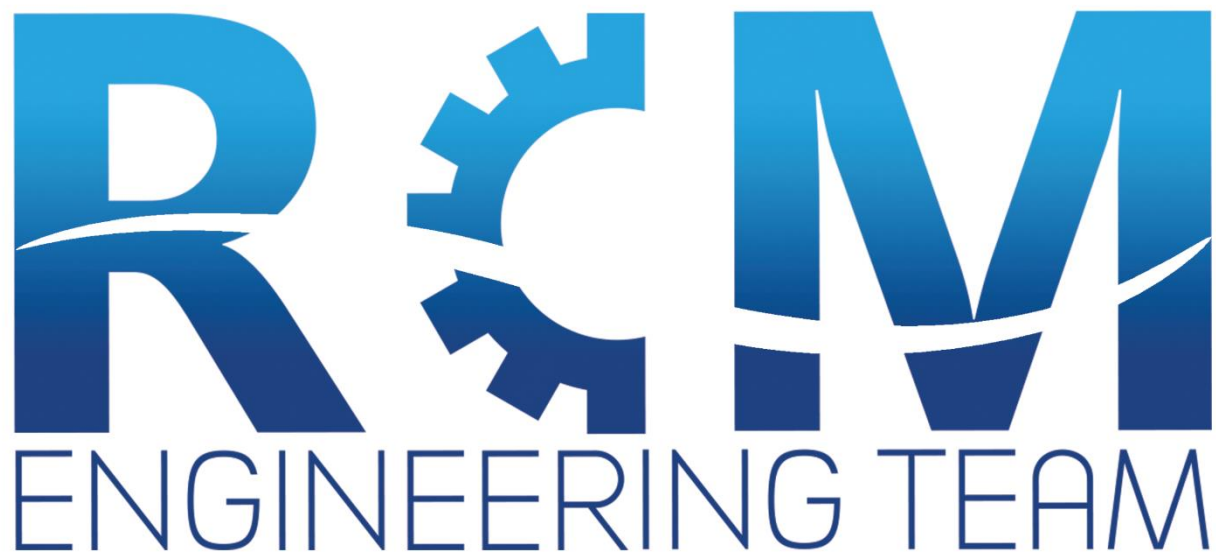




*RCM Engineering Team "IUG Branch"*

*Chapter " 6 "*

Of: Signals



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# C.T Signal analysis : Fourier Series

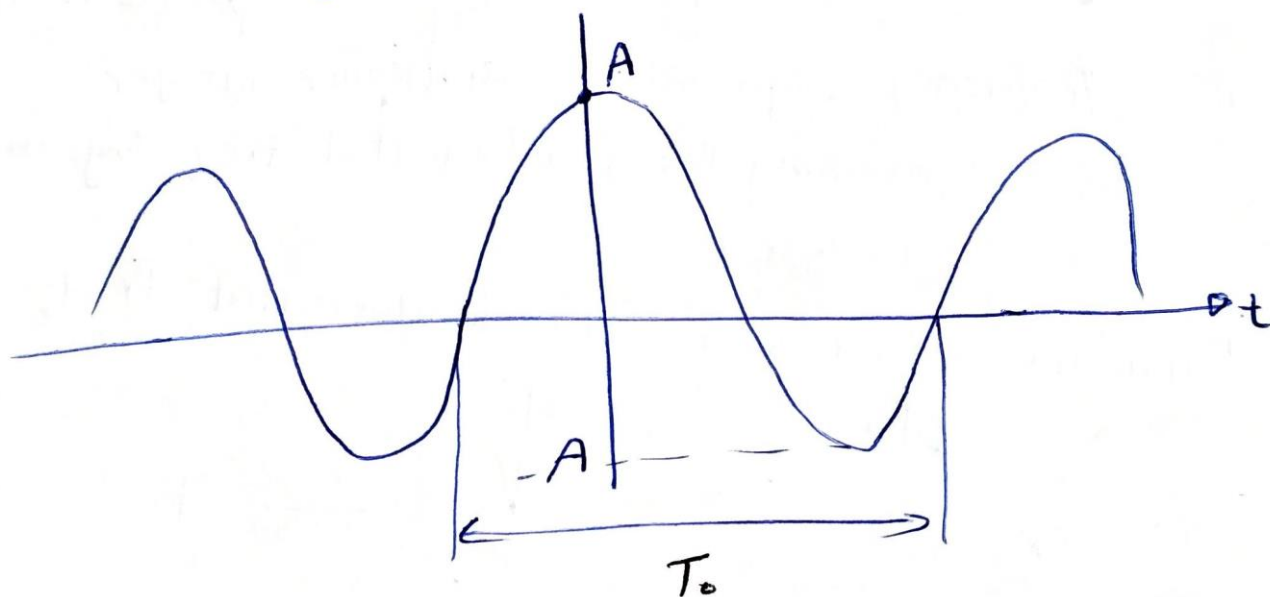
Fourier series :-

\* It's a technique that represents any periodic signal to summation of sin and cos signals or exponential signal

أي إشارة دورية يمكن تمثيلها بطريقة سلسلة فورييه ←  
exponential أو sinusoidal مجموع

\* It is useful to obtain the amplitude and phase spectrum.

\* Sinusoidal signal :-



it has a general form:

$$x(t) = A \sin(\omega_0 t + \phi) \rightarrow \text{its periodic signal}$$

$A \Rightarrow$  amplitude of the signal

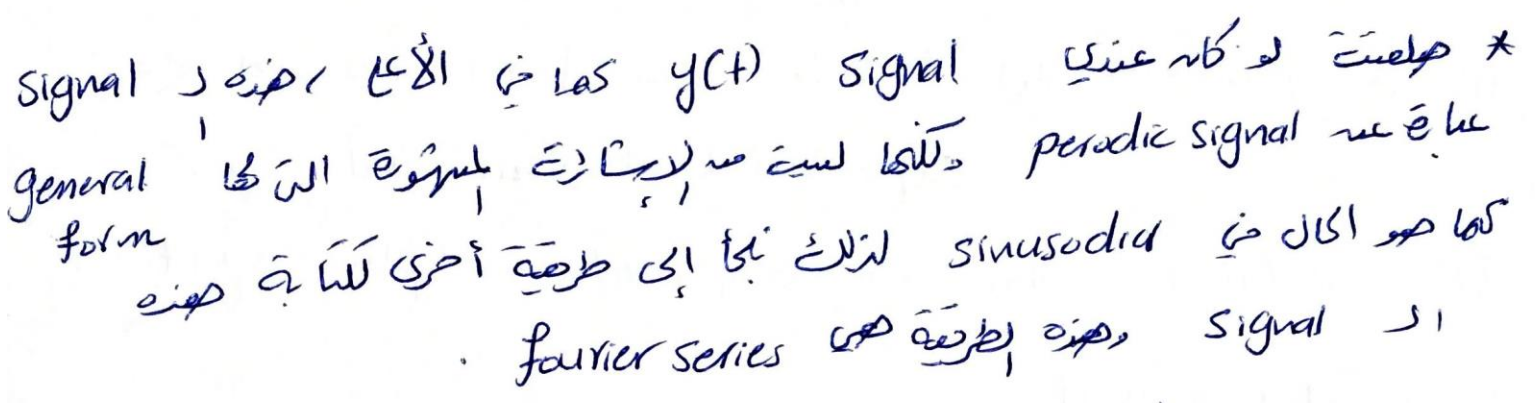
$\omega_0 \Rightarrow$  radian frequency (rad/sec)

$$\omega_0 = 2\pi f \quad (f \Rightarrow \text{Hz})$$

$\phi \Rightarrow$  phase shift (radian)

$T_0 \Rightarrow$  The periodic of the signal

$$\omega_0 = \frac{2\pi}{T_0}$$



\* harmonic: frequency component at multiplier integer (that multiply the fundamental freq. by an integer)

A hand-drawn graph of a periodic function, likely a sine wave, plotted on a coordinate system. The curve oscillates above and below a horizontal axis. The label  $2f_0$  is written near the end of the curve.

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## Time domain Model

Given time domain  
signal  $x(t)$



Find the fourier series  
coefficients  $a_0, a_n, b_n$ .

Converting signal from time domain to frequency domain

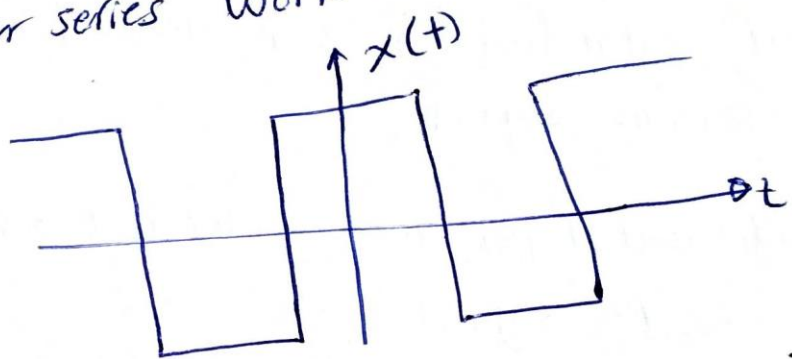
\* General representation of periodic signal :-

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$$

$n \Rightarrow$  Called harmonic order

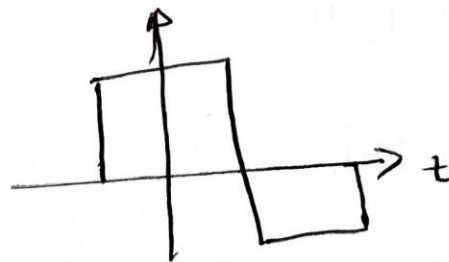
\* Now, how fourier series work :-

periodic C.T signal ⇐

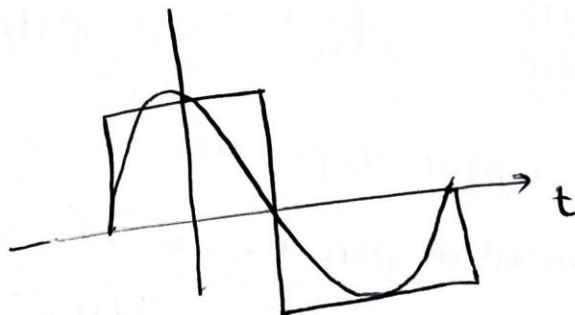


periodic signal } Fourier series // repetitive  
~~periodic~~ } Fourier transform ≠ ~~non periodic~~

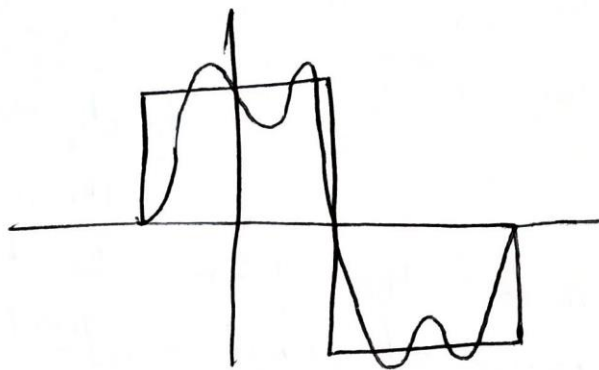
take one periodic ⇒



take the 1st harmonic  
at  $n=1$



Take the second harmonic at  $n = 2$



$\Rightarrow$  any signal that satisfy this equation  $x(t) = x(t + T_0)$  is called periodic.

$\Rightarrow$  fundamental frequency: is defined as the lowest frequency of a periodic signal.  $f_0$

$\Rightarrow$  fundamental period: is defined as the smallest period of a periodic signal  $T_0$

Example #1  $x(t) = \cos \frac{t}{3} + \cos \frac{t}{4}$

Two periodic signals  $2\pi$  is  $1/3$  and  $1/4$   $\leftarrow$

$$T_1 = \frac{2\pi}{1/3} = \boxed{6\pi}$$

$$T_2 = \frac{2\pi}{1/4} = \boxed{8\pi}$$

$$\frac{T_1}{T_2} = \frac{6\pi}{8\pi} = \boxed{\frac{3}{4}} \Rightarrow \text{Rational number} = \frac{\text{int}}{\text{int}}$$

$\Rightarrow$  The signal is periodic

$$\text{The fundamental period} = 3T_2 = 4T_1$$

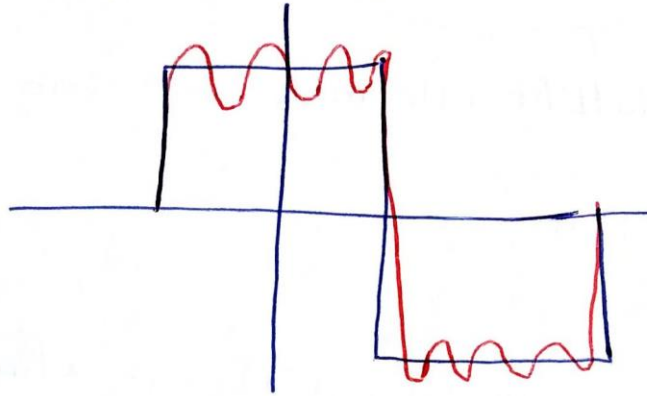
$$\boxed{T_0 = 24\pi}$$

$$\text{The fundamental freq.} = f_0 = \frac{1}{T_0} = \boxed{\frac{1}{24\pi} \text{ Hz}}$$

$\boxed{4}$

\* ملاحظة // كلما زادت فترة  $T_0$  كلما أصغرَت السعة من السعة الأصلية

و زادت الدقة



then find the summation for  $n$ -harmonics, then we get the function we need.

\* ملاحظة // Fourier Series  $\subseteq$  Periodic Signal  $\subseteq$  دالة دورية

[1] Trigonometric form:—

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos \omega_n t + b_n \sin \omega_n t$$

$a_0, a_n, b_n \Rightarrow$  Fourier series coefficient.

$$a_0 = \frac{1}{T_0} \int_{T_0} f(t) dt \quad (\text{DC component})$$

$$\left. \begin{aligned} a_n &= \frac{2}{T_0} \int f(t) \cdot \cos(n\omega_0 t) dt \\ b_n &= \frac{2}{T_0} \int f(t) \cdot \sin(n\omega_0 t) dt \end{aligned} \right\} \text{Trigonometric coefficient} \\ \text{معاملات}$$

\* ملاحظة // السائل في المعادلة السابقة على فترة واحدة (one period)

$$\left\{ \begin{array}{l} T_0 \leftarrow 0 \text{ أي } \\ -\frac{T_0}{2} \rightarrow \frac{T_0}{2} \text{ أو } \end{array} \right.$$

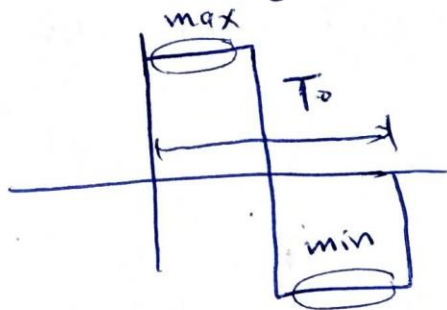
\* Dirichlet conditions (to apply Fourier series)

[1]  $x(t)$  is absolutely integrable over one period

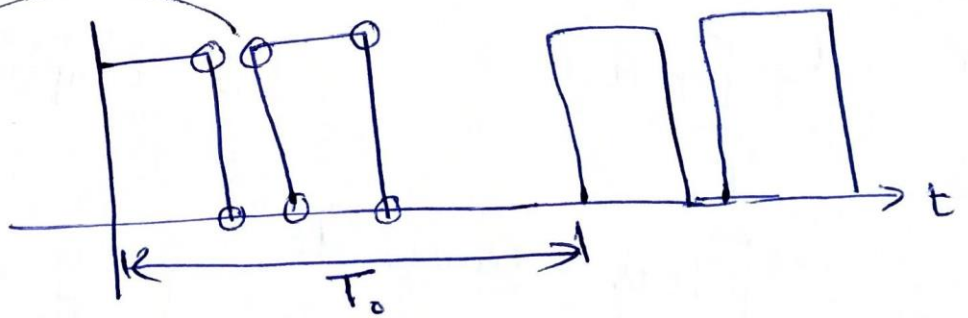
$$\int_{T_0} |x(t)| dt < \infty$$

[2] In a finite time interval,  $x(t)$  has a finite number of maximum and minimum.

Ex<sub>1</sub>  $x(t) = \sin\left(\frac{2\pi}{T} t\right) \quad 0 \leq t < 1$



[3] In a finite time interval,  $x(t)$  has only a finite number of discontinuities



\* properties of Fourier series:-

[1] Even function symmetry  $\Rightarrow x(t) = x(-t)$  (symmetry about y-axis)

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t)$$

$$b_n = 0$$

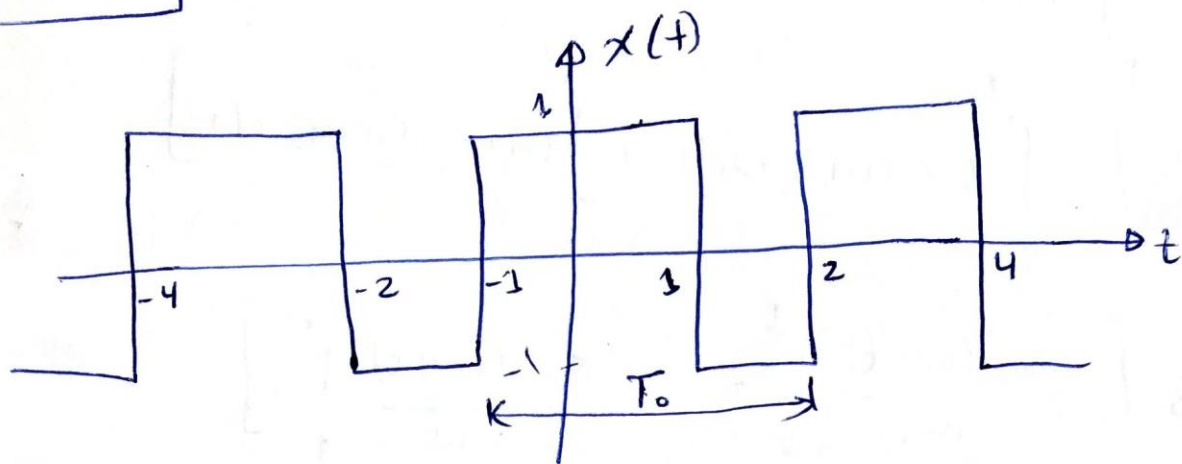
because sin is odd

[2] odd function symmetry  $\Rightarrow x(t) = -x(-t)$   
(symmetry about origin)

$$f(t) = \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

$$a_0 = a_n = 0$$

Example # Find Fourier series of  $x(t)$ .



Solution #

① Find the fundamental period  $T_0$ .

$$\underline{T_0} = 2 - (-1) = \boxed{3 \text{ sec}}, \quad \omega = \frac{2\pi}{T_0} = \boxed{\frac{2\pi}{3} \text{ rad/s}}$$

② Find  $x(t)$  values over one period

$$x(t) = \begin{cases} 1 & , -1 < t < 1 \\ -1 & , 1 < t < 2 \end{cases}$$

③ Find the trigonometric Fourier series coefficient:  
*even function* *we take  $x(t)$  as  $\cos$  not  $\sin$*

So,  $b_n = 0$

$$a_0 = \frac{1}{T} \int_{T_0} f(t) dt \Rightarrow a_0 = \frac{1}{3} \left[ \int_{-1}^1 1 dt + \int_1^2 (-1) dt \right]$$

$$a_0 = \frac{1}{3} \left[ t \Big|_{-1}^1 + -t \Big|_1^2 \right]$$

$$\frac{1}{3} \left( (1 - (-1)) + -(2 - 1) \right) = \boxed{\frac{1}{3}}$$

$$\Rightarrow \boxed{a_0 = \frac{1}{3}}$$

$$a_n = \frac{2}{T} \int_{T_0} f(t) \cdot \cos(n\omega_0 t) dt$$

$$= \frac{2}{3} \left[ \int_{-1}^1 1 \cdot \cos(n\omega_0 t) dt + \int_1^2 (-1) \cos(n\omega_0 t) dt \right]$$

$$= \frac{2}{3} \left[ \frac{\sin(n\omega_0 t)}{n\omega_0} \Big|_{-1}^1 + - \frac{\sin(n\omega_0 t)}{n\omega_0} \Big|_1^2 \right]$$

$$= \frac{2}{3} \left[ \left( \frac{\sin(n\omega_0) - \sin(-n\omega_0)}{n\omega_0} \right) - \left( \frac{\sin(2n\omega_0) - \sin(n\omega_0)}{n\omega_0} \right) \right]$$

$$= \frac{2}{3} \left[ \left( \frac{\sin(n\omega_0) + \sin(n\omega_0)}{n\omega_0} \right) - \frac{\sin(2n\omega_0) + \sin(n\omega_0)}{n\omega_0} \right]$$

$$= \frac{2}{3} \left[ \frac{\sin\left(\frac{2\pi}{3}n\right) + \sin\left(\frac{2\pi}{3}n\right) - \sin\left(\frac{4\pi}{3}n\right) + \sin\left(\frac{2\pi}{3}n\right)}{\left(\frac{2\pi}{3}n\right)} \right]$$

$$= \left[ \frac{\sin\left(\frac{2\pi}{3}n\right) + \sin\left(\frac{2\pi}{3}n\right) - \sin\left(\left(\frac{4\pi}{3} - 2\pi\right)n\right) + \sin\left(\frac{2\pi}{3}n\right)}{\pi n} \right]$$

$$= \left[ \frac{\sin\left(\frac{2\pi}{3}n\right) + \sin\left(\frac{2\pi}{3}n\right) - \overset{\text{odd}}{\sin\left(-\frac{2\pi}{3}n\right)} + \sin\left(\frac{2\pi}{3}n\right)}{\pi n} \right]$$

$$a_n = \left[ \frac{4 \sin\left(\frac{2\pi}{3}n\right)}{\pi n} \right]$$

$$b_n = \text{Zero}$$

(even function)

$$x(t) = \frac{1}{3} + \sum_{n=1}^{\infty} \frac{4 \sin\left(\frac{2\pi}{3}n\right)}{\pi n} \cdot \cos\left(\frac{2\pi}{3}nt\right) \quad \#$$

$a_n$

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Excercise: verify that  $b_n$  in the previous example equal zero

المطلوب إثبات أن  $b_n = 0$  في كل الحالات

$$\begin{aligned}
 b_n &= \frac{2}{T_0} \int_{T_0} f(t) \sin(n\omega_0 t) dt \\
 &= \frac{2}{3} \left[ \int_{-1}^1 1 \sin(n\omega_0 t) dt + \int_1^2 -\sin(n\omega_0 t) dt \right] \\
 &= \frac{2}{3} \left[ \frac{-\cos(n\omega_0) + \cos(-n\omega_0)}{n\omega_0} + \frac{\cos(2n\omega_0) - \cos(n\omega_0)}{n\omega_0} \right] \\
 &= \frac{2}{3} \left[ \frac{-\cos(n\omega_0) + \cos(n\omega_0)}{n\omega_0} + \frac{\cos(2n\omega_0) - \cos(n\omega_0)}{n\omega_0} \right] \\
 &= \frac{2}{3} \left[ \frac{\cos(2n\frac{2\pi}{3}) - \cos(\frac{2\pi}{3}n)}{\frac{2\pi}{3}n} \right]
 \end{aligned}$$

نلاحظ،  
ملاحظة

$$\frac{\cos\left(\left(\frac{4\pi}{3} - 2\pi\right)n\right) - \cos\left(\frac{2\pi}{3}n\right)}{\pi n}$$

$$\frac{\cos\left(-\frac{2\pi}{3}n\right) - \cos\left(\frac{2\pi}{3}n\right)}{\pi n}$$

even function  $\cos\left(-\frac{2\pi}{3}n\right) = \cos\left(\frac{2\pi}{3}n\right)$

$$\frac{\cancel{\cos\left(\frac{2\pi}{3}n\right)} - \cancel{\cos\left(\frac{2\pi}{3}n\right)}}{\pi n} = 0$$

$$\boxed{b_n = 0} \quad \checkmark$$

# المطلوب

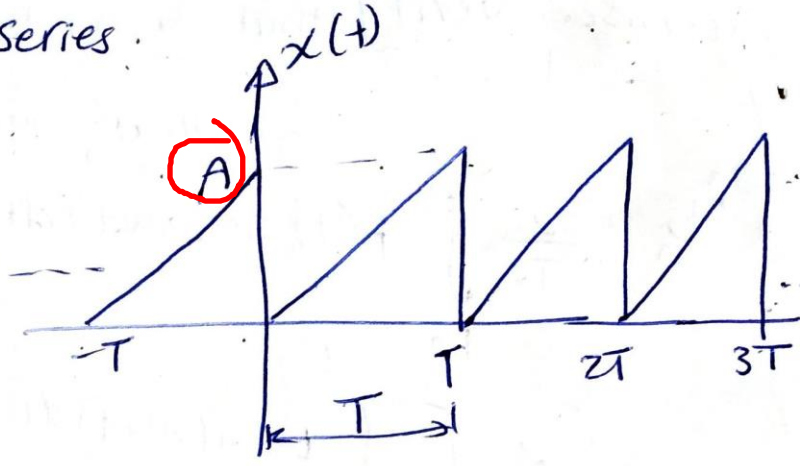
Example #2 | Find Fourier series:

Solution :-

1] fundamental period.

$$T_o = T$$

$$\omega = \frac{2\pi}{T_0} = \left[ \frac{2\pi}{T} \right]$$



2]  $x(t) = \frac{A}{T} t$  for  $0 < t < T$  (over one period)

$\rightarrow$   $x_1, y_1, x_2, y_2$  point, is, also no

$(0, 0), (T, A)$

$$\underline{S} = \frac{A - 0}{T - 0} = \left[ \frac{A}{T} \right] \rightarrow \text{slope}$$

$$\boxed{3} \quad a_0 = \frac{1}{T} \int_{T_0} f(t) dt \Rightarrow \frac{1}{T} \int_0^T \frac{A}{T} t dt$$

$$\Rightarrow \frac{1}{T} \begin{bmatrix} \frac{A}{T} \frac{t^2}{2} & 1 \\ 0 & T \end{bmatrix} \Rightarrow \frac{A}{2T^{3/2}} (T^2 - 0) \Rightarrow \boxed{\frac{A}{2}}$$

$$\Rightarrow \boxed{a_0 = \frac{A}{2}}$$

$$a_n = \frac{2}{T_0} \int_T f(t) \cdot \cos(n\omega_0 t) dt$$

$$= \frac{2}{T} \int_0^T \frac{A}{T} t \cdot \cos(\omega_0 t) dt$$

$$= \frac{2A}{T^2} \int_0^T t \cdot \cos(n\omega t) dt$$

$$= \frac{2A}{T^2} \left[ \begin{matrix} t \sin(\omega_y t) \\ \omega_y n \end{matrix} \right] + \frac{C_s(\omega_y t)}{\omega_y^2 n^2} \begin{matrix} 1 \\ 0 \end{matrix} \right]$$

$$\begin{aligned} & \frac{t}{1} \quad \frac{\cos(n\omega_0 t)}{\omega_0 n} \\ & \frac{0}{-} \quad \frac{-\cos(n\omega_0 t)}{\omega_0^2 n^2} \end{aligned}$$

$$= \frac{2A}{T^2} \left[ \frac{T \sin \left( n \cdot \frac{2\pi}{T} \cdot T \right) - 0}{\frac{2\pi}{T} n} + \frac{\cos \left( \frac{2\pi}{T} \cdot nT \right) - 1}{\left( \frac{2\pi}{T} n \right)^2} \right]$$

$$= \frac{2A}{T^2} \left[ \frac{T \sin(2\pi n)}{\frac{2\pi n}{T}} + \frac{\cos(2\pi n) - 1}{\left( \frac{2\pi}{T} n \right)^2} \right]$$

$$= \frac{2A}{T^2} \left[ \frac{\cancel{\cos(2\pi n)} - 1}{\left( \frac{2\pi}{T} n \right)^2} \right] = \boxed{0}$$

$$\Rightarrow \boxed{a_n = 0}$$

$$b_n = \frac{2}{T} \int_0^T \frac{A}{T} t \cdot \sin(\omega_n t) dt$$

$$b_n = \frac{2A}{T^2} \int_0^T t \cdot \sin(n\omega_0 t) dt$$

$$b_n = \frac{2A}{T^2} \left[ -\frac{t \cos(n\omega_0 t)}{n\omega_0} \Big|_0^T + \frac{\sin(n\omega_0 t)}{(n\omega_0)^2} \Big|_0^T \right]$$

$$= \frac{2A}{T^2} \left[ \frac{-T \cos \left( n \frac{2\pi}{T} T \right) + 0}{n \frac{2\pi}{T}} + \frac{\sin \left( n \frac{2\pi}{T} T \right) - 0}{\left( n \cdot \frac{2\pi}{T} \right)^2} \right]$$

$$= \frac{2A}{T^2} \left[ \frac{-T \cancel{\cos(2\pi n)}}{n \frac{2\pi}{T}} + \frac{\cancel{\sin(2\pi n)}}{\left( n \frac{2\pi}{T} \right)^2} \right]$$

$$= \frac{2A}{T^2} \left[ \frac{-T}{\frac{2\pi n}{T}} \right] \Rightarrow \frac{2A}{T^2} \cdot \frac{-T^2}{2\pi n} \Rightarrow \boxed{\frac{-A}{\pi n}}$$

$$\boxed{b_n = \frac{-A}{\pi n}}$$

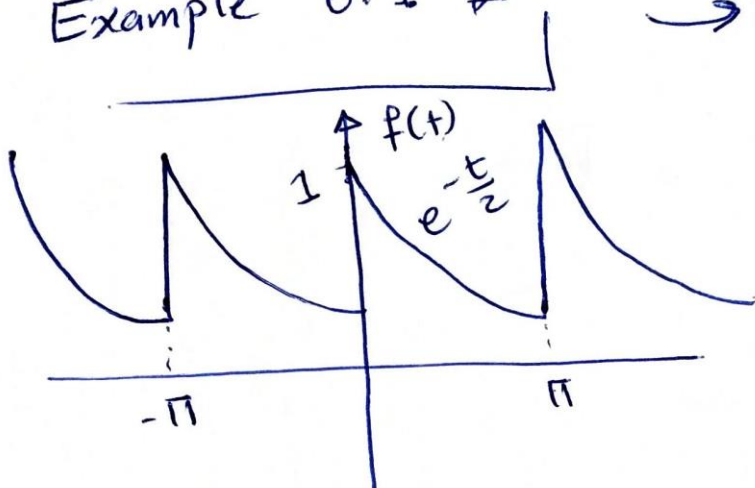
$$\begin{array}{l} t \quad \oplus \quad \sin(n\omega_0 t) \\ 1 \quad \ominus \quad -\frac{\cos(n\omega_0 t)}{n\omega_0} \\ 0 \quad \quad \quad -\frac{\sin(n\omega_0 t)}{(n\omega_0)^2} \end{array}$$

$$x(t) = \frac{A}{2} + \sum_{n=1}^{\infty} \left[ \frac{-A}{\pi n} \cdot \sin(\omega_n t) \right]$$

$$= \frac{A}{2} + \sum_{n=1}^{\infty} \left[ \frac{-A}{\pi n} \sin\left(\frac{2\pi n t}{T}\right) \right] \quad \#$$

Example 6.1 #

سلسلة فورييه، الدالة



Solution: —

1) Fundamental period  $\Rightarrow T_0 = \pi$  s

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{\pi} = 2 \text{ rad/s}$$

2)  $x(t) = e^{-\frac{t}{2}} \quad 0 < t < \pi$

3)  $a_0 = \frac{1}{T} \int_{T_0} f(t) dt$

$$a_0 = \frac{1}{\pi} \int_0^{\pi} e^{-\frac{t}{2}} dt \Rightarrow \frac{1}{\pi} \left( \frac{e^{-\frac{t}{2}}}{(-1/2)} \right) \Big|_0^{\pi}$$

$$= -\frac{2}{\pi} (e^{-\frac{\pi}{2}} - e^0)$$

$$= -\frac{2}{\pi} (e^{-\frac{\pi}{2}} - 1) = 0.504$$

$$\boxed{a_0 = 0.504}$$

$$a_n = \frac{2}{T} \int_0^{\pi} f(t) \cdot \cos(n\omega_0 t) dt$$

$$= \frac{2}{\pi} \int_0^{\pi} e^{-\frac{t}{2}} \left[ \frac{e^{jn\omega_0 t} + e^{-jn\omega_0 t}}{2} \right] dt \quad \omega_0 = 2$$

$$a_n = \frac{1}{\pi} \int_0^{\pi} \left( e^{(-\frac{1}{2} + j2n)t} + e^{(-\frac{1}{2} - j2n)t} \right) dt$$

$$= \frac{1}{\pi} \left[ \frac{e^{(-\frac{1}{2} + j2n)t}}{(-\frac{1}{2} + j2n)} \Big|_0^{\pi} + \frac{e^{(-\frac{1}{2} - j2n)t}}{(-\frac{1}{2} - j2n)} \Big|_0^{\pi} \right]$$

$$= \frac{1}{\pi} \left[ \frac{e^{(-\frac{1}{2} + j2n)\pi} - 1}{-\frac{1}{2} + j2n} + \frac{e^{(-\frac{1}{2} - j2n)\pi} - 1}{(-\frac{1}{2} - j2n)} \right]$$

$$= \frac{2}{\pi} \left[ \frac{e^{-\frac{\pi}{2}} e^{j2n\pi} - 1}{-1 + j4n} + \frac{e^{-\frac{\pi}{2}} e^{-j2n\pi} - 1}{-1 - j4n} \right] \quad \text{قوسية الجانبي}$$

$$= \frac{2}{\pi} \left[ \frac{-e^{-\frac{\pi}{2}} e^{-j2n\pi} + 1 + j4n e^{-\frac{\pi}{2}} e^{-j2n\pi}}{1 + 16n^2} + \frac{-e^{-\frac{\pi}{2}} e^{j2n\pi} - 1 - j4n e^{-\frac{\pi}{2}} e^{j2n\pi}}{1 + 16n^2} \right]$$

$$= \frac{2}{\pi} e^{-\frac{\pi}{2}} \left[ \frac{-e^{-j2n\pi} + \frac{2}{e^{-\frac{\pi}{2}}} + j4n e^{-j2n\pi} - e^{j2n\pi} - j4n e^{j2n\pi}}{1 + 16n^2} \right]$$

$$= \frac{2}{\pi} e^{-\frac{\pi}{2}} \left[ \frac{-\frac{2}{2} (e^{j2n\pi} + e^{-j2n\pi}) - j4n \left( \frac{e^{j2n\pi} - e^{-j2n\pi}}{j2} \right) \cdot j2 + \frac{2}{e^{-\pi/2}}}{(1 + 16n^2)} \right]$$

$$= \frac{2}{\pi} e^{-\frac{\pi}{2}} \left[ \frac{-2 \cos(2n\pi) + 8n \cdot \sin(2n\pi) + 2/e^{-\pi/2}}{(1 + 16n^2)} \right]$$

$$a_n = \boxed{0.504 \left( \frac{2}{1 + 16n^2} \right)}$$

$$b_n = \frac{2}{\pi} \int_0^{\pi} e^{-\frac{t}{2}} \sin(n\omega_0 t) dt$$

$$= \frac{2}{\pi} \int_0^{\pi} e^{-\frac{t}{2}} \left( \frac{e^{jn2t} - e^{-j2nt}}{j2} \right) dt$$

$$\boxed{\omega_0 = 2}$$

$$= \frac{1}{\pi j} \int_0^{\pi} \left( \frac{e^{(-\frac{1}{2} + j2n)t}}{(-\frac{1}{2} + j2n)} - \frac{e^{(-\frac{1}{2} - j2n)t}}{(-\frac{1}{2} - j2n)} \right) dt$$

$$= \frac{1}{\pi j} \left[ \frac{e^{(-\frac{1}{2} + j2n)t}}{(-\frac{1}{2} + j2n)} - \frac{e^{(-\frac{1}{2} - j2n)t}}{(-\frac{1}{2} - j2n)} \right]_0^{\pi}$$

$$= \frac{2}{\pi j} \left[ \frac{e^{(-\frac{1}{2} + j2n)\pi} - 1}{-1 + j4n} - \frac{e^{(-\frac{1}{2} - j2n)\pi} - 1}{-1 - j4n} \right]$$

$$= \frac{2}{\pi j} \left[ \frac{e^{(-\frac{1}{2} + j2n)\pi} - 1}{-1 + j4n} - \frac{e^{(-\frac{1}{2} - j2n)\pi} - 1}{-1 - j4n} \right]$$

$$= \frac{2}{\pi j} \left[ \frac{-e^{-\frac{\pi}{2}} e^{j2n\pi} - (-1 + j4n) + e^{-\frac{\pi}{2}} e^{-j2n\pi} - (-1 - j4n)}{(1 + 16n^2)} \right]$$

$$= \frac{2}{\pi j} e^{-\frac{\pi}{2}} \left[ \frac{-e^{j2n\pi} - j4n + e^{-j2n\pi} + j4n}{(1 + 16n^2)} \right]$$

$$= \frac{2}{\pi j} e^{-\frac{\pi}{2}} \left[ \frac{-2j4n \left( \frac{e^{j2n\pi} + e^{-j2n\pi}}{2} \right) + j8n}{(1 + 16n^2)} \right]$$

$$= \frac{2 e^{-\frac{\pi}{2}}}{j\pi} \left[ \frac{-j8n \cos(n\pi) + j8n}{(1 + 16n^2)} \right]$$

$$= \frac{2 e^{-\frac{\pi}{2}}}{\pi} \left[ \frac{(-8n + 8n e^{-\pi/2})}{1 + 16n^2} \right] \Rightarrow \frac{2}{\pi} \left[ \frac{+8n(-e^{-\frac{\pi}{2}} + 1)}{1 + 16n^2} \right]$$

$$b_n = \frac{2}{\pi} \left[ \frac{8n(1 - e^{-\pi/2})}{1 + 16n^2} \right]$$

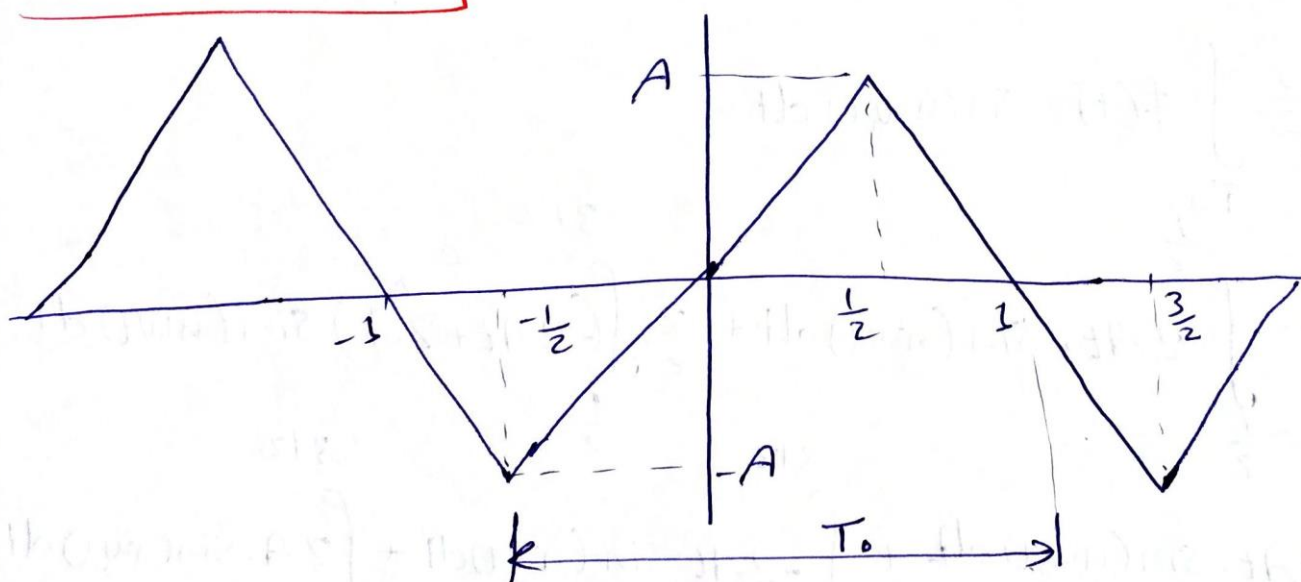
$$b_n = 0.504 \left[ \frac{8n}{1 + 16n^2} \right]$$

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$$

$$= 0.504 \left[ 1 + \sum_{n=1}^{\infty} \frac{2}{1 + 16n^2} (\cos(2nt) + 4 \sin(2nt)) \right]$$

Example # 6.2

مسألة التكرار



$$T_0 = 2, \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2} = \boxed{\pi \text{ rad/s}}$$

لحساب معادلة الاقتران  $f(t)$  نقوم بايجاد الجيب

$$(x_1, y_1), (x_2, y_2)$$

$$-\frac{1}{2} \leq t \leq \frac{1}{2}$$

$$\frac{y+A}{t+\frac{1}{2}} = \frac{A-(-A)}{\frac{1}{2}-(-\frac{1}{2})} \Rightarrow \frac{y+A}{t+\frac{1}{2}} = 2A$$

$$y+A = 2At+A$$

$$\boxed{y = 2At} \quad \checkmark$$

$$-\frac{1}{2} \leq t \leq \frac{1}{2}$$

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$$\left(\frac{x_1}{2}, y_1\right), \left(\frac{x_2}{2}, y_2\right)$$

$$\frac{y-A}{t-\frac{1}{2}} = \frac{-A-A}{\frac{3}{2}-\frac{1}{2}} \Rightarrow \frac{y-A}{t-\frac{1}{2}} = -2A$$

$$y-A = -2At+A \Rightarrow \boxed{y = -2At+2A} \quad \frac{1}{2} \leq t \leq \frac{3}{2}$$

$$f(t) = \begin{cases} 2At & , -\frac{1}{2} \leq t \leq \frac{1}{2} \\ -2At+2A & , \frac{1}{2} \leq t \leq \frac{3}{2} \end{cases}$$

$$\boxed{a_n = 0}, \boxed{a_0 = 0} \quad \leftarrow \text{odd fun. no } a_n$$

$$b_n = \frac{2}{T} \int_{T_0}^{T_0+T} f(t) \cdot \sin(n\omega t) dt$$

$$b_n = \frac{2}{2} \int_{-\frac{1}{2}}^{\frac{1}{2}} 2At \cdot \sin(n\omega t) dt + \frac{2}{2} \int_{\frac{1}{2}}^{\frac{3}{2}} (-2At+2A) \sin(n\omega t) dt$$

$$b_n = \int_{-\frac{1}{2}}^{\frac{1}{2}} 2At \cdot \sin(n\omega t) dt + \int_{\frac{1}{2}}^{\frac{3}{2}} -2At \sin(n\omega t) dt + \int_{\frac{1}{2}}^{\frac{3}{2}} 2A \cdot \sin(n\omega t) dt$$

$\Downarrow$  الجزء الأول       $\Downarrow$  الجزء الثاني       $\Downarrow$  الجزء الثالث

|       |           |                                        |
|-------|-----------|----------------------------------------|
| $2At$ | $\oplus$  | $\sin(n\omega t)$                      |
| $2A$  | $\ominus$ | $-\frac{\cos(n\omega t)}{n\omega}$     |
| $0$   |           | $-\frac{\sin(n\omega t)}{n^2\omega^2}$ |

$$= \left. \frac{-2At}{n\pi} \cos(n\pi t) + \frac{2A}{n^2\omega^2} \sin(n\pi t) \right|_{-\frac{1}{2}}^{\frac{1}{2}}$$

الجزء الأول

$$= -\frac{2A \cdot \frac{1}{2}}{n\pi} \cos\left(\frac{n\pi}{2}\right) + \frac{2A}{n^2\pi^2} \sin\left(\frac{n\pi}{2}\right) + \frac{2A \cdot -\frac{1}{2}}{n\pi} \cos\left(-\frac{n\pi}{2}\right)$$

$$- \frac{2A}{n^2\pi^2} \sin\left(-\frac{n\pi}{2}\right) \Rightarrow$$

$$\underbrace{-\frac{A}{n\pi} \cdot \cos\left(\frac{n\pi}{2}\right)} + \frac{2A}{n^2\pi^2} \sin\left(\frac{n\pi}{2}\right) - \underbrace{\frac{A}{n\pi} \cos\left(\frac{n\pi}{2}\right)} + \frac{2A}{n^2\pi^2} \sin\left(\frac{n\pi}{2}\right)$$

$$= \left[ -\frac{2A}{n\pi} \cos\left(\frac{n\pi}{2}\right) + \frac{4A}{n^2\pi^2} \sin\left(\frac{n\pi}{2}\right) \right]$$

لا تسجل الأول

الكثير الثاني

$$\begin{array}{l} -2At \sin(n\omega t) \\ -2A \begin{array}{l} \oplus \\ \ominus \end{array} \begin{array}{l} \cos(n\omega t) \\ -\frac{\cos(n\omega t)}{n\omega} \\ -\frac{\sin(n\omega t)}{n^2\omega^2} \end{array} \\ 0 \end{array}$$

$$\frac{2At}{n\pi} \cos(n\pi t) - \frac{2A}{n^2\pi^2} \sin(n\pi t) \quad \begin{array}{l} 3/2 \\ 1 \\ 1/2 \end{array}$$

$$\begin{aligned} & \frac{2A \cdot 3}{n\pi} \cos\left(\frac{n\pi \cdot 3}{2}\right) - \frac{2A}{n^2\pi^2} \sin\left(\frac{n\pi \cdot 3}{2}\right) - \frac{2A \cdot \frac{1}{2}}{n\pi} \cos\left(\frac{n\pi}{2}\right) + \frac{2A}{n^2\pi^2} \sin\left(\frac{n\pi}{2}\right) \\ &= \frac{3A}{n\pi} \cos\left(\frac{3\pi n}{2}\right) - \frac{2A}{n^2\pi^2} \sin\left(\frac{3\pi n}{2}\right) - \frac{A}{n\pi} \cos\left(\frac{n\pi}{2}\right) + \frac{2A}{n^2\pi^2} \sin\left(\frac{n\pi}{2}\right) \\ &= \frac{3A}{n\pi} \cos\left(\frac{n\pi}{2}\right) - \frac{2A}{n^2\pi^2} \sin\left(\frac{n\pi}{2}\right) - \frac{A}{n\pi} \cos\left(\frac{n\pi}{2}\right) + \frac{2A}{n^2\pi^2} \sin\left(\frac{n\pi}{2}\right) \end{aligned}$$

$$= \left[ \frac{2A}{n\pi} \cos\left(\frac{n\pi}{2}\right) + \frac{4A}{n^2\pi^2} \sin\left(\frac{n\pi}{2}\right) \right]$$

لا تسجل الثاني

$$= \int_{\frac{1}{2}}^{\frac{3}{2}} 2A \sin(n\omega_0 t) dt$$

$$= \frac{-2A \cos(n\omega_0 t)}{n\omega_0} \Big|_{\frac{1}{2}}^{\frac{3}{2}}$$

$$\Rightarrow \frac{-2A}{n\pi} \left[ \cos\left(n\pi \cdot \frac{3}{2}\right) - \cos\left(n\pi \cdot \frac{1}{2}\right) \right]$$

$$= \frac{-2A}{n\pi} \left[ \cos\left(n\frac{\pi}{2}\right) - \cos\left(n\frac{\pi}{2}\right) \right] = \boxed{0}$$

$$b_n = \cancel{\frac{-2A}{n\pi} \cos\left(n\frac{\pi}{2}\right)} + \frac{4A}{n^2\pi^2} \sin\left(n\frac{\pi}{2}\right) + \cancel{\frac{2A}{n\pi} \cos\left(n\frac{\pi}{2}\right)}$$

$$\frac{4A}{n^2\pi^2} \sin\left(n\frac{\pi}{2}\right)$$

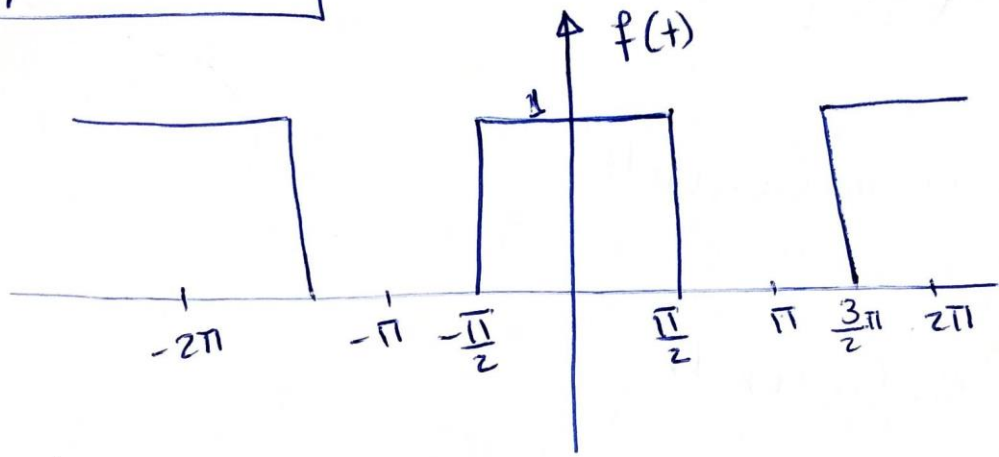
$$\boxed{b_n = \frac{8A}{n^2\pi^2} \sin\left(n\frac{\pi}{2}\right)}$$

$$b_n = \begin{cases} 0, & n \text{ even} \\ \frac{8A}{n^2\pi^2}, & n = 1, 5, 9, 13, \dots \\ -\frac{8A}{n^2\pi^2}, & n = 3, 7, 11, 15, \dots \end{cases}$$

#

# Example 6.3 #1

دالة الجهد الكهربائي



even function  
 $b_n = 0$

Solution:-

[1] Fundamental period  $\Rightarrow \frac{3\pi}{2} - (-\frac{\pi}{2}) = \boxed{2\pi}$

$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2\pi} = \boxed{1 \text{ rad/s}}$

[2]  $f(t) = \begin{cases} 1 & , -\frac{\pi}{2} \leq t < \frac{\pi}{2} \\ 0 & , \frac{3\pi}{2} \leq t < \frac{5\pi}{2} \end{cases}$

[3]  $a_0 = \frac{1}{T} \int_{T_0} f(t) dt \Rightarrow a_0 = \frac{1}{2\pi} \left[ \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1 dt + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 0 dt \right]$

$a_0 = \frac{1}{2\pi} \left[ \frac{\pi}{2} - (-\frac{\pi}{2}) \right] \Rightarrow \frac{\pi}{2\pi} = \boxed{\frac{1}{2}}$

$a_n = \frac{2}{T} \int_{T_0} f(t) \cdot \cos(n\omega_0 t) dt$

$= \frac{2}{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(n\omega_0 t) dt + 0 \Rightarrow \frac{1}{\pi} \frac{\sin(n\omega_0 t)}{n\omega_0} \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$

$\frac{1}{\pi} \cdot \left[ \frac{\sin(n\frac{\pi}{2}) - \sin(-n\frac{\pi}{2})}{n} \right]$

$a_n = \frac{1}{\pi} \left[ \frac{\sin(n\frac{\pi}{2}) + \sin(\frac{n\pi}{2})}{n} \right]$

$$a_n = \frac{2}{\pi n} \sin\left(\frac{n\pi}{2}\right)$$

$$b_n = \frac{2}{T} \int_{T_0}^T f(t) \cdot \sin(n\omega_0 t) dt$$

$$= \frac{2}{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin(n\omega_0 t) dt$$

$$= \frac{1}{\pi} \left[ -\frac{\cos(n\omega_0 t)}{n\omega_0} \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \Rightarrow \frac{1}{\pi} \left[ \frac{-\cos\left(\frac{n\pi}{2}\right) + \cos\left(-\frac{n\pi}{2}\right)}{n} \right]$$

$$b_n = \frac{1}{\pi} \left[ \frac{-\cos\left(\frac{n\pi}{2}\right) + \cos\left(\frac{n\pi}{2}\right)}{n} \right]$$

$$b_n = 0$$

(even function)

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \cancel{b_n \sin(n\omega_0 t)}$$

$$f(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{2}{\pi n} \cos(nt) \cdot \sin\left(\frac{n\pi}{2}\right)$$

$$f(t) = \frac{1}{2} + \frac{2}{\pi} \left[ \frac{1}{1} \cos(t) \cdot \sin\left(\frac{\pi}{2}\right) + \frac{1}{2} \cos(2t) \cdot \cancel{\sin(\pi)} + \frac{1}{3} \cos(3t) \cdot \sin\left(\frac{3\pi}{2}\right) + \frac{1}{4} \cos(4t) \cdot \cancel{\sin(2\pi)} + \frac{1}{5} \cos(5t) \cdot \sin\left(\frac{5\pi}{2}\right) + \dots \right]$$

$$f(t) = \frac{1}{2} + \frac{2}{\pi} \left[ \cos(t) - \frac{1}{3} \cos(3t) + \frac{1}{5} \cos(5t) - \dots \right]$$

The Exponential Fourier series: — (Complex exponential)

$$f(t) = \sum_{n=-\infty}^{\infty} \underline{D_n} e^{jn\omega t}$$

$D_n$  : complex number

$$D_n = \frac{1}{T_0} \int f(t) e^{-jn\omega t} dt \quad \text{at } n \neq 0$$

$$D_0 = \frac{1}{T} \int f(t) dt \quad \text{at } n = 0$$

$$\Rightarrow \underline{D_0 = a_0}$$

$$\Rightarrow a_n = 2 \operatorname{Re} \{ D_n \}$$

$$\Rightarrow b_n = -2 \operatorname{Im} \{ D_n \}$$

$$\underline{D_n = \frac{1}{2} (a_n - jb_n)}$$

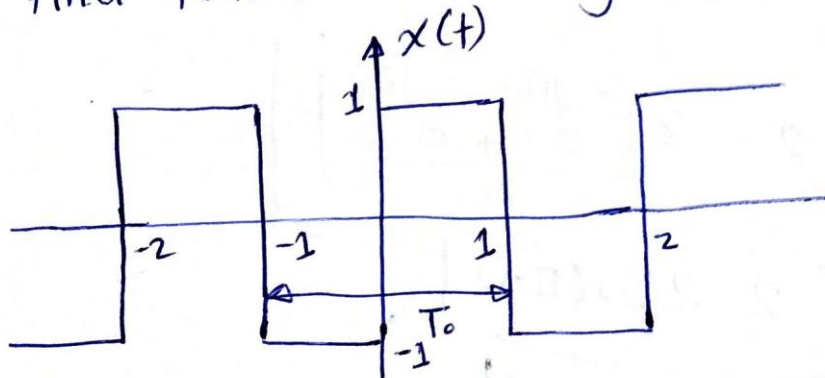
\* Important rules: —

$$e^{j\omega t} = \cos(\omega t) + j \sin(\omega t)$$

$$\cos(\omega t) = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

$$\underline{\sin(\omega t) = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}}$$

Example: Find Fourier series using exponential.



$$T_0 = 1 - (-1) = \underline{2}$$

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2} = \underline{\pi}$$

$$x(t) = \begin{cases} -1, & -1 < t < 0 \\ +1, & 0 < t < 1 \end{cases}$$

$$x(t) = \sum_{n=-\infty}^{\infty} D_n e^{j\omega_n t}$$

$$D_n = \frac{1}{T_0} \int_{T_0} x(t) \cdot e^{-j\omega_n t} dt$$

$$= \frac{1}{2} \left[ \int_{-1}^0 -1 e^{-j\omega_n t} dt + \int_0^1 1 e^{-j\omega_n t} dt \right]$$

$$= \frac{1}{2} \left[ \left. \frac{-e^{-j\omega_n t}}{-j\omega_n} \right|_{-1}^0 + \left. \frac{e^{-j\omega_n t}}{-j\omega_n} \right|_0^1 \right]$$

$$= \frac{1}{2} \left[ \frac{-e^0}{-j\omega_n} + \frac{e^{-j\omega_n(-1)}}{-j\omega_n} + \frac{e^{-j\omega_n(1)}}{-j\omega_n} + \frac{1}{j\omega_n} \right]$$

$$= \frac{1}{2} \left[ \frac{1}{j\omega_n} + \frac{e^{j\omega_n}}{j\omega_n} - \frac{e^{-j\omega_n}}{j\omega_n} + \frac{1}{j\omega_n} \right]$$

$$= \frac{1}{2} \cdot \frac{1}{j\omega_n} \left[ 1 - e^{-j\pi n} - e^{j\pi n} + 1 \right]$$

$$= \frac{1}{2j\pi n} \left[ 2 - e^{j\pi n} - e^{-j\pi n} \right]$$

$$= \frac{1}{2j\pi n} \left[ 2 - 2 \left[ \frac{e^{j\pi n} + e^{-j\pi n}}{2} \right] \right]$$

$$= \frac{1}{2j\pi n} \left[ 2 - 2 \cos(\pi n) \right]$$

$$D_n = \frac{1}{j\pi n} [1 - \cos(\pi n)]$$

$$D_0 = \frac{1}{T_0} \int f(t) dt \quad \text{at } n=0$$

$$D_0 = \frac{1}{2} \left[ \int_{-1}^0 -1 dt + \int_0^1 1 dt \right]$$

$$D_0 = \frac{1}{2} \left[ -t \Big|_{-1}^0 + t \Big|_0^1 \right]$$

$$= \frac{1}{2} \left[ -(0+1) + 1-0 \right]$$

$$= \frac{1}{2} \left[ -1+1 \right] = 0 \Rightarrow \boxed{D_0 = 0}$$

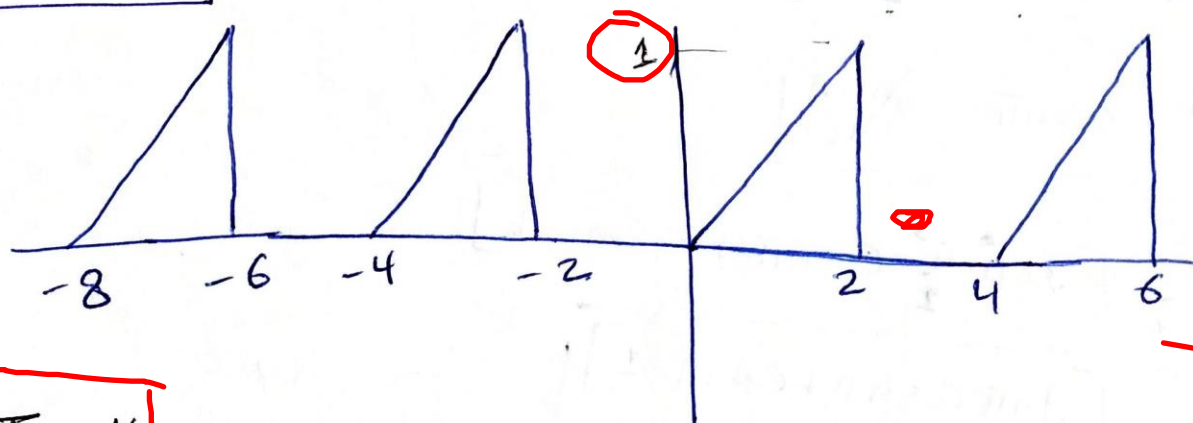
$$x(t) = \sum_{n=-\infty}^{\infty} \frac{1}{j\pi n} [1 - \cos \pi n] e^{jn\omega_0 t}$$

$$\text{If } n \text{ is even} \Rightarrow \cos \pi n = 1$$

$$\text{If } n \text{ is odd} \Rightarrow \cos \pi n = -1$$

$$D_n = \begin{cases} 0, & n \text{ even} \\ \frac{2}{j\pi n}, & n \text{ odd} \end{cases}$$

Example # Find Fourier series using exponential.



$$\boxed{T = 4}$$

$$\omega_0 = \frac{2\pi}{4} = \boxed{\frac{\pi}{2}}$$

$$f(t) = \begin{cases} \frac{1}{2}t, & 0 \leq t < 2 \\ 0, & 2 \leq t < 4 \end{cases}$$

$$D_n = \frac{1}{T} \int_T f(t) e^{-jn\omega_0 t} dt$$

$$= \frac{1}{4} \left[ \int_0^2 \frac{1}{2}t e^{-jn\omega_0 t} dt + 0 \right]$$

$$= \frac{1}{8} \left[ \int_0^2 t e^{-jn\omega_0 t} dt \right]$$

$$= \frac{1}{8} \left[ \frac{t e^{-jn\omega_0 t}}{-jn\omega_0} + \frac{e^{-jn\omega_0 t}}{n^2 \omega_0^2} \right]_0^2$$

$$= \frac{1}{8} \left[ \frac{2 e^{-jn\frac{\pi}{2} \cdot 2}}{-jn\frac{\pi}{2}} - 0 + \frac{e^{-jn\frac{\pi}{2} \cdot 2}}{n^2 \frac{\pi^2}{4}} - \frac{1}{n^2 \frac{\pi^2}{4}} \right]$$

$$= \frac{1}{8} \cdot \frac{1}{n^2 \frac{\pi^2}{4}} \left[ \frac{2 \cdot n^2 \frac{\pi^2}{4} e^{-jn\pi}}{-j} + e^{-jn\pi} - 1 \right]$$

$$= \frac{1}{2\pi^2 n^2} \left[ j 2 n^2 \frac{\pi^2}{4} e^{-jn\pi} + e^{-jn\pi} - 1 \right]$$

$$\boxed{e^{-jn\pi} = \cos n\pi - j \sin n\pi}$$

$$= \frac{1}{2\pi^2 n^2} \left[ j 2 n^2 \frac{\pi^2}{4} \cos n\pi + \cos n\pi - 1 \right]$$

$$\boxed{D_n = \frac{1}{2\pi^2 n^2} \left[ j n \pi \cos n\pi + \cos n\pi - 1 \right]}$$

|   |   |                                             |
|---|---|---------------------------------------------|
| t | ⊕ | $e^{-jn\omega_0 t}$                         |
| 1 | ⊖ | $\frac{e^{-jn\omega_0 t}}{-jn\omega_0}$     |
| 0 |   | $\frac{e^{-jn\omega_0 t}}{(-jn\omega_0)^2}$ |
|   |   | $\downarrow$                                |
|   |   | $-\omega_0^2 n^2$                           |

$n \neq 0$

$$at n=0$$

$$D_0 = \frac{1}{0} = \infty \Rightarrow (\text{undefined})$$

لذلك ننصب الى انقافه الأصلي حساب  $D_0$

$$D_0 = \frac{1}{T} \int_{T_0}^2 f(t) dt$$

$$= \frac{1}{4} \int_0^2 \frac{1}{2} t dt \Rightarrow \frac{1}{8} \left. \frac{t^2}{2} \right|_0^2 \Rightarrow \frac{1}{8} \frac{(4-0)}{2}$$

$$\Rightarrow \boxed{D_0 = \frac{1}{4}}$$

$$\Rightarrow D_n = \begin{cases} \frac{1}{4}, & n=0 \\ \frac{j\pi n}{2\pi^2 n^2}, & n \text{ even} \\ -\frac{1}{2\pi^2 n^2} [j\pi n + 2], & n \text{ odd} \end{cases}$$

$$\boxed{\begin{aligned} \cos n\pi &= 1, n, \text{ even} \\ \cos n\pi &= -1, n \text{ odd} \end{aligned}}$$

$$f(t) = \begin{cases} \frac{1}{4} + \sum_{n=-\infty}^{\infty} \frac{j}{2\pi n} e^{j\frac{\pi}{2}nt} & n \text{ even} \\ \frac{1}{4} + \sum_{n=-\infty}^{\infty} \frac{-1}{2\pi^2 n^2} [j\pi n + 2] e^{j\frac{\pi}{2}nt} & n \text{ odd} \end{cases}$$

$$\boxed{n \neq 0}$$

\* line spectra : ( Amplitude spectrum & phase spectrum )

\* Complex exponential  $e^{jn\omega}$  Amplitude spectrum  
 $D_n$  أو  $C_n$  مع تردد  $\omega$  مع  $n$  (تردد  $n$  لأن  $\omega$  الزمان  $t$  مع  $n$ )  
 كما جأ مع  $\omega$   $C_n$  مع  $n$  phase spectrum  $\angle D_n$  مع  $n$  phase  
 مع  $n$  exponential مع  $n$

\* مواضع

$$\boxed{1} \quad \begin{aligned} \sin n\pi &= 0 \\ \sin 2n\pi &= 0 \end{aligned}$$

$$\boxed{2} \quad \begin{aligned} \cos n\pi &= (-1)^n \\ \cos 2n\pi &= 1 \end{aligned}$$

$$\boxed{3} \quad \begin{aligned} e^{-jn\pi} &= \cos n\pi - j\sin n\pi \\ &= (-1)^n - 0 \Rightarrow e^{-jn\pi} = (-1)^n \end{aligned}$$

$$\boxed{4} \quad \begin{aligned} e^{-j2n\pi} &= \cos 2n\pi - j\sin 2n\pi \\ &= 1 - 0 \\ e^{-j2n\pi} &= \boxed{1} \end{aligned}$$

\* Amplitude spectrum  $\Rightarrow$  even (symmetric about Y-axis)

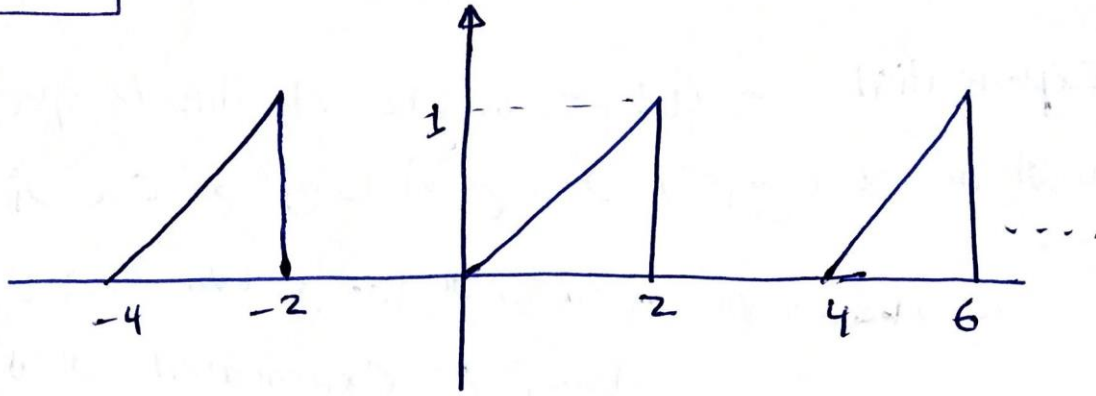
$$|D_n| = |D_{-n}|$$

\* phase spectrum  $\Rightarrow$  odd (symmetric about origin)

$$\left\{ \angle D_n = -\angle D_{-n} \right\}$$

Example #

Sketch line spectrum



Solution:—

$$T_0 = 4, \quad \omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$f(t) = \begin{cases} \frac{1}{2}t & 0 < t < 2 \\ 0 & 2 < t < 4 \end{cases}$$

$$\underline{D_n} = \frac{1}{T_0} \int_0^{T_0} f(t) e^{-jn\omega_0 t} dt$$

$$= \frac{1}{4} \int_0^2 \frac{1}{2}t e^{-jn\omega_0 t} dt + \int_2^4 0 dt$$

$$= \frac{1}{8} \int_0^2 t e^{-jn\omega_0 t} dt$$

$$= \frac{1}{8} \left[ \frac{-t e^{-jn\omega_0 t}}{jn\omega_0} + \frac{e^{-jn\omega_0 t}}{\omega_0^2 n^2} \right]_0^2$$

$$= \frac{1}{8} \left[ \frac{-2 e^{-j\frac{\pi}{2}n \cdot 2}}{j\frac{\pi}{2}n} + \frac{e^{-j\frac{\pi}{2}n \cdot 2}}{(\frac{\pi}{2}n)^2} - \left( \frac{1}{(\frac{\pi}{2}n)^2} \right) \right]$$

$$= \frac{1}{8} \left[ \frac{j2 e^{-j\pi n}}{(\frac{\pi}{2}n)} + \frac{e^{-j\pi n}}{(\frac{\pi}{2}n)^2} - \frac{1}{(\frac{\pi}{2}n)^2} \right]$$

$$\begin{array}{l} t \oplus \\ 1 \ominus \\ 0 \end{array} \begin{array}{l} e^{-jn\omega_0 t} \\ e^{-jn\omega_0 t} \\ e^{-jn\omega_0 t} \end{array} \begin{array}{l} \\ -jn\omega_0 \\ (-jn\omega_0)^2 \end{array}$$

$(j)^2 = -1$

$-\omega_0^2 n^2$

$$= \frac{1}{8} \cdot \frac{1}{\left(\frac{\pi n}{2}\right)^2} \left[ j\pi e^{-j\pi n} \cdot \frac{\pi n}{2} + e^{-j\pi n} - 1 \right]$$

$$= \frac{1}{8} \cdot \frac{1}{\frac{\pi^2 n^2}{4}} \left[ j\pi n e^{-j\pi n} + e^{-j\pi n} - 1 \right]$$

$$= \frac{1}{2\pi^2 n^2} \left[ j\pi n e^{-j\pi n} + e^{-j\pi n} - 1 \right]$$

$$\Rightarrow \boxed{e^{-j\pi n} = \cos \pi n - j \sin \pi n}$$

$$= \frac{1}{2\pi^2 n^2} \left[ j\pi n \cdot \cos \pi n + \cos \pi n - 1 \right]$$

$$\boxed{D_n = \frac{1}{2\pi^2 n^2} \left[ j\pi n \cos \pi n + \cos \pi n - 1 \right]}$$

at  $n=0$

$$D_0 = \frac{1}{0} = \infty \quad (\text{undefined})$$

عند  $n=0$  نحتاج للقانون الآخر كما يلي

$$D_0 = \frac{1}{T} \int_{T_0} f(t) dt$$

$$D_0 = \frac{1}{4} \int_0^2 \frac{1}{2} t dt \Rightarrow \frac{1}{4} \left( \frac{1}{4} t^2 \Big|_0^2 \right)$$

$$\frac{1}{4} \left( \frac{1}{4} \times 4 - 0 \right) = \boxed{\frac{1}{4}}$$

$$D_n = \begin{cases} \frac{1}{4} & , n=0 \\ \frac{j\pi n}{2\pi^2 n^2} & , n \text{ even} \\ \frac{-1}{2\pi^2 n^2} [j\pi n + 2] & , n \text{ odd} \end{cases}$$

$$\cos \pi n = 1 \quad n, \text{ even}$$

$$\cos \pi n = -1 \quad n, \text{ odd}$$

$$f(t) = \begin{cases} \frac{1}{4} + \sum_{n=-\infty}^{\infty} \frac{j}{2\pi n} e^{\frac{j\pi n t}{2}} & , n \text{ even } n \neq 0 \\ \frac{1}{4} + \sum_{n=-\infty}^{\infty} \frac{-1}{2\pi^2 n^2} [j\pi n + 2] e^{\frac{j\pi n t}{2}} & , n \text{ odd } n \neq 0 \end{cases}$$

line spectra \*  
 \* Complex ~  $|D_n|$  /  $D_n$  \*  
 ... ,  $n = \pm 2$  ,  $n = \pm 1$  ,  $n = 0$  \*  
 $D_n$  \*  
 \*  
 \*  
 \*

$$D_0 = \boxed{\frac{1}{4} \angle 0^\circ}$$

$$D_1 = \frac{-1}{2\pi^2(1)^2} [j\pi \times 1 + 2] = \frac{-j}{2\pi} - \frac{2}{2\pi^2} = \boxed{0.1886 \angle -122.48^\circ}$$

$$D_2 = \frac{j\pi(2)}{2\pi^2(2)^2} = \boxed{0.0795 \angle 90^\circ}$$

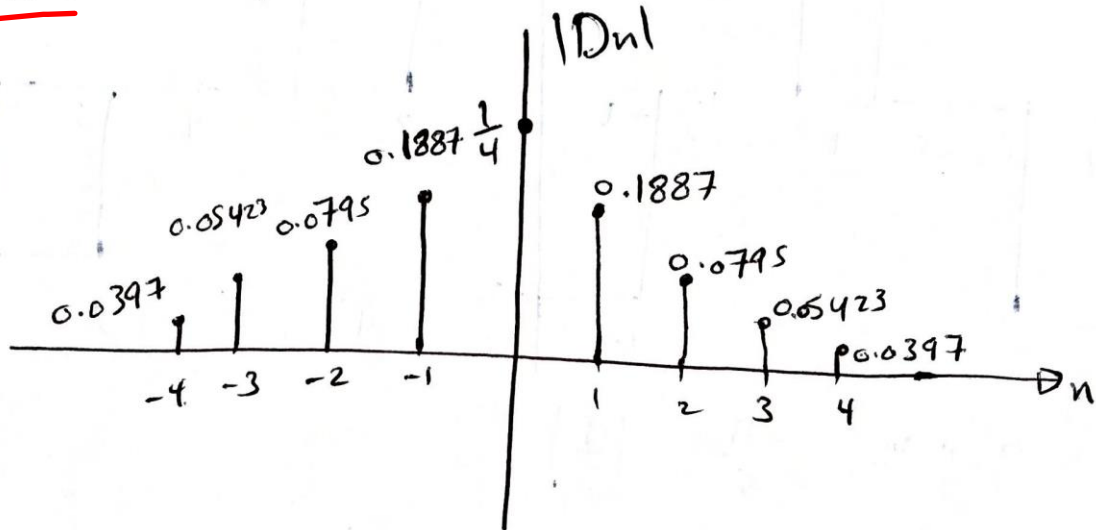
$$D_3 = \frac{-1}{2\pi^2(9)} [j\pi \times 3 + 2] = \boxed{0.05423 \angle -161.98^\circ}$$

$$D_4 = \frac{j\pi(4)}{2\pi^2(16)} = \boxed{0.0397 \angle 90^\circ}$$

ملاحظة عند حساب الزوايا يجب مراعاة الأبراج وذلك مع حساب إشارة كل حد  
 اكن الحصة والقياس هو الـ Complex number ( $D_n$ )

\* عنوان رسم Amplitude spectrum : تأخذ الـ amplitude  $|D_n|$

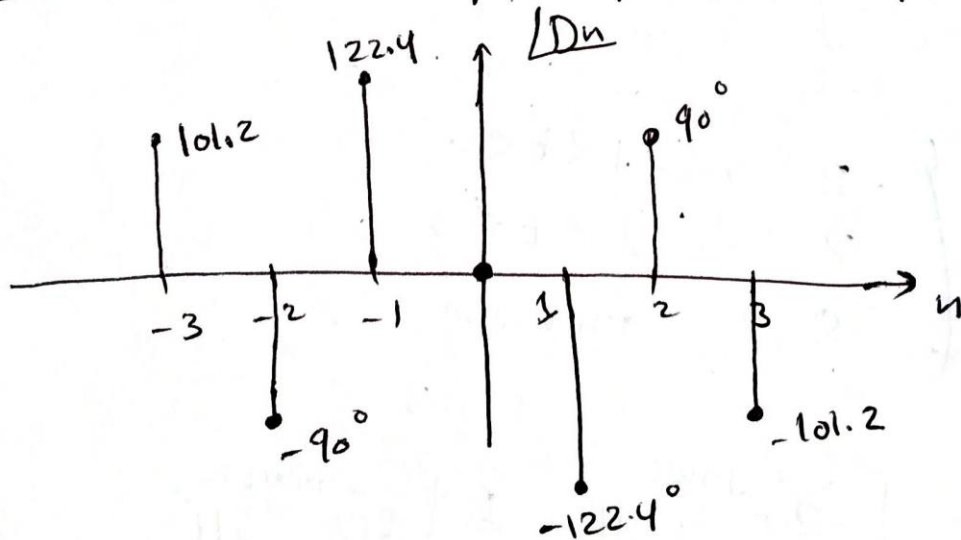
" even "



$$|D_n| = |D_{-n}|$$

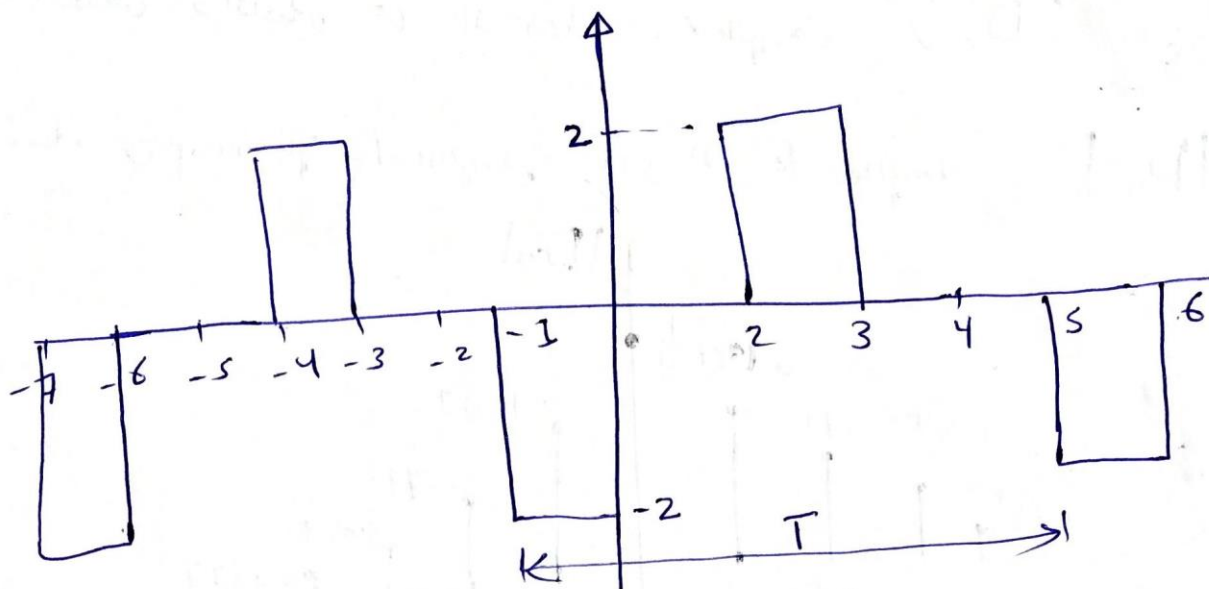
\* نلاحظ أنه الرسم متماثلة حول محور الصادات

\* عنوان رسم الـ phase spectrum : تأخذ الزوايا  $\angle D_n$



\* نلاحظ أنه الرسم متماثلة حول نقطة الأصل (odd.)

Example # | plot the line spectra ??



$$T_0 = 6, \omega_0 = \frac{2\pi}{6} = \boxed{\frac{\pi}{3}}$$

$$x(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$$

$$D_n = \frac{1}{6} \int_{-1}^5 x(t) e^{-jn\omega_0 t} dt$$

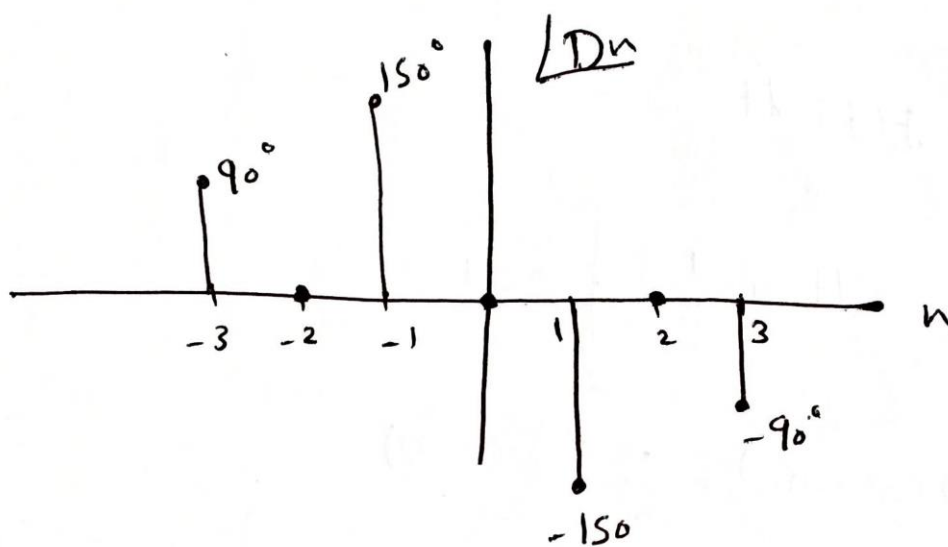
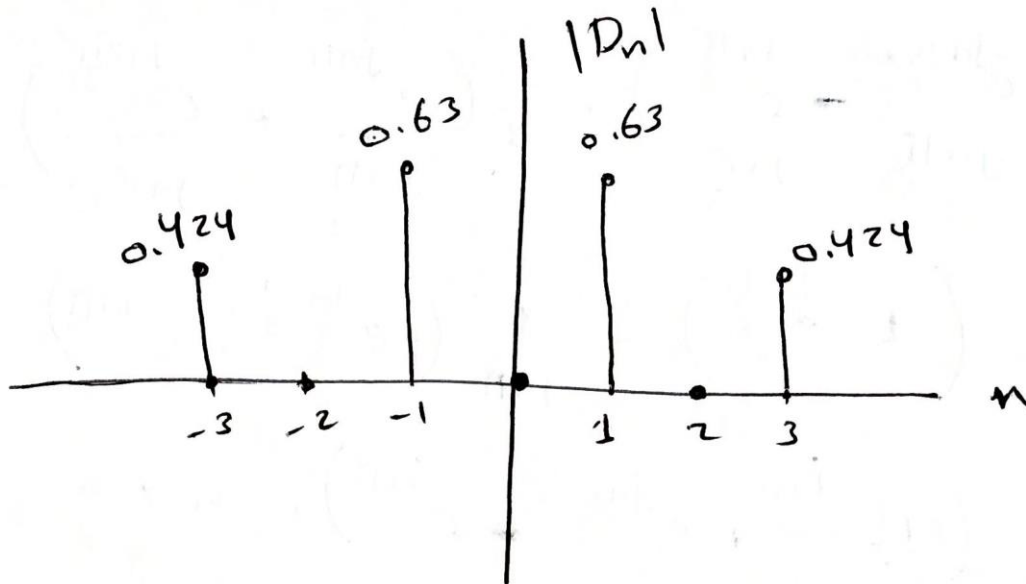
$$x(t) = \begin{cases} -2, & -1 < t < 0 \\ 2, & 2 < t < 3 \\ 0, & \text{otherwise} \end{cases}$$

$$D_n = \frac{1}{6} \int_{-1}^0 -2 \cdot e^{-jn\omega_0 t} dt + \frac{1}{6} \int_2^3 2 e^{-jn\omega_0 t} dt$$

$$= \frac{1}{6} \left( \frac{-2e^{-jn\omega_0 t}}{-jn\omega_0} \right) \Big|_{-1}^0 + \frac{1}{6} \left( \frac{2e^{-jn\omega_0 t}}{-jn\omega_0} \right) \Big|_2^3$$

$$= \frac{1}{6} \left( \frac{-2e^{-jn\frac{\pi}{3} \cdot 0}}{-jn\omega_0} + \frac{2e^{-jn\frac{\pi}{3}(-1)}}{-jn\omega_0} \right) + \frac{1}{6} \left( \frac{2e^{-jn\frac{\pi}{3} \cdot 3}}{-jn\omega_0} - \frac{2e^{-jn\frac{\pi}{3} \cdot 2}}{-jn\omega_0} \right)$$





Compact trigonometric :-

$$f(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n)$$

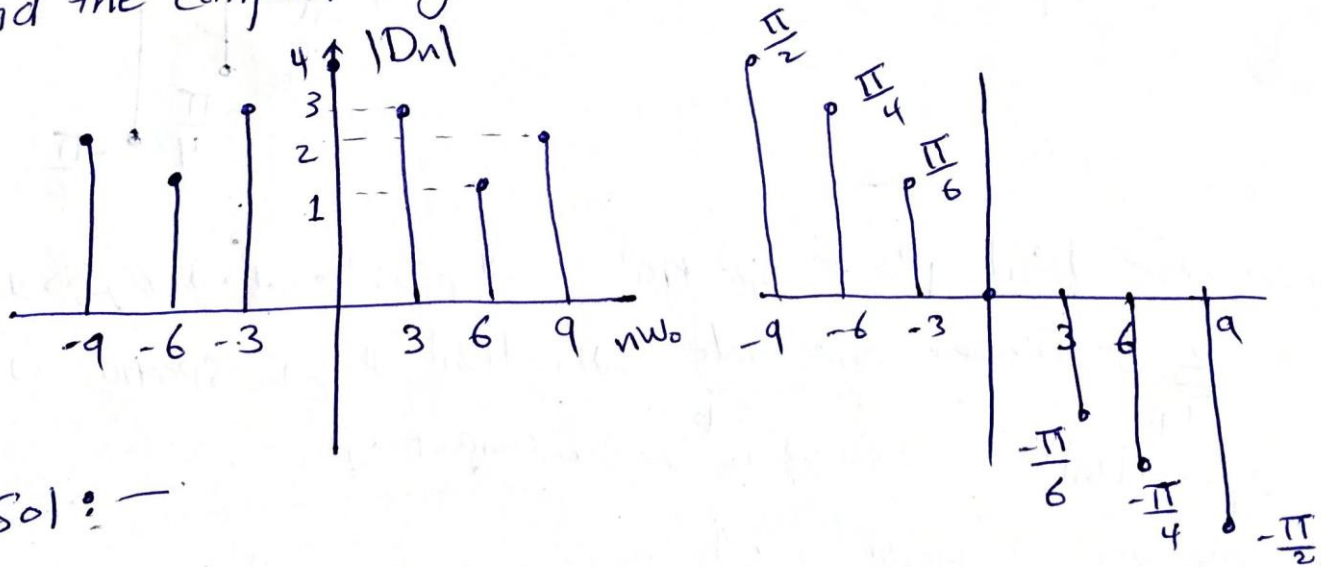
$$C_0 = a_0, \quad C_n = \sqrt{a_n^2 + b_n^2}, \quad \theta_n = \tan^{-1} \left( -\frac{b_n}{a_n} \right)$$

$$C_0 = D_0$$

$$C_n = 2|D_n| \quad n \geq 1$$

$$\theta_n = \frac{1}{D_n}$$

Example :- If you have the exponential Fourier spectra  
Find the compact trigonometric Fourier series.



Sol: -

\* هنا السؤال مريض في Fourier series او في compact  
مطلوب في أمثلة trigonometric compact exponential

$$C_0 = D_0 = 4$$

$w_2 = 3 \Rightarrow$  اضافہ سے کل ضربیں 6 ہیں۔

$$C_n = 2|D_n|, \quad n \geq 1, \quad \theta_n = \angle D_n$$

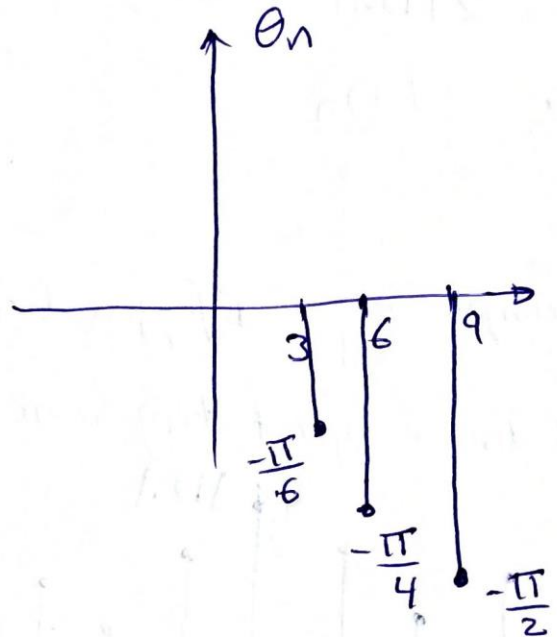
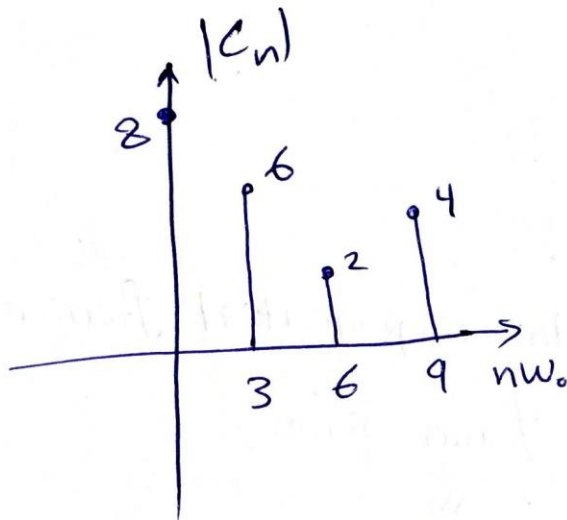
$$x(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n)$$

$$= 4 + 6 \cos\left(3t - \frac{\pi}{6}\right) + 2 \cos\left(6t - \frac{\pi}{4}\right) + 4 \cos\left(9t - \frac{\pi}{2}\right)$$

$$|C_1| = 2|D_1|$$

$$|C_2| = 2|D_2|$$

$$|C_3| = 2|D_3|$$



\* ملاحظة // لو حلت هنا رسم ال Spectra بتستخدم  $|C_n|$  يكون فيه

ال one-side spectrum و  $|D_n|$  و لكن one-side spectrum في 2

$C_n = 2|D_n|$  و  $\theta_n$  Compact مزودة عن  $n$  الموجبة

الملاحظة // للقول من Cartesian الى Polar في Complex number

$$Re + j Im.$$

Real part

imaginary part

$$D_n = |D_n| \angle \theta_n$$

$$|D_n| = \sqrt{(Re)^2 + (Im)^2}$$

$$\theta_n = \tan^{-1} \left( \frac{Im}{Re} \right)$$

Example #  $f(x) = \pi^2 - x^2$  for  $x \in (-\pi, \pi)$

Period =  $2\pi$ .

Find Fourier series.

$$\omega_0 = \frac{2\pi}{2\pi} = \boxed{1}$$

Solution: —

$$f(-x) = \pi^2 - (-x)^2 = \pi^2 - x^2$$

$f(-x) = f(x) \Rightarrow f$  is even function

$$b_n = 0$$

$$a_0 = \frac{1}{T} \int f(t) dt$$

$$a_0 = \frac{1 \cdot 2}{2\pi} \int_0^{\pi} (\pi^2 - x^2) dx$$

$$= \frac{1}{\pi} \left( \pi^2 x \Big|_0^{\pi} - \frac{1}{3} x^3 \Big|_0^{\pi} \right)$$

$$= \frac{1}{\pi} \left( \pi^2 \cdot \pi - 0 - \frac{1}{3} (\pi^3 - 0^3) \right)$$

$$= \frac{1}{\pi} \left( \pi^3 - \frac{1}{3} \pi^3 \right) = \frac{1}{\pi} \left( \frac{2\pi^3}{3} \right) = \boxed{\frac{2\pi^2}{3}}$$

$$a_n = \frac{2}{T} \int_{T_0} f(t) \cos(n\omega_0 t) dt$$

$$a_n = \frac{2}{2\pi} \int_{-\pi}^{\pi} (\pi^2 - x^2) \cos n\omega_0 x \cdot dx$$

$$= \frac{1 \cdot 2}{\pi} \left[ \int_0^{\pi} \pi^2 \cos nx dx - \int_0^{\pi} x^2 \cos nx dx \right]$$

$$= \frac{2}{\pi} \left( \pi^2 \frac{\sin nx}{n} \Big|_0^{\pi} - \int_0^{\pi} x^2 \cos nx dx \right)$$

تقسیم السطح الأجزاء

$$\int_0^{\pi} x^2 \cdot \cos nx \, dx$$

$$= \left. x^2 \frac{\sin nx}{n} + 2x \frac{\cos nx}{n^2} - \frac{2 \sin nx}{n^3} \right|_0^{\pi}$$

$$= \left( \cancel{\frac{\pi^2 \sin n\pi}{n}} + \frac{2\pi \cdot \cos n\pi}{n^2} - \cancel{\frac{2 \sin n\pi}{n^3}} \right) -$$

$$\left( 0 + 0 - \cancel{\frac{2 \sin 0}{n^3}} \right)$$

$$\Rightarrow \int_0^{\pi} x^2 \cdot \cos x \, dx = \boxed{\frac{2\pi}{n^2} \cdot \cos n\pi}$$

← نتایج استنتاجی و به دست آمده:  $a_n$

$$a_n = \frac{2}{\pi} (\pi^2) \cdot \left[ \cancel{\frac{\sin n\pi}{n}} - 0 \right] - \frac{2}{\pi} \left( \frac{2\pi}{n^2} \cdot \cos n\pi \right)$$

$$a_n = \frac{-4}{n^2} * \cos n\pi$$

$$\boxed{a_n = \frac{-4}{n^2} * (-1)^n}$$

#

|            |                        |
|------------|------------------------|
| $x^2$      | $\cos nx$              |
| $\swarrow$ | $\oplus$               |
| $2x$       | $\frac{\sin nx}{n}$    |
| $\swarrow$ | $\ominus$              |
| $2$        | $-\frac{\cos nx}{n^2}$ |
| $\swarrow$ | $\oplus$               |
| $0$        | $-\frac{\sin nx}{n^3}$ |

The graph shows a periodic triangular wave  $x(t)$  plotted against time  $t$ . The wave has a period of  $2\pi$  and an amplitude of  $A$ . The peaks occur at  $t = 0, 2\pi, 4\pi, \dots$  and the troughs occur at  $t = \pi, 3\pi, 5\pi, \dots$ . The wave is zero at  $t = \pi/2, 3\pi/2, 5\pi/2, \dots$ .

$$T_0 = 2\pi, \quad \omega_0 = \frac{2\pi}{2\pi} = 1$$
$$\therefore a_n = a_0 = \text{Zero.}$$
$$x(t) = -x(t - \frac{T_0}{2})$$
$$b_n = \frac{2}{T} \int_{-\pi/2}^{\pi/2} f(t) \cdot \sin(n\omega_0 t) dt$$

$$= \frac{2}{2\pi} \cdot 4 \int_0^{\pi/2} f(t) \cdot \sin(n\omega_0 t) dt$$

$$m = \text{slope} = \frac{A-0}{\frac{\pi}{2}-0} = \frac{2A}{\pi} \Rightarrow \boxed{x = \frac{2A}{\pi} t}$$

$$b_n = \frac{8}{2\pi} \int_0^{\pi/2} \frac{2A}{\pi} t \cdot \sin(n\omega_0 t) dt$$

$$b_n = \frac{8A}{\pi^2} \int_0^{\pi/2} t \cdot \sin(nt) dt.$$

$$b_n = \frac{8A}{\pi^2} \int_0^{\pi/2} t \cdot \sin nt \, dt$$

$$= \frac{8A}{\pi^2} \left[ -\frac{t \cos nt}{n} + \frac{\sin nt}{n^2} \right]_0^{\pi/2}$$

$$= \frac{8A}{\pi^2} \left[ -\frac{\pi}{2} \cdot \frac{\cos n \frac{\pi}{2}}{n} + \frac{\sin \frac{\pi}{2}}{n^2} + \cancel{0} - \cancel{\sin 0} \right]$$

$$b_n = \frac{8A}{\pi^2 n^2} \left[ -\frac{\pi}{2} n \cos n \frac{\pi}{2} + \sin n \frac{\pi}{2} \right]$$

If  $n$  odd  $\Rightarrow \cos(n \frac{\pi}{2}) = 0$

$$\sin n \frac{\pi}{2} = \begin{cases} 1, & n = 1, 5, 9, \dots \Rightarrow b_n = \frac{8A}{\pi^2 n^2} \\ -1, & n = 3, 7, 11, \dots \Rightarrow b_n = -\frac{8A}{\pi^2 n^2} \end{cases}$$

$$x(t) = \frac{8A}{\pi^2} \left( \sin \omega t - \frac{1}{9} \sin 3t + \frac{1}{25} \sin 5t + \dots \right)$$

$$f(t) = \frac{8A}{\pi^2} \sum_{\substack{n-\text{odd} \\ n \neq 0}} (-1)^{\frac{(n-1)}{2}} \cdot \frac{1}{n^2} \sin n \omega t$$

If  $n$  even  $\Rightarrow \sin n \frac{\pi}{2} = 0$

$\Rightarrow$

|     |                        |
|-----|------------------------|
| $t$ | $\sin nt$              |
| 1   | $\frac{-\cos nt}{n}$   |
| 0   | $\frac{-\sin nt}{n^2}$ |

$$\cos n\frac{\pi}{2} = \begin{cases} 1, & n=0, 4, 8, \dots \Rightarrow b_n = -\frac{4A}{\pi n} \\ -1, & n=2, 6, 10, \dots \Rightarrow b_n = \frac{4A}{\pi n} \end{cases}$$

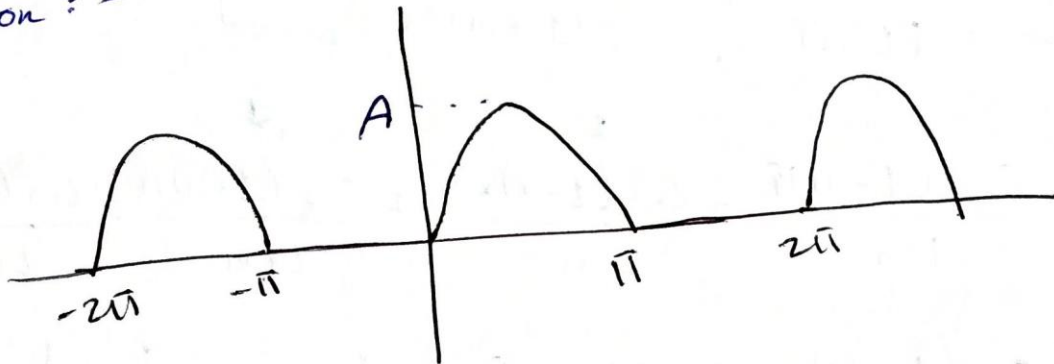
$$x(t) = \frac{4A}{\pi} \left( \frac{1}{2} \sin 2\omega t - \frac{1}{4} \sin 4\omega t + \frac{1}{6} \sin 6\omega t + \dots \right)$$

Example: —

$$x(t) = \begin{cases} A \sin \omega t & , 0 < \omega t < \pi \\ 0 & , \pi < \omega t < 2\pi \end{cases}$$

$\omega = 1$   
Find Fs.

Solution? —



$$T_0 = 2\pi, \omega = 1.$$

$$\Rightarrow a_0 = \frac{1}{T} \int_{T_0}^T f(t) dt$$

$$= \frac{1}{2\pi} \int_0^\pi A \sin(\omega t) dt \Rightarrow \frac{A}{2\pi} \left[ -\frac{\cos(\omega t)}{\omega} \right]_0^\pi$$

$$a_0 = \frac{A}{2\pi} \left[ -\cos\left(\frac{\pi \times 1}{1}\right) + \frac{\cos(0)}{1} \right] = \frac{A}{2\pi} (2)$$

$$\boxed{a_0 = \frac{A}{\pi}}$$

$\Rightarrow$  اوسط قدر کی بنا پر  $a_n$  کی جگہ پر صفر آئے گا

$$\Rightarrow a_n = \frac{2}{T} \int_0^\pi f(t) \cos(n\omega t) dt$$

$$a_n = \frac{2}{2\pi} \int_0^\pi A \sin(\omega t) \cdot \cos(n\omega t) dt$$

from table of integrals

$$\boxed{\int (\sin m x) \cos(n x) dx = -\frac{\cos(m-n)x}{2(m-n)} - \frac{\cos(m+n)x}{2(m+n)}}_{m^2 \neq n^2}$$

$$a_n = \frac{A}{\pi} \left[ \left. -\frac{\cos(1-n)t}{2(1-n)} \right|_0^\pi - \left. \frac{\cos(1+n)t}{2(1+n)} \right|_0^\pi \right]$$

$$a_n = \frac{-A}{2\pi} \left[ \left. \frac{\cos(1-n)t}{(1-n)} \right|_0^\pi + \left. \frac{\cos(1+n)t}{(1+n)} \right|_0^\pi \right]$$

$$a_n = \frac{-A}{2\pi} \left[ \frac{\cos(1-n)\pi}{1-n} - \frac{\cos(1-n) \times 0}{1-n} + \frac{\cos(1+n)\pi}{1+n} - \frac{\cos(1+n) \times 0}{1+n} \right]$$

$$a_n = \frac{-A}{2\pi} \left[ \frac{\cos(1-n)\pi}{1-n} + \frac{\cos(1+n)\pi}{1+n} - \frac{1}{1-n} - \frac{1}{1+n} \right]$$

$$a_n = \frac{-A}{2\pi} \left( \left[ \frac{\cos(\pi - n\pi)}{1-n} + \frac{\cos(\pi + n\pi)}{1+n} \right] - \left[ \frac{1}{1-n} + \frac{1}{1+n} \right] \right)$$

$$\boxed{\cos(x \pm y) = \cos x \cdot \cos y \mp \sin x \cdot \sin y} \quad \text{قانون}$$

$$\cos(\pi \pm n\pi) = \cos \pi \cdot \cos n\pi \pm \sin \pi \cdot \sin n\pi$$

$$\cos(\pi - n\pi) = -\cos n\pi$$

$$a_n = \frac{-A}{2\pi} \left( \frac{-\cos n\pi}{1-n} - \frac{\cos n\pi}{1+n} - \left( \frac{1+n+1-n}{1-n^2} \right) \right)$$

$$a_n = \frac{-A}{2\pi} \left( \frac{-\cos n\pi}{1-n} - \frac{\cos n\pi}{1+n} - \frac{2}{1-n^2} \right)$$

$$a_n = \frac{-A}{2\pi} \left( \frac{-\cos n\pi - n\cos n\pi - \cos n\pi + n\cos n\pi - 2}{1-n^2} \right)$$

$$a_n = \frac{A}{2\pi} \left( \frac{\cos n\pi + n\cos n\pi + \cos n\pi - n\cos n\pi + 2}{1-n^2} \right)$$

$$a_n = \frac{A}{2\pi} \left[ \frac{2\cos n\pi + 2}{1-n^2} \right]$$

$$a_n = \frac{A}{\pi} \left[ \frac{\cos n\pi + 1}{1-n^2} \right] \quad n \neq 1$$

$a_n$  is not defined at  $n=1$ , then we must compute  $a_1$

\*  $a_1$  حسب  $n$  نرجع للقانون الأول  $a_n$   $n$   $\neq 1$   $\Rightarrow$   $a_1$   $\neq 1$

$$a_1 = \frac{2}{2\pi} \int A \sin \omega t \cdot \cos n \omega t \, dt$$

$$a_1 = \frac{A}{\pi} \int_0^{\pi} \sin t \cdot \cos t \, dt$$

$$\boxed{\int \sin(ax) \cdot \cos(ax) \, dx = \frac{1}{2a} (\sin ax)^2}$$

قانون

$$a_1 = \frac{A}{\pi} \cdot \frac{1}{2} (\sin t)^2 \Big|_0^{\pi}$$

$$a_1 = \frac{A}{2\pi} \left[ (\sin/\pi)^2 - (\sin(0))^2 \right] \Rightarrow \boxed{a_1 = 0}$$

$$\Rightarrow b_n = \frac{2}{T} \int f(t) \cdot \sin(n\omega t) \, dt$$

$$b_n = \frac{2}{2\pi} \int A \sin \omega t \cdot \sin(n\omega t) \, dt$$

$$b_n = \frac{A}{\pi} \int_0^{\pi} \sin t \cdot \sin nt \, dt$$

$$\boxed{\int \sin(mx) \cdot \sin(nx) \, dx = \frac{\sin(m-n)x}{2(m-n)} - \frac{\sin(m+n)x}{2(m+n)}} \quad m^2 \neq n^2$$

$$b_n = \frac{A}{\pi} \cdot \frac{1}{2} \left[ \frac{\sin(1-n)t}{1-n} - \frac{\sin(1+n)t}{1+n} \right] \Big|_0^\pi$$

$$= \frac{A}{2\pi} \left[ \frac{\sin(1-n)\pi}{1-n} - \frac{\sin(1+n)\pi}{1+n} - 0 + 0 \right]$$

$$= \frac{A}{2\pi} \left[ \frac{\sin(\pi - n\pi)}{1-n} - \frac{\sin(\pi + n\pi)}{1+n} \right] \quad n \neq 1$$

$$\begin{aligned} \sin(x+y) &= \sin x \cos y + \cos x \sin y \\ \sin(x-y) &= \sin x \cos y - \cos x \sin y \end{aligned}$$

نحتاج

$$= \frac{A}{2\pi} \left[ \frac{(\cancel{\sin \pi} \cdot \cos n\pi - \cos \pi \cdot \cancel{\sin n\pi})}{1-n} - \frac{(\cancel{\sin \pi} \cos n\pi + \cos \pi \cancel{\sin n\pi})}{1+n} \right]$$

$$= \frac{A}{2\pi} \left[ \frac{-\cos \pi \cdot \sin n\pi}{1-n} - \frac{\cos \pi \cdot \sin n\pi}{1+n} \right]$$

$$= \frac{A}{2\pi} \left[ \frac{-\cos \pi \sin n\pi - n \cos \pi \sin n\pi - \cos \pi \sin n\pi + n \cos \pi \sin n\pi}{1-n^2} \right]$$

$$= \frac{A}{2\pi} \left[ \frac{-2 \cos \pi \sin n\pi}{1-n^2} \right]$$

$$= \frac{A}{2\pi} \left[ \frac{2 \sin n\pi}{1-n^2} \right], \quad n \neq 1$$

$\sin^2 t = \frac{1}{2} (1 - \cos 2t)$

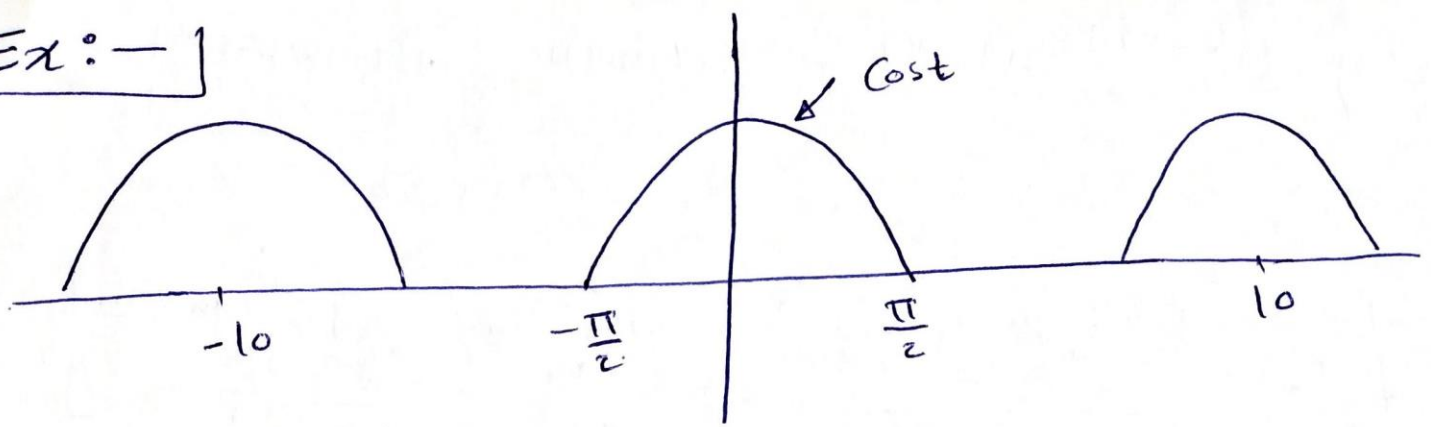
(نحتاج إلى معرفة قيمة  $\sin n\pi$ )  $\sin n\pi = 0$  // نلاحظ

$$b_n = 0, \quad n \neq 1$$

n = 1 نحتاج إلى معرفة قيمة  $b_1$  كما أن  $a_n = 0$

$$\begin{aligned} b_1 &= \frac{A}{\pi} \int_0^\pi (\sin t)^2 dt = \frac{A}{\pi} \left[ t/2 - \frac{\sin 2t}{4} \right] \Big|_0^\pi \\ &= \frac{A}{\pi} \left[ \frac{\pi}{2} - \frac{\sin 2\pi}{4} \right] = \frac{A}{2} \end{aligned}$$

Ex: —



Sol: —

$$T_0 = 10, \quad \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{10} = \boxed{\frac{\pi}{5}}$$

$$x(t) = \begin{cases} \cos t, & -\frac{\pi}{2} < t < \frac{\pi}{2} \\ 0, & \text{otherwise.} \end{cases}$$

$$D_n = \frac{1}{10} \int_{-\pi/2}^{\pi/2} \cos t \cdot e^{-j\omega_0 n t} dt$$

$$= \frac{1}{10} \int_{-\pi/2}^{\pi/2} \left( \frac{e^{jt} + e^{-jt}}{2} \right) e^{-j\omega_0 n t} dt$$

$$= \frac{1}{20} \int_{-\pi/2}^{\pi/2} \left( e^{jt} \cdot e^{-j\omega_0 n t} + e^{-jt} \cdot e^{-j\omega_0 n t} \right) dt$$

$$= \frac{1}{20} \int_{-\pi/2}^{\pi/2} e^{j(1-n\omega_0)t} dt + \frac{1}{20} \int_{-\pi/2}^{\pi/2} e^{-j(1+n\omega_0)t} dt$$

$$= \frac{1}{20} \left[ \frac{e^{j(1-n\omega_0)t}}{j(1-n\omega_0)} \right]_{-\pi/2}^{\pi/2} + \frac{1}{20} \left[ \frac{e^{-j(1+n\omega_0)t}}{-j(1+n\omega_0)} \right]_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{20} \left[ \frac{e^{j(1-n\omega_0)\frac{\pi}{2}} - e^{j(1-n\omega_0)\frac{-\pi}{2}}}{j(1-n\omega_0)} + e^{j(1+n\omega_0)\frac{\pi}{2}} - e^{j(1+n\omega_0)\frac{-\pi}{2}} \right]$$

$$= \frac{1}{20} \left[ \frac{e^{j\frac{\pi}{2}} - e^{-jn\omega_0\frac{\pi}{2}}}{j(1-n\omega_0)} - \frac{e^{-j\frac{\pi}{2}} - e^{jn\omega_0\frac{\pi}{2}}}{-j(1+n\omega_0)} \right]$$

$$\Rightarrow \boxed{e^{j\frac{\pi}{2}} = j}, \quad \boxed{e^{-j\frac{\pi}{2}} = -j} \quad \underline{\text{Evaluating}}$$

$$= \frac{1}{20} \left[ \frac{j e^{-jn\omega_0\frac{\pi}{2}} + j e^{jn\omega_0\frac{\pi}{2}}}{j(1-n\omega_0)} \right] + \frac{1}{20} \left[ \frac{-j e^{-jn\omega_0\frac{\pi}{2}} - j e^{jn\omega_0\frac{\pi}{2}}}{-j(1+n\omega_0)} \right]$$

$$= \frac{1}{20} \left[ \frac{e^{-jn\omega_0\frac{\pi}{2}} + e^{jn\omega_0\frac{\pi}{2}}}{1-n\omega_0} \right] + \frac{1}{20} \left[ \frac{e^{-jn\omega_0\frac{\pi}{2}} + e^{jn\omega_0\frac{\pi}{2}}}{1+n\omega_0} \right]$$

$$= \frac{1}{20} \frac{2 \cos(n\omega_0\frac{\pi}{2})}{1-n\omega_0} + \frac{1}{20} \frac{2 \cos(n\omega_0\frac{\pi}{2})}{(1+n\omega_0)}$$

$$\frac{1}{10} \left[ \frac{\cos(n\omega_0\frac{\pi}{2})}{1-n\omega_0} + \frac{\cos(n\omega_0\frac{\pi}{2})}{1+n\omega_0} \right]$$

$$\frac{\cos(n\omega_0\frac{\pi}{2})}{10} \left[ \frac{1}{1-n\omega_0} + \frac{1}{1+n\omega_0} \right] \Rightarrow \frac{\cos(n\omega_0\frac{\pi}{2})}{10} \left( \frac{2}{1-n^2\omega_0^2} \right)$$

$$\boxed{D_n = \frac{\cos\left(\frac{n\pi^2}{10}\right)}{5(1-n^2\frac{\pi^2}{25})}} \quad \text{at } n=0 \Rightarrow \boxed{D_0 = \frac{1}{5}} \quad \text{البُزْجَة}$$

$$\boxed{x(t) = \frac{1}{5} + \sum \frac{\cos\left(\frac{n\pi^2}{10}\right)}{5(1-n^2\frac{\pi^2}{25})} \cdot e^{jn\frac{\pi}{5}t}} \quad \#$$