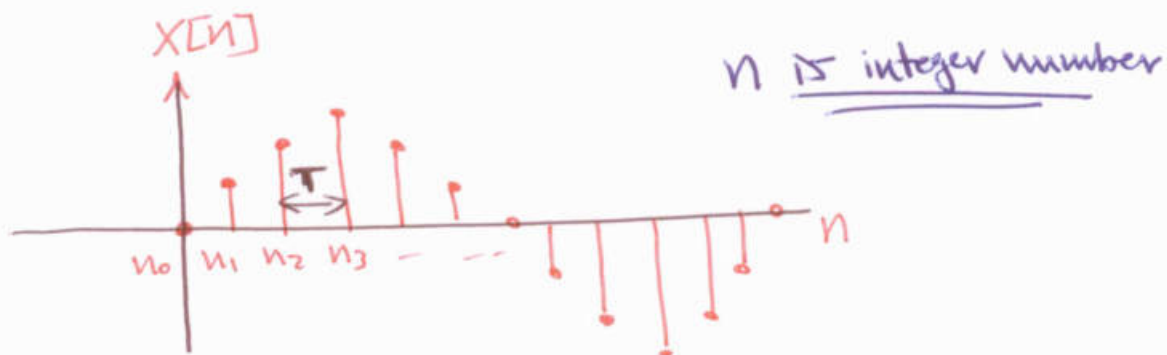


* Time Domain analysis of DT system *

- As, we see All software program processing and generate a signals as Discrete signal not continuous
- So if the All processing in computer done as Discrete so i need to find an interface between external environment and computer which called (A/D & D/A) converter.

* What is a discrete-time (DT) signal ?

→ is a sequence of numbers indexed by integers.

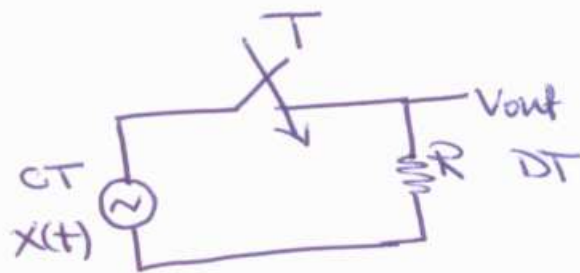


where n equal (nT) , where T is sampling time

- so if the inputs & outputs for a system are DT so this system called (DT system)
- we can convert any CT signal into DT signal by using sampling & holding system or by using switching technique

$T \Rightarrow$ sampling interval

$$X(t) \Big|_{t=T} \Rightarrow X(nT) = X[n]$$



if $X(t) = \sin(t)$ & $T = 0.2 \text{ sec}$

so the discrete signal is $X(nT) = \sin(nT)$

$$X[n] = \sin(0.2n)$$

* Size of DT signal

Any signal can be measured by two ways as we see before

[1] Energy if the signal is energy & Amp $\rightarrow 0$ at $|n| \rightarrow \infty$
(aperiodic)

[2] Power if the signal is periodic signal

$$E_x = \sum_{n=-\infty}^{\infty} |X[n]|^2 < \infty$$
$$P_x = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{-N}^N |X[n]|^2$$

Notes

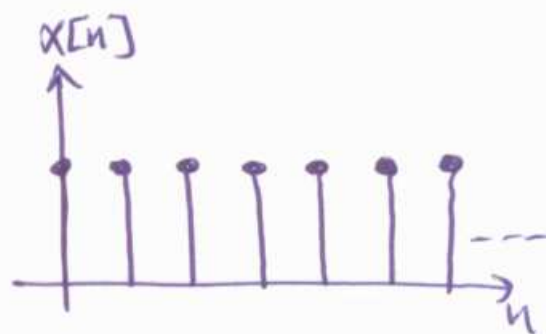
- we use integral with CT signals.
- & use summation with DT signals.

* useful signal operations

(3)

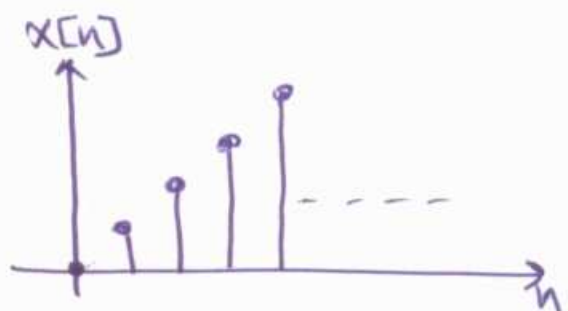
[1] Step - Function 2 (unit)

$$x[n] = \begin{cases} 0 & , n < 0 \\ 1 & , n \geq 0 \end{cases}$$



[2] Ramp - Function 2

$$x[n] = \begin{cases} 0 & , n < 0 \\ n & , n \geq 0 \end{cases}$$



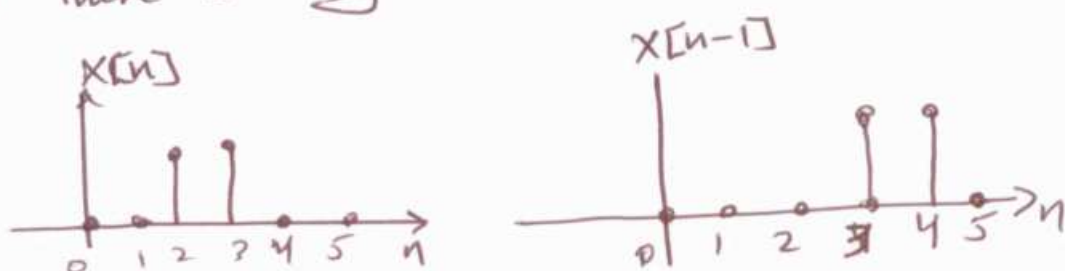
[3] Unit - Impulse Function 2

$$\delta[n] = \begin{cases} 0 & , n \neq 0 \\ 1 & , n = 0 \end{cases}$$



→ The signal operation in CT signal (scaling, shifting, flipping) it is the same of signal operation in DT signal.

→ There is only one different (one in DT & another in CT)



* The general Form for Sinusoids in DTL

4

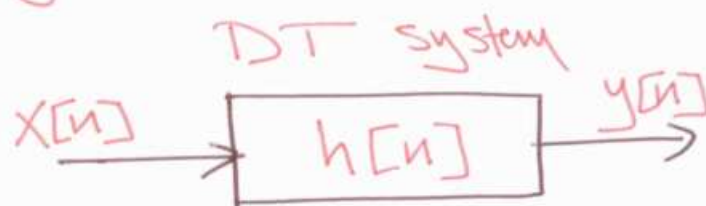
$$x[n] = A \sin(\Omega n + \theta)$$

Ω in radian and equal $\boxed{\Omega = \frac{2\pi}{T} = 2\pi F}$

$$\boxed{\Omega = \omega T}$$

* Classification of DT Systems:-

- 1) linear - system
- 2) Time - invariant system
- 3) causal system
- 4) memory system
- 5) stable system



A linear and non-linear system:-

Steps:-

$$x_1[n] \longrightarrow y_1[n]$$

$$x_2[n] \longrightarrow y_2[n]$$

$$ax_1[n] + bx_2[n] \stackrel{?}{\longrightarrow} ay_1[n] + by_2[n]$$

if true then the system is linear

if false then the system is non-linear

non-linear $\Rightarrow$ if the output is not equal to the sum of the outputs of the individual inputs, then the system is non-linear.

Ex2 $y[n] = x[2n]$

[5]

$$x_1[2n] \rightarrow y_1[n]$$

$$x_2[2n] \rightarrow y_2[n]$$

$$a x_1[2n] + b x_2[2n] = a y_1[n] + b y_2[n]$$

$$x_3[2n] = y_3[n]$$

$\therefore$ the system is linear

[B] Time Varying & Time invariant:-

- A system is Time-invariant if a delay in the input signal causes the same amount of delay in the output signal

- إذا تأخرت الإشارة المدخلة تأخرت الإشارة الخارجة

- إذا كان التأخير ثابتاً مع تغير n فإن النظام ثابت زمنياً

Ex1 $y[n] = x[2n]$

Shift the input by n_0

$$\cancel{x[2n]} x[2(n-n_0)] \stackrel{?}{\Rightarrow} y[n-n_0]$$

$$y_1[n] = x_1[2n] = x[2(n-n_0)]$$

$$\text{if } y_1[n] = x[2(n-n_0)] \Rightarrow \text{Time invariant} \\ = y[n-n_0]$$

[C] Causal and non-causal systems

[6]

- A system is called a causal system if its output depends on past or current inputs only.

$$y[n] = x[n] \rightarrow \text{causal.} \quad \underline{\text{RHP}}$$

$$y[n] = x[n-1] \rightarrow \text{causal.} \quad \underline{\text{RHP}}$$

$$y[n] = x[n+1] \rightarrow \text{non-causal.} \quad \underline{\text{LHP}}$$

[D] Memory and memory less

↓
Dynamic

↓
static

if the system output depends on current input so it called memory less.

$$y[n] \rightarrow x[n] \quad \text{Memory less}$$

$$y[n] \rightarrow \begin{matrix} x[n-1] \\ x[n+1] \end{matrix} \quad \text{Memory less}$$

Notes every memory less systems are causal but not

Vice Versa.

Examples

7

1) $y[n] = nx[2n]$

- memory, because the output value do not come from current input $y[1] = x[2]$
not the same index. (n)
- non-causal, because the the output at $n=1$ need to know the input at $n=2$
- linear system
- time varying (because it multiply by n)

2) $y[n] + 2y[n-1] = x[n+1]$

called difference equation.

- linear $y_1 + 2y_1[n-1] \rightarrow x_1[n+1]$

$$y_2 + 2y_2[n-1] \rightarrow x_2[n+1]$$

$$ax_1[n+1] + bx_2[n+1] = x_3[n] = ax_1[n+1] + bx_2[n+1]$$

$$ay_1 + 2ay_1[n-1] + by_2 + 2by_2[n-1]$$

$$y_3[n] = (a+b)y_3 + 2(a+b)y_3[n-1]$$

- time invariant, non causal, memory

$$3) \quad y[n+1] + 4y[n] = 3x[n+1] - x[n]$$

(8)

- Time invariant system

- So, you can rewrite it as

$$y[n] + 4y[n-1] = 3x[n] - x[n-1]$$

- 1) linear 2) time invariant 3) causal
4) memory

⊗ Discrete-time system equation:

- In DT $x[n]$ & $y[n]$ are related by difference equation.

- There are two general form to write difference eqn.

1) Advance form

2) Delay form

Advance:

$$a_N y[n+N] + a_{N-1} y[n+N-1] + \dots + a_0 y[n] = b_M x[n+M] + \dots + b_0 x[n]$$

To be causal $M \leq N$

Delay:

$$a_N y[n-N] + a_{N-1} y[n-N+1] + \dots + a_0 y[n] = b_M x[n-M] + \dots + b_0 x[n]$$

its order equal $(\max(N, M))$

⑧ Convolution

[9]

$$\begin{aligned} y[n] &= x[n] * h[n] = \sum_{k=-\infty}^{\infty} x(k) h(n-k) \\ &= \sum_{k=-\infty}^{\infty} x(n-k) h(k) \end{aligned}$$

⑧ properties of convolution:-

- 1) $x[n] * h[n] = h[n] * x[n]$
- 2) $x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x[n] * h_2[n]$
- 3) $x[n] * \delta[n] = x[n]$
- 4) $x[n] * \delta[n - n_0] = x[n - n_0]$

Notes if $x[n]$ has length L_1 , & $h[n]$ has length L_2
then the $y[n]$ length equal $(L_1 + L_2 - 1)$

→ we have two way to compute the convolution

- 1) Mathematical
- 2) graphical.

11 Mathematical 2

110

$$- \sum_{k=m}^n r^k = \frac{r^{n+1} - r^m}{r - 1} \quad r \neq 1$$

$$- u[k]u[n-k] = 1 \quad \text{at } 0 < k < n$$

Ex:

$$x[n] = \left(\frac{1}{3}\right)^n u[n], \quad h[n] = \delta[n] + \left(\frac{1}{2}\right)^n u[n]$$

$$\Rightarrow y[n] = x[n] * h[n]$$

$$y[n] = x[n] * h_1[n] + x[n] * h_2[n]$$

$$y_1[n] = \left(\frac{1}{3}\right)^n u[n] * \delta[n] = \left(\frac{1}{3}\right)^n u[n]$$

$$y_2[n] = \sum_{k=-\infty}^{\infty} \left(\frac{1}{3}\right)^k u[k] \cdot \left(\frac{1}{2}\right)^{n-k} u[n-k]$$

$$= \sum_{k=0}^n \left(\frac{1}{3}\right)^k \cdot (2)^k \cdot \left(\frac{1}{2}\right)^n$$

$$y_2[n] = \left(\frac{1}{2}\right)^n \sum_{k=0}^n \left(\frac{2}{3}\right)^k = \left(\frac{1}{2}\right)^n \cdot \frac{\left(\frac{2}{3}\right)^{n+1} - 1}{\frac{2}{3} - 1} u[n]$$

$$y_T[n] = y_1[n] + y_2[n]$$

AP

2) $x[n] = u[n]$, $h[n] = 2(0.8)^n u[n]$ (11)

$$y[n] = \sum_{k=-\infty}^{\infty} u[n-k] \cdot 2(0.8)^k u[k]$$

$$= \sum_{k=0}^n 2(0.8)^k = 2 \cdot \frac{(0.8)^{n+1} - 1}{0.8 - 1}$$

$$= -10 [(0.8)^{n+1} - 1]$$

$$= -10 \cdot 0.8^n \cdot 0.8 + 10$$

$$y[n] = -\cancel{8} (0.8)^n + 10 \quad \#$$

[2] Graphical Convolution:

- [1] write both as function of k : $x[k]$ & $h[k]$
- [2] Flipping $h[k]$ with respect to $k=0 \rightarrow h[-k]$
- [3] shifting $h[-k]$ to n to yield $h[n-k]$
- [4] multiply $h[n-k]$ and $x[k]$ for all k
- [5] summing up all non zero $h[n-k] \cdot x[k]$ to yield $y[n]$

at $n=0 \Rightarrow y[n] = -1 \times -2 = 2$

at $n=1 \Rightarrow y[n] = -1 \times -1 + 0 \times -2 = 1$

at $n=2 \Rightarrow y[n] = -1 \times 0 + 0 \times -1 + 1 \times -2 = -2$

at $n=3 \Rightarrow y[n] = -1 \times 1 + 0 \times 0 + 1 \times -1 + -2 \times 2 = -6$

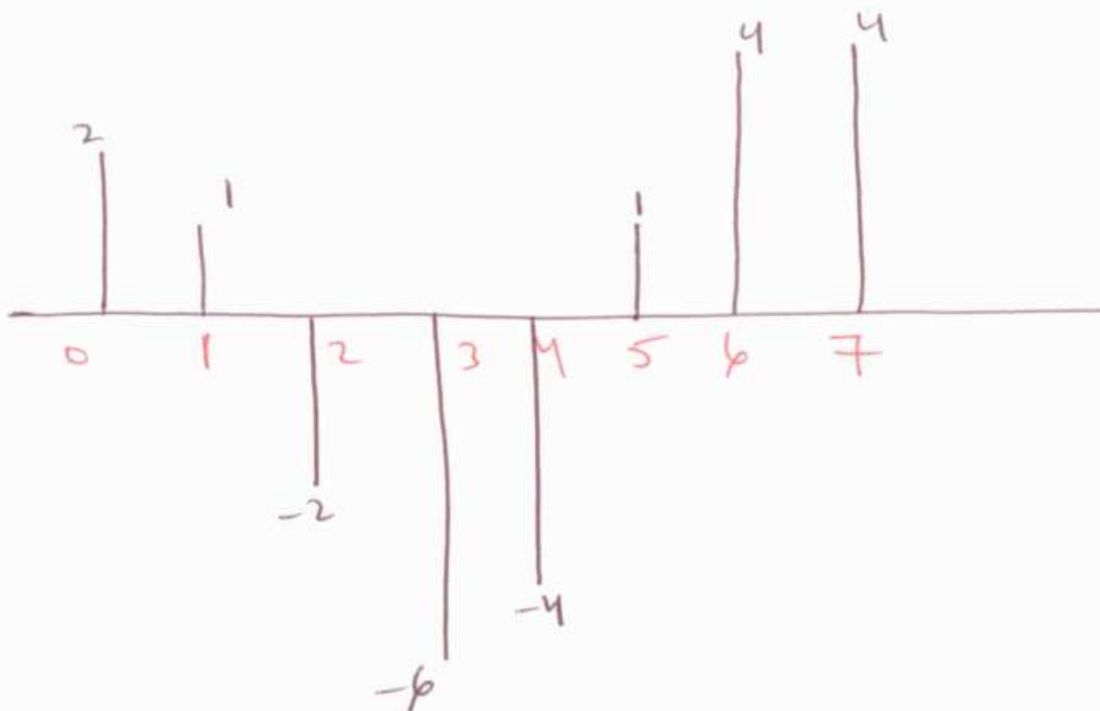
at $n=4 \Rightarrow y[n] = -1 \times 2 + 0 \times 1 + 0 + 2 \times -1 + 0 = -4$

at $n=5 \Rightarrow y[n] = 0 + 1 + 0 + 0 = 1$

at $n=6 \Rightarrow y[n] = 2 + 2 = 4$

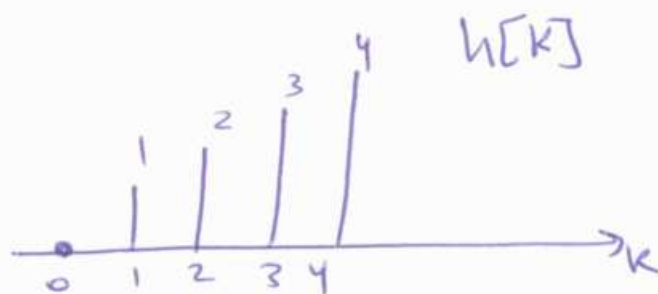
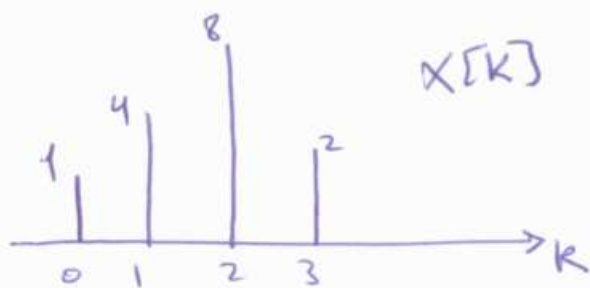
at $n=7 \Rightarrow y[n] = 2 \times 2 = 4$

$y[n] = \{ 2, 1, -2, -6, -4, 1, 4, 4 \}$
 $\uparrow$



2) $x[n] = [1, 4, 8, 2]$, $h[n] = [0, 1, 2, 3, 4]$ 14

both start at $n=0$ Find $y[n]$



start $= 0 + 0 = 0$

end $= 3 + 4 = 7$

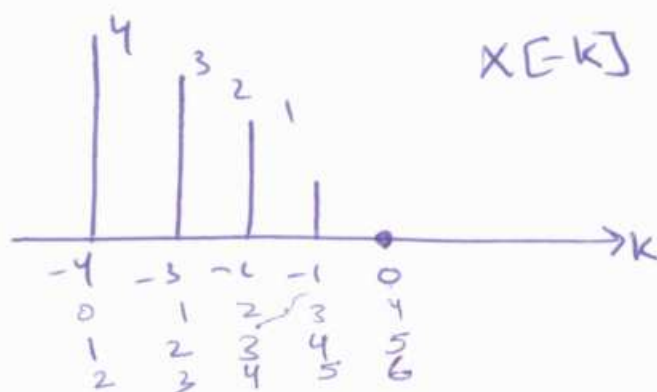
128

$n=0 \Rightarrow y[n] = 0$

$n=1 \Rightarrow y[n] = 1$

$n=2 \Rightarrow y[n] = 6$

$n=3 \Rightarrow y[n] = 19$



$n=4 \Rightarrow y[n] = 34$

$n=5 \Rightarrow y[n] = 44$

$n=6 \Rightarrow y[n] = 38$

$n=7 \Rightarrow y[n] = 8$

$y[n] = \{ \underset{\uparrow}{0}, 1, 6, 19, 34, 44, 38, 8 \}$

⊗ DT Impulse Response:-

— $h[n] \rightarrow$ called impulse Response

} All initial value equal zero

$$y[n] = h[n] \text{ at } x[n] = \delta[n]$$



Ex 2

$$y[n] = 2x[n] + 3x[n-1] - 1.5x[n-3]$$

$x[n] = \delta[n]$ Find the output $y[n]$?

$$\delta[n] = \begin{cases} 1 & n=0 \\ 0 & n \neq 0 \end{cases}$$

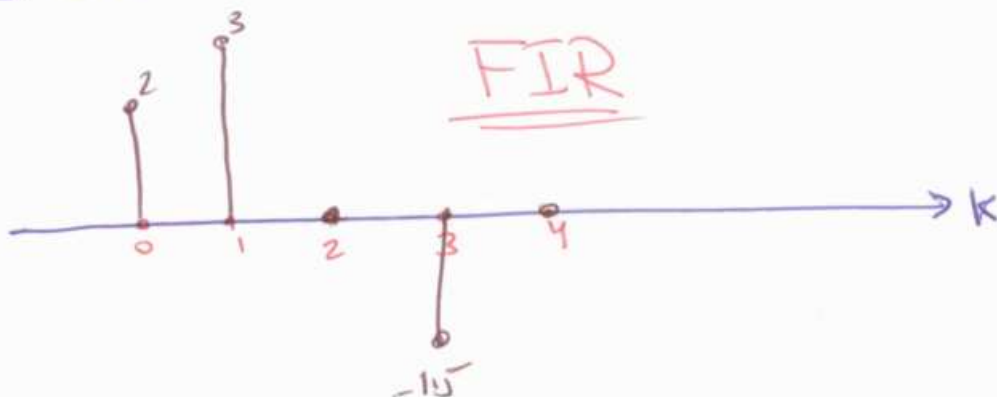
$$y[0] = 2\delta[0] + 3\delta[-1] - 1.5x[-3] = 2$$

$$y[1] = 2\delta[1] + 3\delta[0] - 1.5x[-2] = 3$$

$$y[2] = 2\delta[2] + 3\delta[1] - 1.5\delta[-1] = 0$$

$$y[3] = 2\delta[3] + 3\delta[2] - 1.5\delta[0] = -1.5$$

$$y[4] = 0$$



2) Find the Impulse Response for DT system

$$y[n] + 0.2y[n-1] = x[n] - x[n-1]$$

$$x[n] = \delta[n] \quad \& \quad y[-1] = 0$$

$$y[0] = x[0] - x[-1] - 0.2y[-1] = 1$$

$$y[1] = x[1] - x[0] - 0.2y[0] = -0.2$$

$$y[2] = x[2] - x[1] - 0.2y[1] = 0.04$$

$$y[3] = x[3] - x[2] - 0.2y[2] = -0.008$$

⋮

IIR

لا يوجد noncausal ← system في الوقت الماضي لا يوجد في الوقت الحاضر أو المستقبل لأن الوقت يبدأ من الصفر ويزداد مع الوقت الذي نحسب فيه الاستجابة لأن الوقت يبدأ من الصفر ويزداد مع الوقت الذي نحسب فيه الاستجابة لأن الوقت يبدأ من الصفر ويزداد مع الوقت الذي نحسب فيه الاستجابة

* FIR and IIR system

- DT LTI system is called FIR (Finite impulse response) system if its impulse response has finite number of non-zero entries.
- DT LTI system is called IIR (Infinite impulse response) system if its impulse response has infinite number of non-zero entries.

⋆

⊗ Interconnected Systems

17

Cascade



$$y[n] = x[n] * h_1[n] * h_2[n]$$

Parallel



$$y[n] = (x[n] * h_1[n] + x[n] * h_2[n])$$

Ex1

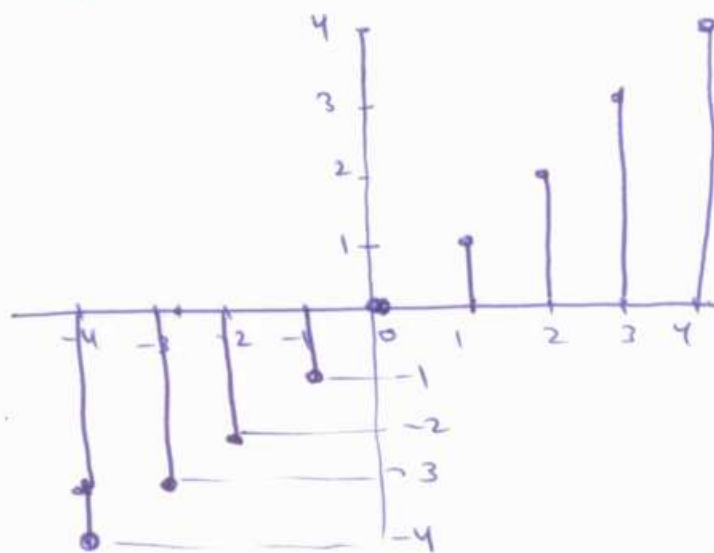
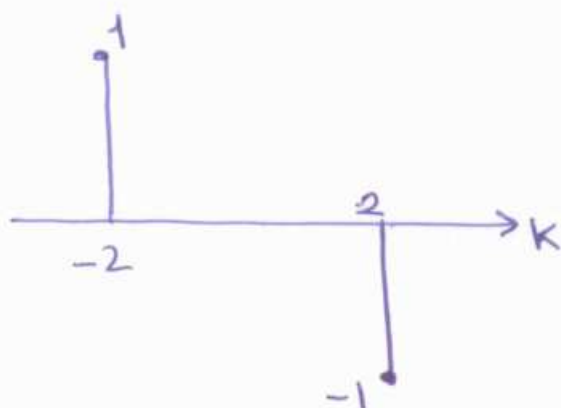
$$h_1[n] = \delta[n+2] - \delta[n-2]$$

two pulses

$$h_2[n] = n(u[n+4] - u[n-4])$$

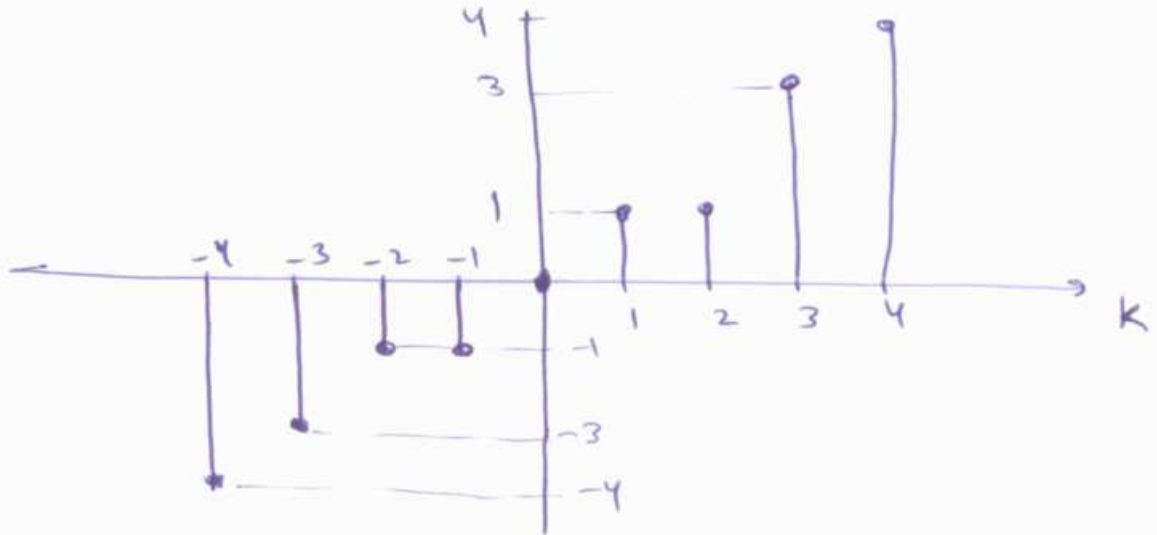
Window (-4, 4)

a) sketch $h_1[n]$ & $h_2[n]$ $-10 \leq n \leq 10$



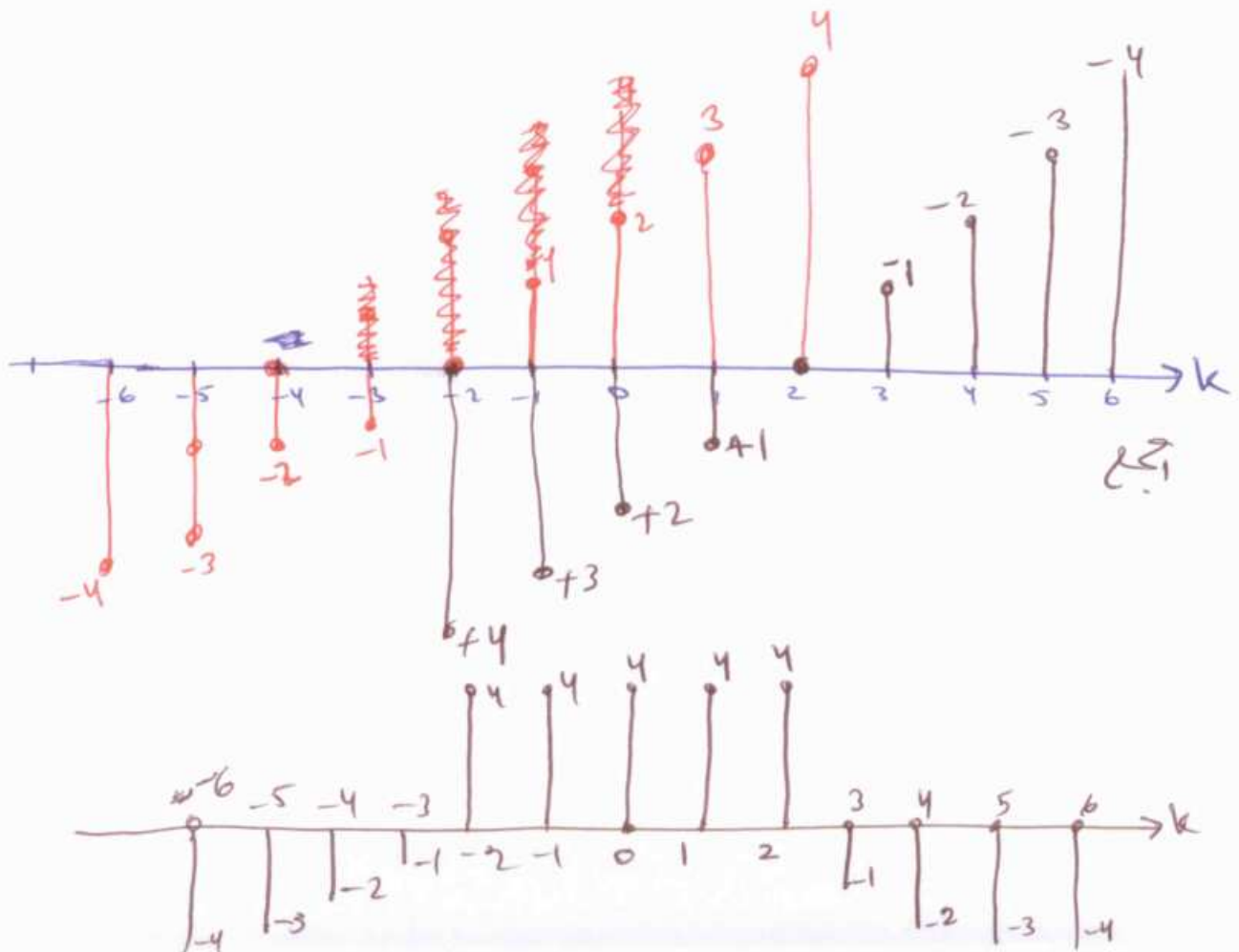
b) Assume the two systems are parallel find Total System 18

$$h_p[n] = h_1[n] + h_2[n]$$



c) Assume the two systems are cascade find $h_c[n]$

$$h_c[n] = h_1[n] * h_2[n]$$



* System stability 2

19

$$S = \sum_{k=-\infty}^{\infty} |h[k]| \text{ is finite}$$

So the LTID is stable

look at example is slide 58, 59 in lecture