unilateral

Roc $\Rightarrow$ Region of Converges

* The range of value for the complex variable s for which the Laplace transform converge is called the Roc (Region of Convergence)

Ex #1 $x(t) = e^{-at} u(t) \Rightarrow x(s) = \frac{1}{s+a}$ $\xrightarrow{s+a=0} \boxed{s=-a}$

Roc: $\text{Re}\{s\} > -a$

3

المعادلة في الصورة في ROC في الجدول (x)

$$\text{if } \int_0^{\infty} e^{\alpha t} e^{-st} dt = \int_0^{\infty} e^{\alpha t} \cdot e^{-(\delta+j\omega)t} dt$$

$$= \int_0^{\infty} e^{\alpha t} \cdot e^{-\delta t} \cdot e^{-j\omega t} dt$$

$$= \int_0^{\infty} e^{-(\alpha-\delta)t} e^{-j\omega t} dt$$

$-\alpha+\delta > 0$ لكي نحصل على Convergence

$$\Rightarrow \delta > \alpha$$

Real part $\Rightarrow \delta$

$X(s) = e^{-bt} u(t)$ Find L.T?

$$X(s) = \int_{-\infty}^{\infty} e^{-bt} e^{-st} u(t) dt = \int_0^{\infty} e^{-bt} e^{-st} dt$$

$$= \int_0^{\infty} e^{-(b+s)t} dt \Rightarrow \left. \frac{e^{-(b+s)t}}{-(b+s)} \right|_0^{\infty}$$

$$\Rightarrow \frac{e^{-\infty}}{-(b+s)} - \frac{e^0}{-(b+s)} \Rightarrow \boxed{\frac{1}{s+b}}$$

$s = -b$

$\text{Roc} \Rightarrow \text{Re}\{s\} > -b$

$$x(t) = -e^{-bt} u(-t) \quad , \text{ find } x(s)$$

$$x(s) = - \int_{-\infty}^0 e^{-bt} e^{-st} dt = - \int_{-\infty}^0 e^{-(b+s)t} dt$$

$$= - \left. \frac{e^{-(b+s)t}}{-(b+s)} \right|_{-\infty}^0 = \frac{-e^0}{-(b+s)} + \frac{e^{-(b+s)(-\infty)}}{(b+s)}$$

$$= \frac{1}{b+s} + \frac{e^{(b+s)\infty}}{(b+s)}$$

هذا الجزء ياتي
من حالة $b+s < 0$

$$= \frac{1}{s+b} \quad , \quad \text{Roc } \text{Re}\{s\} < -b$$

$\sigma < -b$

ملاحظة ←

نلاحظها كهاضفة
L.T

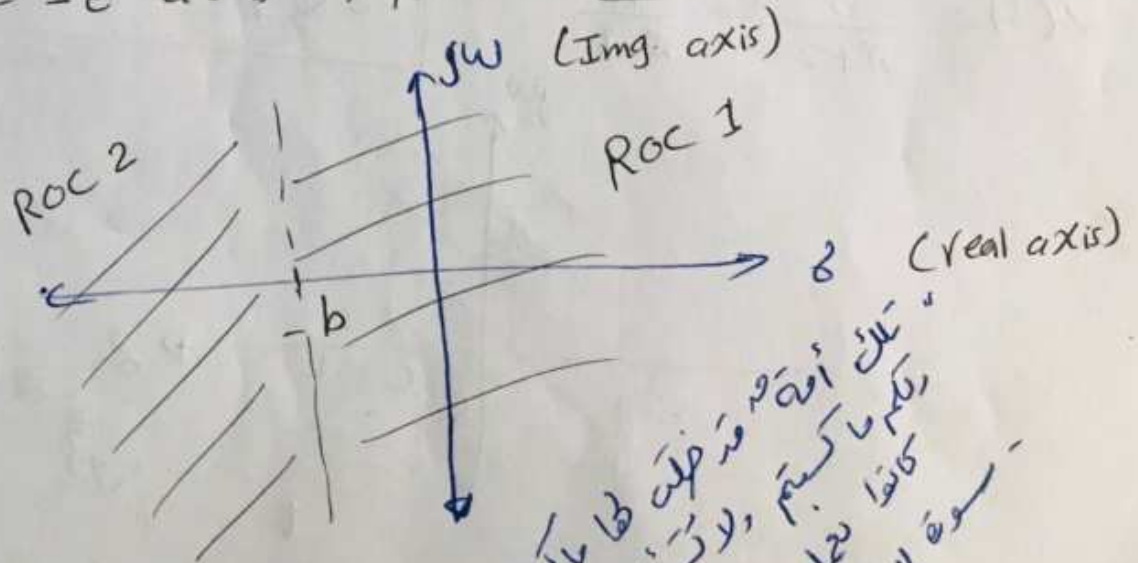
$$\begin{cases} x(t) = e^{-bt} u(t) \\ x(t) = -e^{-bt} u(-t) \end{cases}$$

ولكنه الاخرى في Roc

$$x(t) = e^{-bt} u(t) \quad , \quad \text{Roc 1 } \boxed{\sigma > -b}$$

$$x(t) = -e^{-bt} u(-t) \quad , \quad \text{Roc 2 } \boxed{\sigma < -b}$$

* S-plan:



تلك امة قد خلت لها ما كبت
رأيت ما كبت ولا تسألوه عما
كانوا يفعلوه
سورة البقرة

$$x(t) = 3e^{-2t}u(t) + 4e^t u(-t) \quad \text{Find } x(s)$$

$$x_1(s) = \int_0^{\infty} 3e^{-2t} e^{-st} dt \Rightarrow 3 \int_0^{\infty} e^{-(2+s)t} dt$$

$$\Rightarrow \frac{3e^{-(2+s)t}}{-(2+s)} \Big|_0^{\infty} \Rightarrow \frac{3e^{-\infty}}{-(2+s)} + \frac{3e^0}{(2+s)}$$

$$x_1(s) = \frac{3}{s+2} \quad \boxed{\text{Roc: } \text{Re}\{s\} > -2}$$

$$x_2(s) = \int_{-\infty}^0 4e^t e^{-st} dt \Rightarrow 4 \int_{-\infty}^0 e^{-(s-1)t} dt$$

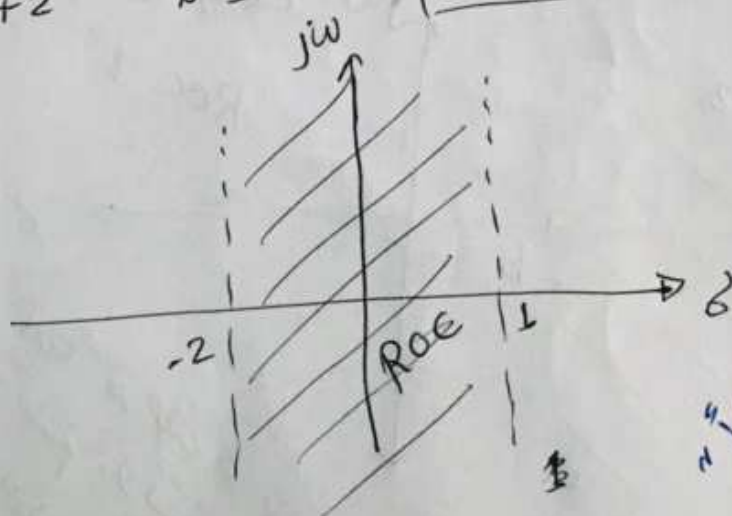
$$\frac{4e^{-(s-1)t}}{-(s-1)} = \frac{4}{-(s-1)} \left[e^0 - \underline{e^{-(s-1)\infty}} \right]$$

$$\frac{4}{1-s} [1 - 0]$$

سأني صفر اذا كان $s-1 < 0$

$$x_2(s) = -\frac{4}{s-1}, \quad \boxed{\text{Roc: } \text{Re}\{s\} < 1}$$

$$x(s) = \frac{3}{s+2} - \frac{4}{s-1} \quad \boxed{-2 < \text{Re}\{s\} < 1} \quad \text{Roc}$$



"منطقة التقاطع"
"منطقة التقاطع"

Find the Laplace transform for the following signals:-

[1] $x(t) = \delta(t)$

$$X(s) = \int_0^{\infty} \delta(t) e^{-st} dt = e^0 = [1] \quad \text{Roc: } \text{Re}\{s\} > 0$$

[2] $x(t) = u(t)$

$$X(s) = \int_{-\infty}^{\infty} u(t) e^{-st} dt = \int_0^{\infty} e^{-st} dt = \left. \frac{e^{-st}}{-s} \right|_0^{\infty}$$

$$X(s) = 0 - \frac{1}{-s} = \left[\frac{1}{s} \right] \quad \text{Roc: } \text{Re}\{s\} > 0$$

[3] $x(t) = e^{2t} u(t)$, find $x(s)$

$$X(s) = \int_0^{\infty} e^{2t} \cdot e^{-st} dt = \left. \frac{e^{-(s-2)t}}{-(s-2)} \right|_0^{\infty}$$

$$X(s) = \frac{e^{-(s-2)(\infty)}}{-(s-2)} + \frac{1}{(s-2)}$$

$$\boxed{X(s) = \frac{1}{s-2}}, \quad \text{Roc: } \text{Re}\{s\} > 2$$

[4] $x(t) = e^{jat} u(t)$

$$X(s) = \int_{-\infty}^{\infty} e^{jat} u(t) \cdot e^{-st} dt \Rightarrow X(s) = \int_0^{\infty} e^{-(s-j a)t} dt$$

$$\left. \frac{e^{-(s-j a)t}}{-(s-j a)} \right|_0^{\infty} = \frac{e^{-(s-j a)(\infty)}}{-(s-j a)} + \frac{e^0}{-(s-j a)} = \frac{1}{s-j a} \times \frac{s+j a}{s+j a}$$

$$\frac{s+j a}{s^2+a^2} \quad \text{Roc: } \text{Re}\{s\} > 0$$

[7]

[5] $x(t) = \cos \omega_0 t u(t)$

$$e^{j\omega_0 t} = \cos(\omega_0 t) + j\sin(\omega_0 t)$$

$$\cos(\omega_0 t) = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$$

$$\int_0^{\infty} e^{j\omega_0 t} e^{-st} dt = \int_0^{\infty} e^{-(s-j\omega_0)t} dt$$

$$= \left. \frac{e^{-(s-j\omega_0)t}}{-(s-j\omega_0)} \right|_0^{\infty} = \frac{e^{-(s-j\omega_0)(\infty)}}{-(s-j\omega_0)} - \frac{e^0}{-(s-j\omega_0)}$$

$$= \frac{1}{s-j\omega_0} \cdot \frac{(s+j\omega_0)}{(s+j\omega_0)} = \boxed{\frac{s+j\omega_0}{s^2+\omega_0^2}} \quad \text{Roc: } \text{Re}\{s\} > 0$$

$$\int_0^{\infty} e^{-j\omega_0 t} e^{-st} dt = \left. \frac{e^{-(j\omega_0+s)t}}{-(j\omega_0+s)} \right|_0^{\infty} = 0 - \frac{1}{-(j\omega_0+s)}$$

$$= \frac{1}{s+j\omega_0} \cdot \frac{(s-j\omega_0)}{(s-j\omega_0)} \Rightarrow \boxed{\frac{s-j\omega_0}{s^2+\omega_0^2}}$$

$$\cos(\omega_0 t) u(t) \xrightarrow{\mathcal{L}} \frac{1}{2} \left[\frac{s+j\omega_0}{s^2+\omega_0^2} + \frac{s-j\omega_0}{s^2+\omega_0^2} \right]$$

$$\frac{1}{2} \left[\frac{s}{s^2+\omega_0^2} + \frac{j\omega_0}{s^2+\omega_0^2} + \frac{s}{s^2+\omega_0^2} - \frac{j\omega_0}{s^2+\omega_0^2} \right] = \boxed{\frac{s}{s^2+\omega_0^2}}$$

Roc: $\text{Re}\{s\} > 0$ #

[6] $x(t) = \sin \omega_0 t u(t)$

$$\sin(\omega_0 t) = \frac{1}{2j} (e^{j\omega_0 t} - e^{-j\omega_0 t})$$

$$= \frac{1}{2j} \left(\frac{s}{s^2+\omega_0^2} + \frac{j\omega_0}{s^2+\omega_0^2} - \frac{s}{s^2+\omega_0^2} + \frac{j\omega_0}{s^2+\omega_0^2} \right)$$

$$= \frac{1}{2j} \left(\frac{2j\omega_0}{s^2+\omega_0^2} \right) = \boxed{\frac{\omega_0}{s^2+\omega_0^2}}$$

Roc: $\text{Re}\{s\} > 0$

[8]

Properties of Laplace transform :-

[1] Linearity: $x_1(t) \longleftrightarrow x_1(s)$
 $x_2(t) \longleftrightarrow x_2(s)$

$$a x_1(t) + b x_2(t) \longleftrightarrow a x_1(s) + b x_2(s)$$

[2] Time shifting:

$$x(t) \longleftrightarrow x(s)$$

(Shift in time) $x(t-t_0) \longleftrightarrow e^{-j s t_0} x(s)$

منه الى

(Shift in frequency) $e^{s_0 t} x(t) \longleftrightarrow x(s-s_0)$

عكس الى

[3] Time scaling :-

$$x(t) \longleftrightarrow x(s)$$

$$x(at) \longleftrightarrow \frac{1}{a} x\left(\frac{s}{a}\right)$$

[4] Differentiator and integrator in time domain:-

$$x(t) \longleftrightarrow x(s)$$

$$\frac{d^n x}{dt^n} \longleftrightarrow s^n x(s) - s^{n-1} x(0) - s^{n-2} x'(0) - \dots - x^{(n-1)}(0)$$

$$\int_0^t x(t) dt \longleftrightarrow \frac{1}{s} x(s)$$

منه الى
 $\frac{1}{s}$
 $x(0)$

Example #1 $y''(t) + 3y'(t) + 2y(t) = 0$ $y(0) = 3, y'(0) = 1$

Find $y(t)$. (convert to s-domain)

$$[s^2 y(s) - s y(0) - y'(0)] + 3[s y(s) - y(0)] + 2 y(s) = 0$$

$$s^2 y(s) - 3s - 1 + 3s y(s) - 3(3) + 2 y(s) = 0$$

$$y(s) (s^2 + 3s + 2) = 3s + 1 + 9$$

$$y(s) = \frac{3s + 10}{s^2 + 3s + 2} \quad \#$$

$$Y(s) = \frac{3s+10}{s^2+3s+2}$$

$$y(t) = \mathcal{L}^{-1} \{ Y(s) \}$$

$$Y(s) = \frac{3s+10}{s^2+3s+2} = \frac{A}{(s+1)} + \frac{B}{(s+2)}$$

$$A = \frac{(3s+10)(s+2)}{(s+1)(s+2)} \Big|_{s=-1} \Rightarrow \frac{-3+10}{-1+2} = \boxed{7}$$

$$B = \frac{(3s+10)(s+1)}{(s+1)(s+2)} \Big|_{s=-2} \Rightarrow \frac{(3)(-2)+10}{-2+1} = \boxed{-4}$$

$$Y(s) = \frac{7}{s+1} - \frac{4}{s+2}$$

$$\boxed{y(t) = (7e^{-t} - 4e^{-2t})u(t)} \quad \#$$

Differentiation in s-domain:-

$$x(t) \longleftrightarrow X(s)$$

$$t^n x(t) \longleftrightarrow \frac{d^n X(s)}{ds^n}$$

$$t^n x(t) \longleftrightarrow (-1)^n \frac{d^n X(s)}{ds^n}$$

Find $R(s)$ if $r(t) = tu(t)$

$$u(t) \longleftrightarrow \frac{1}{s}$$

$$tu(t) \longleftrightarrow (-1)^1 \frac{d}{ds} \left(\frac{1}{s} \right)$$

$$tu(t) \longleftrightarrow (-1) \left(-\frac{1}{s^2} \right)$$

$$tu(t) \longleftrightarrow \frac{1}{s^2}$$

$$\boxed{R(s) \xrightarrow{\mathcal{L}} \frac{1}{s^2}}$$

10

Ex # Find $x(s)$ for $x(t) = t^n u(t)$

$$t^n u(t) \longleftrightarrow \frac{n!}{s^{n+1}}$$

علاقة مع قانون
فولت مع الدالة $u(t)$

If $x(t) = t^3 u(t)$ find $x(s)$

$$t^3 u(t) \longleftrightarrow \frac{3!}{s^4} \Rightarrow \frac{3 \times 2 \times 1}{s^4} = \boxed{\frac{6}{s^4}} \neq$$

$x(t) = t^2 e^{-at}$ find $x(s)$

$$e^{-at} \longleftrightarrow \frac{1}{s+a}$$

$$t^2 e^{-at} \longleftrightarrow (-1)^2 \frac{d^2}{ds^2} \left[\frac{1}{s+a} \right]$$

$$\boxed{x(s) = \frac{2}{(s+a)^3}}$$

"لأنها النابه انقوا ربكم الذي خلقكم من نعتي واحدة
رحله حقا زوجها ديت حقا رحله كثير دناد
وانقوا الله الذي شاء لونه، الانعام ان الله
كان عليكم رقيباً
سورة البقرة

توضيح الا شقاه :

$$\frac{d}{ds} \left(\frac{1}{s+a} \right)$$

$$= \frac{-1}{(s+a)^2}$$

$$\frac{d}{ds} \left(\frac{-1}{(s+a)^2} \right)$$

$$= \frac{(+1)(2)(s+a)}{(s+a)^4}$$

$$= \boxed{\frac{2}{(s+a)^3}}$$

* Convolution

$$x(t) \longleftrightarrow x(s)$$

$$h(t) \longleftrightarrow H(s)$$

$$x(t) * h(t) \longleftrightarrow x(s) \cdot H(s)$$

Ex # | $x(t) = e^{-2t} u(t)$, $y(t) = \frac{2}{3} (e^{-t} + e^{-2t} + (-e^{-3t}))$

Find $h(t)$.

Sol: —

$$X(s) = \frac{1}{s+2}$$

$$Y(s) = \frac{2}{3} \left[\frac{1}{s+1} + \frac{1}{s+2} - \frac{1}{s+3} \right]$$

$$Y(s) = \frac{2}{3} \left[\frac{(s+2)(s+3) + (s+1)(s+3) - (s+2)(s+1)}{(s+1)(s+2)(s+3)} \right]$$

$$Y(s) = \frac{2}{3} \left[\frac{s^2 + 6s + 7}{(s+1)(s+2)(s+3)} \right]$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{\frac{2}{3} \left[\frac{s^2 + 6s + 7}{(s+1)(s+2)(s+3)} \right]}{\left[\frac{1}{s+2} \right]}$$

$$H(s) = \left(\frac{2}{3} \right) \left(\frac{s^2 + 6s + 7}{(s+1)(s+2)(s+3)} \right) \frac{(s+2)}{1}$$

$$H(s) = \frac{2}{3} \left(\frac{s^2 + 6s + 7}{(s+1)(s+3)} \right)$$

Now, we find $h(t)$ by using partial fraction:

$$H(s) = \frac{2}{3} \left(\frac{s^2 + 6s + 7}{(s+1)(s+3)} \right)$$

دالة البت ساي دالة لتمام
لنا نجا للقيمة لعلولة

frequency differentiation: —

$$tx(t) \longleftrightarrow -\frac{d}{ds} X(s)$$

$$x(t) = u(t-5) + e^{5t} u(-t+3) \quad \text{Find } x(s)$$

Without properties:-

$$u(t-5) \Rightarrow \int_5^{\infty} e^{-st} dt \Rightarrow \left. \frac{e^{-st}}{-s} \right|_5^{\infty} = \frac{e^{-\infty}}{-s} - \frac{e^{-5s}}{-s} = \frac{0}{-s} + \frac{e^{-5s}}{s}$$

$$x_1(s) = \frac{e^{-5s}}{s} \quad \text{Roc: } \text{Re}\{s\} > 0$$

$$e^{5t} u(-t+3) \quad \begin{matrix} -t+3 \geq 0 \\ 3 \geq t \end{matrix}$$

$$\int_{-\infty}^3 e^{5t} \cdot e^{-st} dt = \int_{-\infty}^3 e^{-(s-5)t} dt = \left. \frac{e^{-(s-5)t}}{-(s-5)} \right|_{-\infty}^3 = \frac{e^{-(s-5)3}}{-(s-5)} - \frac{e^{-(s-5)(-\infty)}}{-(s-5)}$$

$$\frac{e^{-3(s-5)}}{-(s-5)} \Rightarrow \frac{-e^{-3(s-5)}}{(s-5)}$$

$$\text{Roc: } \text{Re}\{s\} < 5$$

$$x(s) = \frac{e^{-5s}}{s} - \frac{e^{-3(s-5)}}{(s-5)} \quad \begin{matrix} -s+5 \geq 0 \\ s \geq 5 \end{matrix}$$

$$0 < \text{Re}\{s\} < 5$$

By using properties:-

$$u(t-5) \xleftrightarrow{L} \frac{1}{s} e^{-5s}$$

$$e^{5t} u(-t+3) \longleftrightarrow ??$$

$$u(t) \leftrightarrow \frac{1}{s}$$

$$u(-t) \leftrightarrow -\frac{1}{s}$$

$$u(-t+3) \leftrightarrow -\frac{1}{s} e^{-3s} \quad (\text{exp. time shift})$$

$$e^{5t} u(-t+3) \leftrightarrow \left[-\frac{1}{s-5} e^{-3(s-5)} \right]$$

$$x(s) = \frac{e^{-5s}}{s} - \frac{1}{s-5} e^{-3(s-5)} \quad \begin{matrix} \text{Region of Conv.} \\ 0 < \text{Re}\{s\} < 5 \end{matrix}$$

(13)

$$f_1(t) = 5e^{-0.3(t-2)} u(t-2)$$

$$5e^{-0.3t} u(t) \longleftrightarrow \frac{5}{s+0.3}$$

$$5e^{-0.3(t-2)} u(t-2) \longleftrightarrow \boxed{\frac{5e^{-2s}}{s+0.3}}$$

u(t) في shift ال
في shift في exp
cos(x)

$$f_2(t) = 5e^{-0.3t} u(t-2)$$

$$f_2(t) = 5e^{-0.3(t-2+2)} u(t-2)$$

$$= 5e^{-0.3(t-2)} u(t-2) \cdot e^{-0.6}$$

$$= \boxed{2.744 \frac{e^{-2s}}{s+0.3}}$$

"أنا ولا تحملينا سلاطنة لنا به واعتف عنا
واعف لنا وارحمنا أنت مولانا خالصنا
ع، لعمرك الكاذبة
مسودة البقرة"

$$x(t) = u(t-1) \cdot e^{-2t} u(t-1)$$

$$u(t) \longleftrightarrow \frac{1}{s}$$

$$u(t-1) \longleftrightarrow \boxed{\frac{e^{-s}}{s}}$$

$$e^{-2t} u(t-1) \Rightarrow e^{-2(t-1+1)} u(t-1)$$

$$\Rightarrow e^{-2(t-1)} u(t-1) \cdot e^{-2}$$

$$e^{-2} \cdot e^{-2(t-1)} u(t-1) \longleftrightarrow e^{-2} \cdot \frac{e^{-s}}{s+2} \Rightarrow \boxed{\frac{e^{-(s+2)}}{s+2}}$$

$$x(s) = \frac{e^{-s}}{s} \cdot \frac{e^{-(s+2)}}{s+2} = \boxed{\frac{e^{-2(s+1)}}{s(s+2)}}$$

هو المطلوب

$$f(t) = t \frac{d}{dt} (e^{-t} \cos t \cdot u(t))$$

$$\cos(t) u(t) \longleftrightarrow \frac{s}{s^2 + 1}$$

$$e^{-t} \cos(t) u(t) \longleftrightarrow \frac{s+1}{(s+1)^2 + 1}$$

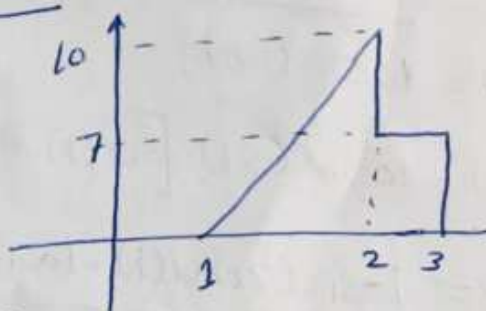
$$\frac{d}{dt} (e^{-t} \cos(t) \cdot u(t)) \longleftrightarrow s \left(\frac{s+1}{(s+1)^2 + 1} \right)$$

$$t \frac{d}{dt} (e^{-t} \cos(t) u(t)) \longleftrightarrow -\frac{d}{ds} \left(\frac{s^2 + s}{(s+1)^2 + 1} \right)$$

$$X(s) = - \left(\frac{s^2 + 4s + 2}{(s^2 + 2s + 2)^2} \right) \quad \#$$

Find $f(s)$:

$$f(t) = 10r(t-1) - 10r(t-2) - 3u(t-2) - 7u(t-3)$$



$$f(t) = 10(t-1)u(t-1) - 10(t-2)u(t-2) - 3u(t-2) - 7u(t-3)$$

$$f(s) = \frac{10}{s^2} e^{-s} - \frac{10}{s^2} e^{-2s} - \frac{3}{s} e^{-2s} - \frac{7}{s} e^{-3s} \quad \#$$

$$u(t) \longleftrightarrow \frac{1}{s}$$

ملاحظة

$$tu(t) \longleftrightarrow \frac{1}{s^2}$$

تحلوه حل آخر بتقديم الاشتقاق:

$$\frac{df(t)}{dt} = 10u(t-1) - 10u(t-2) - 3\delta(t-2) - 7\delta(t-3)$$

$$s \cdot F(s) = \frac{10}{s} e^{-s} - \frac{10}{s} e^{-2s} - 3e^{-2s} - 7e^{-3s}$$

$$F(s) = \frac{10e^{-s}}{s^2} - \frac{10e^{-2s}}{s^2} - \frac{3e^{-2s}}{s} - \frac{7e^{-3s}}{s} \quad \#$$

15

$$x(t) = \frac{d^2}{dt^2} [e^{-3(t-2)} u(t-2)]$$

$$e^{-3t} u(t) \longleftrightarrow \frac{1}{s+3}$$

$$e^{-3(t-2)} u(t-2) \longleftrightarrow \frac{e^{-2s}}{s+3}$$

$$\frac{d^2}{dt^2} (e^{-3(t-2)} u(t-2)) \longleftrightarrow s^2 \left(\frac{e^{-2s}}{s+3} \right) - 0$$

$$\boxed{X(s) = \frac{s^2 e^{-2s}}{s+3}} \quad \#$$

$$x(t) = 10 \sin(\omega t)$$

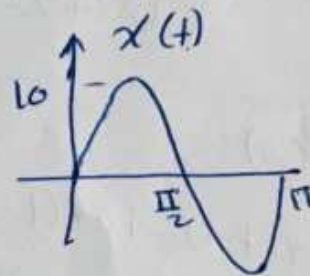
$$x(t) = 10 \sin(2t) [u(t) - u(t-\pi)]$$

$$x(t) = 10 \sin(2t) u(t) - 10 \sin(2t) u(t-\pi)$$

$$\boxed{X(s) = \frac{(10)(2)}{s^2+4} - \frac{(10)(2) e^{-\pi s}}{s^2+4}} \quad \#$$

shift في الـ t مع unit step
 shift في الـ t مع unit step

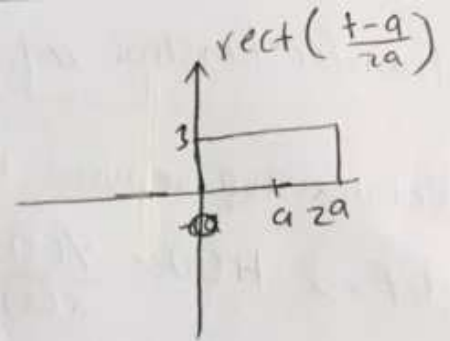
$$\boxed{X(s) = \frac{20}{s^2+4} - \frac{20 e^{-\pi s}}{s^2+4}} \quad \#$$



$$\omega = \frac{2\pi}{T} = \frac{2\pi}{\pi} = \boxed{2}$$

$$\begin{aligned} & 10 \sin(2t) u(t-\pi) \\ &= 10 \sin(2(t-\pi+\pi)) u(t-\pi) \\ &= 10 \sin(2(t-\pi) + 2\pi) u(t-\pi) \\ &= 10 \sin(2(t-\pi)) u(t-\pi) \end{aligned}$$

$$x(t) = \text{rect}\left(\frac{t-a}{2a}\right), \text{ Find } x(s)$$



$$x(t) = u(t) - u(t-2a)$$

$$u(t) \xrightarrow{L} \frac{1}{s}$$

$$u(t-2a) \xrightarrow{L} e^{-2as} \cdot \frac{1}{s}$$

$$x(s) = \frac{1}{s} - \frac{e^{-2as}}{s} \quad \#$$

$$y(0) = 3, y'(0) = 1$$

$$x(t) \Rightarrow y''(t) + 3y'(t) + 2y(t) = 0$$

Find $y(t)$?

$$[s^2 y(s) - sy(0) - y'(0)] + 3[sy(s) - y'(0)] + 2y(s) = 0$$

$$y(s) [s^2 + 3s + 2] = 3s + 1 + 9$$

$$y(s) = \frac{3s + 10}{s^2 + 3s + 2}$$

$$y(s) \Rightarrow \frac{3s + 10}{s^2 + 3s + 2} = \frac{A}{(s+1)} + \frac{B}{(s+2)}$$

$$A = \frac{(3s+10)(s+2)}{(s+1)(s+2)} \Big|_{s=-1} = \boxed{7}$$

$$B = \frac{(s+2)(3s+10)}{(s+1)(s+2)} \Big|_{s=-2} = \boxed{-4}$$

$$y(s) = \frac{7}{s+1} - \frac{4}{s+2}$$

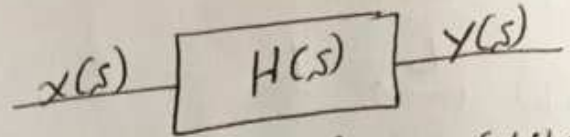
$$y(t) = (7e^{-t} - 4e^{-2t})u(t) \quad \#$$

قالت غلاة يا زها الفل اي الفل
اي كتاب كرم * دينة سليمان
وانه بسم الله الرحمن الرحيم
سورة المنزل

Transfer function #1

Relationship between the input and output of the system $H(s)$

$$T.F \Rightarrow H(s) = \frac{Y(s)}{X(s)}$$



$H(s)$: It is useful to determine the stability of the system

$\Rightarrow$ To find $H(s)$, put all initial conditions Zeros

Ex #1

$$2y''(t) + 3y'(t) + 5y(t) = x'(t) - 2x(t)$$

$$2s^2y(s) + 3sy(s) + 5y(s) = sx(s) - 2x(s)$$

$$(2s^2 + 3s + 5)y(s) = x(s)(s - 2)$$

$$\boxed{\frac{y(s)}{x(s)} = H(s) = \frac{(s - 2)}{(2s^2 + 3s + 5)}} \quad \#$$

Initial Value theorem

$$x(0) = \lim_{s \rightarrow \infty} s x(s)$$

" frequency domain "

$$x(0) = \lim_{t \rightarrow 0} x(t)$$

" time domain "

Ex #1 $x(s) = \frac{-3s^2 + 2}{s^3 + s^2 + 3s + 2}$ find $x(0)$

$$x(0) = \lim_{s \rightarrow \infty} s x(s) \Rightarrow \lim_{s \rightarrow \infty} \frac{s(-3s^2 + 2)}{s^3 + s^2 + 3s + 2}$$

$$x(0) = \lim_{s \rightarrow \infty} \frac{-3 + \frac{2}{s^2}}{1 + \frac{1}{s} + \frac{3}{s^2} + \frac{2}{s^3}} = \boxed{-3} \quad \#$$

نقسم على أكبر أس في المقام

هذه الطريقة لن نستخدم لإيجاد $x(0)$ في حالة لم تكن عند نقطة 1 في time domain

* Final value theorem: -1

$$x(\infty) = \lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} s x(s)$$

- * Final value theorem exist only if the system is stable
- * We use it to find the value of the output without solving the differential equation.

Ex #1 $x(s) = \frac{5}{s(s^2 + s + 2)}$ find $x(\infty)$

System Stability

poles: $s=0$, $s=-0.5 + j1.3228$, $s=-0.5 - j1.3228$

All poles in LHP $\Rightarrow$ system is stable

$$x(\infty) = \lim_{s \rightarrow 0} s x(s) \Rightarrow \lim_{s \rightarrow 0} \frac{5s}{s(s^2 + s + 2)} = \boxed{\frac{5}{2}}$$

Ex # | $x(s) = \frac{3}{s(s-2)}$, find $x(\infty)$

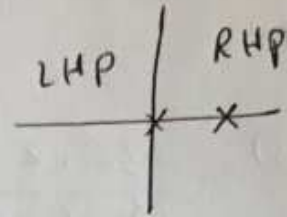
// stability اول اسی تعین کیا جائے گا کہ مستحکم ہوگا یا نہیں

2HP poles (قطب) $s=0, s=2$ stable

poles of $s=0, s=2$

$\Rightarrow$ system is unstable

So, there is no final value



Ex # | $x(s) = \frac{7s+10}{s(s+2)}$ Find $x(0), x(\infty)$

$$x(0) = \lim_{s \rightarrow \infty} \frac{s(7s+10)}{s(s+2)} \Rightarrow \lim_{s \rightarrow \infty} \frac{7 + \frac{10}{s}}{1 + \frac{2}{s}} = \boxed{7}$$

to find final value theorem, we test the stability

poles of $s=0, s=-2 \Rightarrow$ All poles in LHP
 $\Rightarrow$ system is stable.

$$x(\infty) = \lim_{s \rightarrow 0} \frac{s(7s+10)}{s(s+2)} \Rightarrow \frac{10}{2} = \boxed{5}$$

$x(s) = \frac{s-5}{s(s-2)}$, find final value

poles of $s=0, s=2 \Rightarrow$ not stable

there is no final value

$x(s) = \frac{s^3 + 3s^2 + s + 1}{s^2 + 3s + 1}$ find $x(0)$

$$\lim_{s \rightarrow \infty} s \left(\frac{s^3 + 3s^2 + s + 1}{s^2 + 3s + 1} \right) = \infty$$

منقولہ صورت میں

$$x(0) = \lim_{s \rightarrow \infty} s \left((s+1) + \frac{-2s}{s^2 + 3s + 1} \right)$$

lim

$$\begin{array}{r} \frac{s+1}{s^2+3s+1} \frac{s^3+3s^2+s+1}{s^3+2s^2+s} \\ - \frac{s^2+1}{s^2+3s+1} \\ \hline -2s \\ \hline \frac{s+1 + -2s}{s^2+3s+1} \end{array}$$

20

Example #1

$$X(s) = \frac{2s^3 + 2s - 10}{s^3 + 3s^2 - s - 3}$$

Find $x(t)$

Sol: هنا رتبة البسط تساوي رتبة المقام $\Leftarrow$ صفة مطولة مثل اكل

$$X(s) = 2 + \frac{(-6s^2 + 4s - 4)}{s^3 + 3s^2 - s - 3}$$

$$\begin{array}{r} 2 \\ s^3 + 3s^2 - s - 3 \overline{) 2s^3 + 2s - 10} \\ \underline{2s^3 + 6s^2 - 2s - 6} \\ -6s^2 + 4s - 4 \end{array}$$

PF

$$\frac{-6s^2 + 4s - 4}{s^3 + 3s^2 - s - 3} = \frac{K_1}{(s+1)} + \frac{K_2}{(s-1)} + \frac{K_3}{(s+3)}$$

$$K_1 = \frac{(-6s^2 + 4s - 4)(s+1)}{(s+1)(s-1)(s+3)} \Big|_{s=-1} = \frac{-14}{-4} = \boxed{3.5}$$

$$K_2 = \frac{(-6s^2 + 4s - 4)(s-1)}{(s+1)(s-1)(s+3)} \Big|_{s=1} = \boxed{-0.75}$$

$$K_3 = \frac{(-6s^2 + 4s - 4)(s+3)}{(s+1)(s-1)(s+3)} \Big|_{s=-3} = \boxed{-8.75}$$

$$X(s) = 2 + \frac{3.5}{(s+1)} - \frac{0.75}{(s-1)} - \frac{8.75}{s+3}$$

From table

$$x(t) = 2\delta(t) + [3.5e^{-t} - 0.75e^t - 8.75e^{-3t}]u(t)$$

هنا البؤال Real and distinct poles عبارة عن جذور حقيقية مختلفة

$$s^3 + 3s^2 - s - 3 = (s-1)(s+1)(s+3)$$

للتوصيف //

عن طريق التحليل أو مباشرة باستخدام الآلة الكومبيوتر

$$X(s) = \frac{10(s+1)}{s^2+4s+3} \quad \text{find } x(t) \text{ and find final value and initial value}$$

$$X(s) = \frac{10(s+1)}{(s+1)(s+3)} = \frac{10}{s+3}$$

$$x(t) = 10e^{-3t} u(t)$$

to find the final value:

$$\text{poles} \Rightarrow s = -1, s = -3$$

(LHP) $\Rightarrow$ system stable

$$X(0) = \lim_{s \rightarrow \infty} sX(s) \quad \text{"initial value"}$$

$$= \lim_{s \rightarrow \infty} s \left[\frac{10(s+1)}{(s^2+4s+3)} \right] = \lim_{s \rightarrow \infty} \frac{10s^2+10s}{s^2+4s+3} \quad \text{بالقيمة في أكبر من المقام}$$

$$= \lim_{s \rightarrow \infty} \frac{10 + \frac{10}{s}}{1 + \frac{4}{s} + \frac{3}{s^2}} = \boxed{10}$$

$$X(\infty) = \lim_{s \rightarrow 0} sX(s) = \lim_{s \rightarrow 0} \frac{10s^2+10s}{s^2+4s+3} = \boxed{0} \quad \text{"final value"}$$

$$\boxed{3} \quad X(s) = \frac{2s^2+3s-1}{(s+1)(s+2)^3} \quad \text{Real and Repeated}$$

$$X(s) = \frac{2s^2+3s-1}{(s+1)(s+2)^3} = \frac{k_1}{s+1} + \frac{k_2}{(s+2)^3} + \frac{k_3}{(s+2)^2} + \frac{k_4}{(s+2)}$$

$$k_1 = \frac{(2s^2+3s-1)(s+2)^3}{(s+1)(s+2)^3} \Big|_{s=-1} = \boxed{-2}$$

$$k_2 = \frac{(2s^2+3s-1)(s+2)^3}{(s+1)(s+2)^3} \Big|_{s=-2} = \boxed{-1}$$

231

$$K_3 = \frac{1}{1!} \frac{d}{ds} \left(\frac{2s^2 + 3s - 1}{s+1} \right) \Big|_{s=-2}$$

$$= \frac{(s+1)(4s+3) - (2s^2+3s-1)(1)}{(s+1)^2}$$

$$= \frac{4s^2 + 7s + 3 - 2s^2 - 3s + 1}{(s+1)^2} = \frac{2s^2 + 4s + 4}{(s+1)^2} \Big|_{s=-2} = \boxed{4}$$

$$K_4 = \frac{1}{2!} \frac{d^2}{ds^2} \left(\frac{2s^2 + 3s - 1}{s+1} \right) \quad \text{or} \quad \frac{1}{2!} \frac{d}{ds} \left(\frac{2s^2 + 4s + 4}{(s+1)^2} \right)$$

$$= \frac{1}{2} \left(\frac{-4}{(s+1)^3} \right) \Big|_{s=-2} = \boxed{2}$$

$$X(s) = \frac{-2}{s+1} - \frac{1}{(s+2)^3} + \frac{4}{(s+2)^2} + \frac{2}{(s+2)}$$

$$X(t) = \left(-2e^{-t} - \frac{1}{2!} t^2 e^{-2t} + 4t e^{-2t} + 2e^{-2t} \right) u(t)$$

$$\boxed{X(t) = \left(-2e^{-t} - \frac{1}{2} t^2 e^{-2t} + 4t e^{-2t} + 2e^{-2t} \right) u(t)} \quad \#$$

$$X(s) = \frac{100(s+3)}{(s+6)(s^2+6s+25)} \quad \text{find } x(t)$$

poles // $s = -6, s = -3 + j4, s = -3 - j4$

بالقانون العام
أو على الآلة الحاسبة

poles $\Rightarrow$ complex.

$$X(s) = \frac{100(s+3)}{(s+6)(s+3+j4)(s+3-j4)} = \frac{K_1}{(s+6)} + \frac{K_2}{(s+3+j4)} + \frac{K_2^*}{(s+3-j4)}$$

$$K_1 = \frac{100(s+3)(s+6)}{(s+6)(s^2+6s+25)} \Big|_{s=-6} = \boxed{-12}$$

$\boxed{24}$

$$K_2 = \frac{100(s+3)(s+3-j4)}{(s+6)(s+3+j4)(s+3-j4)} \bigg|_{s=-3+j4} = 6-j8 = \boxed{10 \angle -53.13^\circ}$$

$$K_2^* = 6+j8 = 10 \angle 53.13^\circ$$

$$X(s) = \frac{-12}{s+6} + \frac{10 \angle -53.13^\circ}{s+3-j4} + \frac{10 \angle 53.13^\circ}{s+3+j4}$$

الجزء حقيقي من الجذور معًا
Complex, Conjugate

$$\mathcal{L}^{-1} \left\{ \frac{k}{(s+\alpha-j\beta)^r} + \frac{k^*}{(s+\alpha+j\beta)^r} \right\} = \left[\frac{2|k|t^{r-1}}{(r-1)!} e^{-\alpha t} \cos(\beta t + \theta) \right] u(t)$$

$$X(t) = \left[-12e^{-6t} + 20t^{1-1} e^{-3t} \cos(4t - 53.13^\circ) \right] u(t)$$

$$x(t) = \left[-12e^{-6t} + 20e^{-3t} \cos(4t - 53.13^\circ) \right] u(t) \quad \#$$

$$X(s) = \frac{768}{(s^2+6s+24)^2} \quad \text{Find } x(t)$$

$$s^2+6s+24 \Rightarrow$$

$$\frac{-b \pm \sqrt{b^2-4pc}}{2p} = \frac{-6 \pm \sqrt{36-4(1)(24)}}{2}$$

$$= \frac{-6 \pm \sqrt{-60}}{2} = \frac{-3 \pm j3.87}{1} \approx \boxed{-3 \pm j4}$$

$$X(s) = \frac{K_1}{(s+3-j4)^2} + \frac{K_2}{(s+3-j4)} + \frac{K_1^*}{(s+3+j4)^2} + \frac{K_2^*}{(s+3+j4)}$$

"Repeated complex poles"

"أصل في السطوح"
في الزاوية +، أصلي
من الزاوية جهة الخلف
سعة السطوح

25

$$k_1 = \frac{768}{(s+3+j4)^2(s+3-j4)^2} \Big|_{s=-3+j4} = \underline{-12 \angle 0^\circ}$$

$$k_2 = \frac{d}{ds} \left(\frac{768}{(s+3+j4)^2} \right) \Big|_{s=-3+j4}$$

$$k_2 = \frac{-1536}{(s+3+j4)^3} \Big|_{s=-3+j4} = -j3 = \underline{3 \angle -90^\circ}$$

$$k_1^* = -12 \angle 0^\circ, \quad k_2^* = 3 \angle 90^\circ$$

$$X(s) = \left[\frac{-12}{(s+3-j4)^2} - \frac{12}{(s+3+j4)^2} \right] + \left[\frac{3 \angle -90^\circ}{s+3-j4} + \frac{3 \angle 90^\circ}{s+3+j4} \right]$$

$$\mathcal{L}^{-1} \left\{ \frac{k}{(s+\alpha-j\beta)^r} + \frac{k^*}{(s+\alpha+j\beta)^r} \right\} = \left[\frac{2|k| t^{r-1}}{(r-1)!} e^{-\alpha t} \cos(\beta t + \theta) \right] u(t) \quad \text{تقسيم القاسم}$$

$$x(t) = \left[-24t e^{-3t} \cos(4t) + 6 e^{-3t} \cos(4t - 90^\circ) \right] u(t) \quad \#$$

Find $f(t)$ of $f(s) = \frac{e^{-2.5s}(s+3)}{s^3 + 7s^2 + 10s}$

$$\left(\frac{s+3}{s^3 + 7s^2 + 10s} \right) = \frac{k_1}{s} + \frac{k_2}{s+2} + \frac{k_3}{s+5}$$

$$k_1 = \frac{(s+3)s}{s(s^2 + 7s + 10)} \Big|_{s=0} = \underline{0.3}$$

$$k_2 = \frac{(s+3)(s+2)}{s(s+2)(s+5)} \Big|_{s=-2} = \underline{-\frac{1}{6}}$$

261

$$\frac{2}{3} = \frac{(s+3)(s+s)}{s(s+2)(s+s)} \quad \Big|_{s=-5} = \boxed{\frac{2}{15}}$$

$$F(s) = \frac{e^{-2.5s}}{s} (0.3) + \frac{e^{-2.5s}}{s+2} \left(-\frac{1}{6}\right) + \frac{e^{-2.5s}}{s+s} \left(-\frac{2}{15}\right)$$

$$F(t) = 0.3 u(t-2.5) - \frac{1}{6} e^{-2(t-2.5)} u(t-2.5) - \frac{2}{15} e^{-5(t-2.5)} u(t-2.5)$$

$$X(s) = \frac{s+3+5e^{-2s}}{(s+1)(s+2)} + (s+1)$$

$$X(s) = \frac{(s+3)}{(s+1)(s+2)} + \frac{5e^{-2s}}{(s+1)(s+2)} + (s+1)$$

$$\frac{s+3}{(s+1)(s+2)} = \frac{k_1}{(s+1)} + \frac{k_2}{(s+2)}$$

$$k_1 = \frac{(s+1)(s+3)}{(s+1)(s+2)} \Big|_{s=-1} = \boxed{2}$$

$$k_2 = \frac{(s+3)(s+2)}{(s+1)(s+2)} \Big|_{s=-2} = \boxed{-1}$$

الجزء الأول

$$X_1(t) = (2e^{-t} - e^{-2t})u(t)$$

$$\frac{5e^{-2s}}{(s+1)(s+2)} \Rightarrow \text{multiplication} \Rightarrow \frac{s}{(s+1)(s+2)} = \frac{k_3}{(s+1)} + \frac{k_4}{(s+2)}$$

$$k_3 = \frac{(s)(s+1)}{(s+1)(s+2)} \Big|_{s=-1} = \boxed{5}$$

$$k_4 = \frac{(s)(s+2)}{(s+1)(s+2)} \Big|_{s=-2} = \boxed{-5}$$

$$X_2(t) = (5e^{-t} - 5e^{-2t})u(t)$$

$$X_2(t) = (5e^{-(t-2)} - 5e^{-2(t-2)})u(t-2)$$

exp 27

27

$$x(t) = (2e^{-t} - e^{-2t})u(t) + 5 \left[e^{-(t-2)} - e^{-2(t-2)} \right] u(t-2) + \delta'(t)$$

$$\frac{s+1}{s} \xleftrightarrow{\text{ILP}} \delta'(t) + \delta(t)$$

ملحوظة //

Ex # | $x(s) = \frac{6(s+34)}{s(s^2+10s+34)}$

تستخدم لقانونه العام
 $s = -s \pm j3$

$$x(s) = \frac{6(s+34)}{s(s+s+j3)(s+s-j3)}$$

$$\frac{6(s+34)}{s(s+s+j3)(s+s-j3)} = \frac{A}{s} + \frac{B}{s+s-j3} + \frac{B^*}{s+s+j3}$$

$$A = \frac{6(s+34)}{s(s^2+10s+34)} \Big|_{s=0} = \boxed{6}$$

$$B = \frac{6(s+s-j3)(s+34)}{(s+s-j3)(s+s+j3)} \Big|_{s=-s+j3} = 3-j29 = 29.154 \angle -84.09^\circ$$

$$B^* = 3+j29 = 29.154 \angle 84.09^\circ$$

$$x(s) = \frac{6}{s} + \frac{29.154 \angle -84.09^\circ}{s+s-j3} + \frac{29.154 \angle 84.09^\circ}{s+s+j3}$$

$$x(t) = 6u(t) + \left[58.308 e^{-st} \cos(3t - 84.09^\circ) \right] u(t)$$

"أنا عرضنا الرقعة على السموات والأرض، الجبال
 فبأسه أن تحملنا، حملها الآن
 إنه كان ظلوماً جهولاً"
 - سورة الأخرس -

Solution of Differential equations //

Ex# $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y(t) - \frac{dx}{dt} + x(t) = 0$

$y(0) = 2, y'(0) = 1$ and input $x(t) = e^{-4t}u(t)$

$$[s^2y(s) - sy(0) - y'(0)] + 5[sy(s) - y(0)] + 6y(s) - [sx(s) - x(0)] + x(s) = 0$$

$$\boxed{x(s) = \frac{1}{s+4}}, \quad x(0^-) = 0$$

$$y(s)[s^2 + 5s + 6] = 2s + 1 + 10 + \cancel{x(0)} + \frac{s}{s+4} - \frac{1}{s+4}$$

$$y(s)[s^2 + 5s + 6] = 2s + 11 + \frac{s-1}{s+4}$$

$$y(s) = \frac{2s^2 + 20s + 43}{(s^2 + 5s + 6)(s+4)} = \frac{k_1}{(s+4)} + \frac{k_2}{(s+2)} + \frac{k_3}{(s+3)}$$

$$k_1 = \frac{(s+4)(2s^2 + 20s + 43)}{(s+4)(s^2 + 5s + 6)} \Big|_{s=-4} = \boxed{-2.5}$$

$$k_2 = \frac{(s+2)(2s^2 + 20s + 43)}{(s+4)(s^2 + 5s + 6)} \Big|_{s=-2} = \boxed{5.5}$$

$$k_3 = \frac{(s+3)(2s^2 + 20s + 43)}{(s+4)(s^2 + 5s + 6)} \Big|_{s=-3} = \boxed{-1}$$

$$y(s) = \frac{-2.5}{s+4} + \frac{5.5}{s+2} - \frac{1}{s+3}$$

$$\boxed{y(t) = (-2.5e^{-4t} + 5.5e^{-2t} - e^{-3t})u(t)}$$

#

29

$$\frac{d^2 y(t)}{dt^2} + 4 \frac{dy(t)}{dt} + 3y(t) = 2 \frac{d^2 x(t)}{dt^2} - 4 \frac{dx(t)}{dt} - x(t)$$

$$y(0) = -2, y'(0) = 1, x(t) = u(t), \text{ Find } H(s), y(t)$$

$$[s^2 y(s) - sy(0) - y'(0)] + 4[sy(s) - y(0)] + 3y(s) =$$

$$2[s^2 x(s) - sx(0) - \dot{x}(0)] - 4[sx(s) - x(0)] - x(s)$$

$$\Rightarrow s^2 y(s) + 2s - 1 + 4sy(s) + 8 + 3y(s) = 2s^2 x(s) - \frac{4s}{s} x(s) - x(s)$$

$$y(s) [s^2 + 4s + 3] = -2s + 1 - 8 + [2s^2 - 4s - 1] x(s)$$

$$y(s) [s^2 + 4s + 3] = -2s - 7 + \frac{(2s^2 - 4s - 1)}{s} x(s)$$

$$y(s) = \frac{-(11s+1)}{s(s^2+4s+3)} = \frac{A}{s} + \frac{B}{(s+1)} + \frac{C}{(s+3)}$$

$$A = -\frac{(11s+1)(s)}{s(s^2+4s+3)} \Big|_{s=0} = \boxed{-\frac{1}{3}}$$

$$B = -\frac{(11s+1)(s+1)}{s(s+1)(s+3)} \Big|_{s=-1} = \boxed{-5}$$

$$C = -\frac{(11s+1)(s+3)}{s(s+1)(s+3)} \Big|_{s=-3} = \boxed{\frac{-16}{3}}$$

$$y(s) = \frac{-1/3}{s} - \frac{5}{(s+1)} - \frac{16/3}{(s+3)}$$

$$y(t) = \left(-\frac{1}{3} - 5e^{-t} - \frac{16}{3}e^{-3t} \right) u(t)$$

To Find $H(s)$ (I.C = 0)

$$s^2 y(s) + 4sy(s) + 3y(s) = 2s^2 x(s) - 4sx(s) - x(s)$$

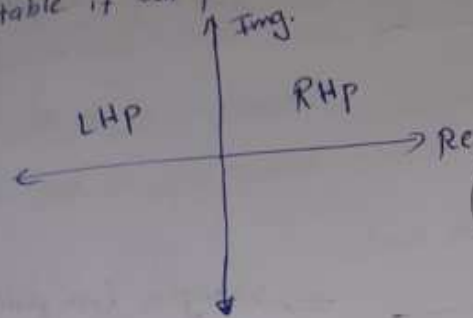
$$y(s) (s^2 + 4s + 3) = (2s^2 - 4s - 1) x(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{(2s^2 - 4s - 1)}{s^2 + 4s + 3}$$

”اذا مثل لهم دافعا كما افسدنا من اهل
قالوا انفسه كما افسد السهال الى
البحرهم هم السهال، ولكن لا يحيطون
- سورة البقرة -

Stability #1

The system is stable if all poles in the Left half plane (LHP)

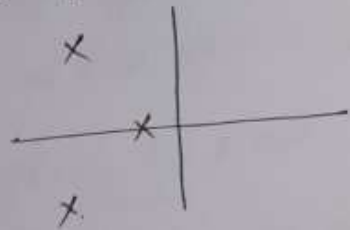


Eng. Hadit Jamal Hatten
Part #2
Signal & System
Ch #4

* If one pole in RHP $\Rightarrow$ system is unstable or poles repeated in JW axis

* margin system $\Rightarrow$ If at least one single pole exist on JW axis, but no repeated poles on JW

Examples: -

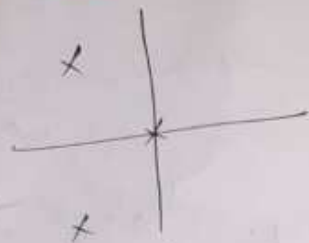


$\Rightarrow$ stable (all poles in the LHP)

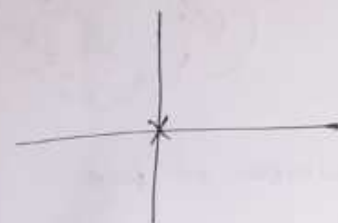


$\Rightarrow$ Stable (all poles in the LHP)

إذا وجدت
سعة القوة
في نقطة



$\Rightarrow$ Stable (all poles in the LHP)



$\Rightarrow$ margin (no poles in LHP, only one poles in Jw)



$\Rightarrow$ margin system (single poles on Jw)



$\Rightarrow$ margin system



$\Rightarrow$ unstable "exist pole on RHP"



$\Rightarrow$ un stable (repeated poles on Jw)



$\Rightarrow$ unstable (repeated pole on Jw)

How we can find the poles of the system:-

$$H(s) = \frac{(s+a)(s+b)\dots}{(s+c)(s+d)(s+e)\dots}$$

$\leftarrow$ zeros
 $\leftarrow$ poles

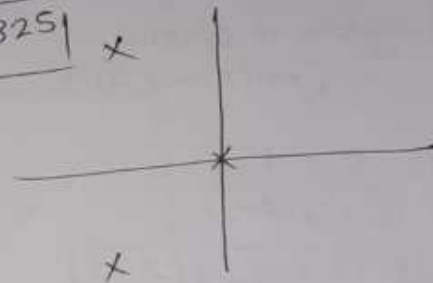
* لنجد أصفاء البسط $\rightarrow$ zeros ويرمز لها بـ 0
 * لنجد أصفاء المقام $\rightarrow$ poles ويرمز لها بـ x

Ex 4 | $X(s) = \frac{5}{s(s^2+s+2)}$ find $x(\infty)$

$x(\infty) \Rightarrow$ final value theorem
 يعني متى لزم انخفض او stability وذلك من خلال معرفة poles

$$s(s^2+s+2) = 0$$

$s = 0$, $s = -0.5 \pm j1.325$



$\Rightarrow$ system is stable
 $\Rightarrow$ final value is exist

$$\begin{aligned}
 x(\infty) &= \lim_{s \rightarrow 0} s X(s) \\
 &= \lim_{s \rightarrow 0} \frac{s(5)}{s(s^2+s+2)} = \boxed{\frac{5}{2}}
 \end{aligned}$$

" وفي الليل نقدر ان نأكله " لك على انه
 بيتك ان ضاها " محموداً "

$$X(s) = \frac{6}{s(s^2+2s)}$$

$$s(s^2+2s)=0$$

$$s=0, s=\pm j5$$

The system is margin stable so that, there is no final value

final $x(\infty)$
system is marginally stable poles $s=0, \pm j5$



* Test the stability:-

$$H(s) = \frac{s+1}{(s+2)(s+3)}$$

$$\text{poles: } s=-2, s=-3$$

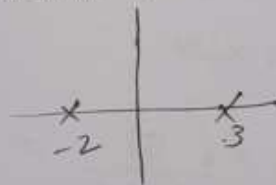
⇒ system is stable.
(poles in LHP)



$$H(s) = \frac{s+1}{(s+2)(s-3)}$$

$$\text{poles: } s=-2, s=3$$

⇒ system is unstable because there is one pole in RHP



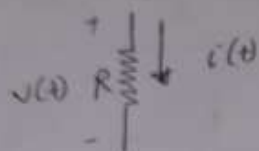
لينا وداينا ما ودينا على الله
فقرنا يوم القيامة انك
تخلف لحياد
العران.

Analysis of Electrical Networks :-

- Notes // The Laplace transform method gives the total response which includes Zero input and Zero state components
- * It is possible to separate the two components if we so desire

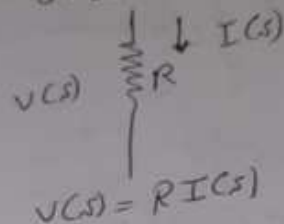
Time domain

[1]



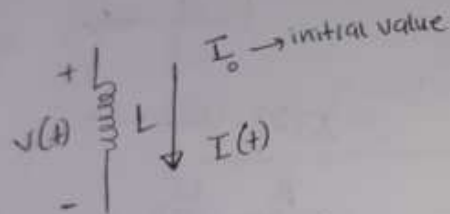
$$v(t) = R i(t)$$

S-domain



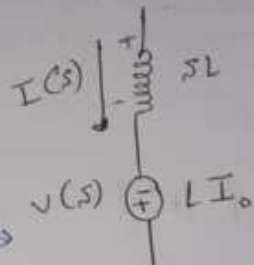
$$V(s) = R I(s)$$

[2]



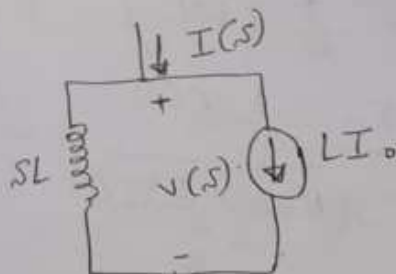
$$v(t) = L \frac{di}{dt}$$

$$V(s) = L (s I(s) - i(0))$$



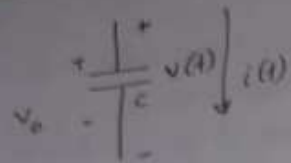
$$V(s) = L [s I(s) - I_0]$$

OR



$$I(s) = \frac{V(s)}{sL} + \frac{I_0}{s}$$

مبدأ التراكب
يستخدم في الدوائر الخطية
لحل الدوائر التي تحتوي على
مصادر مستقلة ومتعددة
في نفس الوقت.



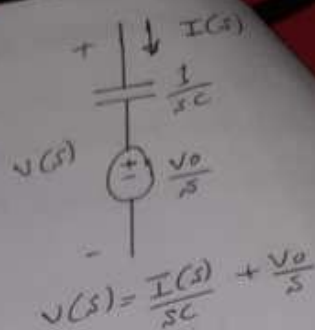
$$i = C \frac{dv}{dt}$$

$$v = \frac{1}{C} \int_0^t i(t) dt + v_0$$

$$i(t) = C \frac{dv}{dt}$$

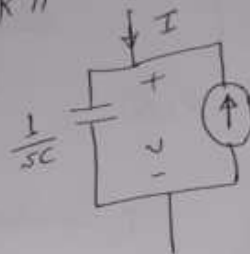
$$i(s) = C [sV(s) - v(0)]$$

$$= [sCV(s) - Cv(0)]$$



$$v(s) = \frac{I(s)}{sC} + \frac{v_0}{s}$$

OR //



$$I(s) = sCV - Cv_0$$

Ex # Find $H(s)$?

$$H(s) = \frac{y(s)}{x(s)}$$

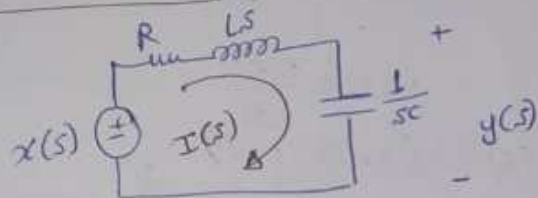
$$x(s) = \left(R + Ls + \frac{1}{sC}\right) I(s)$$

$$y(s) = \frac{1}{sC} I(s)$$

$$H(s) = \frac{\frac{1}{sC} I(s)}{\left(R + Ls + \frac{1}{sC}\right) I(s)}$$

$$H(s) = \frac{1}{Rsc + Ls^2 + 1}$$

$$H(s) = \frac{1/Lc}{s^2 + \frac{R}{L}s + \frac{1}{Lc}}$$



$$= \frac{\frac{1}{sC}}{R + Ls + \frac{1}{sC}} \cdot \frac{sC}{sC}$$

المقام به المقام
 $\frac{1}{sC} \sim \frac{1}{Lc}$
 $\frac{1}{Lc} \sim \frac{1}{Lc}$

$H(s)$ Transfer function لا تأخذ إلا قيم عددية
 هي نسبة الإخراج على الدخل $\left(\frac{\text{output}}{\text{input}} \right)$ نسبة
 التردد في frequency domain إلى مخرج استقر في time domain

في Transfer function يجب أن initial condition
 تكون صفرية (أيها) بالسؤال بحيثها من معرفة

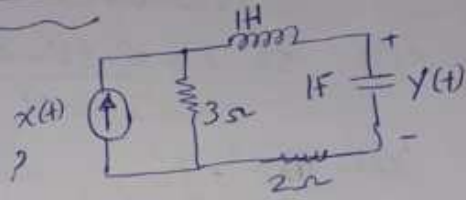
$$H(s) = \frac{Y(s)}{X(s)} = \frac{\text{output}}{\text{input}} \quad \begin{array}{l} I.C = 0 \\ \downarrow \\ \text{initial condition} \end{array}$$

Ex #1

$$V_C(0) = 1V$$

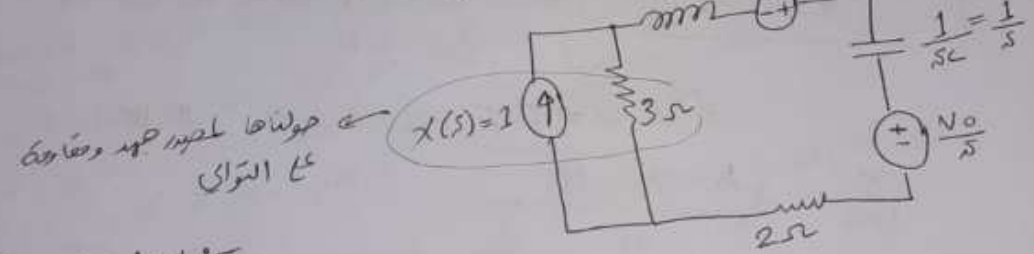
$$V_C(0) = 2A$$

$$x(t) = \delta(t), \text{ find } y(t)?$$



Sol:

في frequency domain يكون الدارة

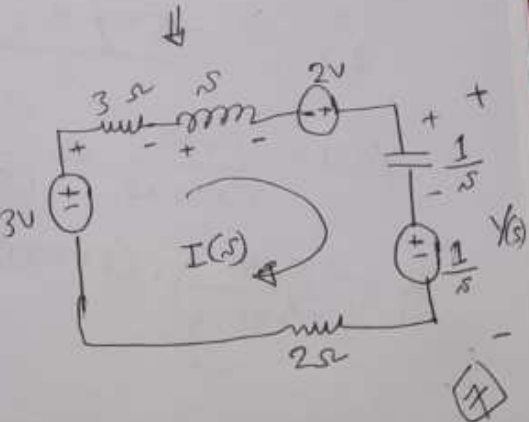


By using KVL:-

$$-3 + 3I(s) + sI(s) - 2 + \frac{1}{s}I(s) + \frac{1}{s} + 2I(s) = 0$$

$$I(s) \left(3 + s + \frac{1}{s} + 2 \right) = 3 + 2 - \frac{1}{s}$$

$$I(s) = \frac{5 - \frac{1}{s}}{s + \frac{1}{s} + 5}$$



$$I(s) = \frac{5s-1}{s^2+5s+1} \Rightarrow \boxed{I(s) = \frac{s(5s-1)}{s(s^2+5s+1)}}$$

$$Y(s) = \underbrace{\frac{1}{s}}_{\text{علاقة}} \underbrace{I(s)}_{\text{علاقة}} + \underbrace{\frac{1}{s}}_{\text{علاقة}}$$

$$Y(s) = \left(\frac{5s-1}{s^2+5s+1} \right) \cdot \frac{1}{s} + \frac{1}{s}$$

$$Y(s) = \frac{5s^2-1}{s(s^2+5s+1)} + \frac{1}{s}$$

$$Y(s) = \frac{5s^2-1 + s^3+5s^2+s}{s^2(s^2+5s+1)}$$

$$Y(s) = \frac{10s^2+s^3}{s^2(s^2+5s+1)} \Rightarrow \frac{s^2(s+10)}{s^2(s^2+5s+1)}$$

$$\boxed{Y(s) = \frac{s+10}{s^2+5s+1}}$$

Now, we used partial fraction to find $y(t)$

* نبدأ كل ال poles نكتب قيم λ لقانون العاكس

$$s_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$s_{1,2} = \frac{-5 \pm \sqrt{25-4 \times 1}}{2} \Rightarrow s_{1,2} = \boxed{-0.21}, \boxed{-4.79}$$

$$Y(s) = \frac{s+10}{s^2+5s+1} = \frac{K_1}{(s+0.21)} + \frac{K_2}{(s+4.79)}$$

$$K_1 = \left(\frac{s+10}{s^2+5s+1} \right) \cdot (s+0.21) \Big|_{s=-0.21}$$

نضرب $Y(s)$ في البسط K_1
، نعوض عند قيمة $s = -0.21$
ونحصل على قيمة الثوابت

$$\frac{(s+10)(s+0.21)}{(s+0.21)(s+4.79)} \Big|_{s=-0.21} \Rightarrow \boxed{K_1 = 2.138}$$

$$K_2 = \frac{(s+10)(s+4.79)}{(s+0.21)(s+4.79)} \Big|_{s=-4.79} \Rightarrow \boxed{K_2 = -1.138}$$

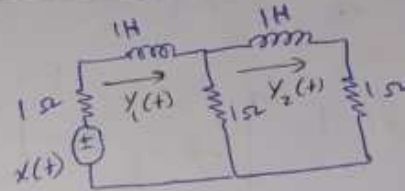
$$Y(s) = \frac{2.138}{s+0.21} + \frac{-1.138}{s+4.79}$$

$$y(t) = \left[2.138 e^{-0.21t} - 1.138 e^{-4.79t} \right] u(t) \quad \neq$$

Ex #1 find $y_1(t), y_2(t)$ for $t \geq 0$

$$y_1(0) = 2, y_2(0) = 1$$

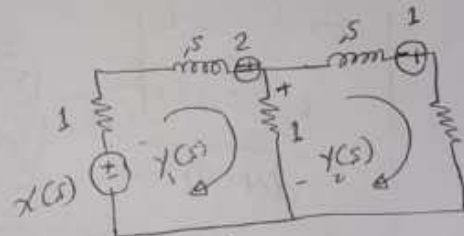
$$\text{for } t \geq 0, x(t) = 6u(t)$$



أولاً نحول الدارة إلى frequency domain

initial condition

نلاحظ أن هناك شرط ابتدائي $y_1(0) = 2$ و $y_2(0) = 1$ وهذا يعني أن الدارة ليست في حالة مستقرة عند $t=0$ ، لذا نحتاج إلى أخذ هذه الشروط في الاعتبار عند حل الدارة.



$$X(s) = \frac{6}{s}$$

(طريقة حل المعادلات درجتها عادة الدوائر الكهربائية)

⇒ By using mesh current

$$-\frac{6}{s} + (1+s+1)Y_1(s) - Y_2(s) - 2 = 0$$

$$Y_1(s) + sY_1(s) + Y_1(s) - Y_2(s) = 2 + \frac{6}{s}$$

$$\boxed{Y_1(s)(s+2) - Y_2(s) = 2 + \frac{6}{s}} \quad \dots \quad (1)$$

معادلة

مطابقة

دست المعادله و نکات استخراج طرفه کریم

$$Y_1(s+2) - Y_2(s) = 2 + \frac{6}{s}$$

$$-Y(s) + Y_2(s)(s+2) = 1$$

$Ax = b$

- مصفوفة المصفوفة
- مصفوفة المتغيرات
- مصفوفة الثوابت

$$\begin{bmatrix} s+2 & -1 \\ -1 & s+2 \end{bmatrix} \begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} 2 + \frac{6}{s} \\ 1 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} s+2 & -1 \\ -1 & s+2 \end{vmatrix} = (s+2)(s+2) - (-1)(-1) = \boxed{s^2 + 4s + 3}$$

$$Y_1(s) = \frac{\begin{vmatrix} 2 + \frac{6}{s} & -1 \\ 1 & s+2 \end{vmatrix}}{\det(A)} = \frac{\left(\frac{2s+6}{s}\right)(s+2) + 1}{s^2 + 4s + 3}$$

$$= \frac{2s^2 + 10s + 12}{s} + 1 \Rightarrow \frac{s + 2s^2 + 10s + 12}{s(s^2 + 4s + 3)}$$

$$Y_1(s) = \frac{2s^2 + 11s + 12}{s(s^2 + 4s + 3)}$$

$$Y_2(s) = \frac{\begin{vmatrix} s+2 & 2+\frac{6}{s} \\ -1 & 1 \end{vmatrix}}{\det(A)} = \frac{(s+2) + (2+\frac{6}{s})}{s^2+4s+3}$$

$$= \frac{(s+2) + \left(\frac{2s+6}{s}\right)}{s^2+4s+3}$$

$$Y(s) = \frac{s^2 + 4s + 6}{s(s^2 + 4s + 3)}$$

By find inverse Laplace transform //

$$Y(s) = \frac{2s^2 + 11s + 12}{s(s^2 + 4s + 3)} = \frac{A}{(s+1)} + \frac{B}{(s+3)} + \frac{C}{s}$$

$$A = \frac{(2s^2 + 11s + 12)(s+3)}{s(s+1)(s+3)} \Big|_{s=-1} = \boxed{-\frac{3}{2}}$$

$$B = \frac{(2s^2 + 11s + 12)(s+1)}{s(s+1)(s+3)} \Big|_{s=-3} = \boxed{-\frac{1}{2}}$$

$$C = \frac{(2s^2 + 11s + 12)s'}{s(s+1)(s+3)} \Big|_{s=0} = \boxed{4}$$

$$Y(s) = \frac{-3/2}{s+1} - \frac{1/2}{s+3} + \frac{4}{s}$$

$$y_1(t) = \left(-\frac{3}{2}e^{-t} - \frac{1}{2}e^{-3t} + 4 \right) u(t) \quad \#$$

لدينا كسور من البسط، المقام، المقام
مما نتيج طالعنا
الذي هو المقام
سواء البسط

$$Y(s) = \frac{s^2 + 4s + 6}{s(s^2 + 4s + 3)} = \frac{A}{(s+1)} + \frac{B}{(s+3)} + \frac{C}{s}$$

$$A = \frac{(s^2 + 4s + 6)(s+3)}{s(s+1)(s+3)} \Big|_{s=-1} = \boxed{-\frac{3}{2}}$$

$$B = \frac{(s^2 + 4s + 6)(s+1)}{s(s+1)(s+3)} \Big|_{s=-3} = \boxed{\frac{1}{2}}$$

$$C = \frac{(s^2 + 4s + 6)s'}{s(s+1)(s+3)} \Big|_{s=0} = \boxed{2}$$

$$y_2(t) = \left(-\frac{3}{2}e^{-t} + \frac{1}{2}e^{-3t} + 2 \right) u(t) \quad \#$$

لدينا كسور من البسط، المقام، المقام
مما نتيج طالعنا
الذي هو المقام
سواء البسط

Ex #1 $v_c(0) = 1$, $i_L(0) = 2$, $x(t) = u(t)$

find $y(t)$?

Sol:
By using node voltage

$$\frac{V - 4 - \frac{1}{s}}{2s} + \frac{V}{1} + \frac{V - \frac{1}{s}}{1/2s} = 0$$

$$y(s) = V$$

$$\frac{y(s) - 4 - \frac{1}{s}}{2s} + y(s) + \frac{y(s) - \frac{1}{s}}{1/2s} = 0$$

$$\frac{y(s)}{2s} - \frac{4}{2s} - \frac{1}{2s^2} + y(s) + 2sy(s) - 2 = 0$$

$$y(s) \left(\frac{1}{2s} + 1 + 2s \right) = 2 + \frac{4}{2s} + \frac{1}{2s^2}$$

$$y(s) = \frac{2 + \frac{4}{2s} + \frac{1}{2s^2}}{\frac{1}{2s} + 1 + 2s} = \frac{4s^2 + 4s + 1}{2s^2(4s^2 + 2s + 1)}$$

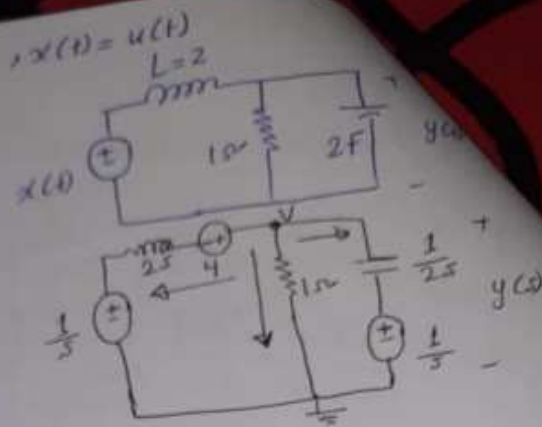
$$y(s) = \frac{4s^2 + 4s + 1}{s(4s^2 + 2s + 1)} = \frac{k_1}{s} + \frac{k_2}{s + 0.25 - j0.433} + \frac{k_2^*}{s + 0.25 + j0.433}$$

$$k_1 = \frac{(4s^2 + 4s + 1)(s)}{s(4s^2 + 2s + 1)} \Big|_{s=0} = 1$$

$$k_2 = \frac{(4s^2 + 4s + 1)(s + 0.25 - j0.433)}{s(s + 0.25 - j0.433)(s + 0.25 + j0.433)} \Big|_{s = -0.25 + j0.433}$$

$$k_2 = 2.31 \angle -90^\circ, \quad k_2^* = 2.31 \angle 90^\circ$$

$$y(t) = u(t) + 4.618 e^{-0.25t} \cos(0.433t - 90^\circ) u(t)$$



#

12

Ex 17.1 Given the circuit shown find $y(t)$

$$y(0^-) = 2 \text{ (in inductor)}$$

$$V_L(0) = (2)(2) = 2V$$

$$-\frac{10}{s} + sY(s) - 2\left(1 + \frac{5}{s}Y(s)\right) + \frac{10}{s} = 0$$

$$\frac{10}{s} + 2 - \frac{10}{s} = \left(s + \frac{5}{s} + 2\right)Y(s)$$

$$Y(s) = \frac{2}{s + \frac{5}{s} + 2}$$

$$Y(s) = \frac{2s}{s^2 + 5s + 2s}$$

$$Y(s) = \frac{2s}{s^2 + 2s + s} = \frac{K_1}{s+1-j2} + \frac{K_1^*}{s+1+j2}$$

$$K_1 = \frac{(2s)(s+1-j2)}{(s+1-j2)(s+1+j2)} \Big|_{s=-1+j2} = 1 + j0.5 = 1.118 \angle 26.56^\circ$$

$$K_1^* = 1.118 \angle -26.56^\circ$$

$$y(t) = \left[2.236 e^{-t} \cos(2t + 26.56^\circ) \right] u(t) \quad \#$$

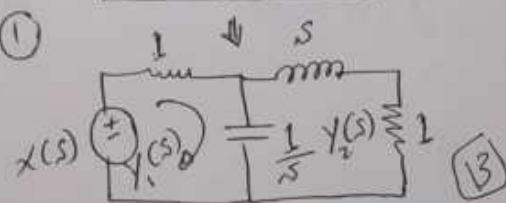
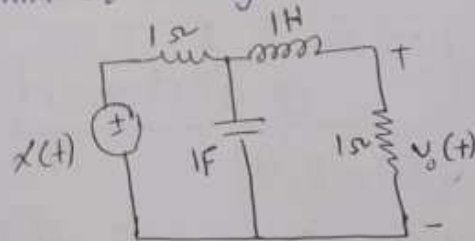
Find the Zero state response $v_o(t)$ if the input voltage is $v(t)$

$= te^{-t}u(t)$. Find T.F, write differential eq. relating $v_o(t)$ to $x(t)$

by using mesh current method :-

$$X(s) = \left(1 + \frac{1}{s}\right)Y_1(s) - \frac{1}{s}Y_2(s)$$

$$\left[\frac{1}{(s+1)^2}\right] = \left(1 + \frac{1}{s}\right)Y_1(s) - \frac{1}{s}Y_2(s) \quad \dots (1)$$



$$\left(\frac{1}{s} + s + 1 \right) Y_1(s) - \frac{1}{s} Y_2(s) = 0$$

$$\left[-\frac{1}{s} Y_1(s) + \left(\frac{1}{s} + s + 1 \right) Y_2(s) = 0 \right] \dots (2)$$

$$\begin{bmatrix} \frac{s+1}{s} & -\frac{1}{s} \\ -\frac{1}{s} & \frac{s^2+s+1}{s} \end{bmatrix} \begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} \frac{1}{(s+1)^2} \\ 0 \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} \frac{s+1}{s} & -\frac{1}{s} \\ -\frac{1}{s} & \frac{s^2+s+1}{s} \end{vmatrix} = s + \frac{2}{s} + 2 = \frac{s^2 + 2s + 2}{s}$$

$$Y_1(s) = \frac{\begin{vmatrix} \frac{1}{(s+1)^2} & -\frac{1}{s} \\ 0 & \frac{s^2+s+1}{s} \end{vmatrix}}{\det(A)} = \frac{(s^2+s+1) / s (s+1)^2}{\frac{s^2+2s+2}{s}}$$

$$= \frac{(s^2+s+1) s}{s (s+1)^2 (s^2+2s+2)} = \left[\frac{s^2+s+1}{(s+1)^2 (s^2+2s+2)} \right]$$

$$Y_2(s) = \frac{\begin{vmatrix} \frac{s+1}{s} & \frac{1}{(s+1)^2} \\ -\frac{1}{s} & 0 \end{vmatrix}}{\det(A)} = \frac{1 / s (s+1)^2}{\left(\frac{s^2+2s+2}{s} \right)}$$

$$Y_2(s) = \frac{1}{(s+1)^2 (s^2+2s+2)}$$

$$y_0(t) = y_2(t)$$

$$y_2(t) = ??$$

" ILT signa "

$$= \frac{1}{(s+1)^2(s^2+2s+2)} = \frac{K_1}{(s+1)^2} + \frac{K_2}{(s+1)} + \frac{K_3}{s+1-j} + \frac{K_3^*}{s+1+j}$$

$$K_1 = \frac{(s+1)^2}{(s+1)^2(s^2+2s+2)} \Big|_{s=-1} = \boxed{1}$$

$$K_2 = \frac{d}{ds} \left(\frac{(s+1)^2}{(s+1)^2(s^2+2s+2)} \right) \Big|_{s=-1}$$

$$K_2 = \frac{-(2s+2)}{(s^2+2s+2)^2} \Big|_{s=-1} = \boxed{0}$$

$$K_3 = \frac{(s+1-j)(1)}{(s+1)^2(s+1-j)(s+1+j)} \Big|_{s=-1+j} = j0.5 = 0.5 \angle 90^\circ$$

$$K_3^* = + 0.5 \angle -90^\circ$$

$$Y_2(s) = \frac{1}{(s+1)^2} + \frac{0.5 \angle 90^\circ}{(s+1-j)} + \frac{0.5 \angle -90^\circ}{(s+1+j)}$$

$$y_2(t) = t e^{-t} + (2) \cos \bar{e}^t \cos(\omega t + 90^\circ)$$

$$y_2(t) = [t e^{-t} + \bar{e}^t \cos(\omega t + 90^\circ)] u(t)$$

$$\text{or } \Rightarrow y_2(t) = [t e^{-t} + \bar{e}^t \sin(\omega t)] u(t) \quad \#$$

$$y_o(t) = y_2(t) = [t e^{-t} + \bar{e}^t \sin(\omega t)] u(t)$$

$$H(s) = T \cdot F = \frac{y_o(s)}{x(s)} = \frac{Y_2(s)(1)}{X(s)}$$

$$H(s) = \frac{1 / [(s+1)^2(s^2+2s+2)]}{1 / (s+1)^2}$$

$$H(s) = \frac{1(s+1)^2}{(s+0)^2(s^2+2s+2)}$$

$$\Rightarrow \left[H(s) = \frac{1}{s^2+2s+2} \right] \#$$

دالة المخرج

differential equation //

$$H(s) = \frac{V_o(s)}{X(s)} = \frac{1}{s^2+2s+2}$$

$$s^2 V_o(s) + 2s V_o(s) + 2V_o(s) = X(s)$$

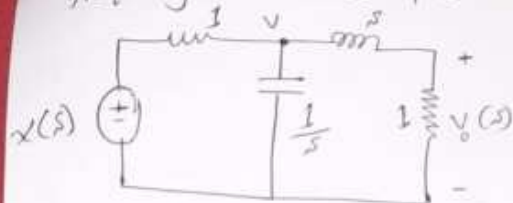
$$\left[\frac{d^2 V_o(t)}{dt^2} + 2 \frac{dV_o(t)}{dt} + 2V_o(t) = X(t) \right] \#$$

أقيم الصلابة لمحرك المصنوع إلى خمسة
التيك ومقارن الفهم
مستورد
منصة الإحصاء

Find the Zero state response $V_o(t)$ if the input voltage $x(t) = t e^{-t} u(t)$. Find T.F, write differential equation relating $x(t)$ to $x(t)$.

Sol:

frequency domain لا نأخذ، بقول



$$\frac{V - x(s)}{1} + \frac{V}{1/s} + \frac{V}{s+1} = 0$$

$$\frac{V}{1} - \frac{x(s)}{1} + \frac{V}{1/s} + \frac{V}{s+1} = 0$$

$$V \left(1 + s + \frac{1}{s+1} \right) = x(s)$$

$$V = \frac{x(s)}{\left(1 + s + \frac{1}{s+1} \right)}$$

$$T.F = H(s) = \frac{V_o(s)}{x(s)}$$

$$V_o(s) = (1) \left(\frac{V}{s+1} \right) = \frac{V}{s+1}$$

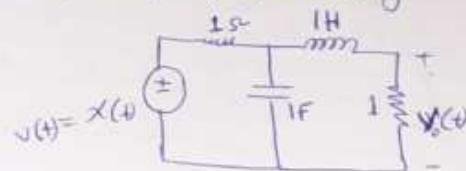
$$H(s) = T.F = \frac{V/(s+1)}{x(s)}$$

$$H(s) = \frac{(x(s) / (1 + s + \frac{1}{s+1}))}{(s+1) x(s)} \Rightarrow \left(\frac{1}{1 + s + \frac{1}{s+1}} \right) \left(\frac{1}{s+1} \right)$$

$$H(s) = \frac{1}{s^2 + 2s + 2} \quad \#$$

* Transfer function *

$$H(s) = \frac{V_o(s)}{x(s)} = \frac{1}{s^2 + 2s + 2}$$



by using
node voltage

$$H(s) = \frac{V_o(s)}{X(s)} = \frac{1}{s^2 + 2s + 2}$$

$$s^2 Y(s) + 2s Y(s) + 2Y(s) = X(s)$$

$$\left[\frac{d^2 y(t)}{dt^2} + 2 \frac{dy(t)}{dt} + 2y(t) = x(t) \right]$$

the differential equation

$$y(t) = t e^{-t} u(t)$$

$$Y(s) = \frac{1}{(s+1)^2}$$

$$V_o = X(s) \cdot H(s)$$

$$Y(s) = \frac{1}{(s+1)^2} \left(\frac{1}{s^2 + 2s + 2} \right)$$

$$= \frac{K_1}{(s+1)^2} + \frac{K_2}{(s+1)} + \frac{K_3^*}{s+1-j} + \frac{K_3^*}{s+1+j}$$

$$K_1 = \frac{(s+1)^2}{(s+1)^2(s^2+2s+2)} \Big|_{s=-1} = \boxed{1}$$

$$K_2 = \frac{d}{ds} \left(\frac{1}{s^2+2s+2} \right) \Big|_{s=-1} = \boxed{0}$$

$$K_3 = \frac{(s+1-j)(2)}{(s+1)^2(s+1-j)(s+1+j)} \Big|_{s=-1+j} = \boxed{j0.5} = \boxed{0.5 \angle 90^\circ}$$

$$K_3^* = 0.5 \angle -90^\circ$$

$$Y(s) = \frac{1}{(s+1)^2} + \frac{0.5 \angle 90^\circ}{(s+1-j)} + \frac{0.5 \angle -90^\circ}{(s+1+j)}$$

$$y(t) = [t e^{-t} + e^{-t} \sin(\omega t)] u(t) \quad \#$$

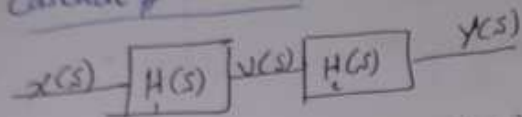
يخرج المخرج (التي) في وقت آخر فأننا
نقول له أنه في وقت " "
سرعة التغير

Diagram

It is convenient to represent a system by suitably interconnected subsystems

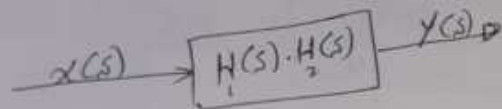
⇒ There are three types of interconnected subsystems

1) Cascade (Series)

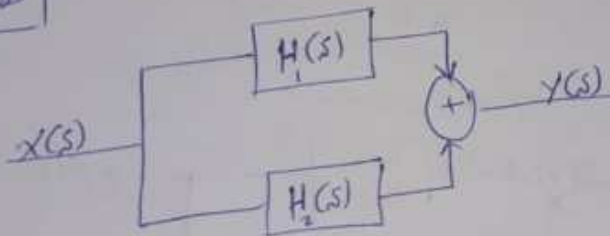


$$\frac{Y(s)}{X(s)} = \frac{V(s)}{X(s)} \cdot \frac{Y(s)}{V(s)} = H_1(s) \cdot H_2(s)$$

المتسلسلة

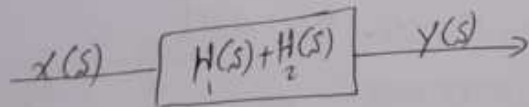


2) parallel



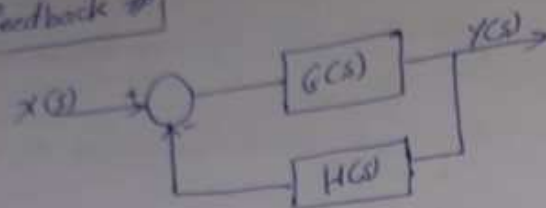
$$H_{\text{total}}(s) = H_1(s) + H_2(s)$$

المتوازية



"سأكون الذي تزلّ الفرفان على
عبدك ليكنه للعالمية تذكراً"
صوت الفرفان 😊

3) feedback

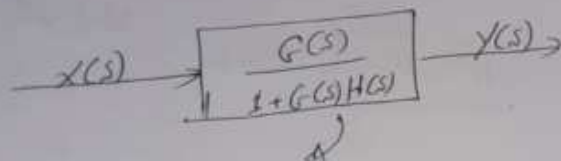


$G(s) \Rightarrow$ open loop TF نقل

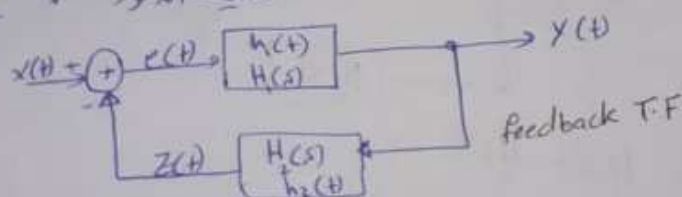
$H(s) \Rightarrow$ feedback Transfer function

$$H(s)_{\text{total}} = \frac{G(s)}{1 + G(s)H(s)}$$

النقل (المجموع) $\frac{1}{1 + G(s)H(s)}$



Example # نقل
Find T-F نقل



$$E(s) = X(s) - Z(s)$$

$$E(s) = X(s) - H_2(s)Y(s)$$

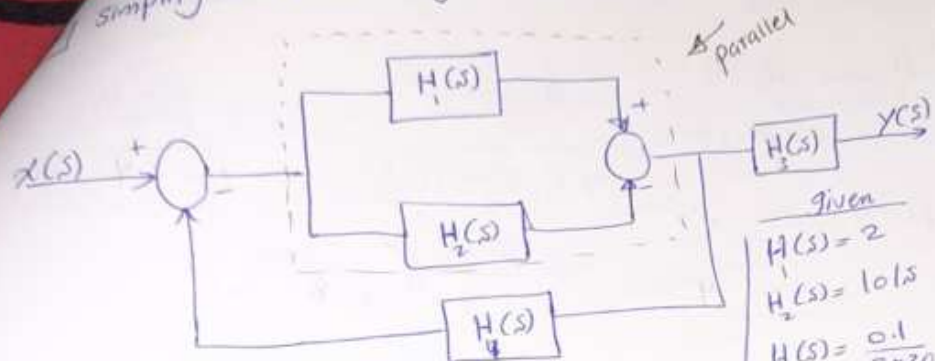
$$Y(s) = H_1(s)E(s) = H_1(s)[X(s) - H_2(s)Y(s)]$$

$$Y(s) = H_1(s)X(s) - H_1(s)H_2(s)Y(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{H_1(s)[X(s) - H_2(s)Y(s)]}{X(s)} = \frac{H_1(s)X(s)}{1 + H_1(s)H_2(s)} \cdot \frac{1}{X(s)}$$

$$H(s) = \frac{H_1(s)}{1 + H_1(s)H_2(s)}$$

simplify the block diagram to find the T.F

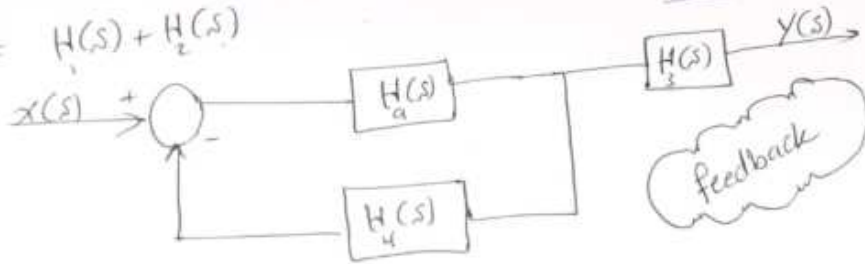


given

$$\begin{aligned} H_1(s) &= 2 \\ H_2(s) &= 10/s \\ H_3(s) &= \frac{0.1}{s+20} \\ H_4(s) &= \frac{2}{s+4} \end{aligned}$$

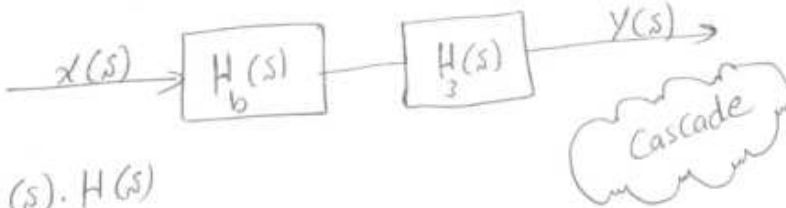
$H_1(s), H_2(s) \Rightarrow$ parallel

$$H_a = H_1(s) + H_2(s)$$



$H_a(s), H_4(s) \Rightarrow$ feedback

$$H_b(s) = \frac{H_a(s)}{1 + H_a(s) \cdot H_4(s)} \Rightarrow H_b(s) = \frac{H_1(s) + H_2(s)}{1 + H_1(s) \cdot H_2(s) \cdot H_4(s)}$$



Total

$$H(s) = H_3(s) \cdot H_b(s) = \left(\frac{H_1(s) + H_2(s)}{1 + (H_1(s) \cdot H_2(s)) H_4(s)} \right) \cdot H_3(s)$$

#

$$H_a(s) = H_1(s) + H_2(s) = 2 + \frac{10}{s} = \frac{2s+10}{s}$$

$$H_b(s) = \frac{H(s)}{1 + H(s)H_4(s)} = \frac{\left(\frac{2s+10}{s}\right)}{1 + \left(\frac{2s+10}{s}\right)\left(\frac{2}{s+4}\right)} \Rightarrow \frac{\left(\frac{2s+10}{s}\right)}{1 + \left(\frac{4s+20}{s^2+4s}\right)}$$

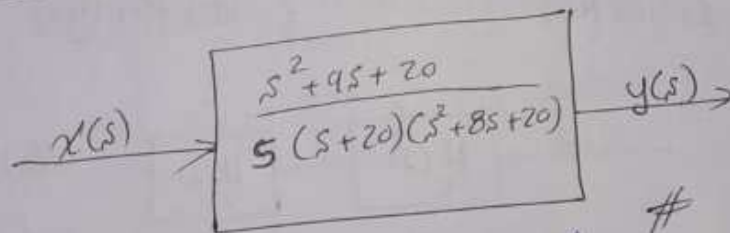
$$H_b(s) = \frac{\left(\frac{2s+10}{s}\right)}{\frac{s^2+4s+4s+20}{s^2+4s}} \Rightarrow \left(\frac{2s+10}{s}\right) \left(\frac{s^2+4s}{s^2+8s+20}\right)$$

$$H_b(s) = \frac{(2s+10)(s+4)(s)}{s(s^2+8s+20)} = \boxed{\frac{(2s+10)(s+4)}{s^2+8s+20}}$$

$$H_{total}(s) = H_b(s) \cdot H_3(s)$$

$$H_{total}(s) = \left[\frac{(2s+10)(s+4)}{s^2+8s+20} \right] \cdot \left(\frac{0.1}{s+20} \right)$$

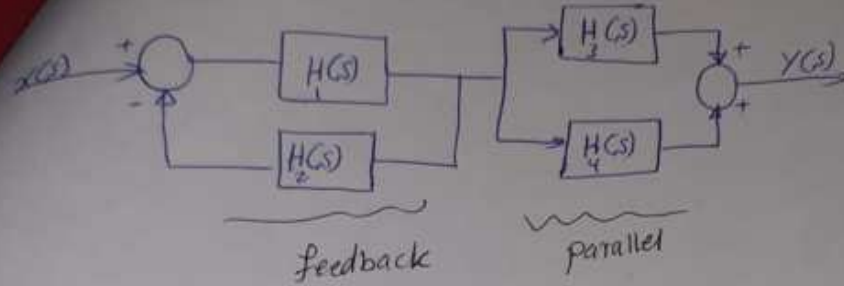
$$H_{total}(s) = \boxed{\frac{s^2+4s+20}{5(s+20)(s^2+8s+20)}} \quad \#$$



این سیستم در سری است
در سری سه قطعه
اولی: $\frac{0.1}{s+20}$
دومی: $\frac{2s+10}{s}$
سومی: $\frac{2}{s+4}$

و بعد از مرحله این به دست می آید
این قطعه هم و اما خارج از
این قطعه قاطعاً سه قطعه
سویق الفزاق

Reduce the block diagram to one block:-



cascade *سلسلة*

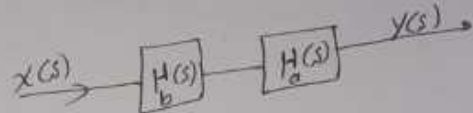
$H_3(s), H_4(s) \Rightarrow$ parallel

$$\Rightarrow \boxed{H_a(s) = H_3(s) + H_4(s)}$$

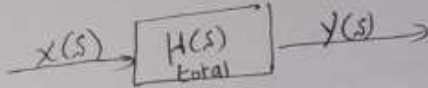


$H_1(s), H_2(s) \Rightarrow$ feedback

$$\Rightarrow \boxed{H_b(s) = \frac{H_1(s)}{1 + H_1(s)H_2(s)}}$$



$H_a(s), H_b(s) \Rightarrow$ cascade



$$H(s) = H_b(s) \cdot H_a(s)$$

$$H_{total}(s) = \left(\frac{H_1(s)}{H_3(s) + H_4(s)} \right) \left(\frac{H_1(s)}{1 + H_1(s)H_2(s)} \right)$$

$$\boxed{H(s)_{total} = \frac{H_1(s) \cdot H_3(s)}{1 + H_1(s)H_2(s)} + \frac{H_4(s)H_1(s)}{1 + H_1(s)H_2(s)}} \quad \# \text{ لا تطرح}$$

$$H(s)_{total} = \frac{(H_1(s)H_3(s))(1 + H_1(s)H_2(s)) + (H_4(s)H_1(s))(1 + H_1(s)H_2(s))}{(1 + H_1(s)H_2(s))(1 + H_1(s)H_2(s))}$$

منها ما داس
هنا القوة
😊