chapter #1

Introduction

→ what is the signal?

- Signals are detectable physical quantities or variables by means which message or information can be transmitted.

- Signals can be represented mathmatically as a function of one or more independent variables. like time

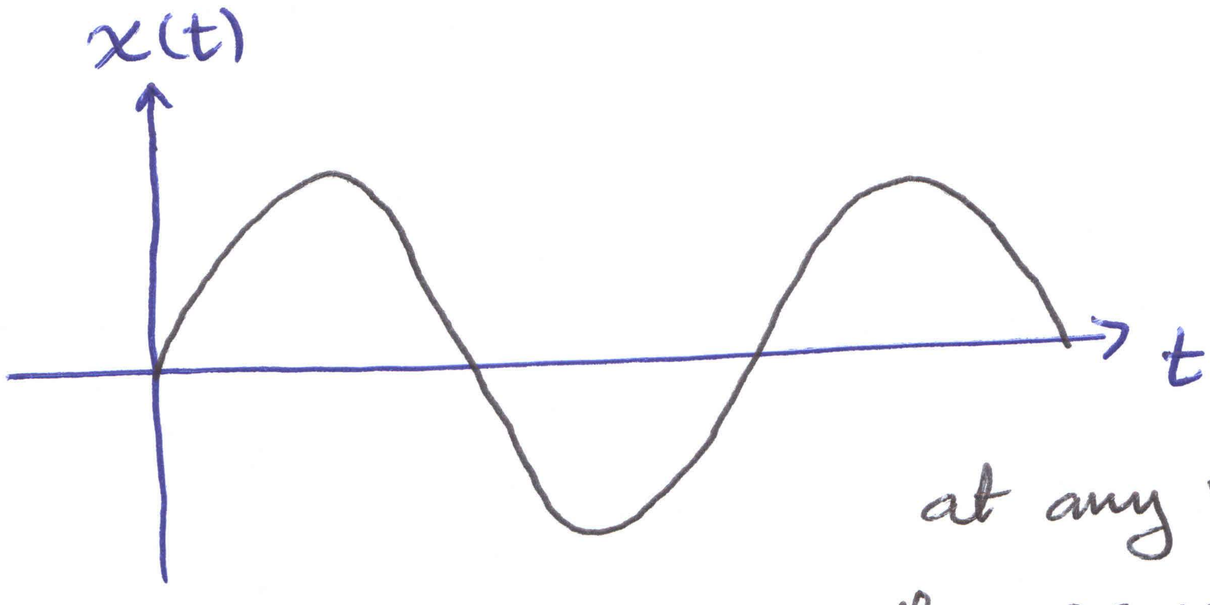
For example:

1) Electrical signals like Voltages and currents in the circuit.

→ we can know the type of signals by determine the type of independent variables. (t)

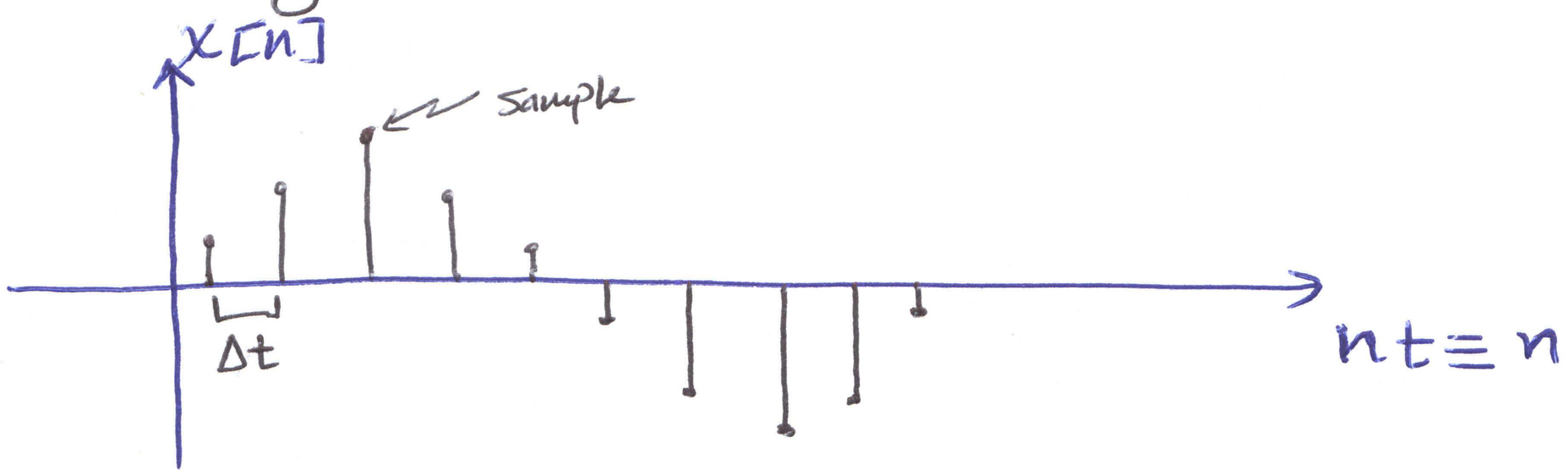
1) if the independent variables is continous so the corresponding signal is continous time (CT)

Example:



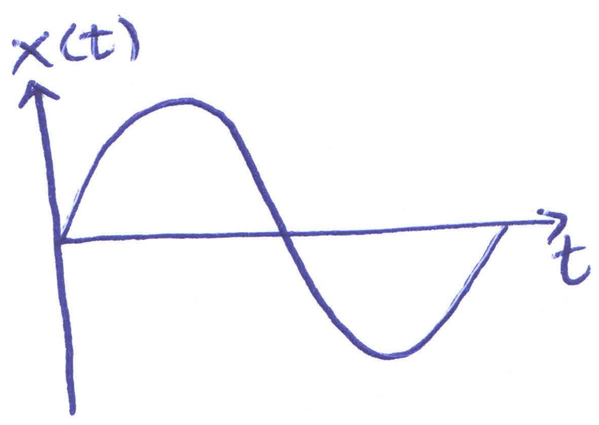
at any value of t
there is value for x(t)

2) if the independent variables is discrete, the corresponding signals is Discrete time (DT)

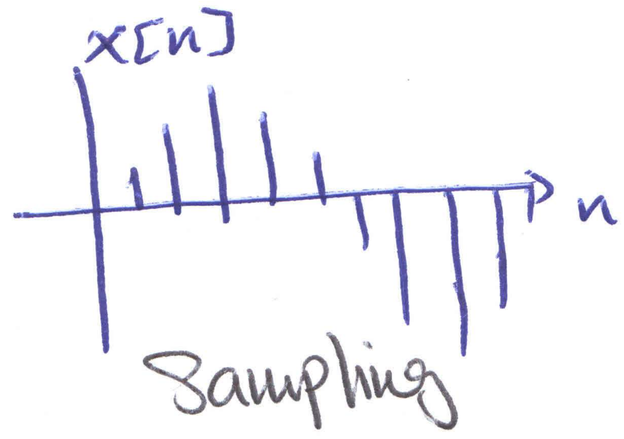


Notes:

- Analoge
  - Continous time
  - Continous value
  - x(t)



- Discrete
  - Discrete time
  - Continous value
  - x[n]

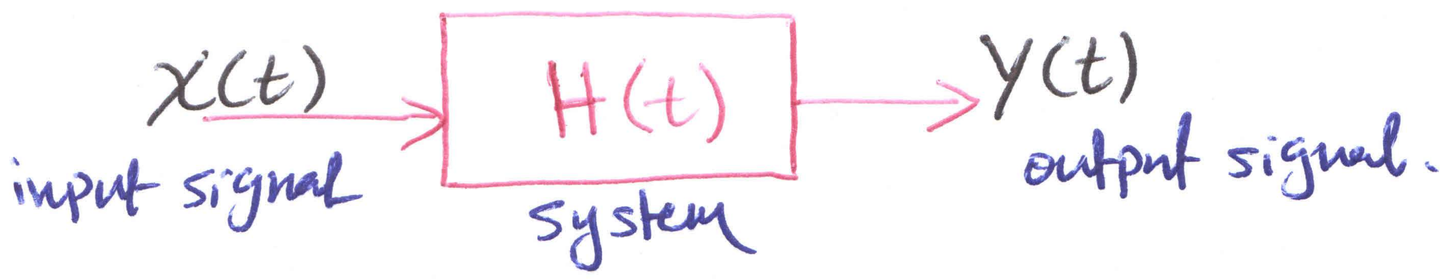


- Digital
  - Discrete time
  - Discrete value
  - x[n]

This operation or processing calles A/D converter

→ what is the system?.

- Signals may be processed futher by systems which may modify them or extract additional information From them.
- System is a black box that transforms input signals to output signals.



1.1 Size of signal :-

- we can use the energy of the signals as a measure for the size of the signal

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

↳ Abs.

- the signal must be finite to find its energy, which means the signal amplitude $\longrightarrow 0$ at $|t| \longrightarrow \infty$ (Converge function or system)

- if the signal is not converge, we find the size using the scale of the power of signal.

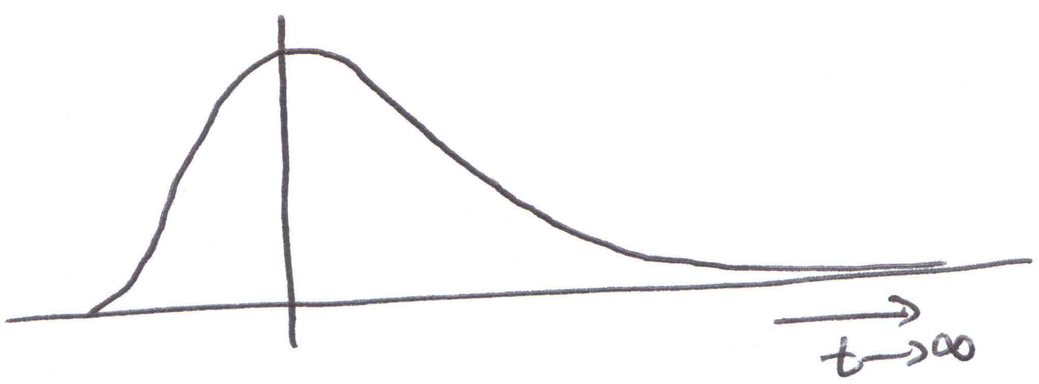
$$P_x = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

$$P_x = \lim_{T\to\infty} \frac{E_x}{2T}$$

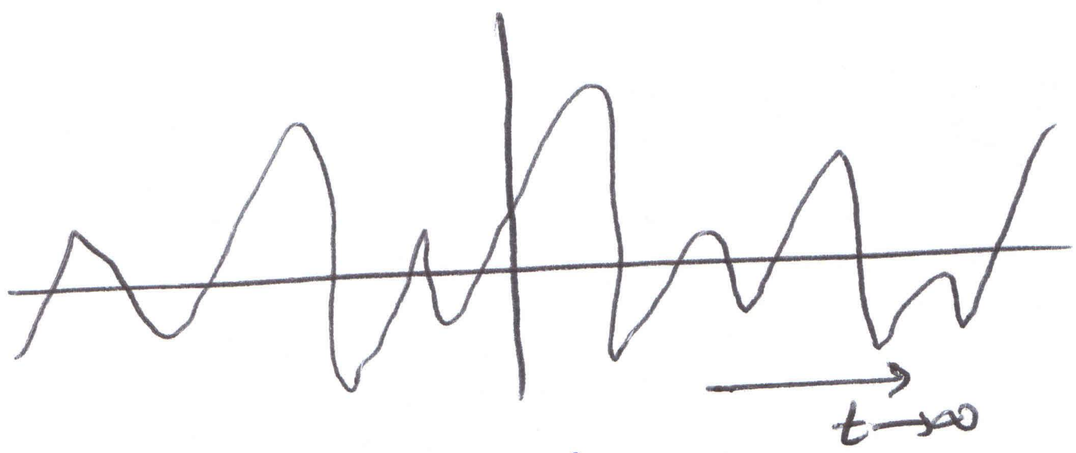
- P_x is the time average of the energy

- in general, P_x exists iff the signal is periodic

Periodic :- that means the signal is repeat it self every specific period of time such as Sin and Cos

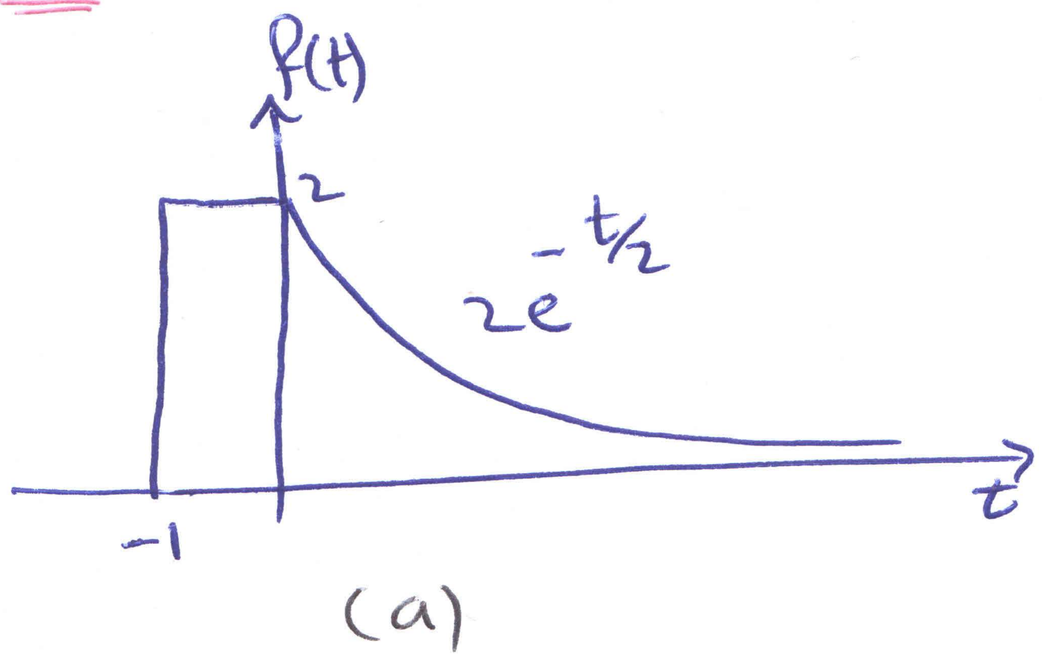


Signal Energy

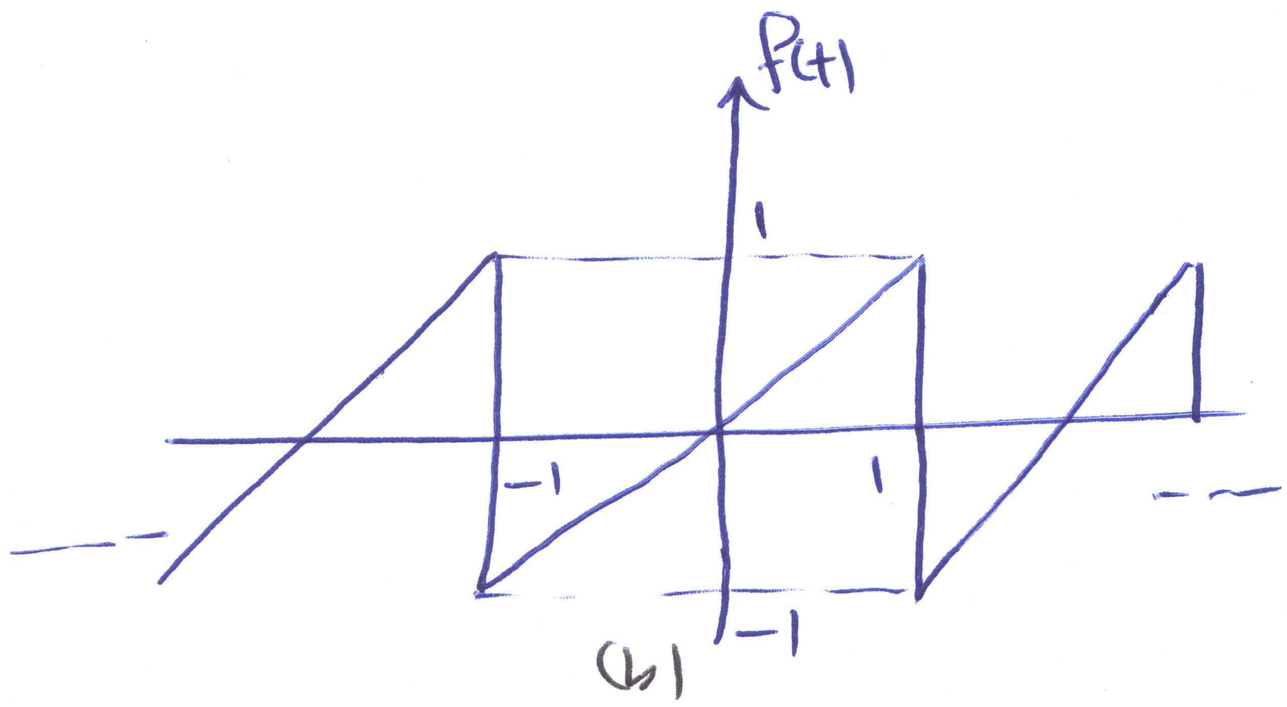


Signal power

Ex: Determine the suitable measures of these signals



(a)



(b)

a) This signal amplitude $\longrightarrow 0$ at $t \longrightarrow \infty$

that mean this signal energy so the measure is the energy

$$E_f = \int_{-\infty}^{\infty} |f(t)|^2 dt = \int_{-1}^{0} (2)^2 dt + \int_{0}^{\infty} 4e^{-t} dt$$

$$= 4t\Big|_{-1}^{0} + \frac{4e^{-t}}{-1}\Big|_{0}^{\infty}$$

$$= 0-(-4) + 0-(-4)$$

$$= \boxed{8}$$

b) This signal is periodic signal so the measure is Power

Period of signal is T

$T = 2$

$$P_f = \frac{1}{T}\int_{-T/2}^{T/2} f(t)^2 dt$$

$$= \frac{1}{2}\int_{-2/2}^{2/2} t^2 dt = \frac{1}{2}\frac{t^3}{3}\Big|_{-1}^{1}$$

$$= 1/2 * \frac{2}{3} = \boxed{\frac{1}{3}}$$

$y = mx$

$$m = \frac{b_1 - b_0}{a_1 - a_0}$$

$$m = \frac{1-(-1)}{1-(-1)} = \frac{2}{2}$$

$= 1$

$f(t) = 1 \cdot t$

Notes 2

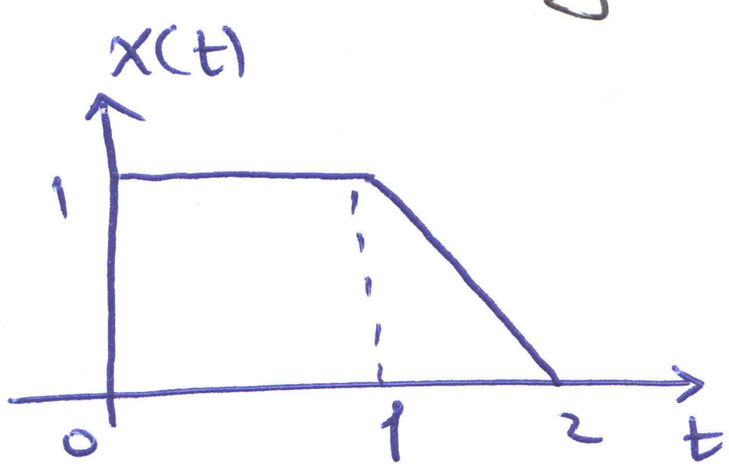
- All periodic signals are power signals but not vice versa.

1.2 Operation on signals :- (Transformation)

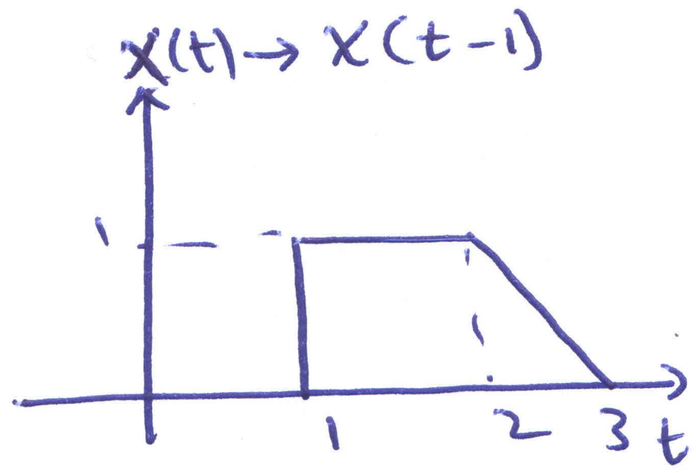
There are 3 possible time transformation :-

1) Time shifting 2) Time scaling 3) Time reversal (reflection)

(1) Time shifting :-

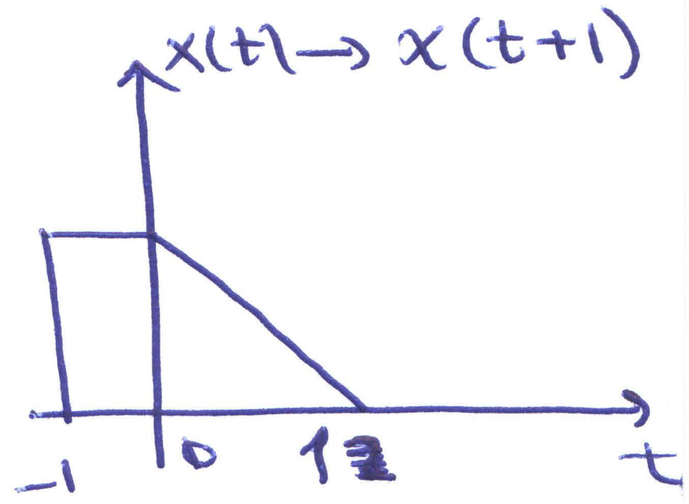


original signal



Shift to right

That means make delay or go to the past

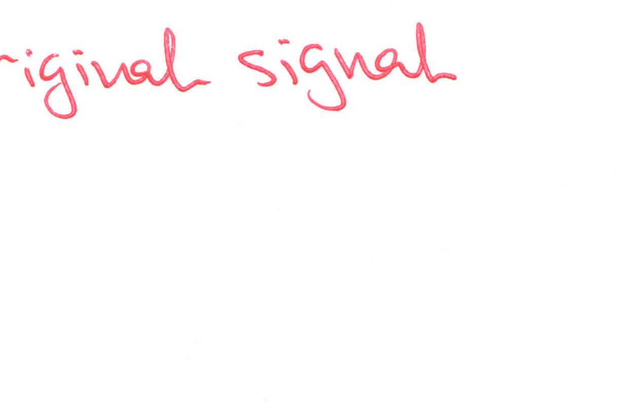


Shift to left

means make Advance or go to the future

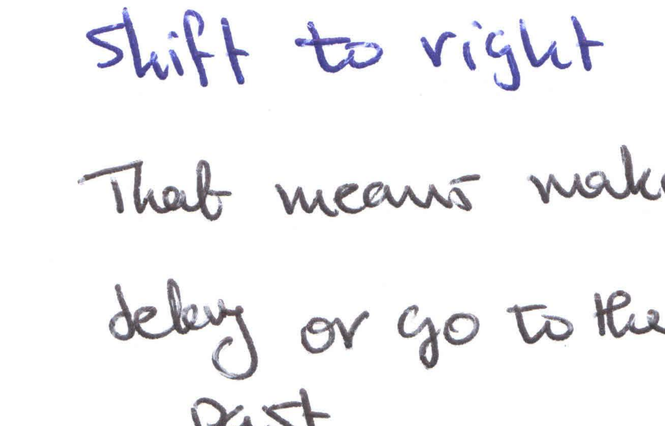
& that impossible in our life

(2) Time scaling :-



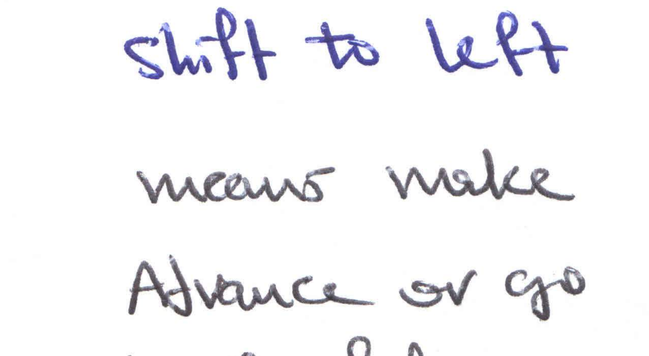
original signal

x(2t)



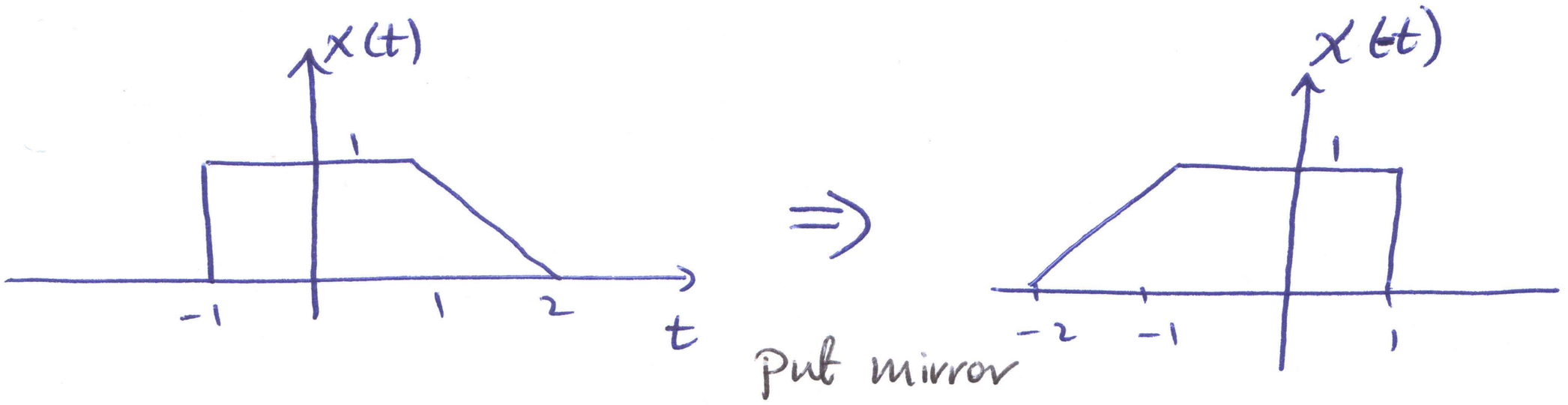
scales signal

x(t/2)



scales signal

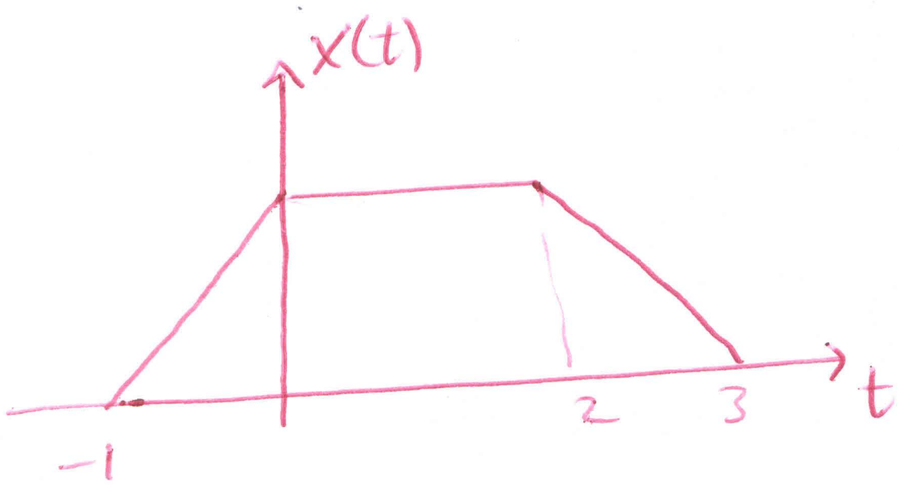
8) Time reversal (reflection)



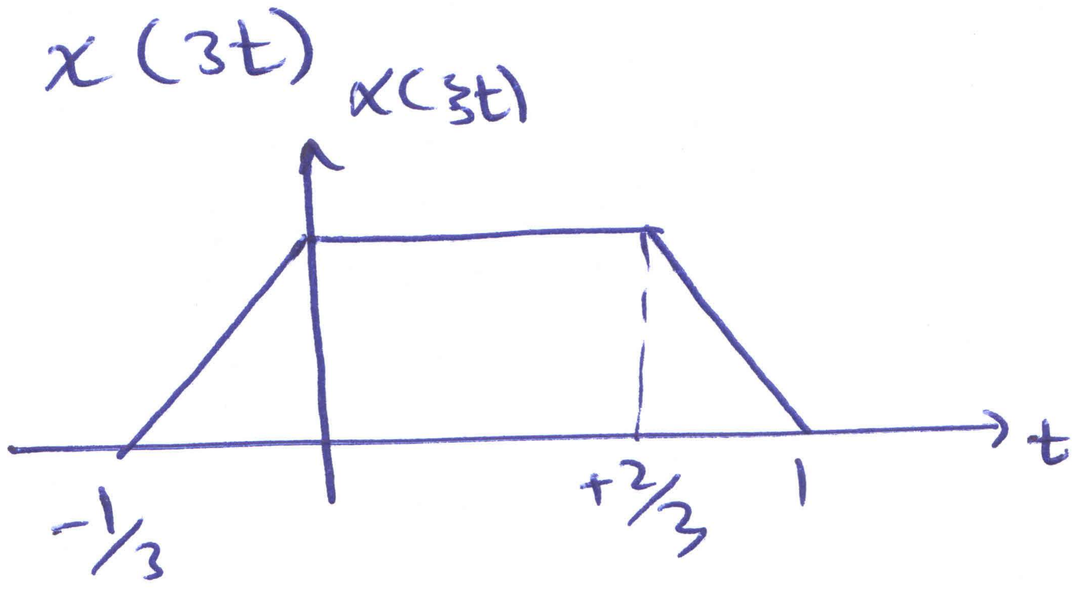
Examples

If you have the original signal $x(t)$ as in figure (a), so find the $x(3t-6)$

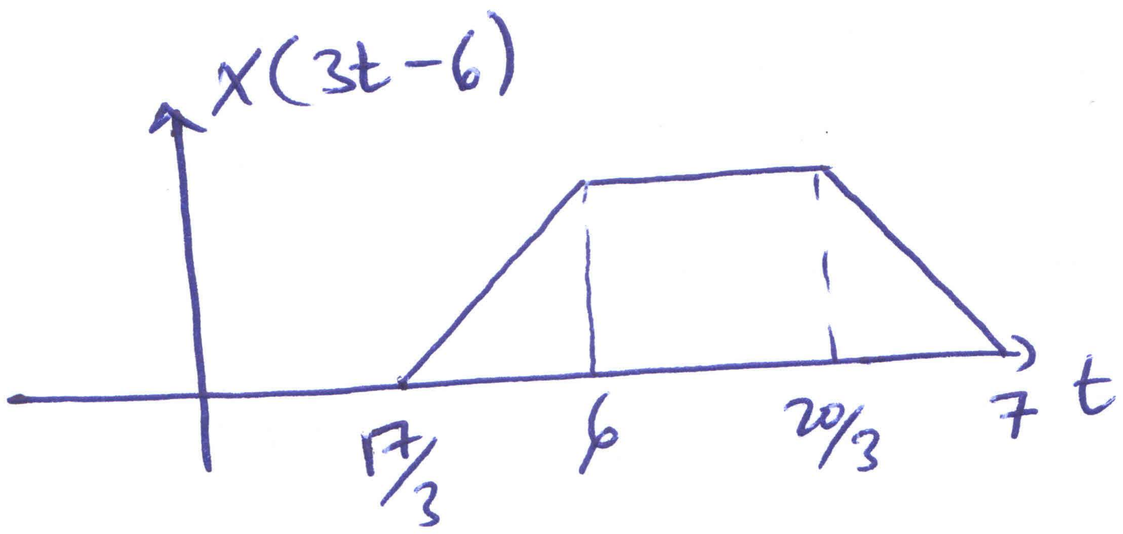
ملاحظة: الأولوية في هذه العمليات كذلك ① الضرب ② القوى ③ الإزاحة



1) $x(3t)$



2) $x(3t-6)$ → shift to right.

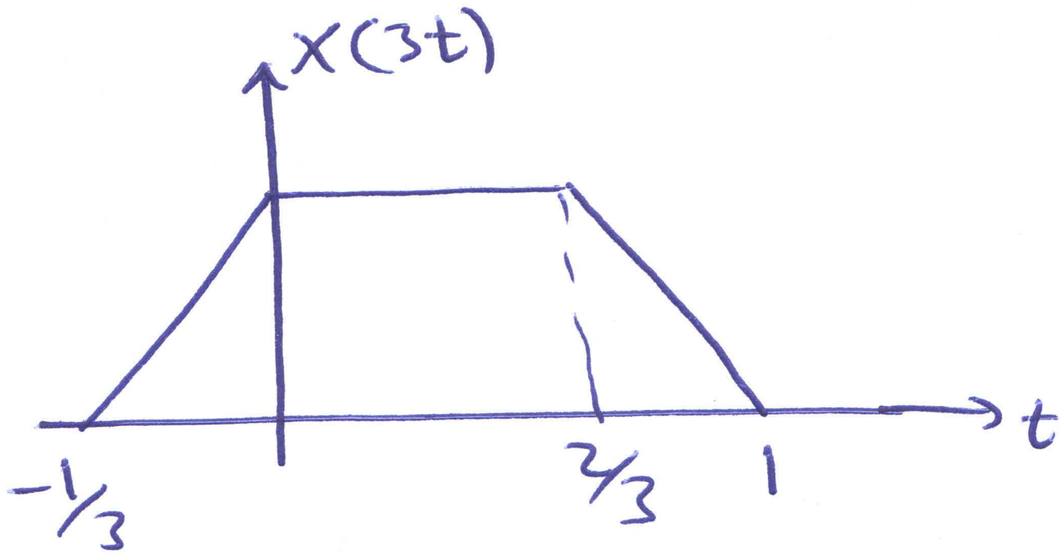


End

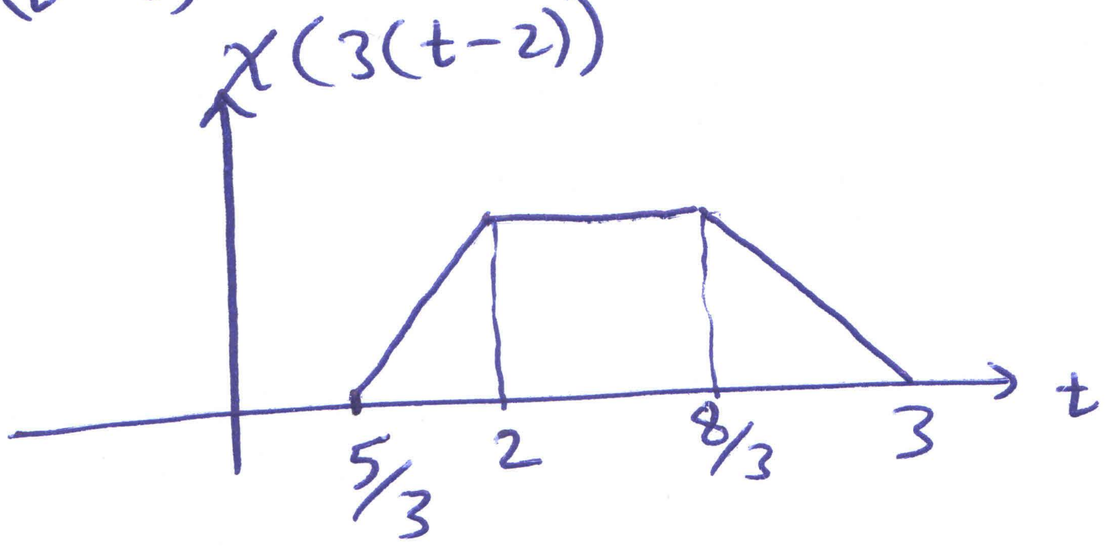
That is wrong solution why ?!
Let we see ---

$x(3t-6) \Rightarrow x(3(t-2))$ ← That is correct expression

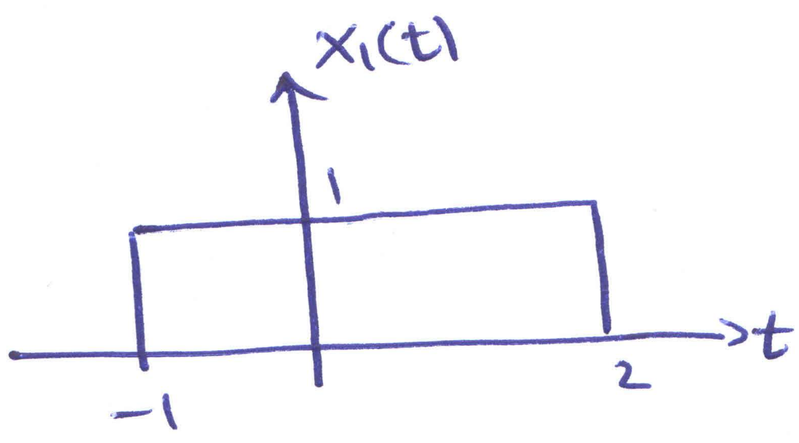
① First make scaling 2

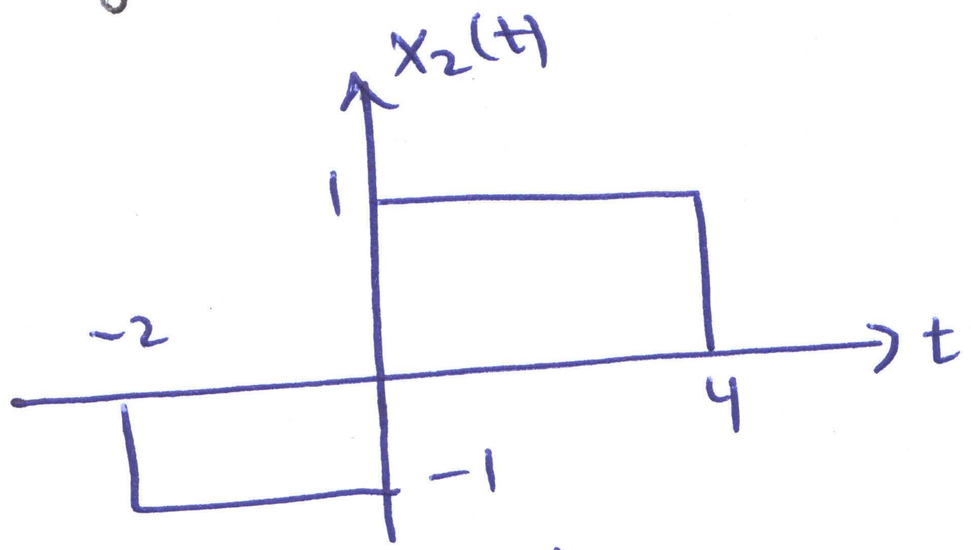


② second make shift to right by 2 → replace ~~t~~ all (t) by (t-2)

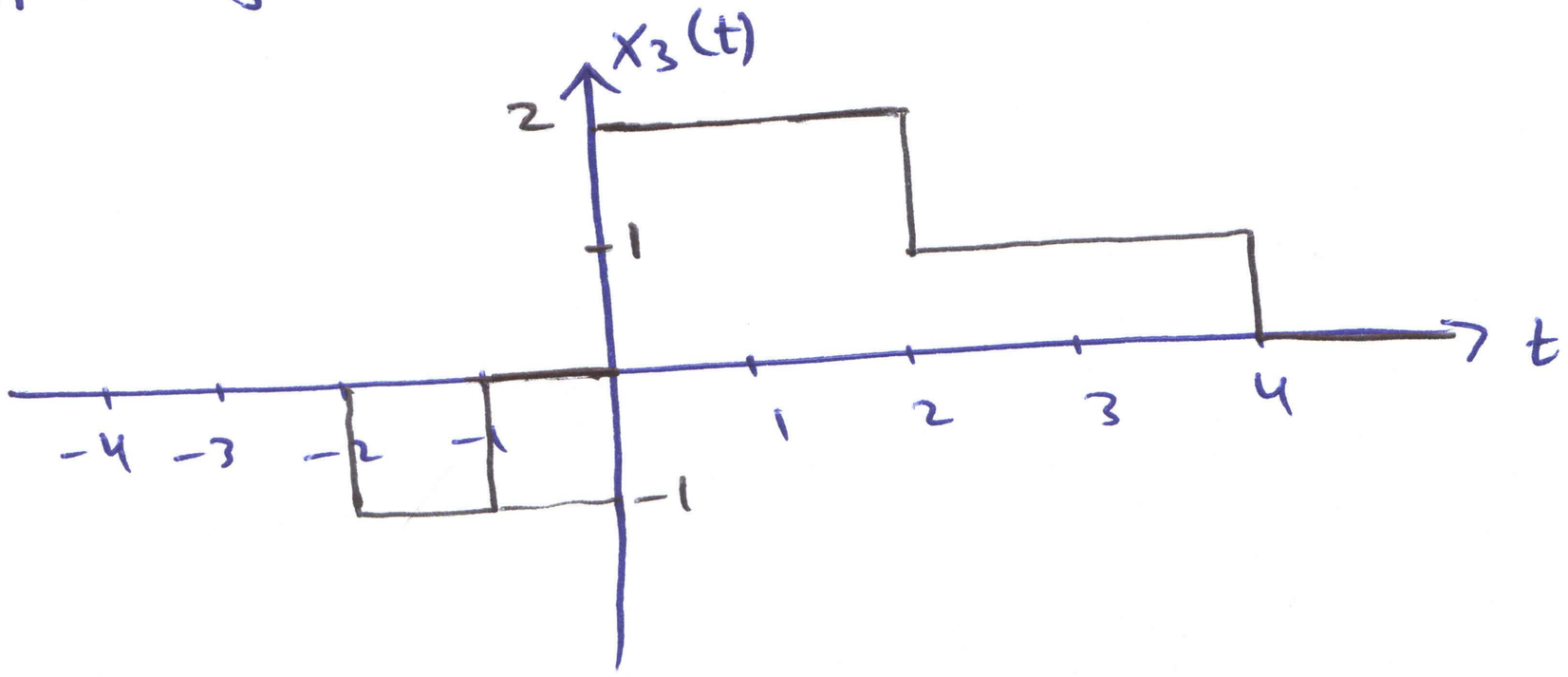


⊛ Multiplication & addition signals 2.

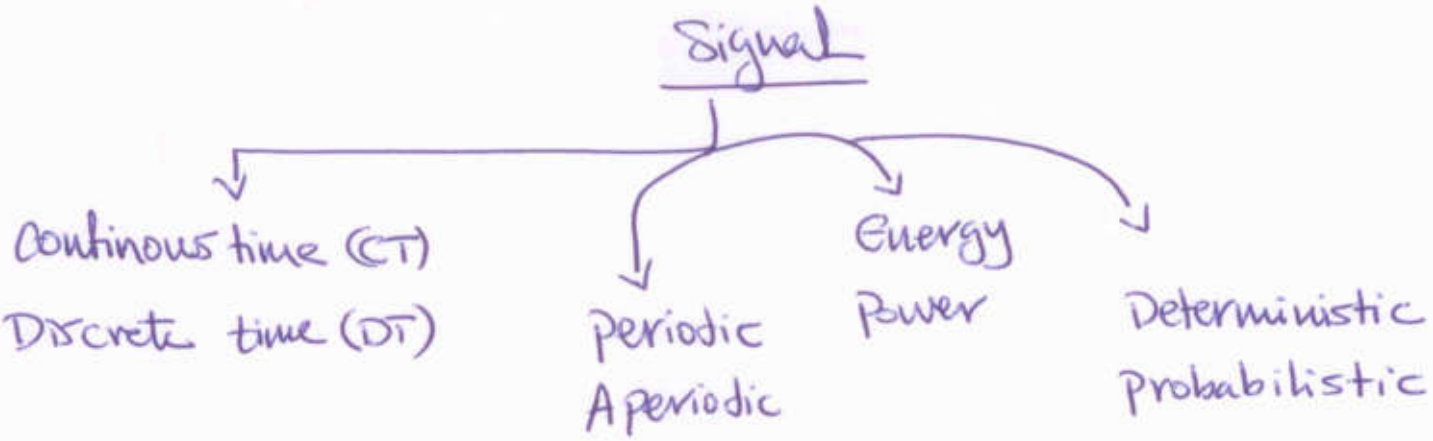




Find the output signal $x_3(t) = x_1(t) + x_2(t)$



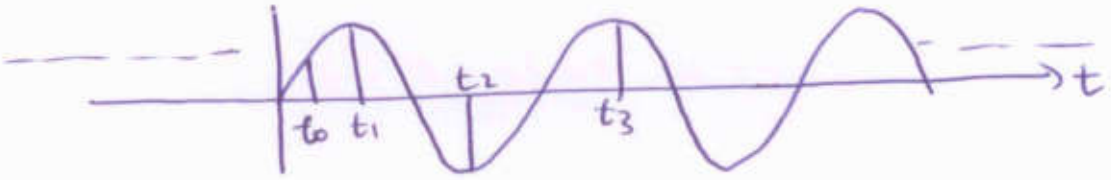
1.3 Classification of signals:



Let we see what Does it mean ?!

[1] CT and DT signals:

- Continous time signal are function of a real argument like time $\Rightarrow$ $x(t) = \sin(t)$

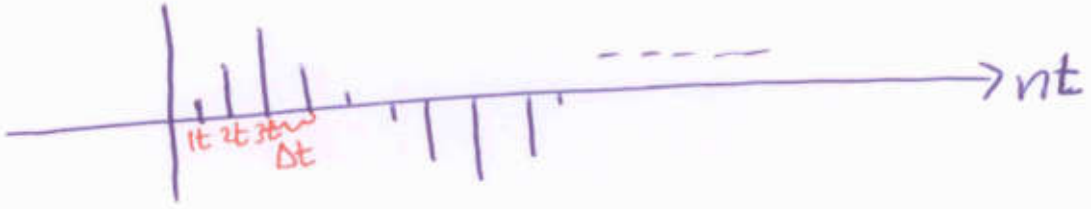


at any value of t_n it has a value

- Discrete time signal are function of an argument that takes values from a discrete set

$$x[n] = \sin(n)$$

↳ Integer value



$$\Delta t = 2t_0 - t_1 = t$$

where n is specific value like $n = [\cdots, -2, -1, 0, 1, 2 \cdots$

(2) Periodic and Aperiodic Signals:

- The condition of periodicity is

$$x(t) = x(t + nT) \qquad n = 0, 1, 2, \dots$$

where T is the period of $x(t)$

NOTES: In continous signal sin & cos are always periodic but in discrete time sin & cos are not.

Examples:

$$x(t) = \underbrace{\sin 2t}_{\text{periodic}} + \underbrace{\sin \pi t}_{\text{periodic}} \Rightarrow \boxed{\sin \omega_0 t \;\; | \;\; P = \frac{2\pi}{\omega_0}}$$

(3) إذا كان المثال مكون من مجموع دالتين periodic فإن الدالة الناتجة تكون Aperiodic إلا إذا تحقق الشرط التالي

$$\frac{P_1}{P_2} = \frac{\text{Int}}{\text{Int}} \quad \text{(rational number)}$$

$$P_1 = \frac{2\pi}{2} = \pi \quad , \quad P_2 = \frac{2\pi}{\pi} = 2$$

$$\therefore \frac{P_1}{P_2} = \frac{\pi}{2} \quad \text{is not rational}$$

$\therefore$ $x(t)$ is aperiodic signal.

Example (2)

$$x(t) = \sin\frac{\pi}{3}t + 2\cos\frac{8\pi}{3}t$$

في الاختبار يطلب منك انك تجد ال periodic الكلي للاشارة يعني انك تفحص اول ال periodicity بعدين تجد ال Period.

$$P_1 = \frac{2\pi}{\frac{\pi}{3}} = 6 \quad , \quad P_2 = \frac{2\pi}{\frac{8\pi}{3}} = \frac{6}{8} = \frac{3}{4}$$

$$\therefore \frac{P_1}{P_2} = \frac{6}{\frac{3}{4}} = \frac{8}{1} = \frac{\text{Int}}{\text{Int}} \text{ rational}$$

$\therefore$ $x(t)$ is periodic, so the period of $x(t)$ equal.

$$P = 8P_2 = P_1 = 6$$

must be equal to be correct answer

- So the last signal is Continous signal like $x(t) = \cos\omega_0 t$ so its period equal $\boxed{P = \frac{2\pi}{\omega_0}}$

- if the signal is Discrete signal how you can solve it

$$x[n] = \cos\omega_0 nT \Rightarrow \text{its period equal}$$

$$\boxed{\frac{N}{K} = \frac{2\pi}{\omega_0 T}}$$ must be rational

$\omega_0 T \Rightarrow 2\pi_0$

$\omega_0 T$: continous, $2\pi_0$: discrete

$$\boxed{\frac{N}{K} = \frac{2\pi}{2\pi_0}}$$

Ex2 $y[n] = \sin[0.3\pi n]$

Discrete signal

$$\frac{N}{K} = \frac{2\pi}{0.3\pi} = \frac{20}{3} = \frac{\text{int}}{\text{int}} \quad \text{rational}$$

∴ its periodic signal & its period $N = 20$

Ex3 $y[n] = \cos\left[\frac{\pi}{4} n\right] + \sin\left[\frac{\pi}{7} n + \frac{1}{2}\right]$

① ②

$$\frac{N_1}{K} = \frac{2\pi}{\pi/4} = \frac{8}{1} \quad \text{rational} \Rightarrow \text{periodic} \quad N_1 = 8$$

$$\frac{N_2}{K} = \frac{2\pi}{\pi/7} = \frac{14}{1} \quad \text{rational} \Rightarrow \text{periodic} \quad N_2 = 14$$

$$\frac{N_1}{N_2} = \frac{8}{14} \quad \text{rational}$$

So $x[n]$ is periodic & its period equal

$$N = \overset{4}{8} N_2 = \overset{7}{14} N_1 = 56$$

VIP:

1) $\sin \omega_0 t = \cos(\omega_0 t - \frac{\pi}{2})$
2) $\cos \omega_0 t = \sin(\omega_0 t + \frac{\pi}{2})$
3) $e^{j\omega_0 t} = \cos \omega_0 t + j \sin \omega_0 t$

↳ it is always periodic & its period equal $P = \frac{2\pi}{\omega_0}$

Ex2

$$x(t) = \sin\frac{2\pi}{5}t \cdot \cos\frac{4\pi}{3}t$$

$$\sin x \cdot \cos y = \frac{1}{2}\{\sin(x+y) + \sin(x-y)\}$$

odd function.

$$x(t) = \frac{1}{2}\left\{\sin\frac{26\pi}{15}t + \sin\left(-\frac{14}{15}\pi\right)t\right\}$$

$$P_1 = \frac{2\pi}{\frac{26\pi}{15}} = \frac{30}{26} = \frac{15}{13}$$

$$P_2 = \frac{2\pi}{\frac{14\pi}{15}} = \frac{30}{14} = \frac{15}{7}$$

$$\frac{P_1}{P_2} = \frac{15/13}{15/7} = \frac{7}{13}$$ is rational

$$P = 7P_2 = 13P_1 = \boxed{15}$$ #

Notes: The exponential function is not periodic

Ex:

$$x(t) = e^{j\frac{\pi}{6}t} + e^{j\frac{4\pi}{5}t}$$

$$P_1 = \frac{2\pi}{\frac{\pi}{6}} = 12$$

$$P_2 = \frac{2\pi}{\frac{4\pi}{5}} = \frac{5}{2}$$

$$\frac{P_1}{P_2} = \frac{12}{5/2} = \frac{24}{5}$$

$$P = 5P_1 = 24 \cdot P_2$$

$$P = 60$$

#

[3] Energy and power signals:-

- if the signal has finite energy so it called energy signal
- if the signal has finite power and nonzero so it called power signal.

The power signal have infinite energy $E=\infty$

[4] Deterministic and probabilistic signals:

- if the signal has an equation or graphical that describe it and i can expect the next values for it its called Deterministic signal.
- if the signal has random value and I can't expect its values so its called probabilistic signal.

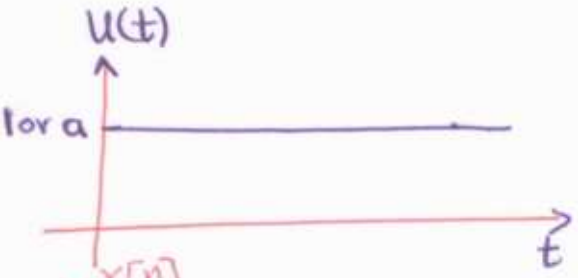
1.4 Some useful signal models:

- Step signal
- ramp signal
- Impulse signal
- Exponential signal

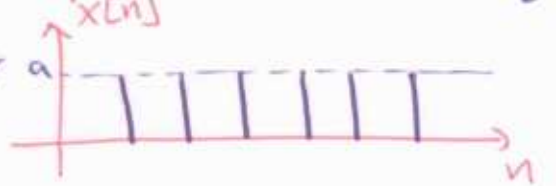
1 Step Function $u(t)$, $u[n]$

unit step — step function

$$u(t) = \begin{cases} 1 \text{ or } a & t \geq 0 \\ 0 & t < 0 \end{cases}$$



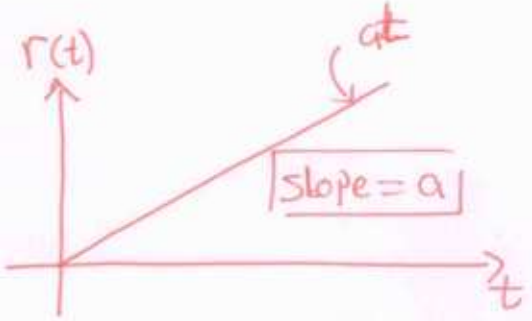
$$u[n] = \begin{cases} 1 \text{ or } a & n \geq 0 \\ 0 & n < 0 \end{cases}$$



2 ramp function :- $r(t)$

m = slope

$$r(t) = \begin{cases} at & t \geq 0 \\ 0 & t < 0 \end{cases}$$



there is relation ship between $r(t)$ & $u(t)$

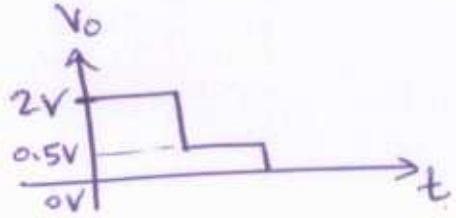
$$r(t) = \int_{0}^{t} u(t)\, dt$$

$$u(t) = \frac{dr(t)}{dt}$$

ملاحظة :

آذا تغيرت قيمة الدالة من ناحية الـ Amplitude فإنها تكبر صغر

الدالة step



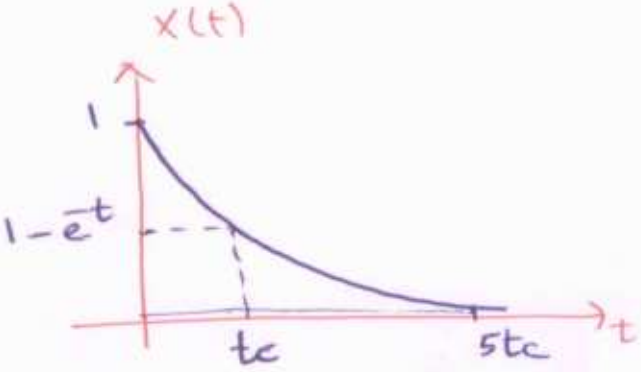
[3] Impulse function :- $\delta(t)$

$$\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & t \neq 0 \end{cases}$$

[4] Real and Complex exponential :-

- Real exponential Function :-

$$x(t) = e^{-at}$$



$$\boxed{t_c = \frac{1}{a}}$$

↳ is called time constant which is the time required to decrease the amplitude $x(t)$ by (e^{-1})

& when ~~x(t)~~ $t = 5t \Rightarrow x(t) = 0$

- Complex exponential Function :-

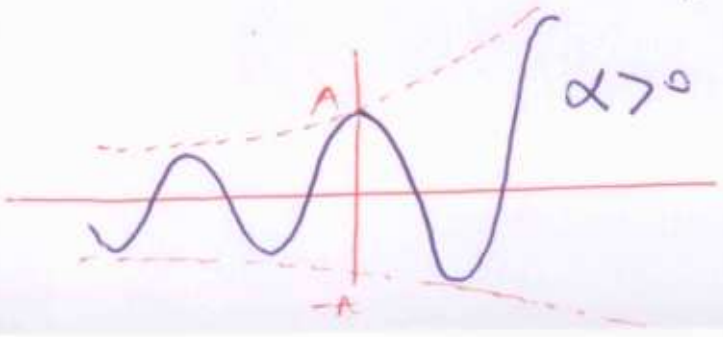
$$x(t) = A e^{j(\omega_0 t + \alpha)t}$$

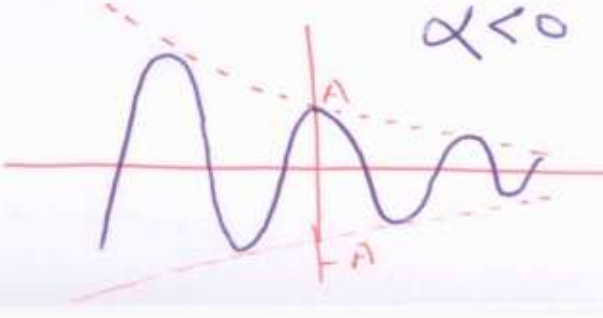
A : Amplitude

α : real part

$$x(t) = A e^{\alpha t} \cdot e^{j\omega_0 t}$$

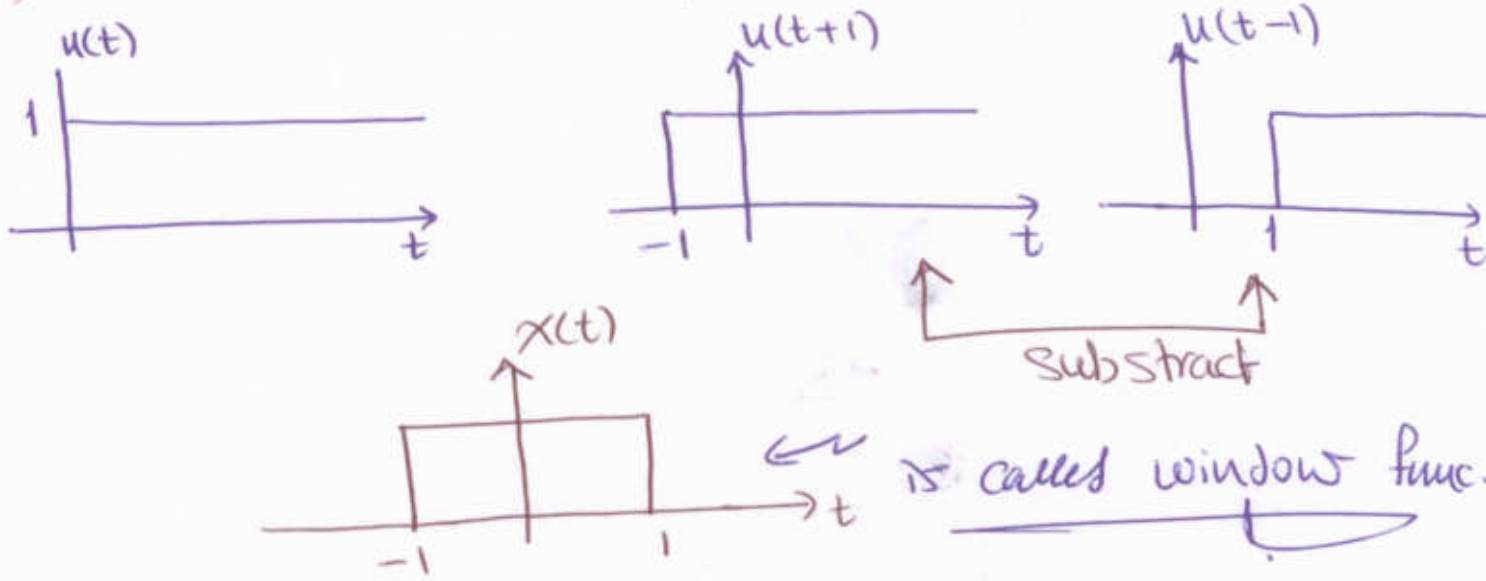
↳ $\cos + j \sin$



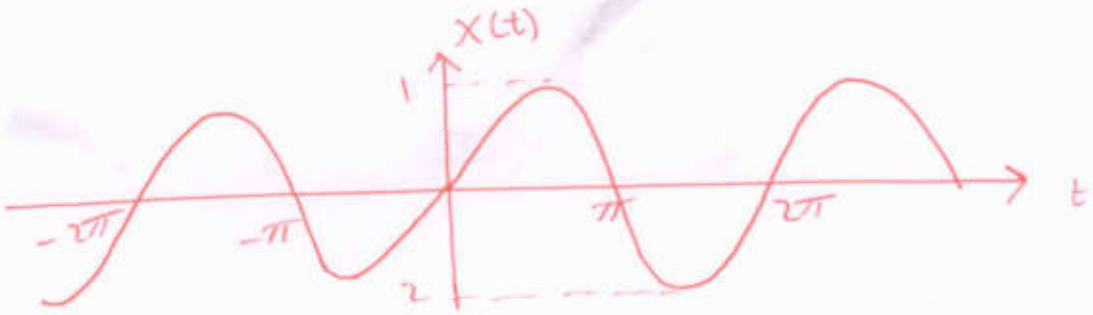


Examples:

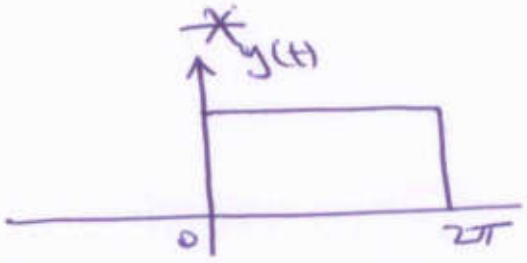
1) $x(t) = u(t+1) - u(t-1)$



2)



Find the sinwave function with 2π period.



$$y(t) = u(t) - u(t-2\pi)$$

$$\therefore \quad x'(t) = \sin\omega t\,[u(t) - u(t-2\pi)]$$

- Some properties for Impluse function:

1) $\int_{-\infty}^{\infty} \delta(t)\,dt = 1$ & $\delta(-t) = \delta(t)$

2) $\int_{0}^{\infty} \delta(t)\,dt = 1$

3) $\int_{5}^{\infty} \delta(t)\,dt = 0$

4) $\int_{5}^{\infty} \delta(t-5)\,dt = 1$

5) $\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0) = x(t_0)$

↳ constant value.

↗ VIP (shifting property) ≡ sampling

Example:

1) $\int_{0}^{\infty} \sin(t)\,\delta(t-3)\,dt$

$= \sin(3)$

2) $\int_{0}^{\infty} \sin(t-2)\,\delta(t-3)\,dt$

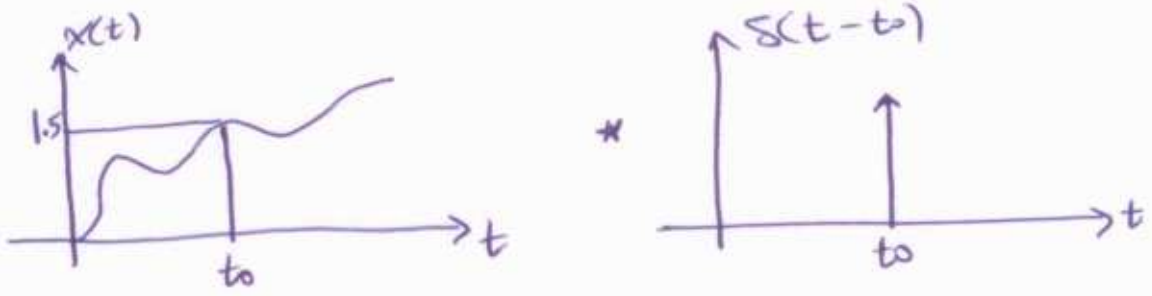
$= \sin(3-2) = \sin(1)$

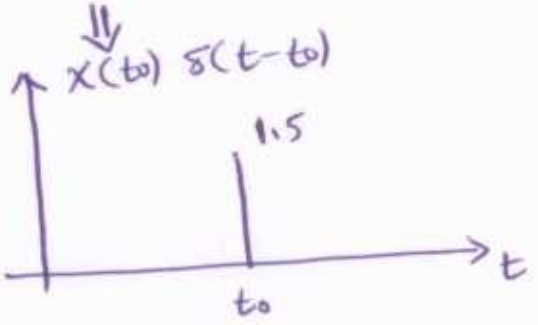
3) $\int_{0}^{\infty} \cos(t-\tau)\,\delta(\tau-2)\,d\tau$ ← هذا هو المتغير $x(\tau)$

$= \cos(t-2)$

#

$$x(t) \cdot \delta(t-t_0) = x(t_0)\,\delta(t-t_0)$$





sampling

$$x[n] = x[0]\,\delta[n-0] + x[1]\,\delta[n-1] + \cdots\cdots$$

$$x[n] = \sum_{k=0}^{\infty} x[k]\,\delta[n-k]$$

⊛ Euler's equations :-

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$

$$\sin\theta = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$

⊛ Signal representation :-

1) The change in the slope at $t = t_0$ means that ramp function is exist.

$$\boxed{\Delta S\ r(t-t_0)}$$

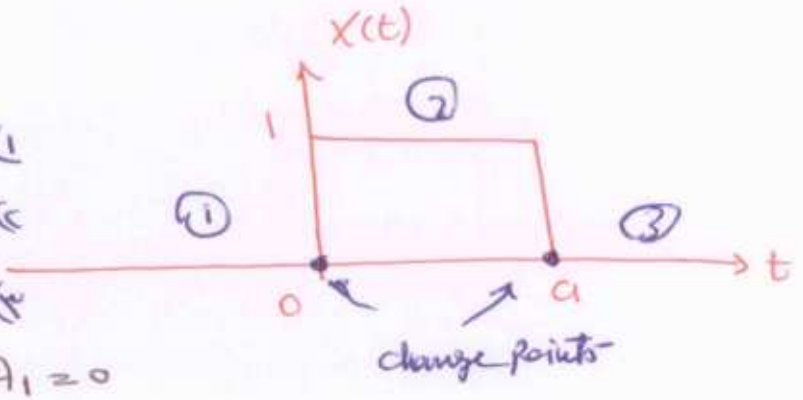
ΔS ← slop difference , t_0 → change point

2) The change in Amplitude at $t = t_0$ means that step function is exist.

$$\boxed{\Delta A\ u(t-t_0)}$$

ΔA ← Amp difference , t_0 → change point

Ex 2



1) حدد نقاط تغير القيم او الميل
2) احسب الـ slope اللي تتغير
3) كذلك الـ Amp
4) عوض في القانون المناسب

$S_1 = 0$, $S_2 = 0$, $S_3 = 0$; $A_1 = 0$, $A_2 = 1$, $A_3 = 0$

at $t = 0$

$$(A_{new} - A_{old})\ u(t - t_0)$$

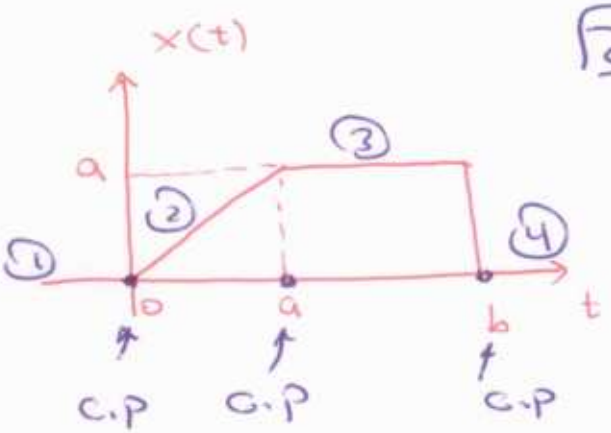
$$(1 - 0)\ u(t-0) \Rightarrow u(t)$$

at $t = a$

$$(0 - 1)\ u(t-a) \Rightarrow -u(t-a)$$

$$\therefore x(t) = u(t) - u(t-a)$$ #.

Ex 1



critically points

$S_1 = 0$ $\quad A_1 = 0$

$S_2 = \frac{a-0}{a-0} = 1$ $\quad A_2 = \frac{1}{2}a$

$S_3 = 0$ $\quad A_3 = a$

$S_4 = 0$ $\quad A_4 = 0$

at $t = 0$

$$(1-0)\,r(t-0) = r(t)$$

at $t = a$

$$(0-1)\,r(t-a) = -r(t-a)$$

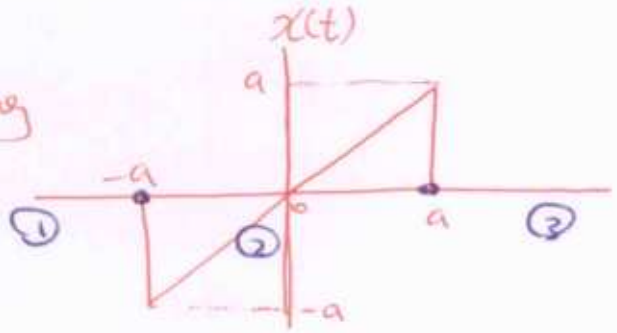
at $t = b$

$$(0-1)\,u(t-b) = -u(t-b)$$

$$\therefore \quad x(t) = r(t) - r(t-a) - u(t-b) \qquad \#.$$

Exam

Find the expression of the following signal.



$S_1 = 0$ $\quad A_1 = 0$

$S_2 = \frac{a+a}{a+a} = 1$ $\quad A_2 = \frac{1}{2}a - a$

$S_3 = 0$ $\quad A_3 = 0$

at $t = -a$

$$(1-0)\,r(t+a) + (-a-0)\,u(t+a) = r(t+a) - a\,u(t+a)$$

at $t = a$

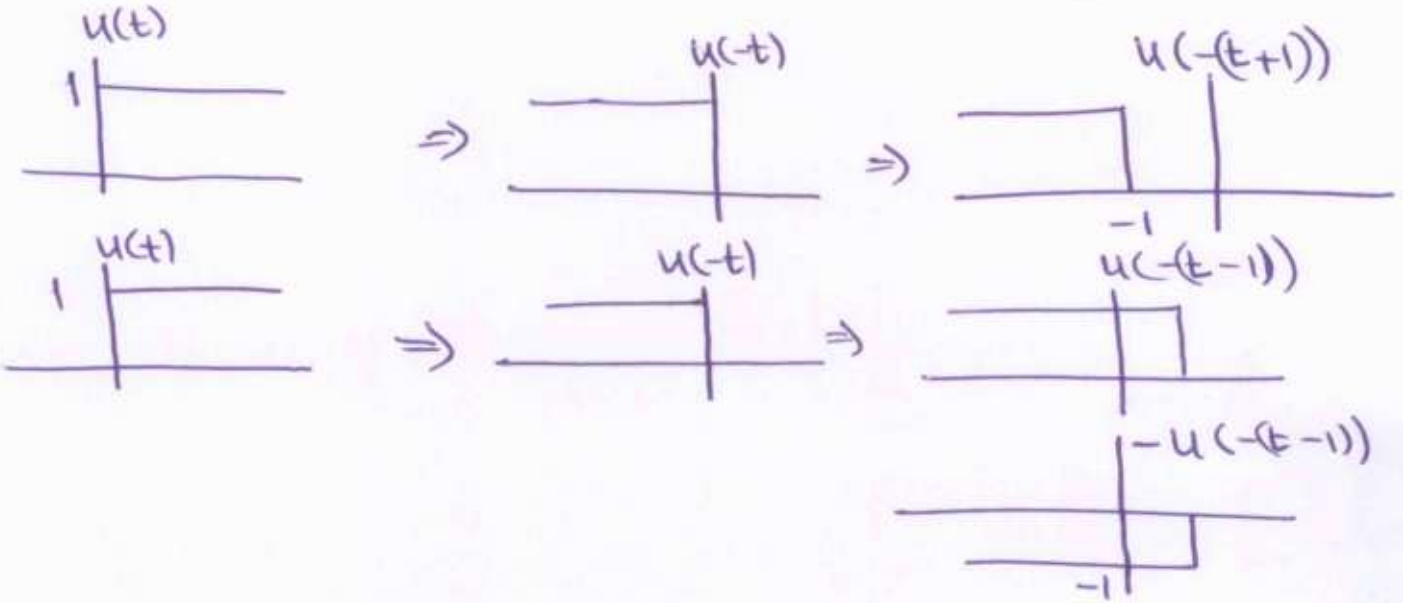
$$(0-1)\,r(t-a) + (0-a)\,u(t-a) = -r(t-a) - a\,u(t-a)$$

$\therefore x(t) = r(t+a) - au(t+a) - r(t-a) - au(t-a)$ [21]

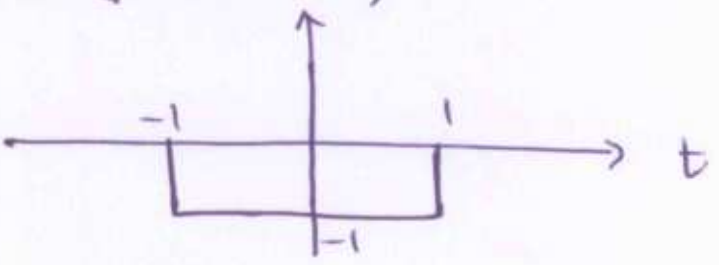
Ex2 Plot the following expression

1) $x(t) = t\,[u(-t-1) - u(-t+1)]$

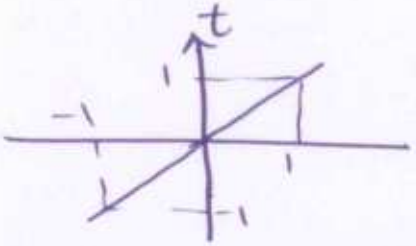
$$x(t) = t\left[u(-(t+1)) - u(-(t-1))\right]$$



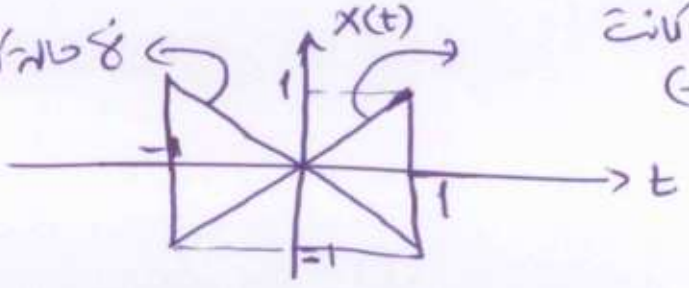
$\therefore u(-(t+1)) + (-u(-(t-1)))$



(t)

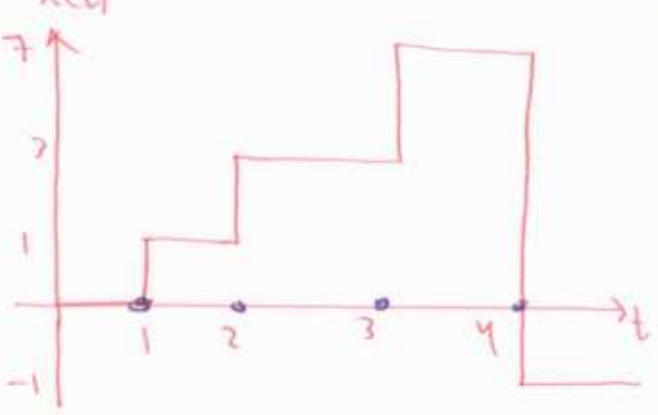


$\therefore$ $x(t)$ (t)ينعكس حول المحور ينعكس حول المحور (-t)



#

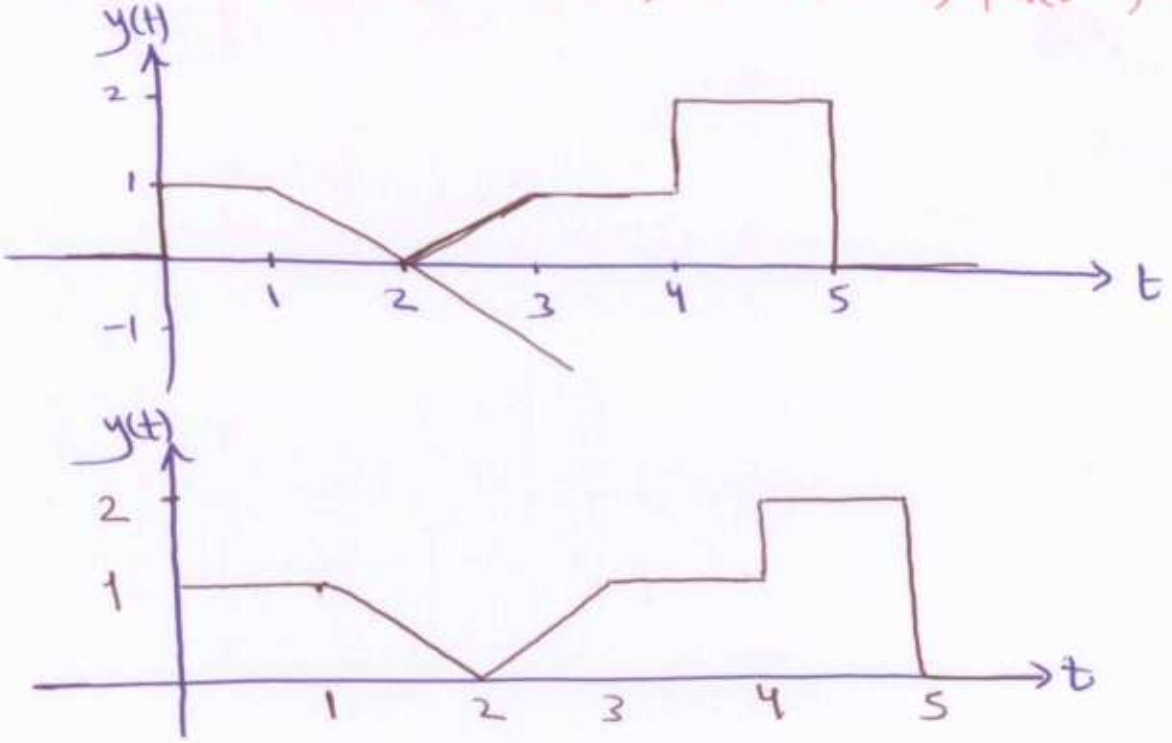
Ex: Find the expression of $x(t)$?



$$x(t) = u(t-1) + 2u(t-2) + 4u(t-3) - 8u(t-4)$$

Ex: plot the following signal.

$$y(t) = u(t) - r(t-1) + 2r(t-2) - r(t-3) + u(t-4) - 2u(t-$$



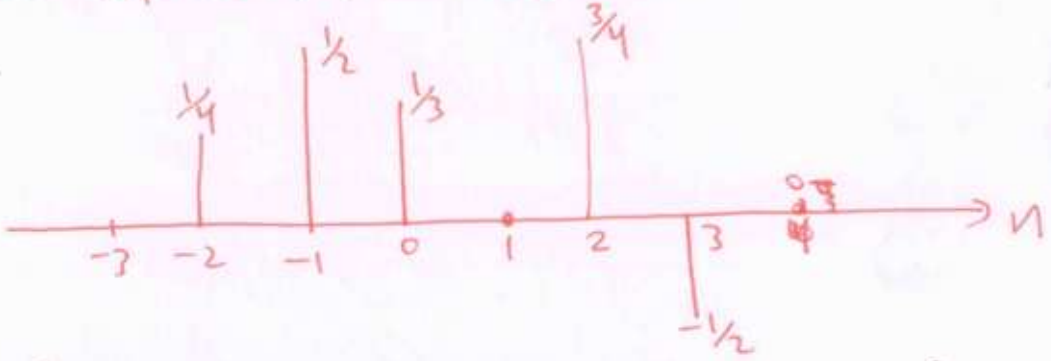
notes in DT signals their some properities :

1) $\sum_{n=0}^{\infty} \delta[n-k] = u[n]$ step function

Discr. في ال $\sum$ تقابل Contn. في ال التكامل

2) $x[n] - x[n-1] = \delta[n]$

Ex2 Find the expression for $x[n]$



$$x[n] = \{ \tfrac{1}{4}, \tfrac{1}{2}, \tfrac{1}{3}, 0, \tfrac{3}{4}, -\tfrac{1}{2} \}$$

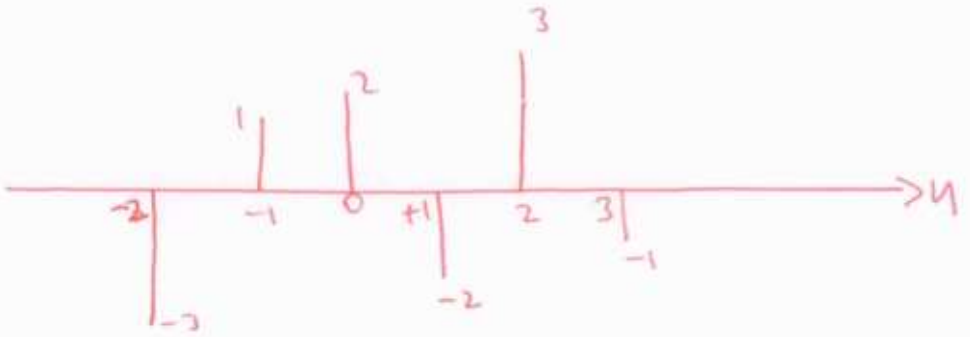
↑

هذه الاشارة تحدد نقطة الصفر في ال x-axis كما في الرسم أيضا
أبدأ من عند أول عنصر

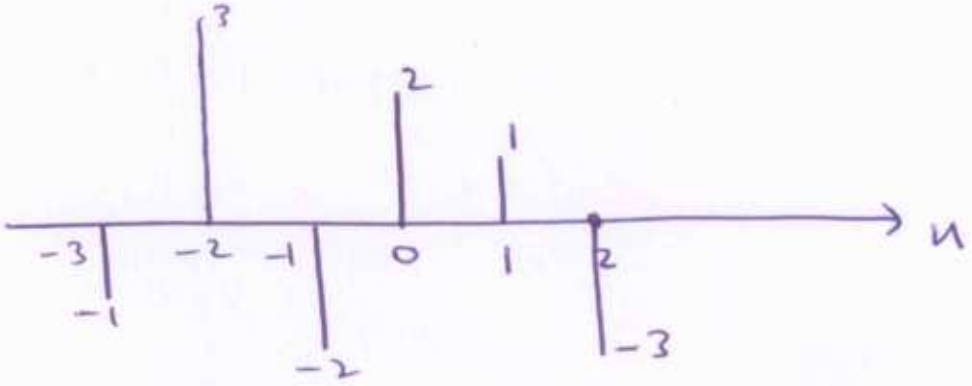
So $x[n] = \frac{1}{4}\delta[n+2] + \frac{1}{2}\delta[n+1] + \frac{1}{3}\delta[n] + \frac{3}{4}\delta[n-2] - \frac{1}{2}\delta[n-3]$

A.P

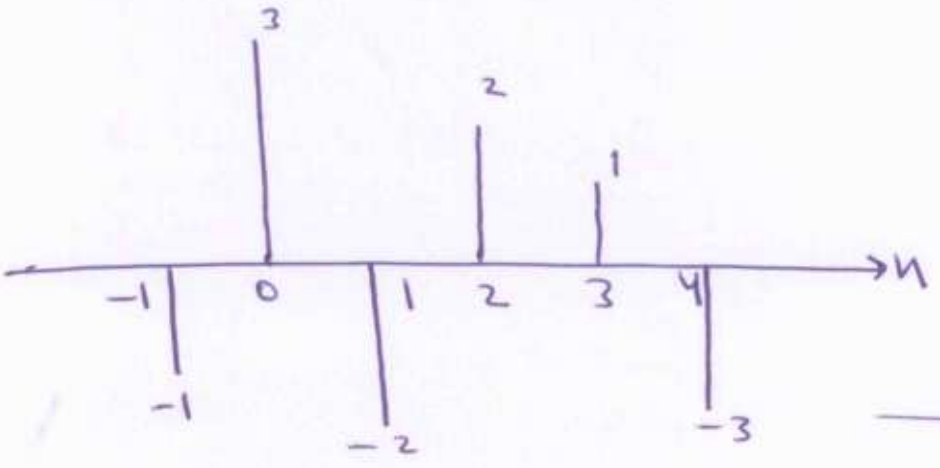
Ex: Find $x[2-n]$, where $x[n]$ shown below



- $x[-n]$ → Reflection about zero point



- $x[-(n-2)]$



1.5 Even and Odd Signals :-

- if $x(t) = x(-t)$, So $x(t)$ is even

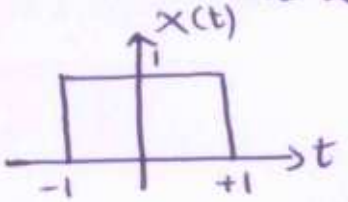
  like $\cos(x) = \cos(-x)$

- if $x(t) = -x(t)$, So $x(t)$ is odd

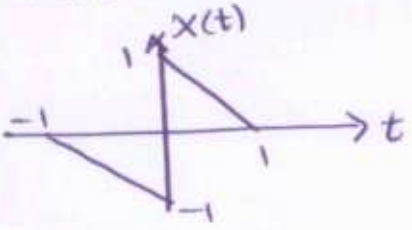
  like $\sin(-x) = -\sin(x)$

- even function x_e is symmetrical about vertical axis

  like window



- odd function x_o is antisymmetrical about vertical axis



- So any signal can be divided into two parts :-

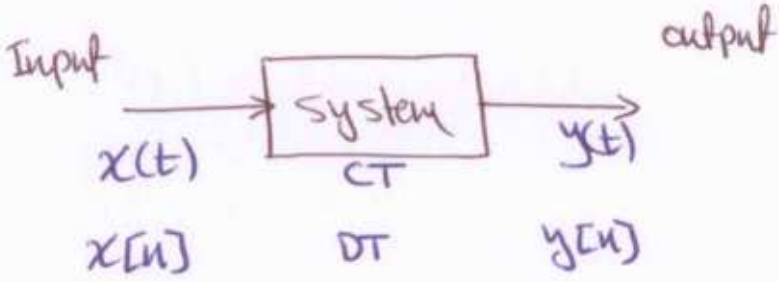
  - $Ev\{x(t)\} = \dfrac{(x(t) + x(-t))}{2}$

  - $od\{x(t)\} = \dfrac{(x(t) - x(-t))}{2}$

$\therefore$ $$\boxed{x(t) = \frac{1}{2}[x(t) + x(-t)] + \frac{1}{2}[x(t) - x(-t)]}$$

even part ↑ odd part ↑

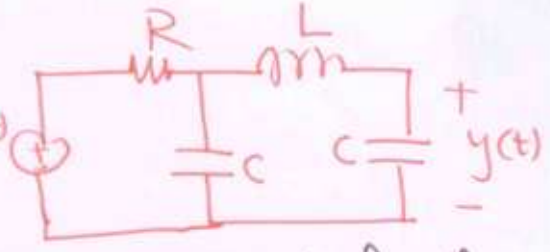
1.6 Systems



→ the system may be physical components (hardware) or may consist of algorithmic that compute the output signal from input so it called (software)

→ A system is characterized by its input, & its output, and the rule of operations adequate to describe its behavior.

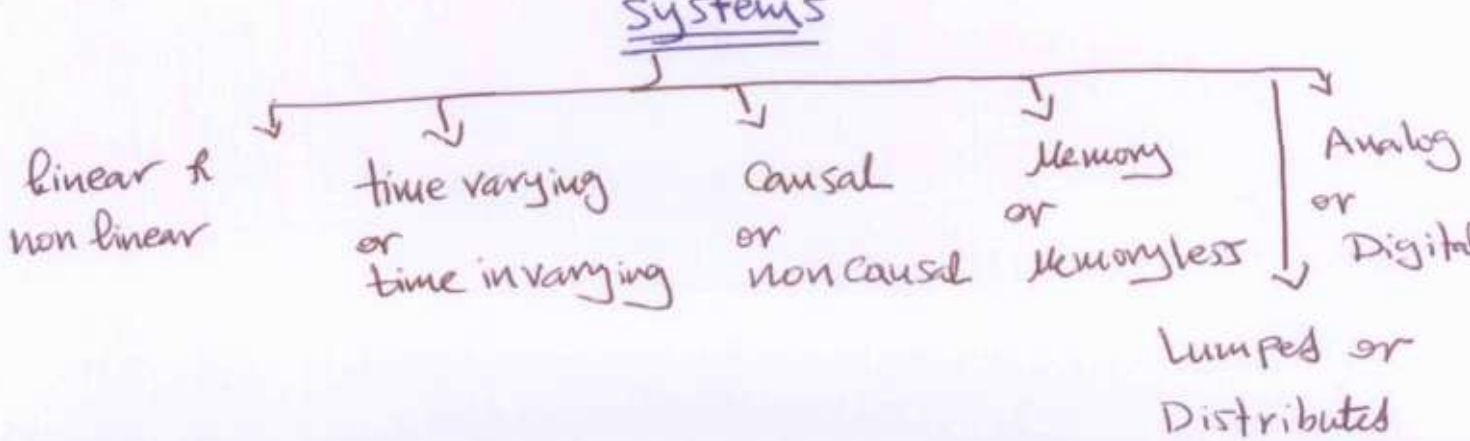
→ Example: RLC circuit



By driving a mathmatical equation that relate the input signal x(t) with output signal y(t), we can find a mathmatical model for the system or transfer function

$$H(S) = \frac{Y(S)}{X(S)}$$

1.7 classification of systems:-



1 linear and non linear systems 2

- To test if the system linear or not that called linearty test.
- There steps for this test which are 2.

1) $x_1(t) \longrightarrow y_1(t) \Rightarrow a x_1 = a y_1$ homogeneity test

2) $x_2(t) \longrightarrow y_2(t)$

3) $x_3(t) = a x_1(t) + b x_2(t) \longrightarrow y_3(t)$ —①

4) if $y_3(t) = a y_1(t) + b y_2(t)$ —②

} Additivity test

if ① = ② ⇒ then the system is linear

Ex2

at Input → output

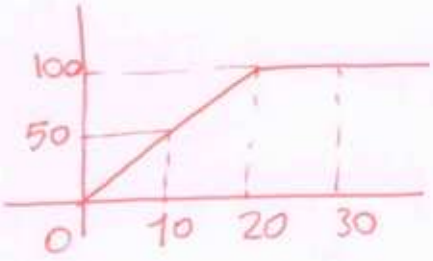
0 → 0

10 → 50

20 → 100

30 → 100

+ → + → 150 ≠



so the system non linear

the super position not hold

Ex2 $y' + 10y + 5 = x$

Is this system linear ?!

let $x_3(t) = a x_1(t) + b x_2(t) \overset{?}{\Rightarrow} y_3(t) = a y_1(t) + b y_2(t)$

$a x_1(t) = a y_1' + 10 a y_1 + 5a$ ——— ①

$b x_2(t) = b y_2' + 10 b y_2 + 5b$ ——— ②

$a x_1 + b x_2 = a y_1' + b y_2' + 10[a y_1 + b y_2] + 5[a+b]$

$x_3(t) = y_3' + 10 y_3 + 5[a+b]$

↓

آذا كانت ممنوع الصرف الطرفين

2- the superposition doesnot hold so the system is not linear

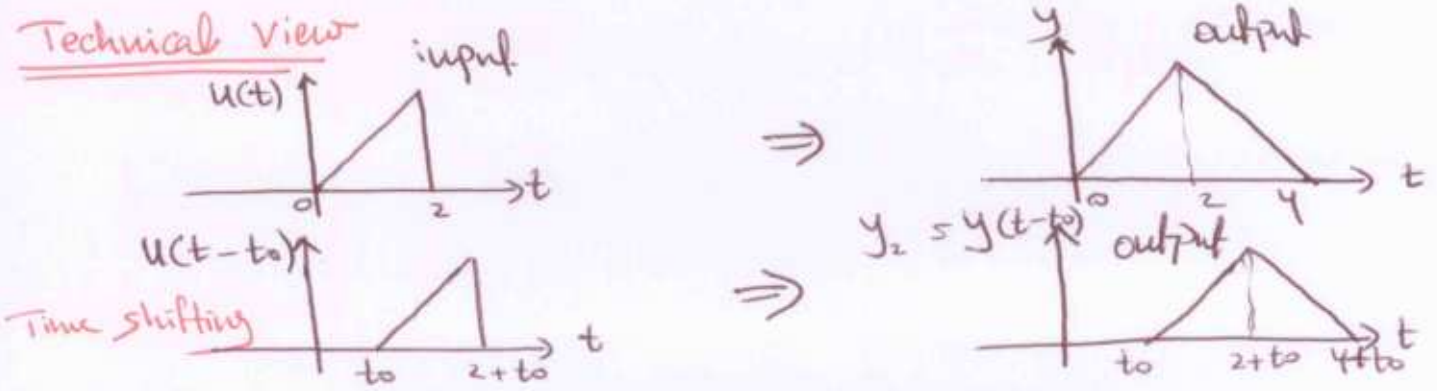
3] Time Varying & Time invariant :-

Physical view :

- if the characteristic of a system do not change with time so this system called time Invariant

- The Fan is Time invarient , but the space rocket is time ~~varient~~.

Technical View



Invarient من output و input لم يتغير ولا تكونات آذا كان آذاحة لا

So if $x(t) \longrightarrow y(t)$

$x(t-t_0) \longrightarrow y(t-t_0)$

So the system called time Invarient

[3] Memory and memoryless :-

- A system is said to be memoryless or instantaneous if the present value of output depends only on the present value of input. [Resistor

like $\boxed{V(t) = R\,i(t)}$

- A system is said to be memory or Dynamic if and only if the present value & future value are affected by past value like [capacitor & inductor]

$$\boxed{y(t) = x(t) + x(t-1)}$$

present value — past value

Ex2

$y(t) = x(t)$ → memoryless

$y(t) = x(t-1)$ → memory

$y(t) = x(t+1)$ → memory

[4] Causal and non causal systems :-

- A system is called a causal or non causal system if its current output depends on past and current inputs but not on future input. or if depends on future respectivilly.

- By another way causility mean the system causal if the output at any time "to" depend only on value of x input signal for $t \leq to$ ⇒ mean present time or past

Examples :

$y(t) = x(t)$	Causal
$y(t) = x(t-1)$	Causal
$y(t) = x(t+1)$	non causal
$y(t) = x(-t)$	non causal لانو فيه زمن بالسالب
$y(t) = x(2t)$	non causal لانه فيه المستقبل

non causal	Causal

Notes : every memoryless systems are causal system but not vice versa

Ex: 1) $y(t) = \cos t$ time variant or Invariant

$\cos(t - t_0) = \cos(t)$ periodic

$y(t - t_0) = y(t)$

∴ system is time invariant

2) $y(t) = t \cos(t)$

$y_1 = t \cos(t)$

$y_2 = t \cos(t_2)$

$t_2 = t - t_0$

$y_2 = t \cos(t - t_0)$ —①

$y(t - t_0) = (t - t_0) \cos(t - t_0)$

∴ $y_2 \neq y(t - t_0)$ ∴ time variant

⊛ Step by step for Determine the time variant or not:

step 1: let $y_1(t)$ output for input $x_1(t)$

step 2: let $x_2(t) = x_1(t - t_0)$ and obtain $y_2(t)$

step 3: from step1 find $y_1(t - t_0)$

step 4: if $y_2(t) = y_1(t - t_0)$

⇒ or equal if yes the system is Time invariant

if No the system is Time variant

Ex: Is the following systems linear, causal, Time varying memory?

1) $y(t) = 10x(t+2) + 5$

- linearity $\quad ax_1 + bx_2 = x_3$

$ay_1 + by_2 = y_3$

$$a y_1 + b y_2 = 10[a x_1(t+2) + b x_2(t+2)] + 5[a+b]$$

$$y_3 = 10\ x_3 + 5[a+b]$$

Superposition not hold $\rightarrow$ non linear system

- non causal because it depend on future $(t+2)$

- Time varient or not

1) $y_1(t) = 10\, x_1(t+2) + 5$

$y_2(t) = 10\, x_1(t - t_0 + 2) + 5$

$y_1(t - t_0) = 10\, x_1(t - t_0 + 2) + 5$

$\therefore y_2(t) = y_1(t - t_0)$ So the system time Invariant.

- With memory

[2] $y(t) = 2x(t) + 3$

- causal
- memory less
- non linear
- time invarient.

[3] $y(t) = A t\, x(t)$

- causal
- memory less
- linear
- time varying.

[4] $y(t) = 2x[n-6] + 6$

- causal
- non linear
- Time invarient
- memory

In our study we just focus on LTI system As we will be see in next chapters

END (prepare yourself to quiz next lecture)